Title: PSI 2018/2019 - Beyond Standard Model - Lecture 8

Speakers: Latham Boyle

Collection: PSI 2018/2019 - Beyond Standard Model (Boyle)

Date: April 03, 2019 - 11:30 AM

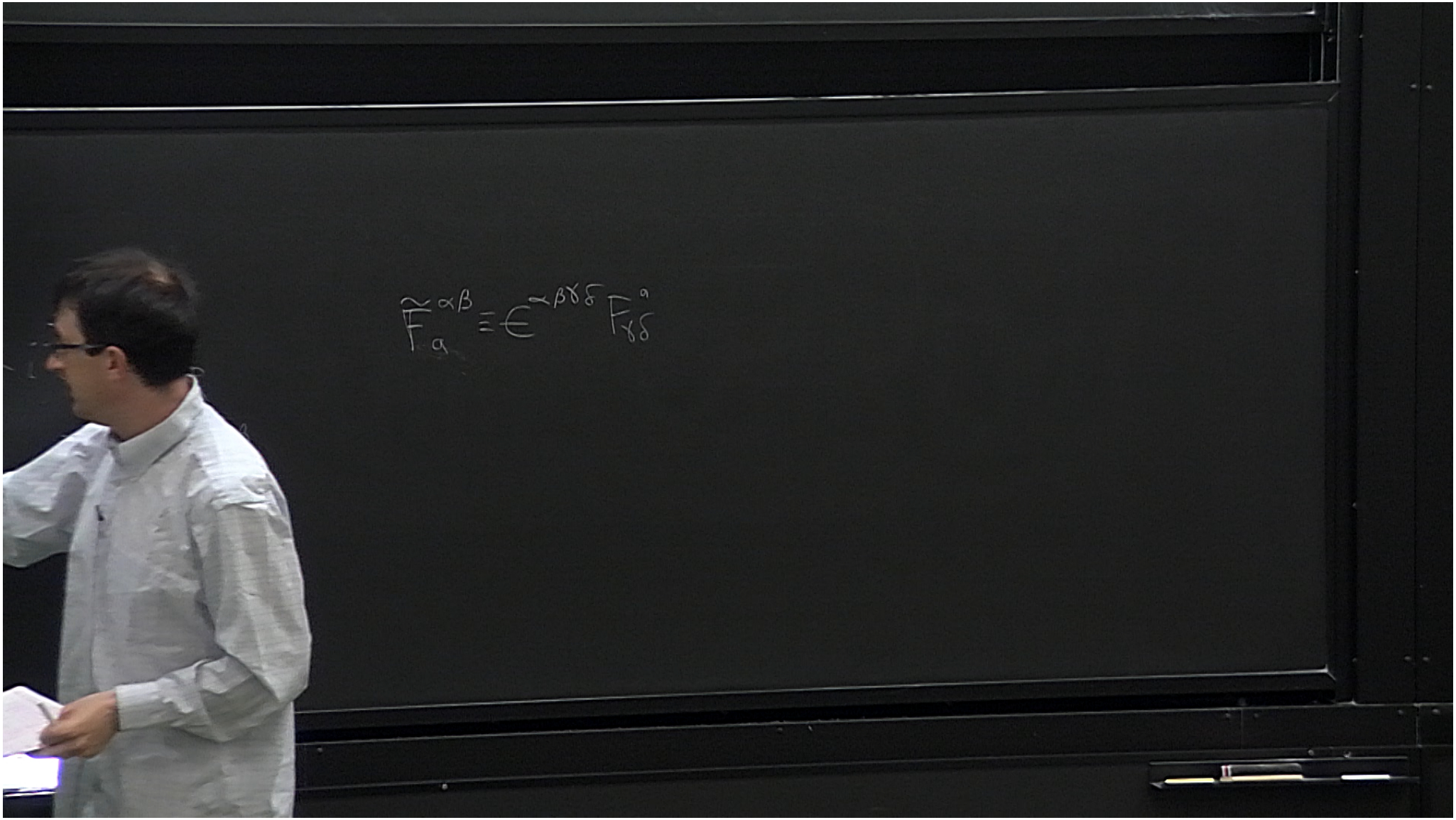
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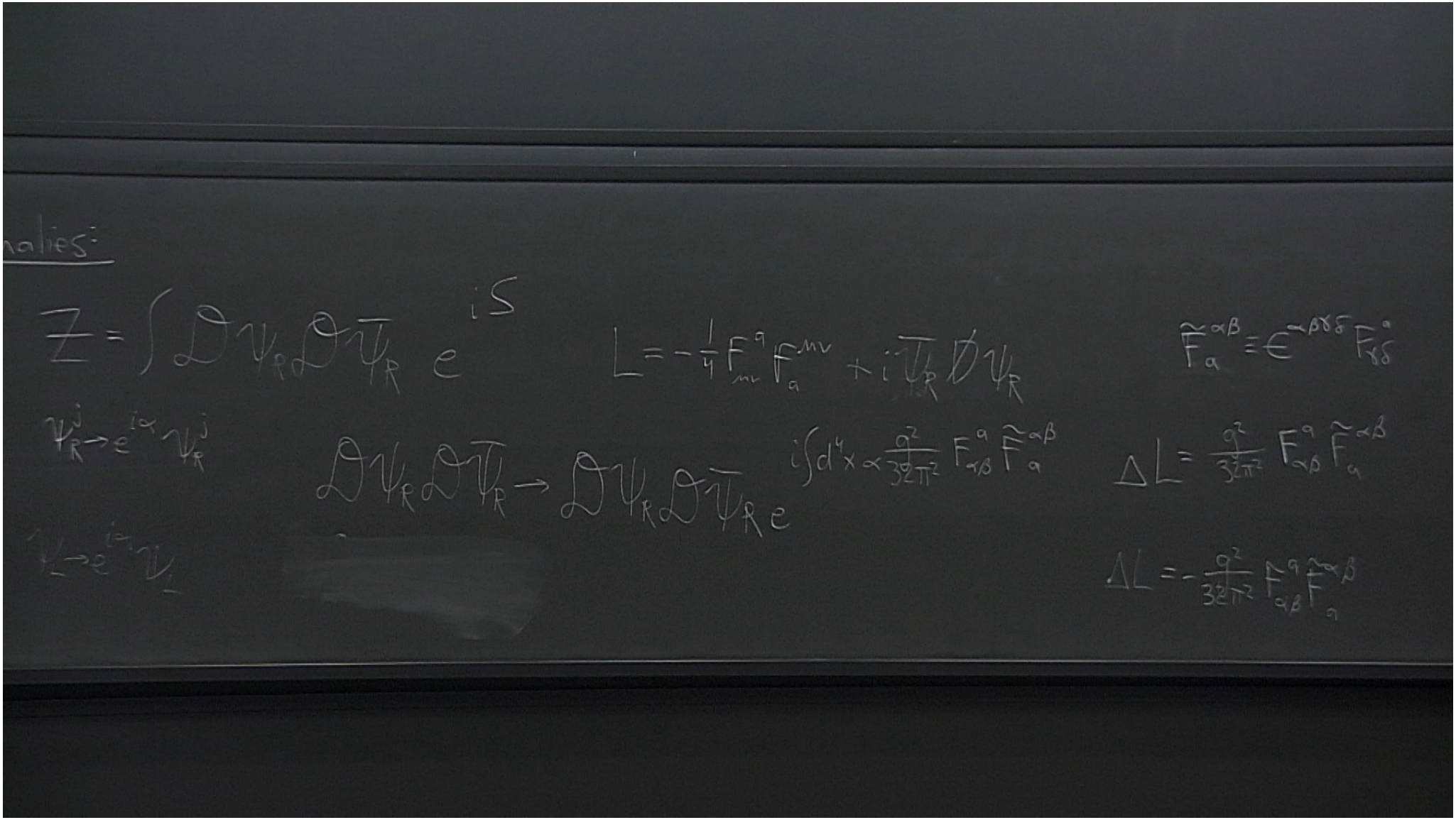
Anomalies:

