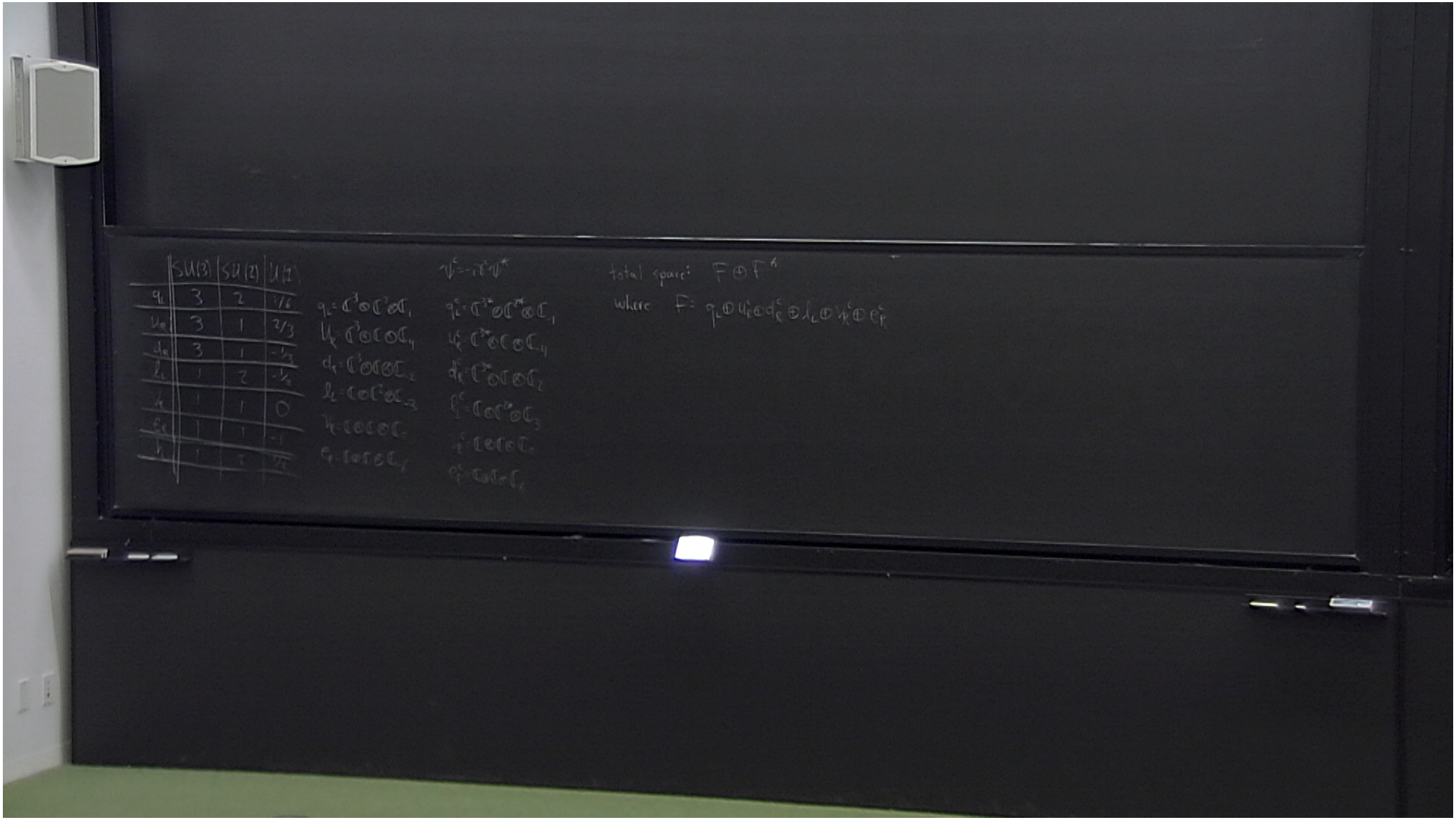
Title: PSI 2018/2019 - Beyond Standard Model - Lecture 10

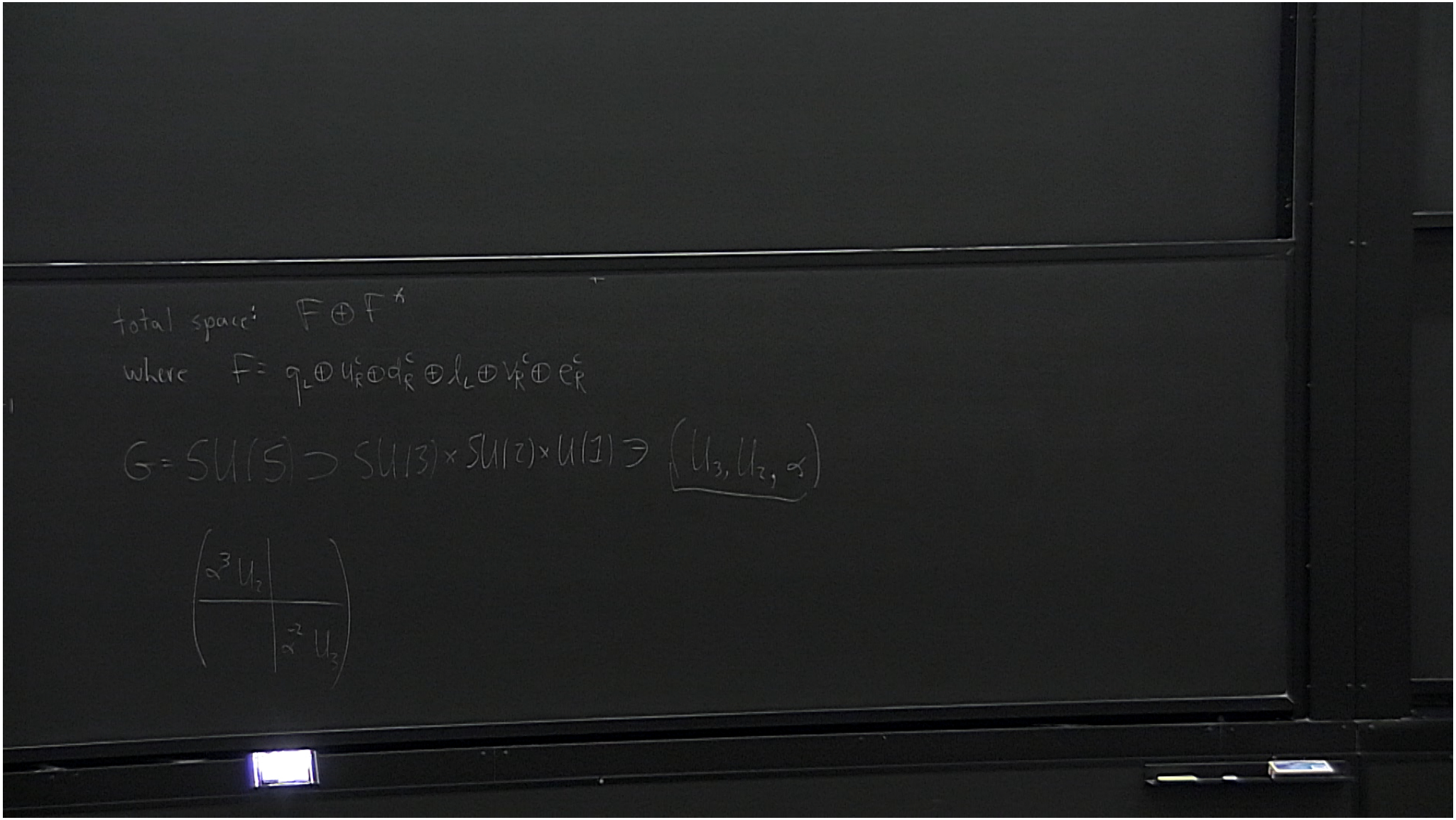
Speakers: Latham Boyle

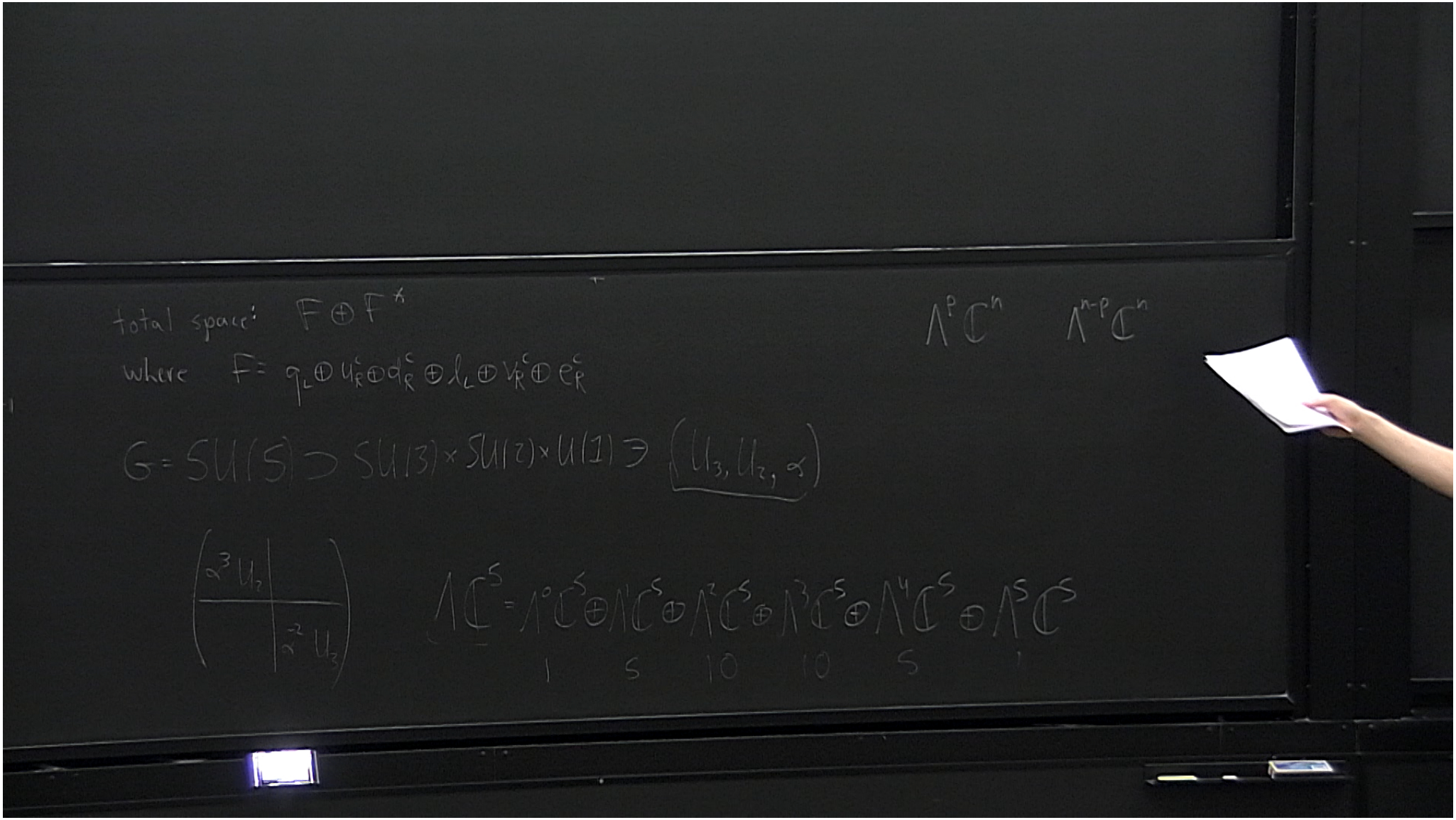
Collection: PSI 2018/2019 - Beyond Standard Model (Boyle)

