Title: PSI 2018/2019 - Beyond Standard Model - Lecture 9

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Collection: PSI 2018/2019 - Beyond Standard Model (Boyle)

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	$SU(3)$	$SU(2)$	$U(1)$
q_L	3	2	1
u_R	3	1	4
d_R	3	1	-2
l_L	1	2	-3
ν_R	1	1	0
e_R	1	1	-6
h	1	2	3

$$\boxed{q_L : \mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1}$$

$$u_R : \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_4$$

$$d_R : \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2}$$

$$l_L : \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-3}$$

$$\nu_R : \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$$

$$e_R : \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_{-6}$$

$q_L^* =$

$u_R^* :$

d_R^*

l_L^*

ν_R^*

e_R^*

$F \oplus$

$q_L : \mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1$

$u_R : \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_4$

$d_R : \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2}$

$l_L : \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-3}$

$\nu_R : \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$

$e_R : \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_{-6}$

$q_L^* : \mathbb{C}^{3*} \otimes \mathbb{C}^{2*} \otimes \mathbb{C}_{-1}$

$u_R^* : \mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_{-4}$

$d_R^* : \mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_2$

$l_L^* : \mathbb{C} \otimes \mathbb{C}^{2*} \otimes \mathbb{C}_3$

$\nu_R^* : \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$

$e_R^* : \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6$

$$SU(3)\times SU(2)\times U(1) \longrightarrow SU(5)$$

$U_3 \quad U_2 \quad \alpha = e^{i\varphi}$

$$\begin{pmatrix} \alpha^3 U_2 & \\ & \alpha^{-2} U_3 \end{pmatrix}$$

$\alpha^{2p}\,\alpha^{3q} = 1$

$$\begin{pmatrix} \alpha^3 I_2 & \\ & \alpha^{-2} I_3 \end{pmatrix}$$

$U_2 = \alpha^{-3} I_2 \qquad \alpha^{-6} = 1$

$U_3 = \alpha^{2} I_3 \qquad \alpha^{6} = 1$

$\alpha = e^{2\pi i m/6}$

$$G = [SU(3)\times SU(2)\times U(1)]/\mathbb{Z}_6$$

$q_L: \alpha^2 \alpha^{-3} \alpha^1 = \alpha^0$ ✓

$u_R: \alpha^2 \alpha^4 = \alpha^6$ ✓

$\{\alpha^2 I_3 ; \alpha^{-3} I_2 , \alpha\} \in SU(3) \times SU(2) \times U(1) / \mathbb{Z}_6$

$SU(5): \quad \underbrace{\Lambda \mathbb{C}^5} \longrightarrow F \oplus F^*$

$SU(n)$ $\quad U^\dagger U = 1 \quad (U^\dagger)^i_j U^j_k = \delta^i_k$

$$\det U = 1 \equiv \epsilon_{i_1 \dots i_n} U^{i_1}_1 U^{i_2}_2 \cdots U^{i_n}_n = 1$$

$$\Rightarrow \epsilon_{i_1 \dots i_n} U^{i_1}_{j_1} \cdots U^{i_n}_{j_n} = \epsilon_{j_1 \dots j_n}$$

$$(U^\dagger)^{j_n}_{k_n} \qquad (U^\dagger)^{j_n}_{k_n}$$

$$\epsilon_{i_1 \dots i_n} U^{i_1}_{j_1} \cdots U^{i_{n-1}}_{j_{n-1}} = \epsilon_{j_1 \dots j_n} (U^\dagger)^{j_n}_{i_n}$$

$\Lambda^2 \mathbb{C}^5$ $\quad \mathbb{C}^5 \otimes \mathbb{C}^5$

$v_1, \dots, v_5$ $\quad v_i \otimes v_j - v_j \otimes v_i = v_i \wedge v_j \leftarrow \binom{5}{2} = 10$

$\Lambda^3 \mathbb{C}^5 \quad \mathbb{C}^5 \otimes \mathbb{C}^5 \otimes \mathbb{C}^5 \quad \binom{5}{3} = 10$

$v_1 \wedge v_2 \wedge v_3 \wedge v_4 \wedge v_5$

$$\boxed{\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C} \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5} = \boxed{F \oplus F^*}$$

$$1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$$

SU

$\mathbb{C}^5 \quad \varphi^i \to U^i_j \varphi^j$

$\Lambda^2 \mathbb{C}^5 \quad \varphi^{ij} \to U^i_k U^j_l \varphi^{kl}$

$\Lambda^n \mathbb{C}^5 \quad \varphi^{i_1 \dots i_n}$

$\left(\Lambda^{(5-n)} \mathbb{C}^5\right)^*$