$$Z = \int \mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \, e^{iS} \qquad \mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + i \bar{\psi}_R \not{D} \psi_R$$

$$\psi_R^j \to e^{i\alpha} \psi_R^j$$

$$\mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \to \mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \, e^{i \int d^4x \, \alpha \frac{g^2}{32\pi^2}}$$


$\tilde{F}_a^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\delta} F_{\gamma\delta}{}^{a}$


nalies:
$Z = \int \mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \, e^{iS}$
$L = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + i\bar{\psi}_R \not{D} \psi_R$
$\tilde{F}^{\alpha\beta}_a \equiv \epsilon^{\alpha\beta\gamma\delta} F^a_{\gamma\delta}$
$\psi^j_R \to e^{i\alpha} \psi^j_R$
$\mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \to \mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R e^{i\int d^4x \, \alpha \frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a}$
$\Delta L = \frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$
$\psi_L \to e^{i\alpha} \psi_L$
$\Delta L = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$

$\psi_R \to e^{i\alpha} \psi_R$

$\psi_L \to e^{i\alpha} \psi_L$

$\mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \to \mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R e^{i\int d^4x \propto \frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a}$

$\Delta L = \frac{g^2}{32\pi^2}$

$\Delta L = -\frac{g^2}{32\pi^2}$

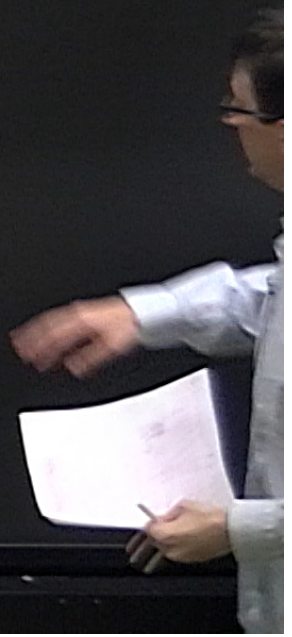
$$\mathcal{L}_{SM} = (\text{bosonic terms}) + \text{fermion kinetic}$$

$$- \left[\bar{q}_L Y_d h d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{l}_L Y_e h e_R + \text{h.c.} \right]$$