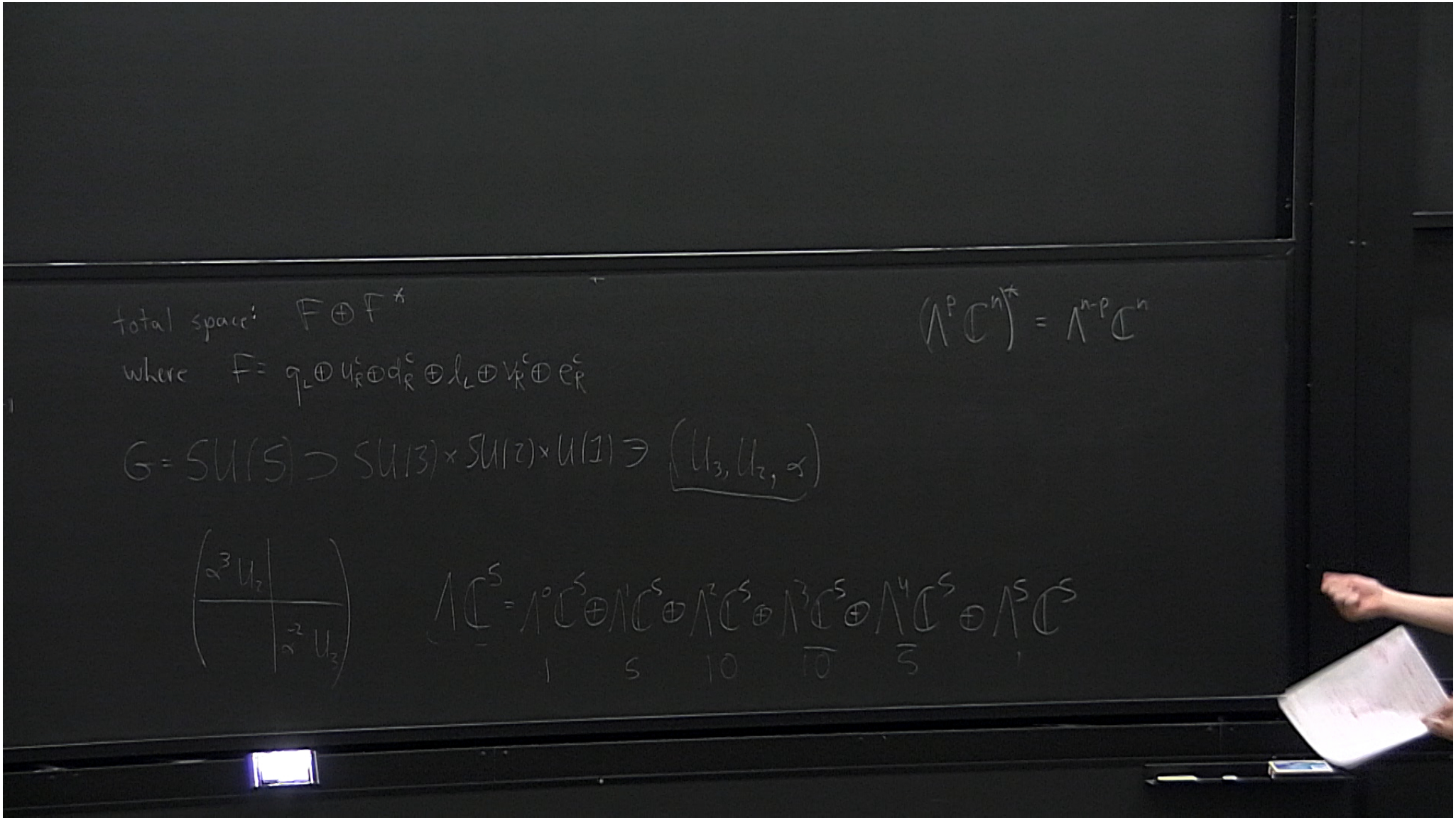
Date: April 05, 2019 - 11:30 AM

URL: http://pirsa.org/19040025


SU(3) SU(2) U(1)
total space: F ⊕ F*
where F:


total space: $F \oplus F^*$
where $F = q_L \oplus u_R^c \oplus d_R^c \oplus l_L \oplus \nu_R^c \oplus e_R^c$
$G = SU(5) \supset SU(3) \times SU(2) \times U(1) \ni (U_3, U_2, \alpha)$
$\begin{pmatrix} \alpha^3 U_2 & \\ & \alpha^{-2} U_3 \end{pmatrix}$


total space: $F \oplus F^*$
where $F = q_L \oplus u_R^c \oplus d_R^c \oplus l_L \oplus \nu_R^c \oplus e_R^c$
$\Lambda^p \mathbb{C}^n \quad \Lambda^{n-p} \mathbb{C}^n$
$G = SU(5) \supset SU(3) \times SU(2) \times U(1) \ni (U_3, U_2, \alpha)$
$\begin{pmatrix} \alpha^3 U_2 & \\ & \alpha^{-2} U_3 \end{pmatrix}$
$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$
1 5 10 10 5 1


total space: $F \oplus F^*$
where $F = q_L \oplus u_R^c \oplus d_R^c \oplus l_L \oplus \nu_R^c \oplus e_R^c$
$(\Lambda^p \mathbb{C}^n)^* = \Lambda^{n-p} \mathbb{C}^n$
$G = SU(5) \supset SU(3) \times SU(2) \times U(1) \ni (U_3, U_2, \alpha)$
$\begin{pmatrix} \alpha^3 U_2 & \\ & \alpha^{-2} U_3 \end{pmatrix}$
$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$
1 5 10 $\overline{10}$ $\overline{5}$ 1

ν_R	1	1	0
e_R	1	1	-1
h	1	2	1/2

$\nu_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$

$e_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_{-6}$

$\ell_L = \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_3$

$\nu_R^c = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$

$e_R^c = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6$

$$\begin{pmatrix} \alpha^3 U_2 & \\ & \alpha^{-2} U_3 \end{pmatrix}$$

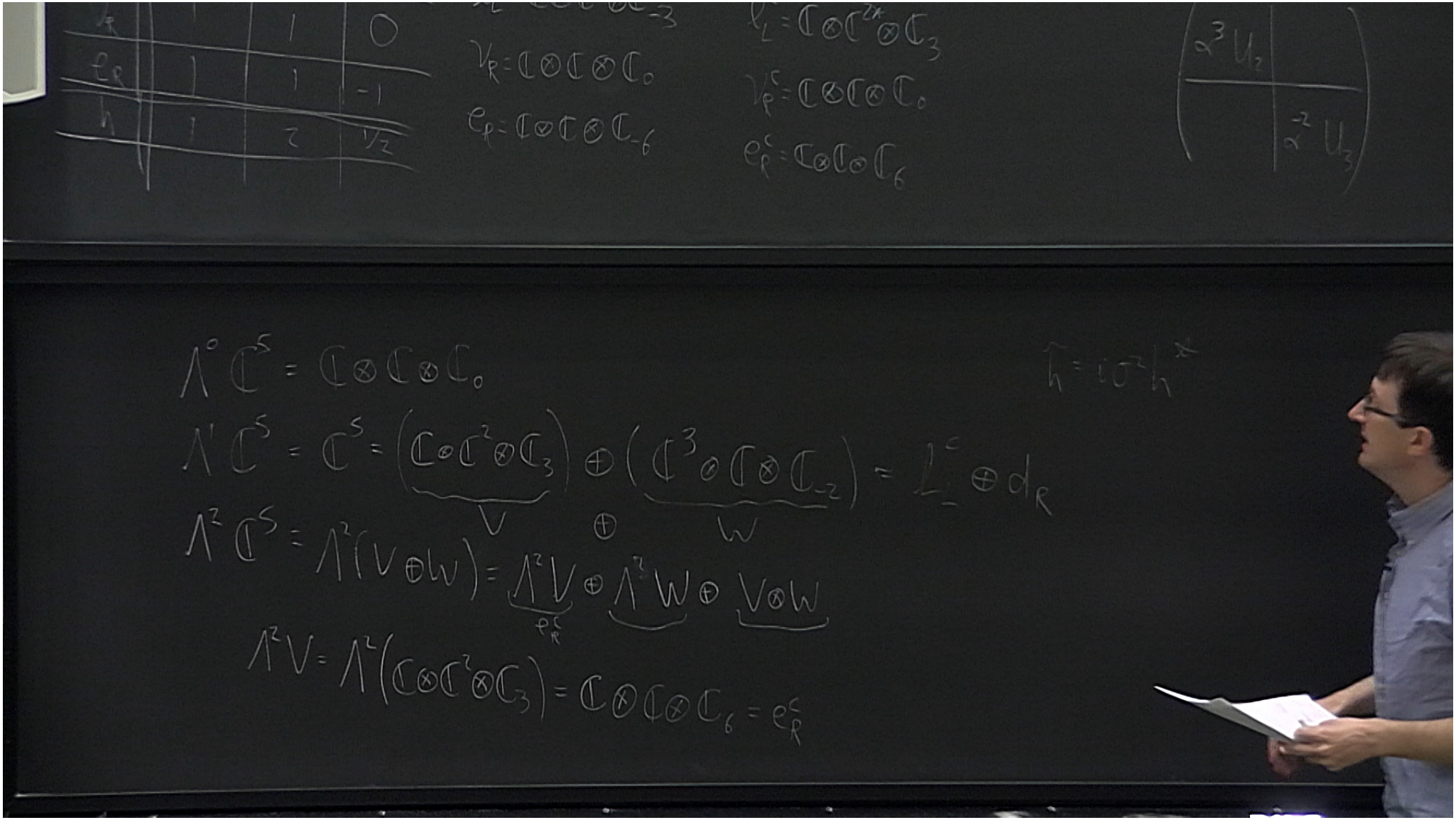
$$\Lambda^0 \mathbb{C}^5 = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$$

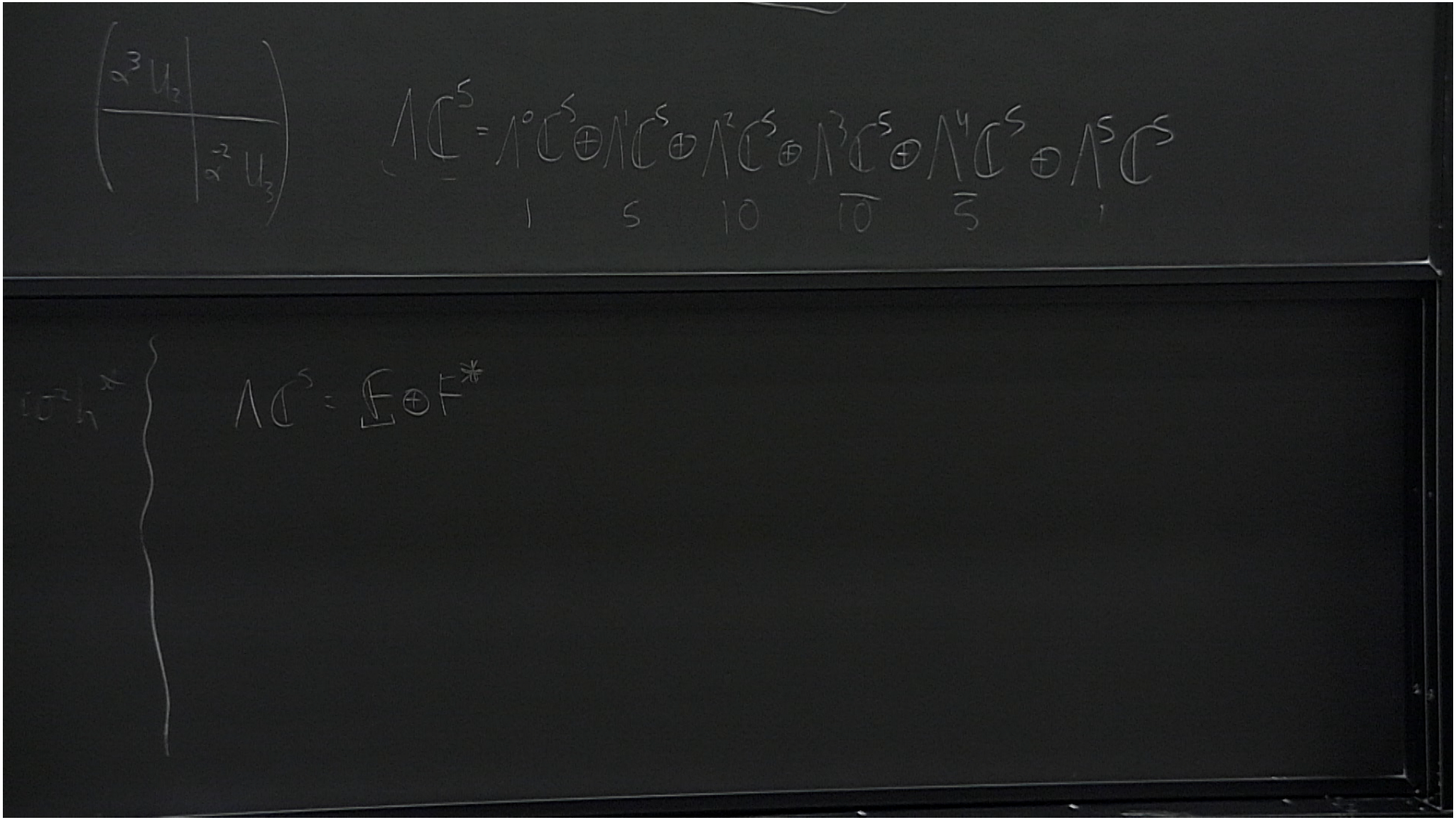
$$\Lambda^1 \mathbb{C}^5 = \mathbb{C}^5 = \underbrace{(\mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_3)}_{V} \oplus \underbrace{(\mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2})}_{W} = L^c \oplus d_R$$

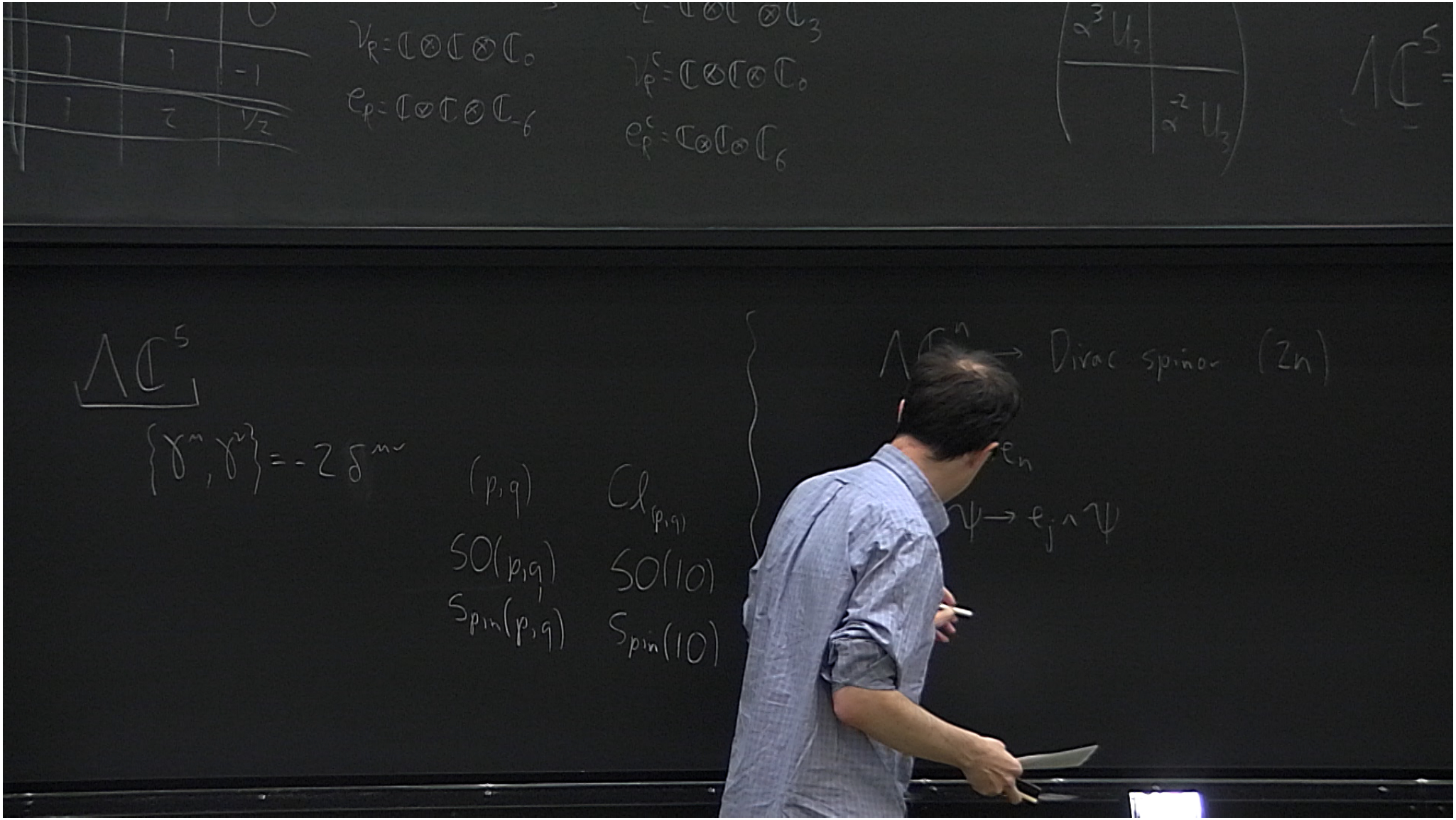
$$\Lambda^2 \mathbb{C}^5 = \Lambda^2(V \oplus W) = \Lambda^2 V \oplus \Lambda^2 W \oplus V \otimes W$$