$Y_d = V_d^\dagger y_d U_d$

$Y_u = V_u^\dagger y_u U_u$

$Y_e = V_e^\dagger y_e U_e$

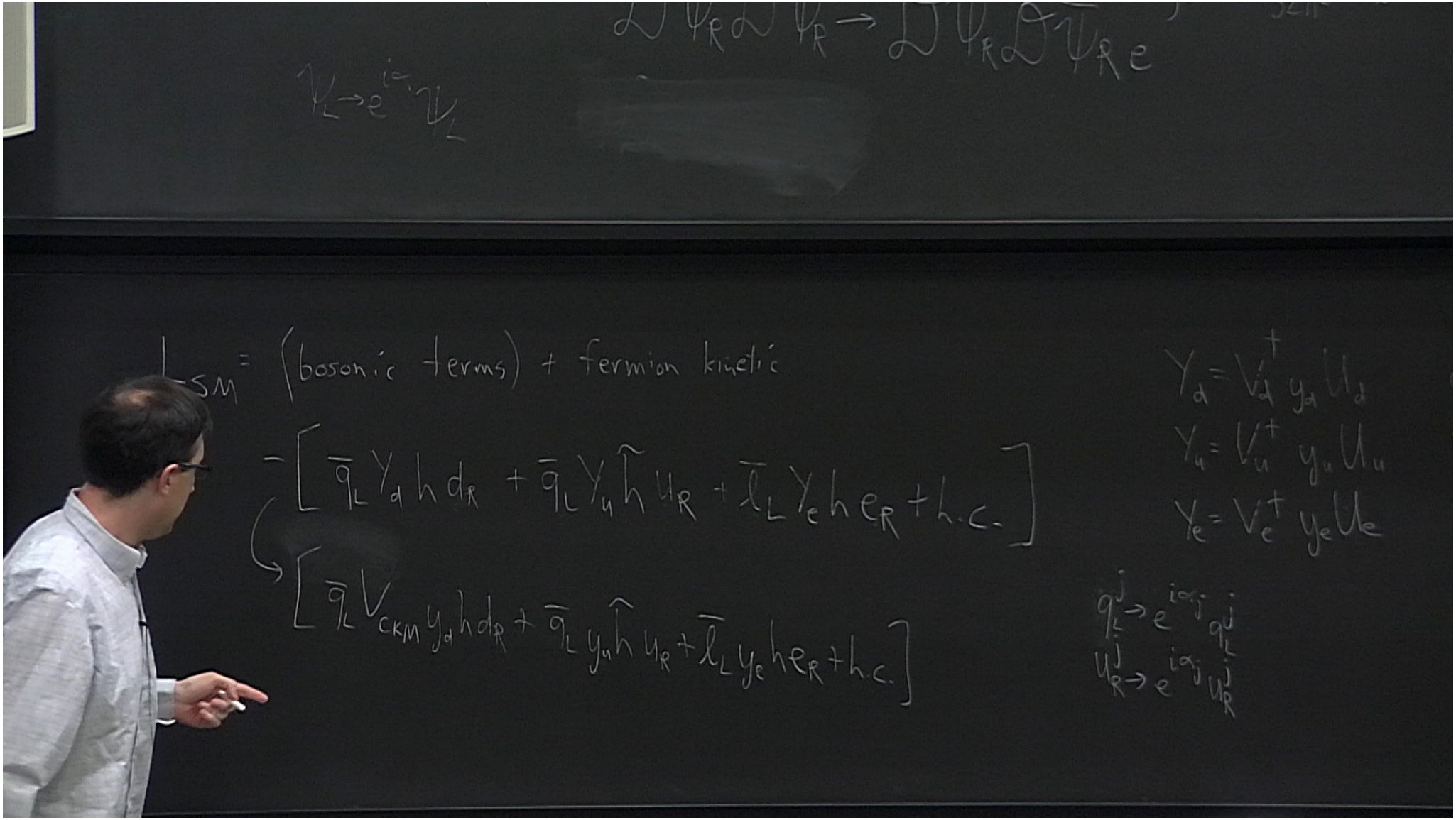


$$\Delta\mathcal{L} = -\frac{g^2}{32\pi^2} F^{a}_{\alpha\beta} \tilde{F}^{\alpha\beta}_{a}$$

$$Y_d = V_d^{\dagger}\, y_d\, U_d \qquad d_R' = U_d\, d_R$$

$$Y_u = V_u^{\dagger}\, y_u\, U_u \qquad u_R' = U_u\, u_R,\ q_L' = V_u\, q_L$$

$$Y_e = V_e^{\dagger}\, y_e\, U_e \qquad e_R' = U_e\, e_R,\ l_L' = V_e\, l_L$$


$\mathcal{L}_{SM}$ = (bosonic terms) + fermion kinetic
$-[\bar{q}_L Y_d h d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{\ell}_L Y_e h e_R + h.c.]$
$[\bar{q}_L V_{CKM} y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{\ell}_L y_e h e_R + h.c.]$
$Y_d = V_d^\dagger y_d U_d$
$Y_u = V_u^\dagger y_u U_u$
$Y_e = V_e^\dagger y_e U_e$
$q_L^j \to e^{i\alpha_j} q_L^j$
$u_R^j \to e^{i\alpha_j} u_R^j$

$$\mathcal{L} \supset \bar{\psi}_R \mathcal{D} \psi_R \to \mathcal{L} \supset \bar{\psi}_R \mathcal{D} \bar{\psi}_R e$$

$$\Delta \mathcal{L} = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$$

terms) + fermion kinetic

$$\ldots d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{l}_L Y_e h e_R + h.c.]$$

$$y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{l}_L y_e h e_R + h.c.]$$

$$Y_d = V_d^\dagger y_d U_d \qquad d'_R = U_d d_R$$

$$Y_u = V_u^\dagger y_u U_u \qquad u'_R = U_u u_R, \; q'_L = V_u q_L$$

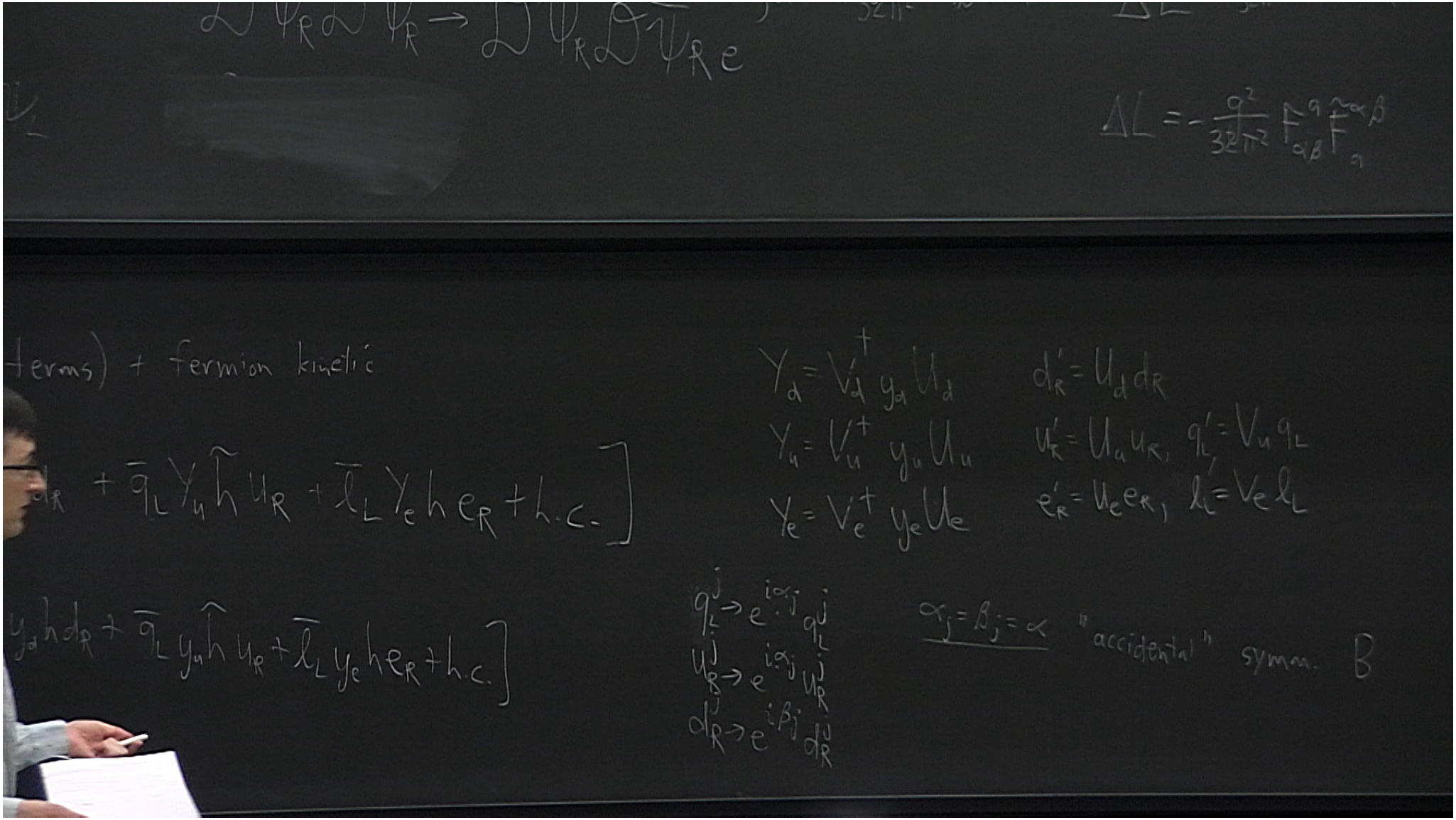
$$Y_e = V_e^\dagger y_e U_e \qquad e'_R = U_e e_R, \; l'_L = V_e l_L$$

$$q_L^j \to e^{i\alpha_j} q_L^j$$

$$u_R^j \to e^{i\alpha_j} u_R^j$$

$$d_R^j \to e^{i\beta_j} d_R^j$$

$$\underline{\alpha_j = \beta_j = \alpha}$$


$\Delta\mathcal{L} = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$
terms) + fermion kinetic
$+ \bar{q}_L Y_u \tilde{h} u_R + \bar{l}_L Y_e h e_R + h.c.]$
$y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{l}_L y_e h e_R + h.c.]$
$Y_d = V_d^\dagger y_d U_d$ $\quad d_R' = U_d d_R$
$Y_u = V_u^\dagger y_u U_u$ $\quad u_R' = U_u u_R,\ q_L' = V_u q_L$
$Y_e = V_e^\dagger y_e U_e$ $\quad e_R' = U_e e_R,\ l_L' = V_e l_L$
$q_L^j \to e^{i\alpha_j} q_L^j$
$u_R^j \to e^{i\alpha_j} u_R^j$
$d_R^j \to e^{i\beta_j} d_R^j$
$\alpha_j = \beta_j = \alpha$ "accidental" symm. B

ψ_{Re}

$$\Delta L = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$$

$$Y_d = V_d^\dagger y_d U_d \qquad d_R' = U_d d_R$$

$$Y_u = V_u^\dagger y_u U_u \qquad u_R' = U_u u_R, \; q_L' = V_u q_L$$

$$Y_e = V_e^\dagger y_e U_e \qquad e_R' = U_e e_R, \; l_L = V_e l_L$$

$$\Delta L = \frac{-g g_2^2}{32\pi^2} W^a_{\mu\nu} \tilde{W}^{\mu\nu}_a$$

$$\frac{g g_1^2}{64\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

$q_L^j \to e^{i\alpha} q_L^j$

$u_R^j \to e^{i\alpha} u_R^j$

$d_R^j \to e^{i\alpha} d_R^j$

$\alpha_j = \beta_j = \alpha$ "accidental" symm. B

$\psi_L \to e^{i\alpha} \psi_L$

$$\mathcal{L}_{SM} = (\text{bosonic terms}) + \text{fermion kinetic}$$

$$- \left[\bar{q}_L Y_d h d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{\ell}_L Y_e h e_R + \text{h.c.} \right]$$

$$\hookrightarrow \left[\bar{q}_L V_{CKM} y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{\ell}_L y_e h e_R + \text{h.c.} \right]$$