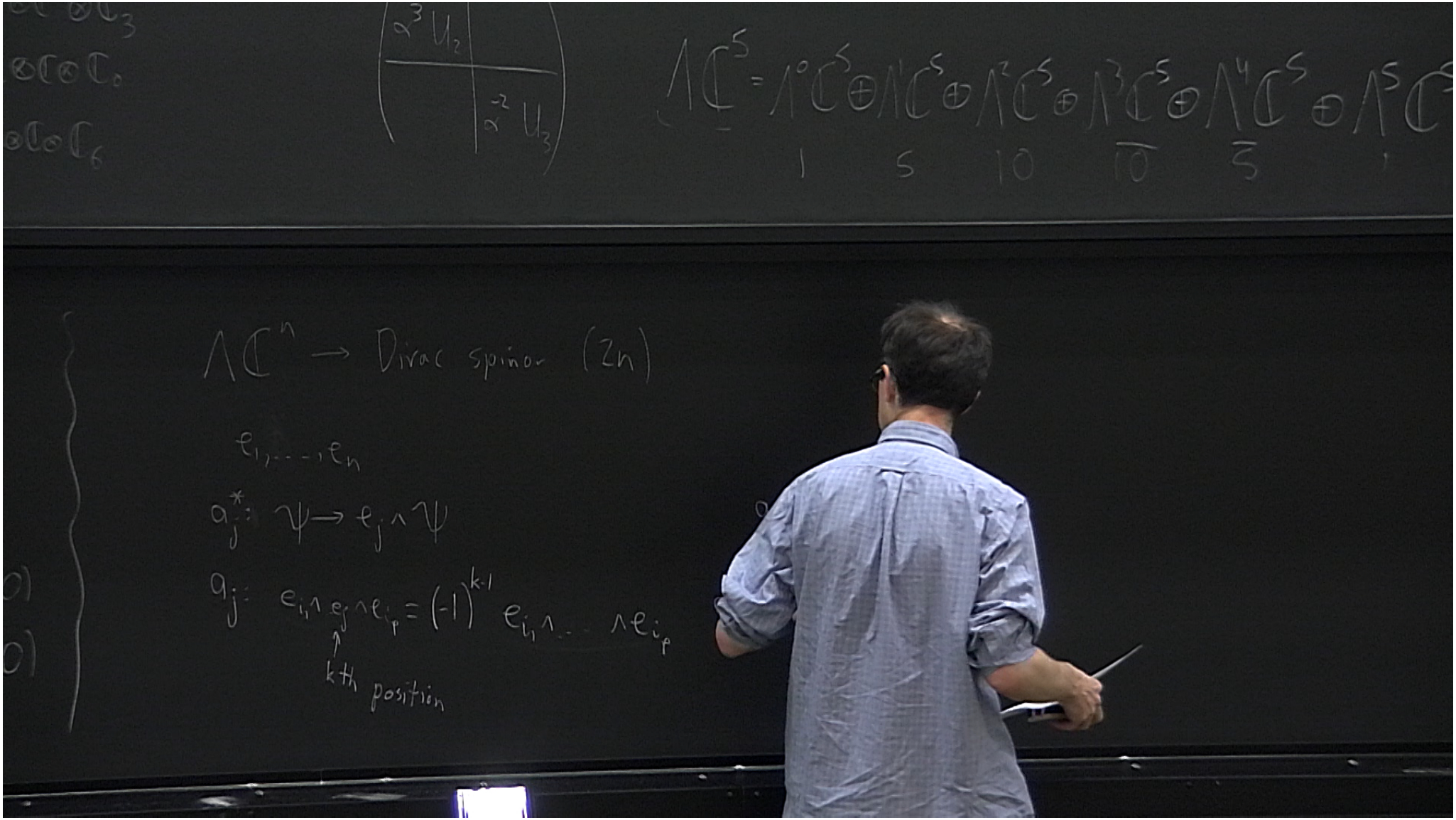
$$\Lambda^2 V = \Lambda^2(\mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_3) = \mathbb{C} \otimes$$

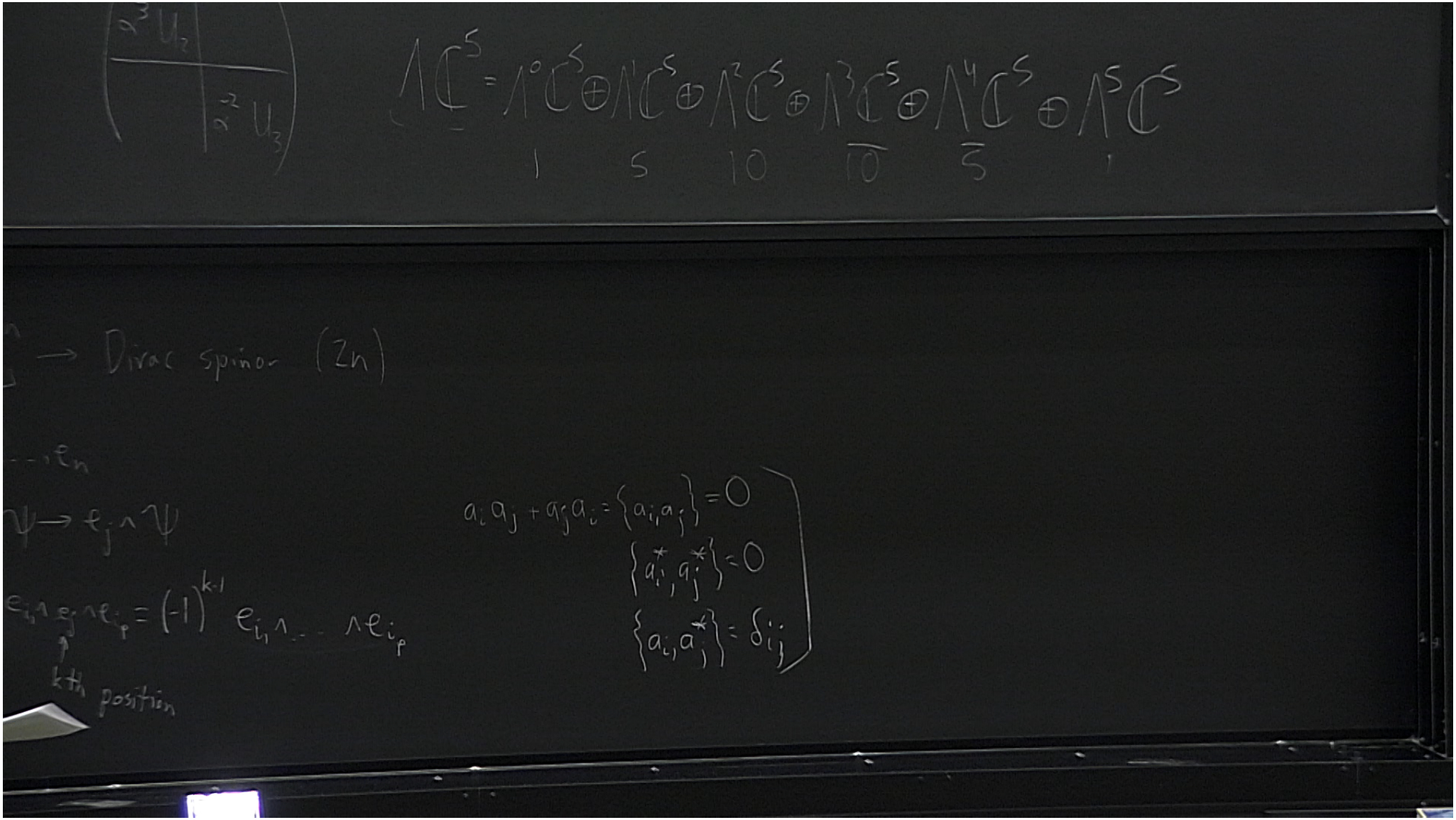
$\overline{h} = \mathbb{C}^2 h^*$

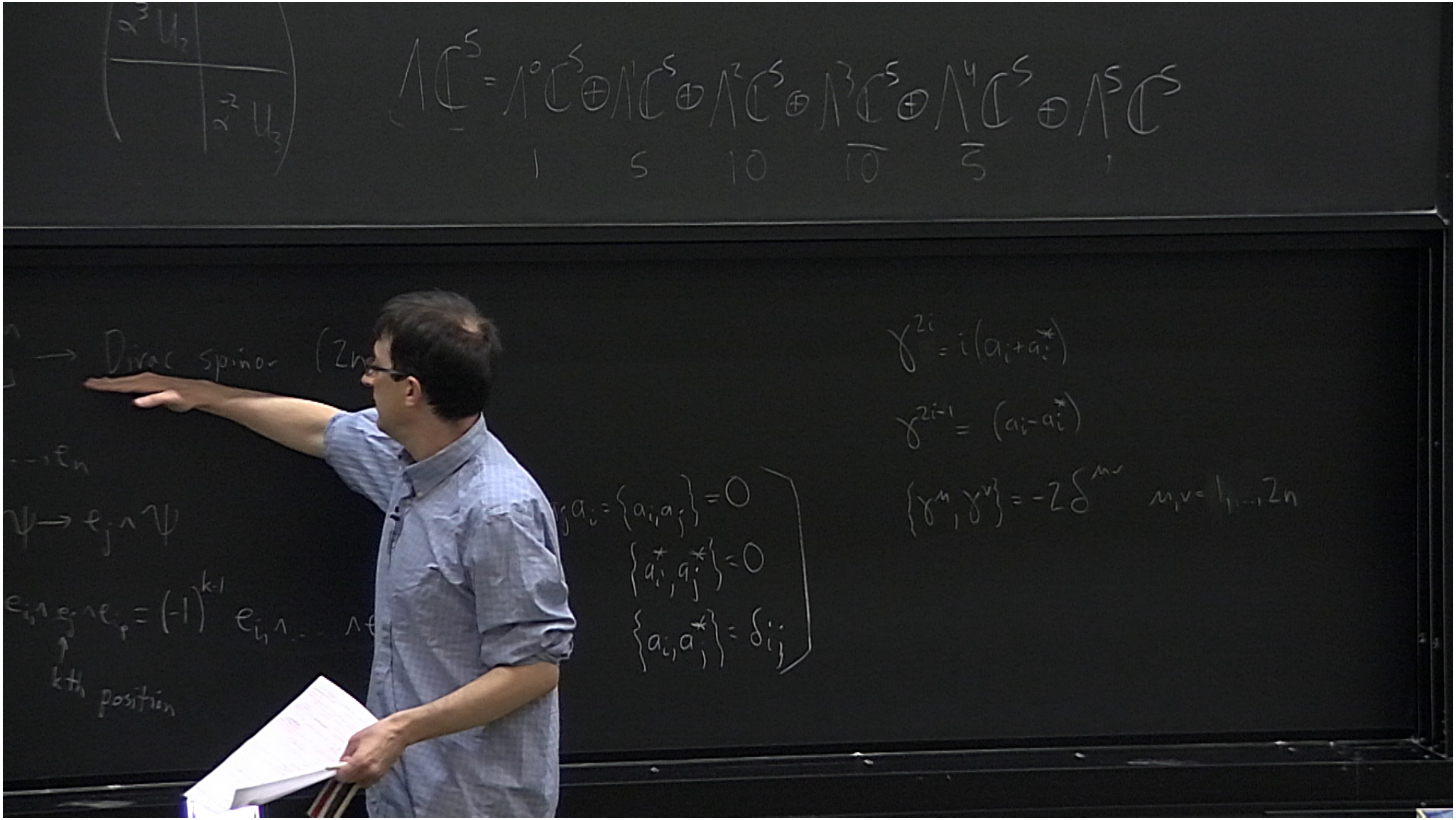

$\nu_R = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0$
$e_R = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_{-6}$
$\nu_L^c = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0$
$e_L^c = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_6$
$\Lambda^0\mathbb{C}^5 = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0$
$\Lambda^1\mathbb{C}^5 = \mathbb{C}^5 = (\mathbb{C}\otimes\mathbb{C}^2\otimes\mathbb{C}_3) \oplus (\mathbb{C}^3\otimes\mathbb{C}\otimes\mathbb{C}_{-2}) = L^c \oplus d_R$
$\Lambda^2\mathbb{C}^5 = \Lambda^2(V\oplus W) = \Lambda^2 V \oplus \Lambda^2 W \oplus V\otimes W$
$\Lambda^2 V = \Lambda^2(\mathbb{C}\otimes\mathbb{C}^2\otimes\mathbb{C}_3) = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_6 = e_R^c$
$\bar{h} = i\sigma^2 h^*$