$$+ \frac{\theta_3 g_3^2}{32\pi^2} G\tilde{G}$$

$Y_d = V_d^\dagger y_d$

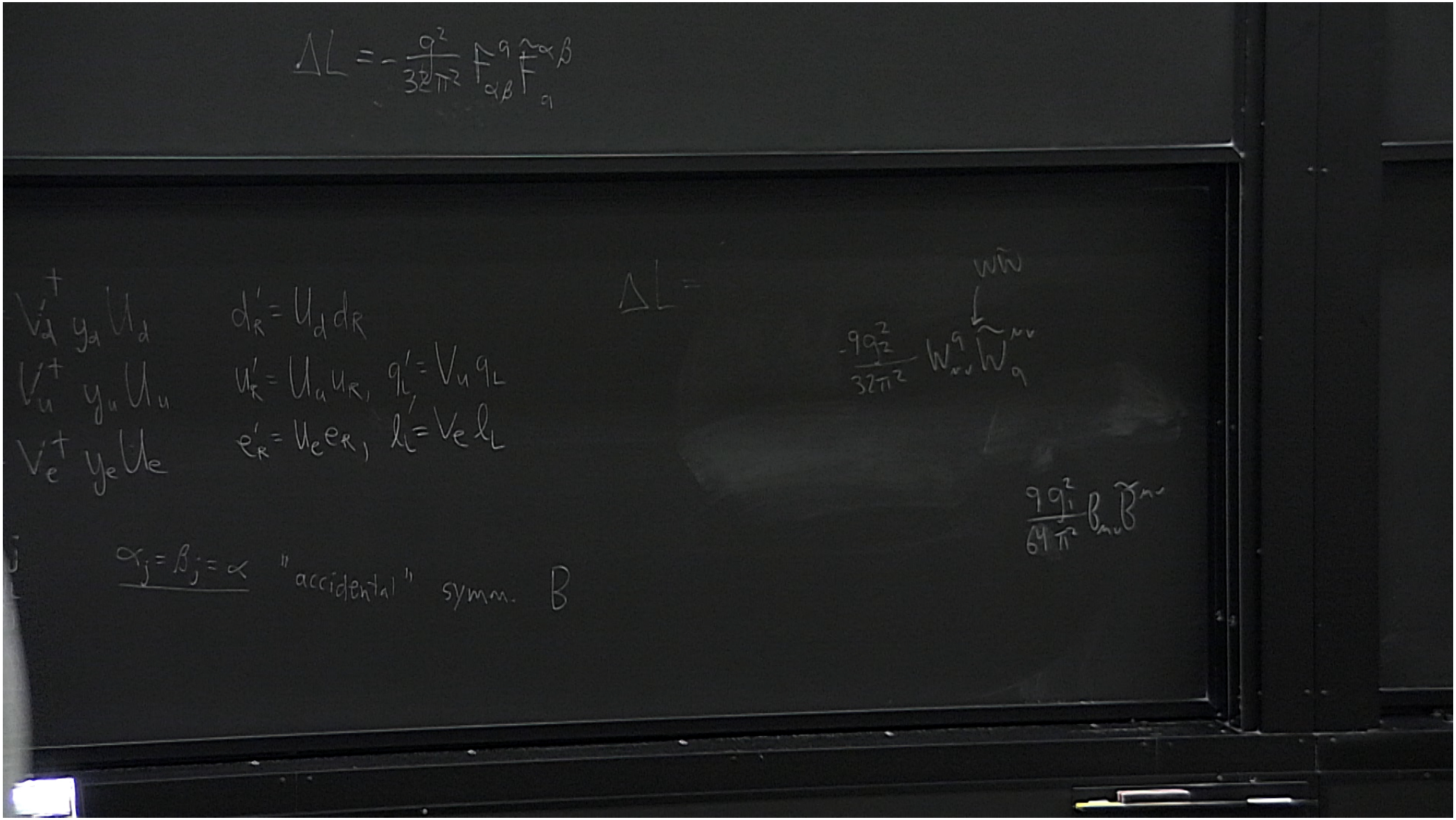
$Y_u = V_u^\dagger y_u$

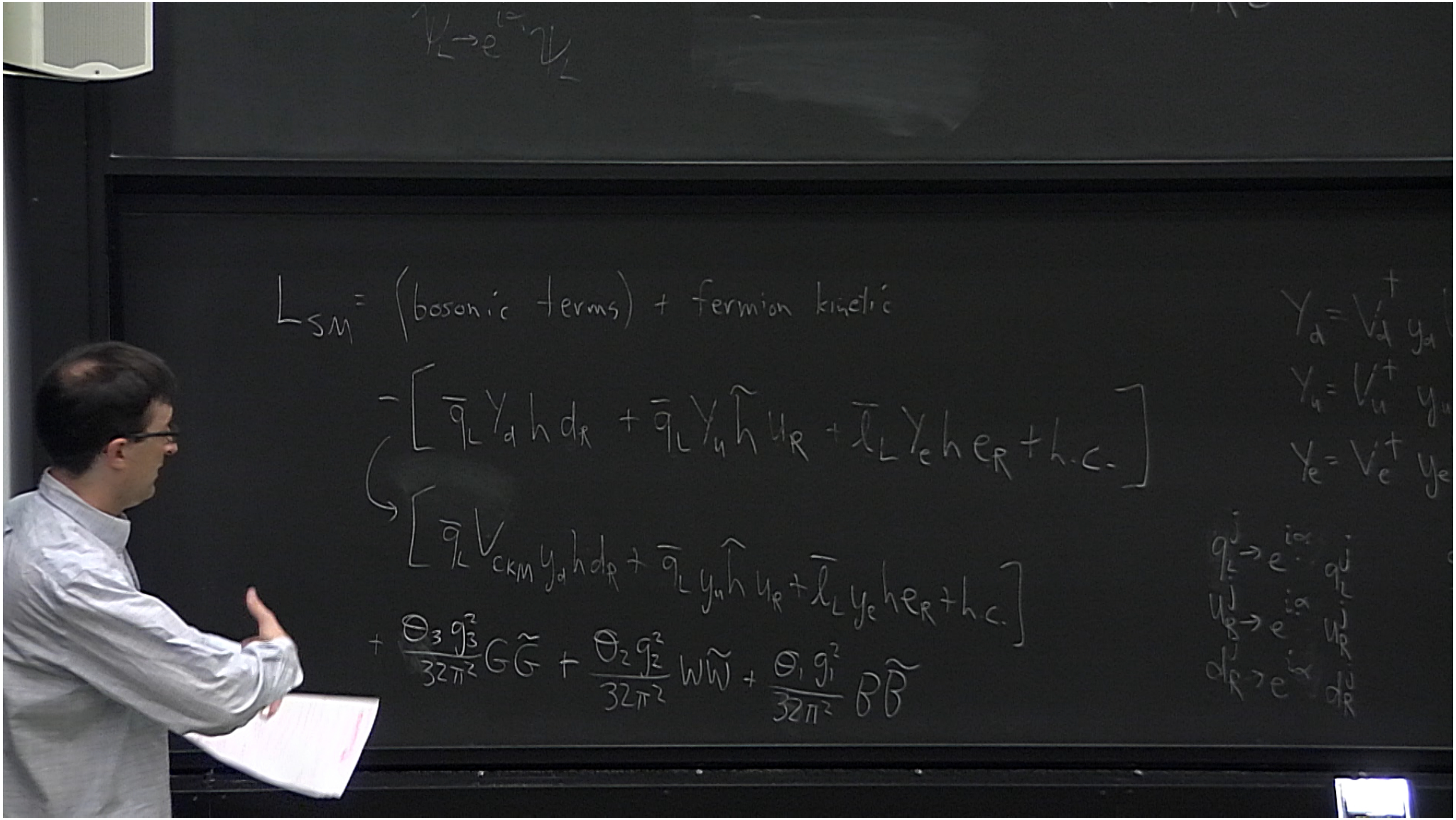
$Y_e = V_e^\dagger y_e$

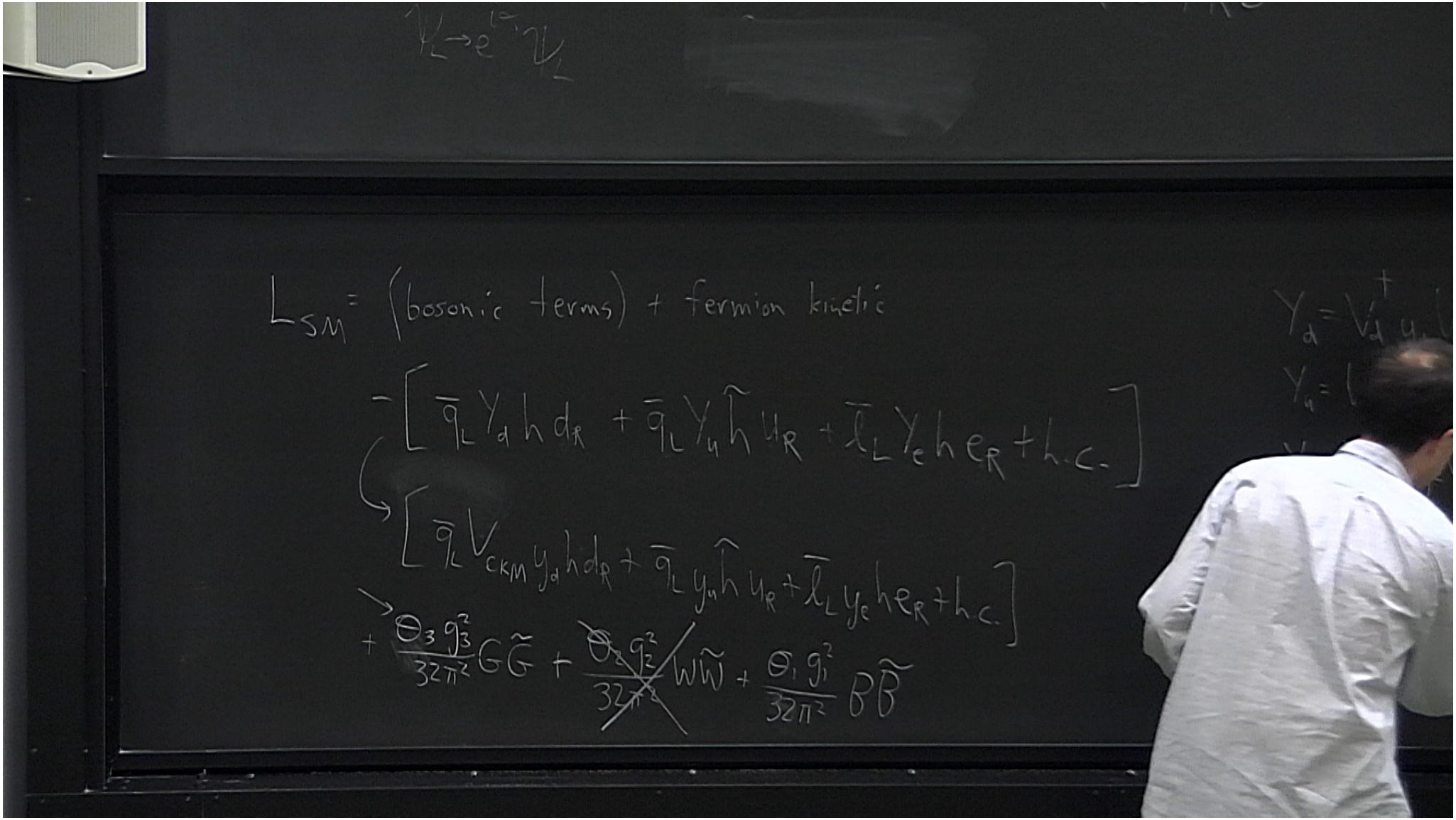
$q_L^j \to e^{i\alpha} q_L^j$

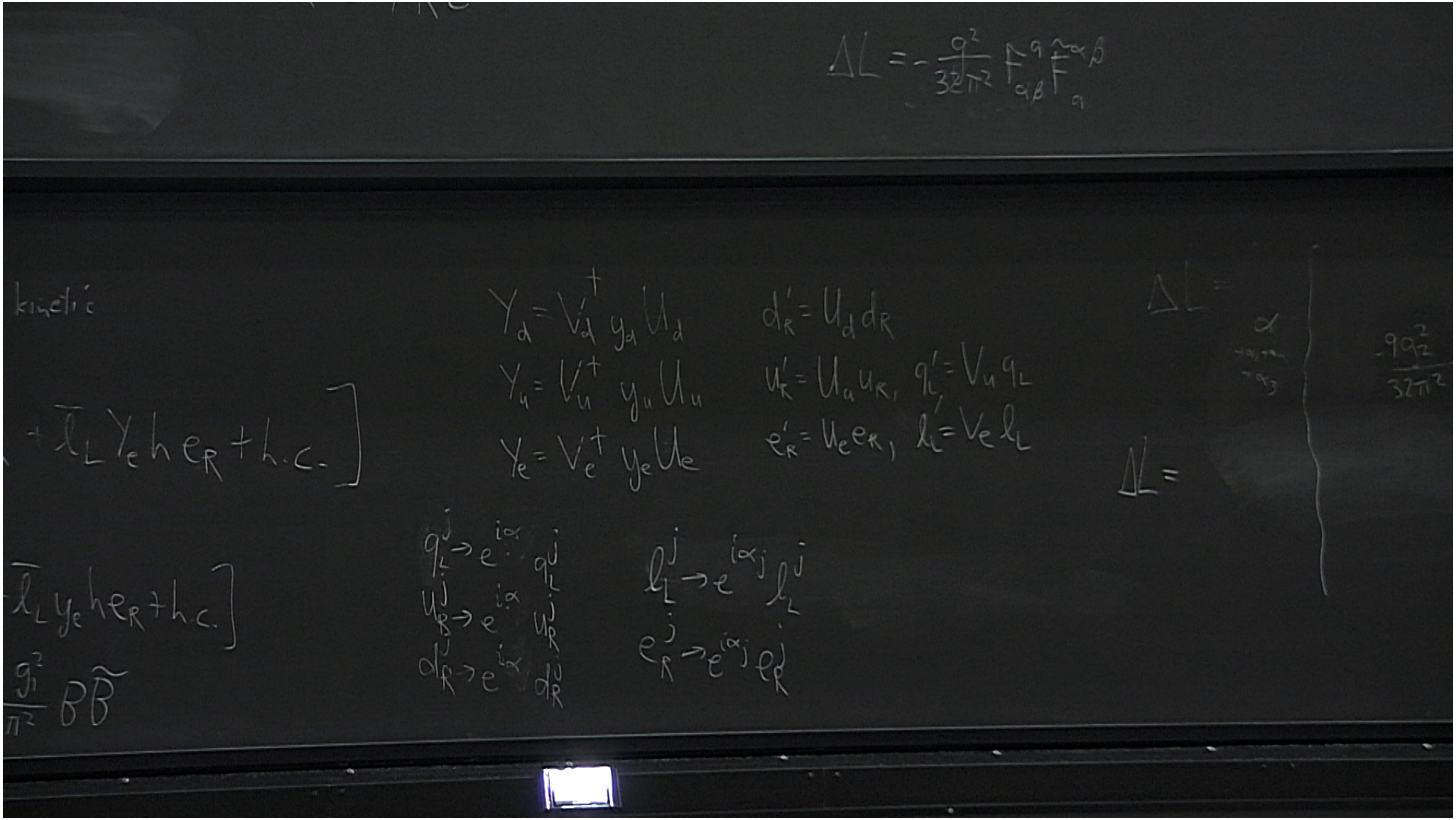
$u_R^j \to e^{i\alpha} u_R^j$

$d_R^j \to e^{i\alpha} d_R^j$


$\Delta L = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$
$V_d^\dagger y_d U_d \quad d_R' = U_d d_R$
$V_u^\dagger y_u U_u \quad u_R' = U_u u_R, \; q_L' = V_u q_L$
$V_e^\dagger y_e U_e \quad e_R' = U_e e_R, \; l_L' = V_e l_L$
$\Delta L =$
$\frac{-9g_2^2}{32\pi^2} W^a_{\mu\nu} \tilde{W}^{\mu\nu}_a$
$W\tilde{W}$
$\frac{9g_1^2}{64\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$
$\alpha_j = \beta_j = \alpha$ "accidental" symm. B


$\psi_L \to e^{i\alpha} \psi_L$
$\mathcal{L}_{SM} = (\text{bosonic terms}) + \text{fermion kinetic}$
$-\left[\bar{q}_L Y_d h d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{l}_L Y_e h e_R + \text{h.c.}\right]$
$\hookrightarrow \left[\bar{q}_L V_{CKM} y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{l}_L y_e h e_R + \text{h.c.}\right]$
$+ \frac{\theta_3 g_3^2}{32\pi^2} G\tilde{G} + \frac{\theta_2 g_2^2}{32\pi^2} W\tilde{W} + \frac{\theta_1 g_1^2}{32\pi^2} B\tilde{B}$
$Y_d = V_d^\dagger y_d$
$Y_u = V_u^\dagger y_u$
$Y_e = V_e^\dagger y_e$
$q_L^j \to e^{i\alpha} q_L^j$
$u_R^j \to e^{i\alpha} u_R^j$
$d_R^j \to e^{i\alpha} d_R^j$