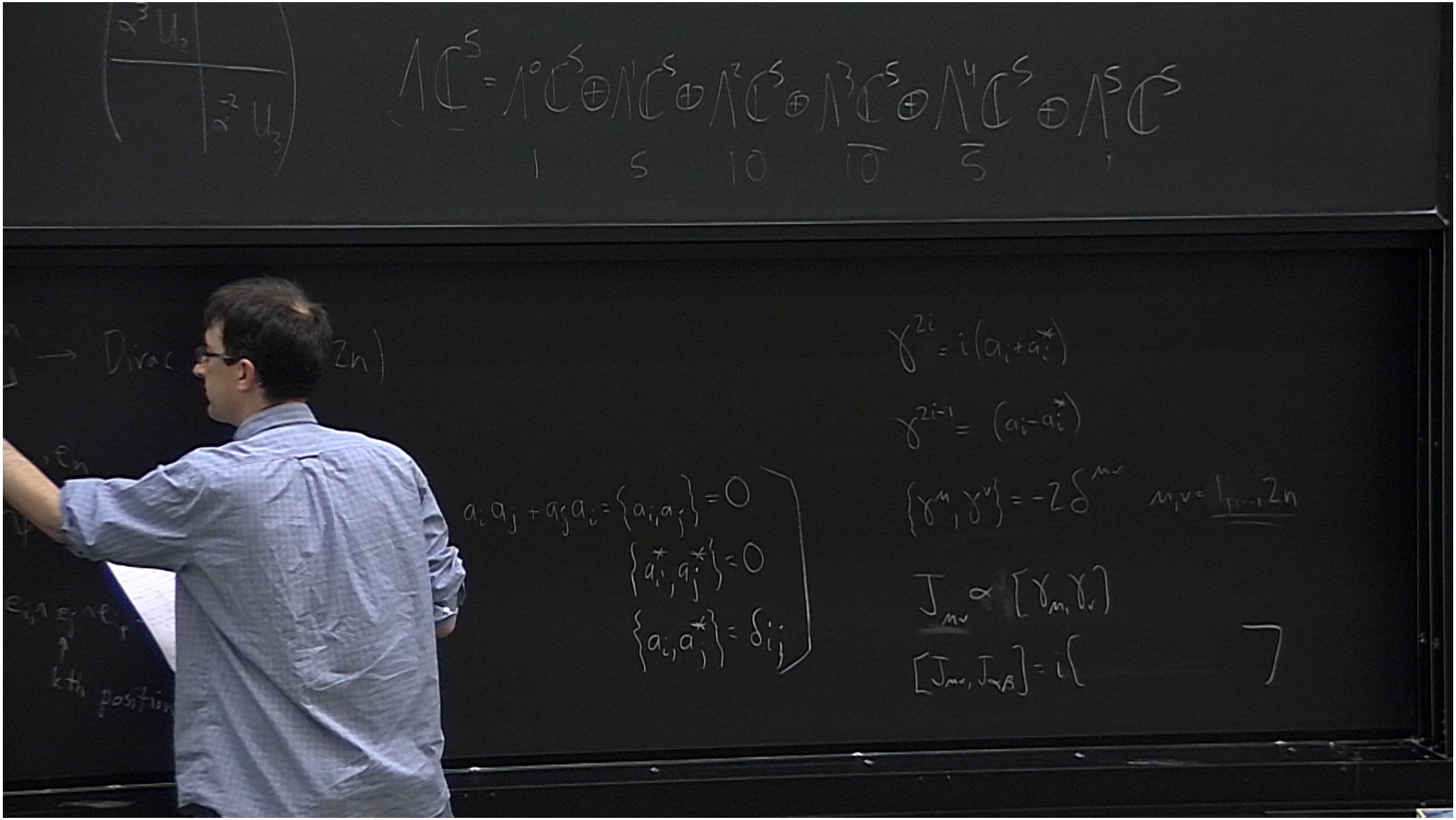

$\begin{pmatrix} a^3 U_2 & \\ & a^{-2} U_3 \end{pmatrix}$
$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$
1 5 10 $\overline{10}$ $\overline{5}$ 1
$\Lambda \mathbb{C}^5 = \Sigma \oplus F^*$


$\nu_R = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0$
$e_R = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_6$
$\nu_R^c = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0$
$e_R^c = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_6$
$\Lambda\mathbb{C}^5$
$\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$
(p,q) $Cl_{(p,q)}$
$SO(p,q)$ $SO(10)$
$Spin(p,q)$ $Spin(10)$
$\Lambda\mathbb{C}^n \to$ Dirac spinor (2^n)
$\psi \to e_j \wedge \psi$


$\mathbb{C}^3 \otimes \mathbb{C}_3$
$\otimes \mathbb{C} \otimes \mathbb{C}_0$
$\otimes \mathbb{C} \otimes \mathbb{C}_6$
α^3 U_2
α^{-2} U_3
$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$
1 5 10 $\overline{10}$ $\overline{5}$ 1
$\Lambda \mathbb{C}^n \to$ Dirac spinor (2^n)
$e_1, \ldots, e_n$
$a_j^*: \psi \to e_j \wedge \psi$
$a_j: e_{i_1} \wedge e_j \wedge e_{i_p} = (-1)^{k-1} e_{i_1} \wedge \ldots \wedge e_{i_p}$
kth position


$\begin{pmatrix} a^3 U_2 & \\ & a^{-2} U_3 \end{pmatrix}$
$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$
1 5 10 $\overline{10}$ $\overline{5}$ 1
$\rightarrow$ Dirac spinor $(\mathbb{Z}_n)$
$\ldots, e_n$
$\psi \rightarrow e_j \wedge \psi$
$e_{i_1} \wedge e_j \wedge e_{i_p} = (-1)^{k-1} e_{i_1} \wedge \ldots \wedge e_{i_p}$
k^{th} position
$a_i a_j + a_j a_i = \{a_i, a_j\} = 0$
$\{a_i^*, a_j^*\} = 0$
$\{a_i, a_j^*\} = \delta_{ij}$


$\Lambda\mathbb{C}^5 = \Lambda^0\mathbb{C}^5 \oplus \Lambda^1\mathbb{C}^5 \oplus \Lambda^2\mathbb{C}^5 \oplus \Lambda^3\mathbb{C}^5 \oplus \Lambda^4\mathbb{C}^5 \oplus \Lambda^5\mathbb{C}^5$
1 5 10 $\overline{10}$ $\overline{5}$ 1
$\gamma^{2i} = i(a_i + a_i^*)$
$\gamma^{2i-1} = (a_i - a_i^*)$
$\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$ $\mu,\nu = 1,\dots,2n$
$\{a_i, a_j\} = 0$
$\{a_i^*, a_j^*\} = 0$
$\{a_i, a_j^*\} = \delta_{ij}$
$\psi \to e_j \wedge \psi$
$e_{i_1} \wedge e_{i_2} \wedge e_{i_3} = (-1)^{k-1} e_{i_1} \wedge \dots$
kth position


$\begin{pmatrix} a^3 U_2 & \\ & a^{-2} U_3 \end{pmatrix}$
$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$
1 5 10 $\overline{10}$ $\overline{5}$ 1
$\gamma^{2i} = i(a_i + a_i^*)$
$\gamma^{2i-1} = (a_i - a_i^*)$
$\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$ $\mu, \nu = 1, \ldots, 2n$
$J_{\mu\nu} \propto [\gamma_\mu, \gamma_\nu]$
$[J_{\mu\nu}, J_{\nu\beta}] = i($
$a_i a_j + a_j a_i = \{a_i, a_j\} = 0$
$\{a_i^*, a_j^*\} = 0$
$\{a_i, a_j^*\} = \delta_{ij}$
$\rightarrow$ Dirac
$2n)$
e_n
$e_i \wedge e_j \wedge e_r$
kth position

$$\begin{pmatrix} a^3 U_2 & \\ \hline & a^{-2} U_3 \end{pmatrix}$$

$$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$$

$$1 \quad 5 \quad 10 \quad \overline{10} \quad \overline{5} \quad 1$$

$\Lambda \mathbb{C}^5 \to$ Dirac spinor $(10) = \underbrace{\Lambda^{\text{even}} \mathbb{C}^n}_{\text{left Weyl Spin(10) SO(2n)}} \oplus \underbrace{\Lambda^{\text{odd}} \mathbb{C}^n}_{\text{right Weyl Spin(10)}}$

$e_1, \dots, e_n$

$a_j^*: \psi \to e_j \wedge \psi$

$a_j: e_{i_1} \wedge e_{i_2} \wedge e_{i_p} = (-1)^{k-1} e_{i_1} \wedge \dots \wedge e_{i_p}$

(kth position)

$$a_i a_j + a_j a_i = \{a_i, a_j\} = 0$$

$$\{a_i^*, a_j^*\} = 0$$

$$\{a_i, a_j^*\} = \delta_{ij}$$

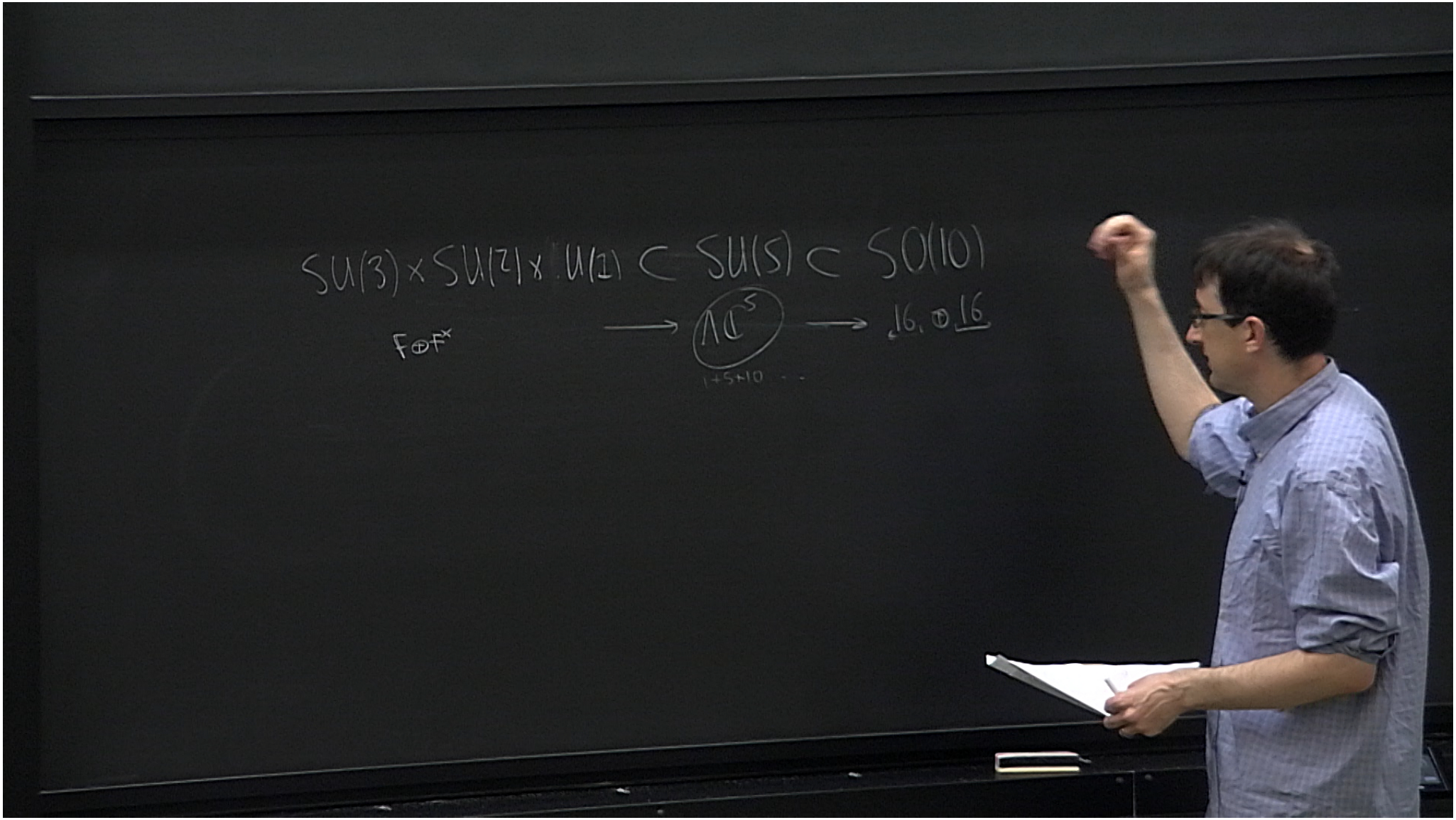
$$\gamma^{2i} = i(a_i + a_i^*)$$

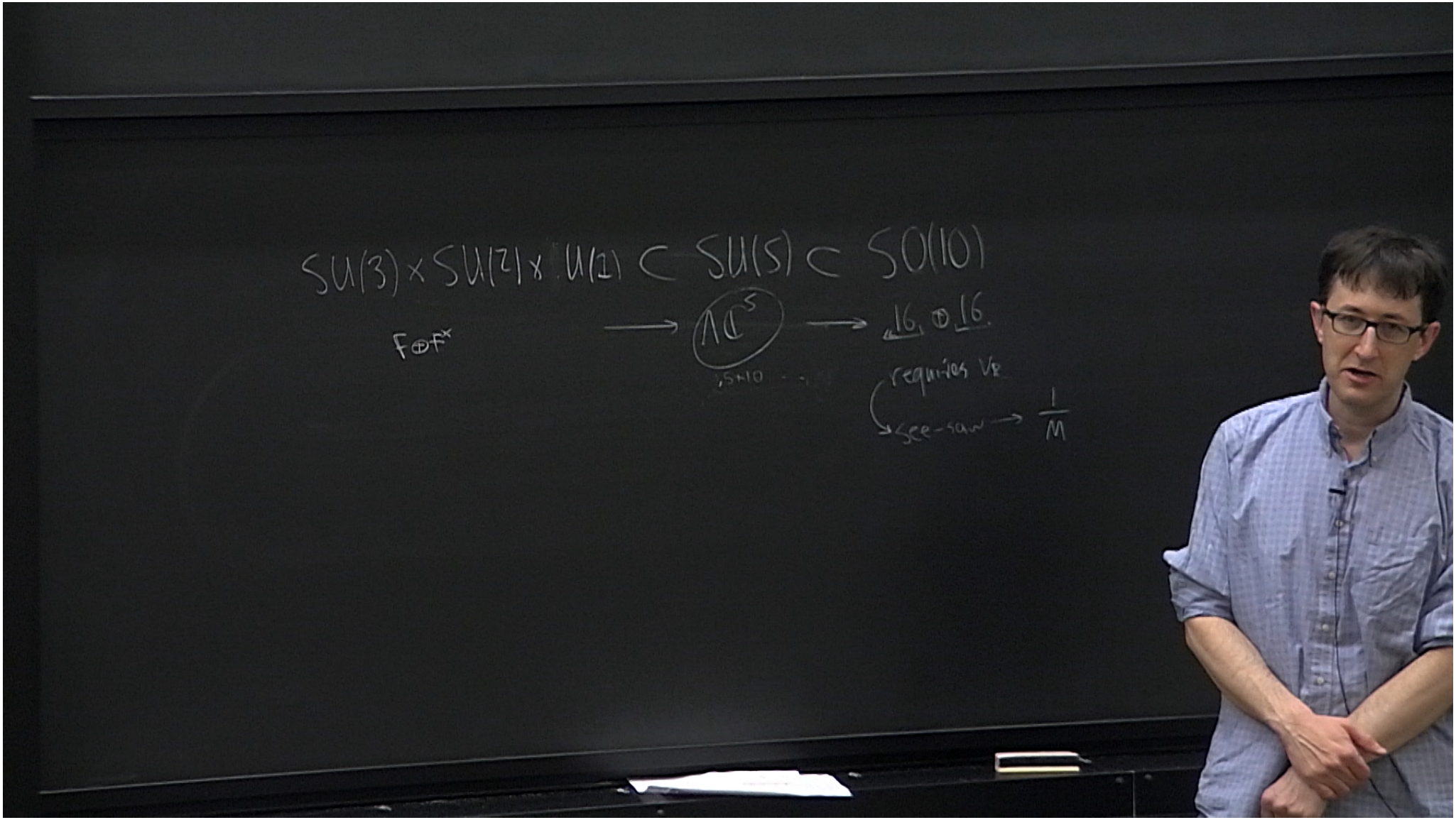
$$\gamma^{2i-1} = (a_i - a_i^*)$$

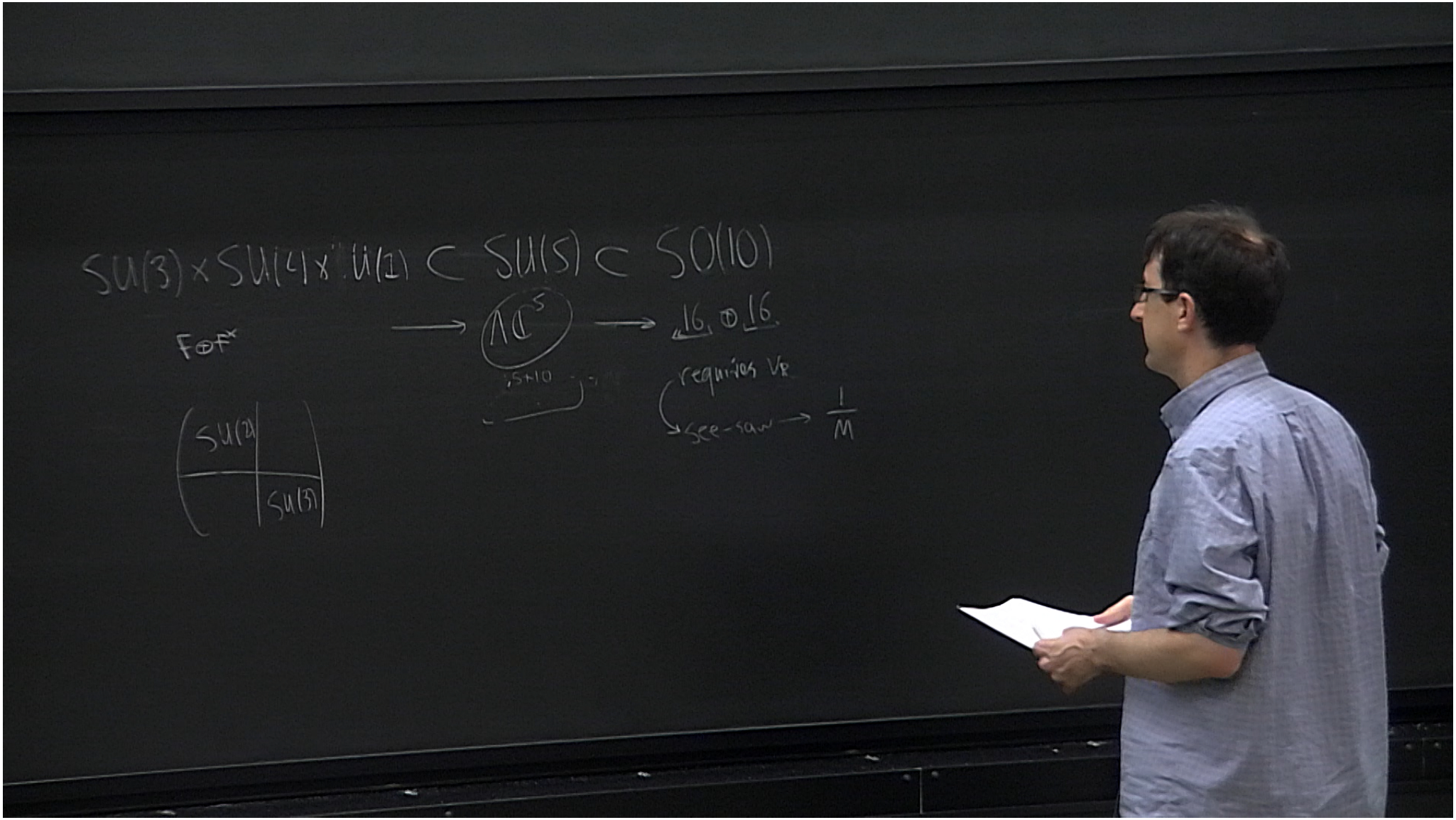
$$\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu} \qquad \mu, \nu = 1, \dots, 2n$$