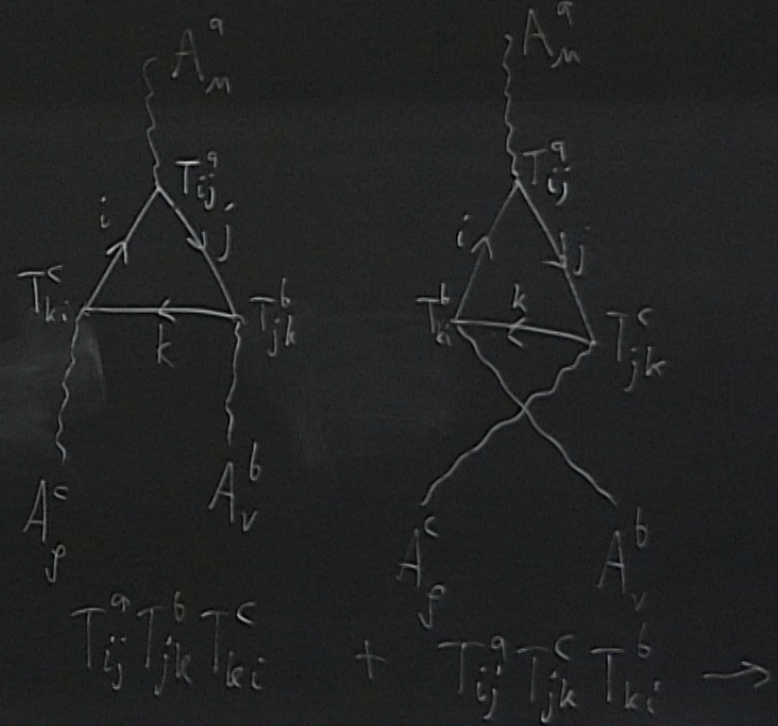

$\psi_L \to e^{i\theta} \psi_L$
$\mathcal{L}_{SM} = (\text{bosonic terms}) + \text{fermion kinetic}$
$-\left[\bar{q}_L Y_d h d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{l}_L Y_e h e_R + \text{h.c.}\right]$
$\left[\bar{q}_L V_{CKM} y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{l}_L y_e h e_R + \text{h.c.}\right]$
$+ \frac{\theta_3 g_3^2}{32\pi^2} G\tilde{G} + \frac{\theta_2 g_2^2}{32\pi^2} W\tilde{W} + \frac{\theta_1 g_1^2}{32\pi^2} B\tilde{B}$
$Y_d = V_d^\dagger$
$Y_u =$


$\Delta L = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$
kinetic
$Y_d = V_d^\dagger y_d U_d$
$Y_u = V_u^\dagger y_u U_u$
$Y_e = V_e^\dagger y_e U_e$
$d'_R = U_d d_R$
$u'_R = U_u u_R$, $q'_L = V_u q_L$
$e'_R = U_e e_R$, $l'_L = V_e l_L$
$\Delta L =$
$-\frac{9g_2^2}{32\pi^2}$
$+ \bar{l}_L Y_e h e_R + h.c.]$
$-\bar{l}_L y_e h e_R + h.c.]$
$\frac{g_1^2}{\pi^2} B\tilde{B}$
$q_L^j \to e^{i\alpha} q_L^j$
$u_R^j \to e^{i\alpha} u_R^j$
$d_R^j \to e^{i\alpha} d_R^j$
$l_L^j \to e^{i\alpha_j} l_L^j$
$e_R^j \to e^{i\alpha_j} e_R^j$