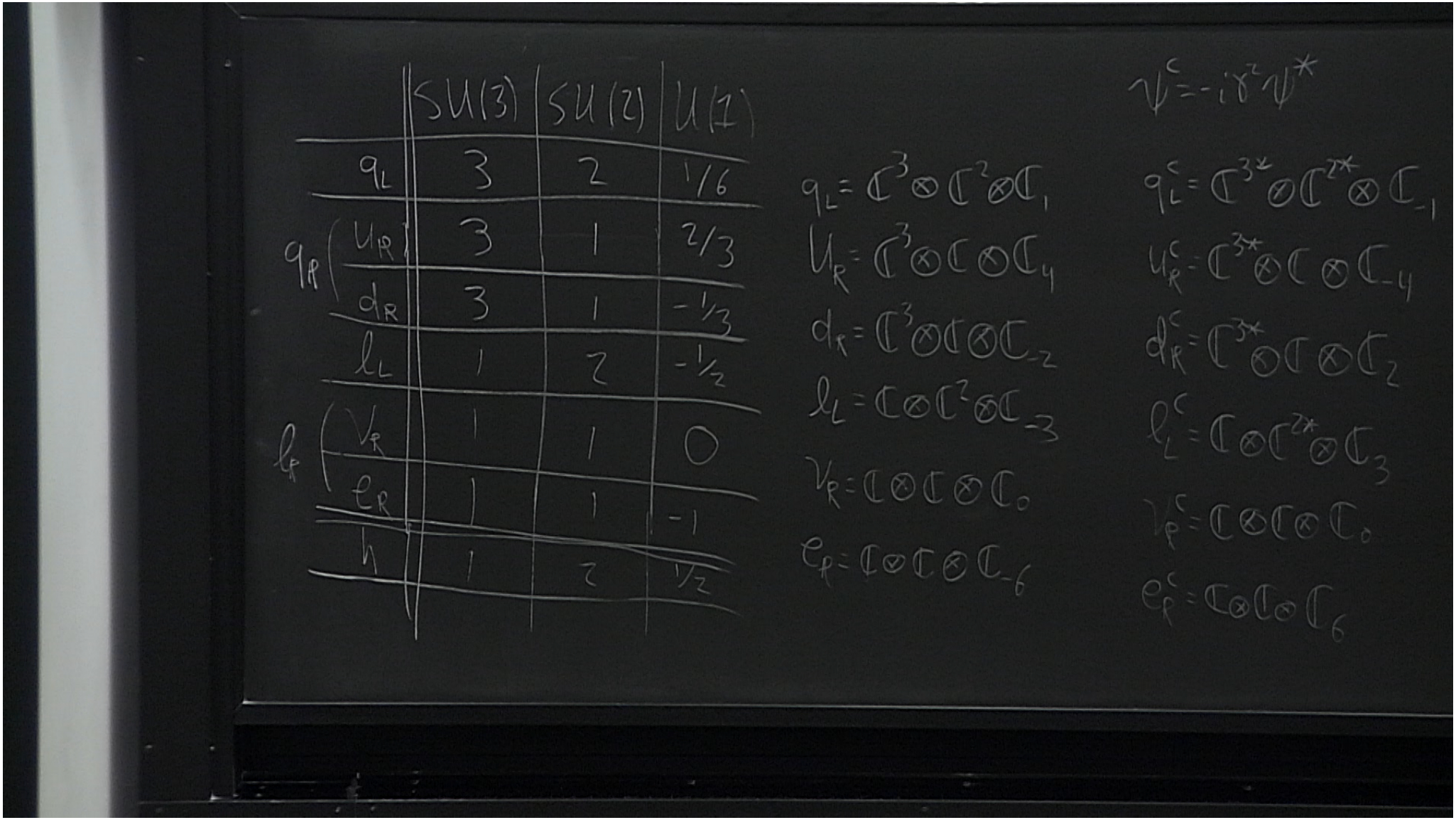
$$J_{\mu\nu} \propto [\gamma_\mu, \gamma_\nu]$$

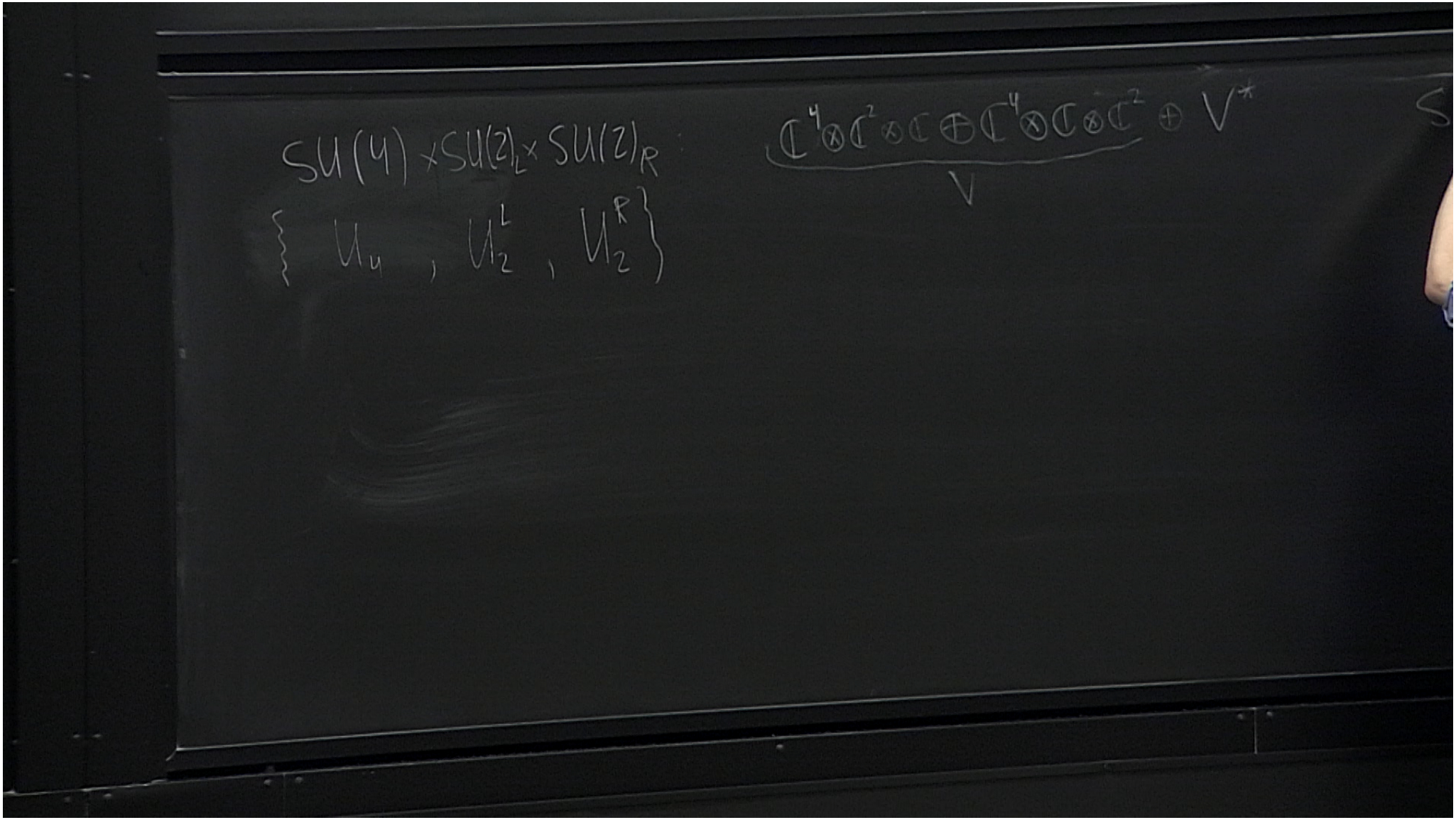
$$[J_{\mu\nu}, J_{\alpha\beta}] = i[\quad]$$

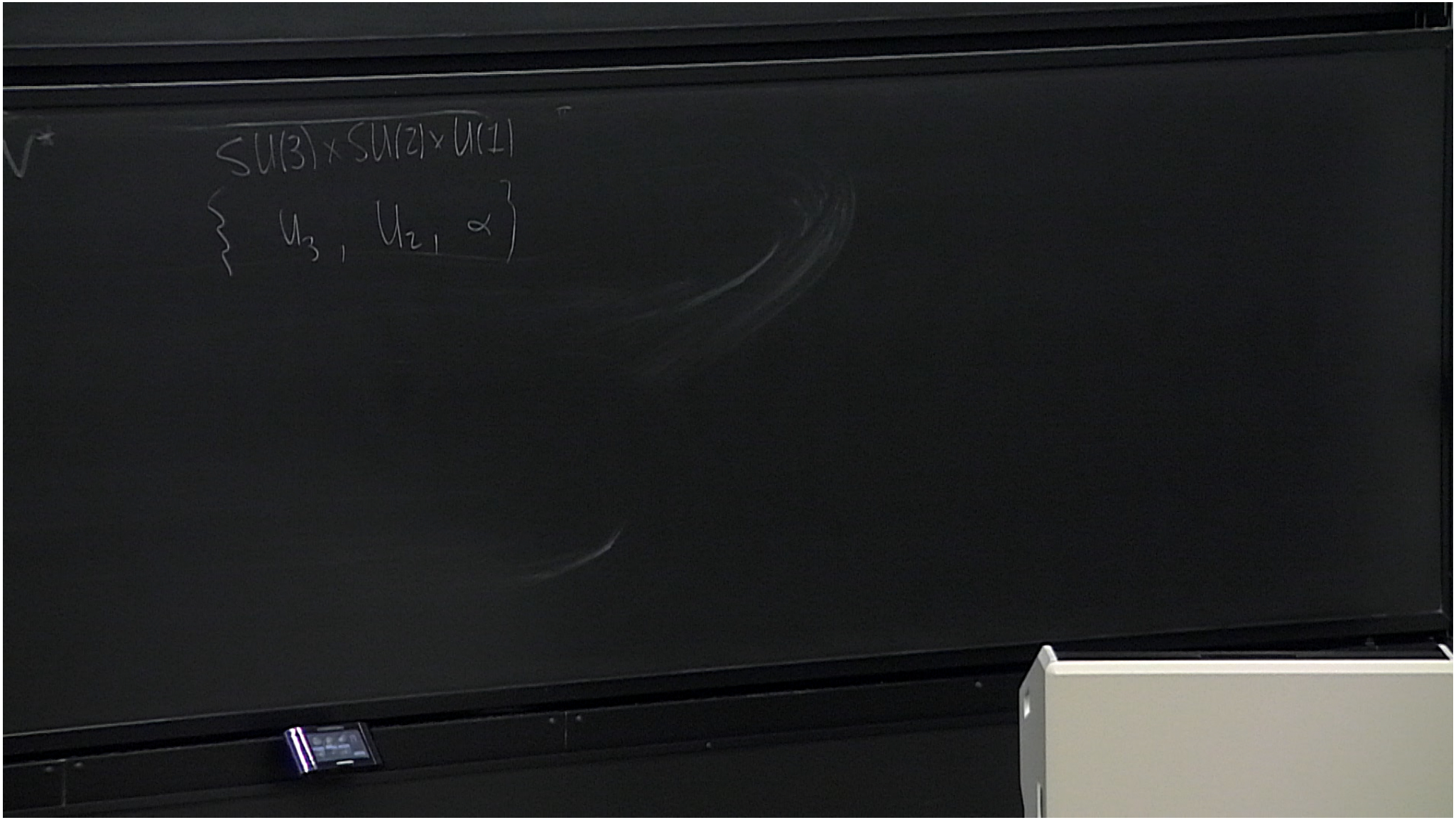

SU(3) × SU(2) × U(1) ⊂ SU(5) ⊂ SO(10)
F ⊕ F*
1 + 5 + 10 . . .
16, ⊕ 16

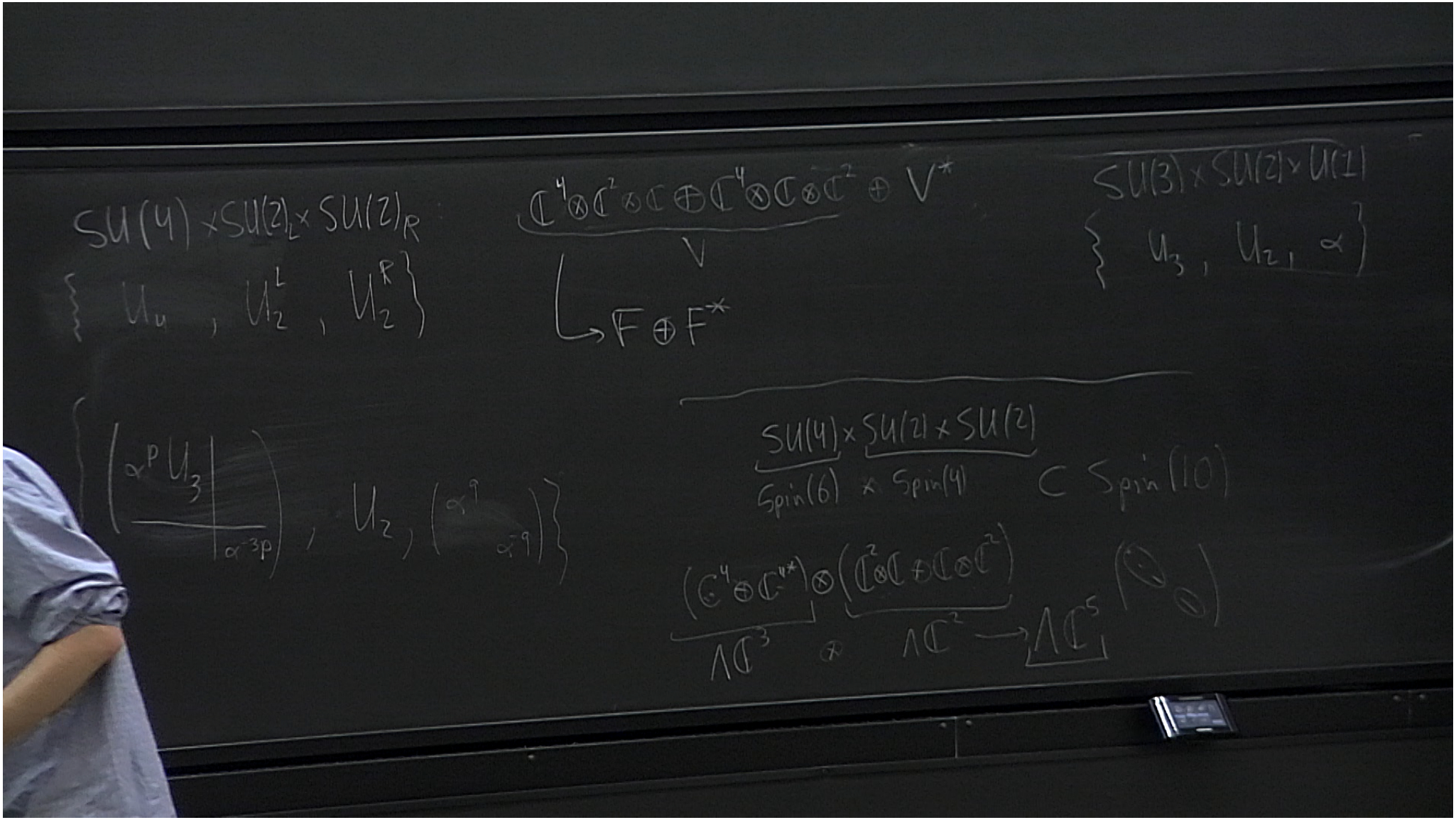

SU(3) × SU(2) × U(1) ⊂ SU(5) ⊂ SO(10)
F ⊕ F*
10 ⊕ 5̄
5+10
16 ⊕ 16
requires νR
see-saw → 1/M


SU(3) × SU(4) × U(1) ⊂ SU(5) ⊂ SO(10)
Fof
10
5×10
16 ⊕ 16
requires νR
see-saw
1/M
SU(2)
SU(3)


	SU(3)	SU(2)	U(1)
q_L	3	2	1/6
q_R u_R	3	1	2/3
q_R d_R	3	1	-1/3
l_L	1	2	-1/2
l_R ν_R	1	1	0
l_R e_R	1	1	-1
h	1	2	1/2
$q_L = \mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1$
$u_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_4$
$d_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2}$
$l_L = \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-3}$
$\nu_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$
$e_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_{-6}$
$\psi^c = -i\gamma^2\psi^*$
$q_L^c = \mathbb{C}^{3^*} \otimes \mathbb{C}^{2^*} \otimes \mathbb{C}_{-1}$
$u_R^c = \mathbb{C}^{3^*} \otimes \mathbb{C} \otimes \mathbb{C}_{-4}$
$d_R^c = \mathbb{C}^{3^*} \otimes \mathbb{C} \otimes \mathbb{C}_2$
$l_L^c = \mathbb{C} \otimes \mathbb{C}^{2^*} \otimes \mathbb{C}_3$
$\nu_R^c = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$
$e_R^c = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6$


$SU(4) \times SU(2)_L \times SU(2)_R$
$\{ U_4 , U_2^L , U_2^R \}$
$\underbrace{\mathbb{C}^4 \otimes \mathbb{C}^2 \otimes \mathbb{C} \oplus \mathbb{C}^4 \otimes \mathbb{C} \otimes \mathbb{C}^2}_{V} \oplus V^*$


SU(3) × SU(2) × U(1)
{ u3 , u2 , α }


$SU(4) \times SU(2)_L \times SU(2)_R$
$\{ U_4 , U_2^L , U_2^R \}$
$\mathbb{C}^4 \otimes \mathbb{C}^2 \otimes \mathbb{C} \oplus \mathbb{C}^4 \otimes \mathbb{C} \otimes \mathbb{C}^2 \oplus V^*$
V
$\hookrightarrow F \oplus F^*$
$SU(3) \times SU(2) \times U(1)$
$\{ U_3 , U_2 , \alpha \}$
$\left\{ \left(\begin{array}{c|c} \alpha^p U_3 & \\ \hline & \alpha^{-3p} \end{array} \right) , U_2 , \left(\begin{array}{cc} \alpha^q & \\ & \alpha^{-q} \end{array} \right) \right\}$
$\frac{SU(4) \times SU(2) \times SU(2)}{Spin(6) \times Spin(4)} \subset Spin(10)$
$(\mathbb{C}^4 \otimes \mathbb{C}^{4*}) \otimes (\mathbb{C}^2 \otimes \mathbb{C} \oplus \mathbb{C} \otimes \mathbb{C}^2)$
$\Lambda \mathbb{C}^3 \quad \Lambda \mathbb{C}^2 \longrightarrow \Lambda \mathbb{C}^5$