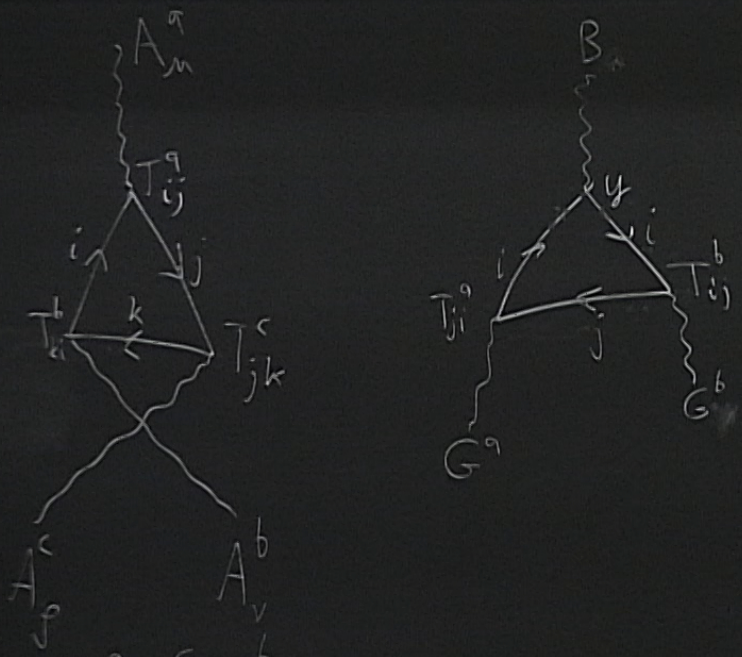
$\psi_L \to e^{i\alpha} \psi_L$

$$\mathrm{Tr}\left[\gamma^5 T^a \{T^b, T^c\}\right] = 0$$

$SU(3) \times SU(2) \times U(1)$



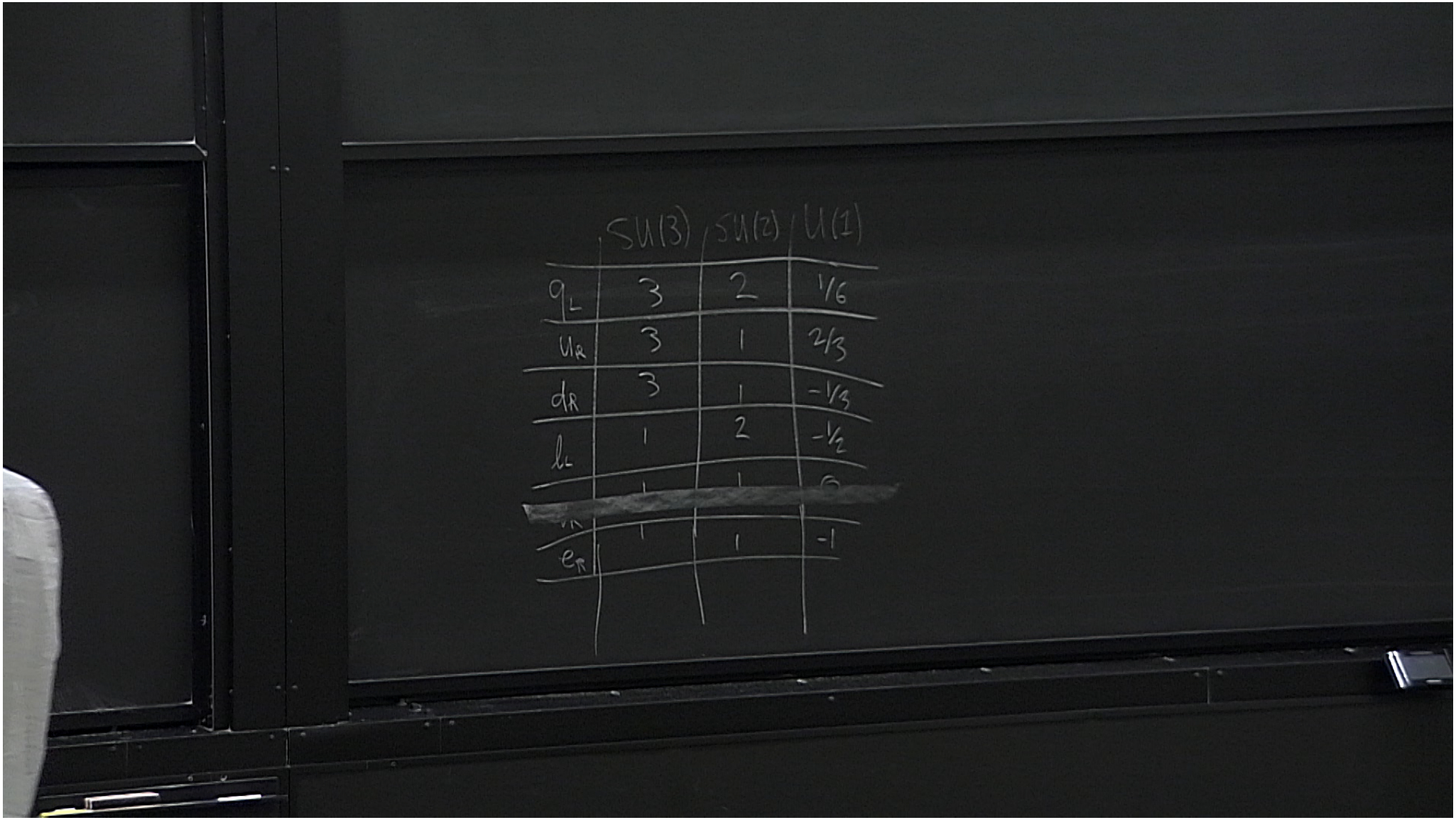
$$\Delta L = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$$



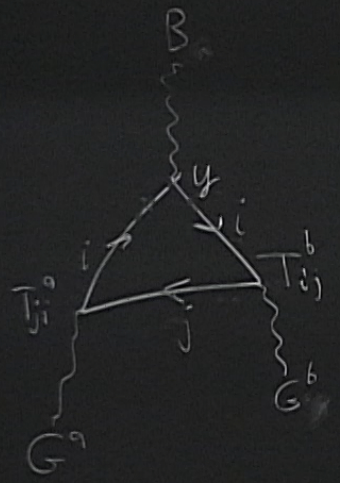
$$Tr\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$$

q_L

$+ \quad T^a_{ij} T^c_{jk} T^b_{ki} \rightarrow$


SU(3) SU(2) U(1)
q_L 3 2 1/6
u_R 3 1 2/3
d_R 3 1 -1/3
l_L 1 2 -1/2
1 1 0
1 1 -1
e_R

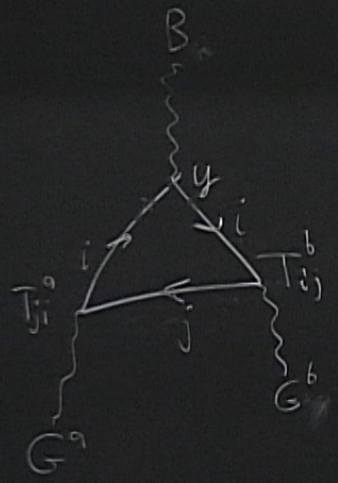
$$\Delta L = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$$



$$\mathrm{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$$

$$\underset{q_L}{-3\times 2\left(\frac{1}{6}\right)} + \underset{u_R}{3\times\left(\frac{2}{3}\right)} + \underset{d_R}{3\times\left(-\frac{1}{3}\right)} = 0 \checkmark$$

$$\Delta L = -\frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a$$



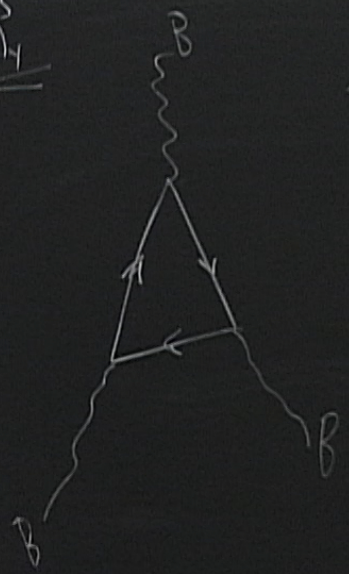
$$\mathrm{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$$

$U(1)_Y SU(3)^2$

$$\underset{q_L}{-3\times 2\left(\frac{1}{6}\right)} + \underset{u_R}{3\times\left(\frac{2}{3}\right)} + \underset{d_R}{3\times\left(-\frac{1}{3}\right)} = 0 \checkmark$$

e_R

$(U(1)_Y)^3$

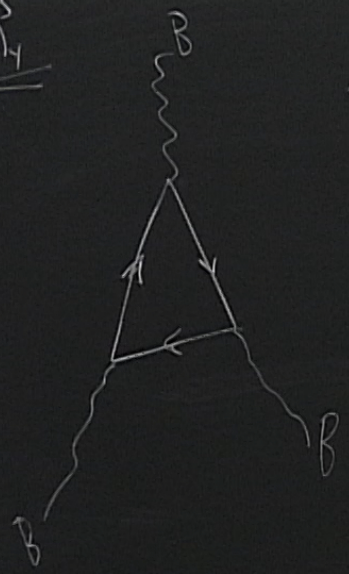


$$3\times\Big[-3\times2\Big(\tfrac{1}{6}\Big)^3+3\Big(\tfrac{2}{3}\Big)^3+3\Big(-\tfrac{1}{3}\Big)^3-2\Big(-\tfrac{1}{2}\Big)^3+1(-1)^3\Big]=0$$

q_L u_R d_R l_L e_R

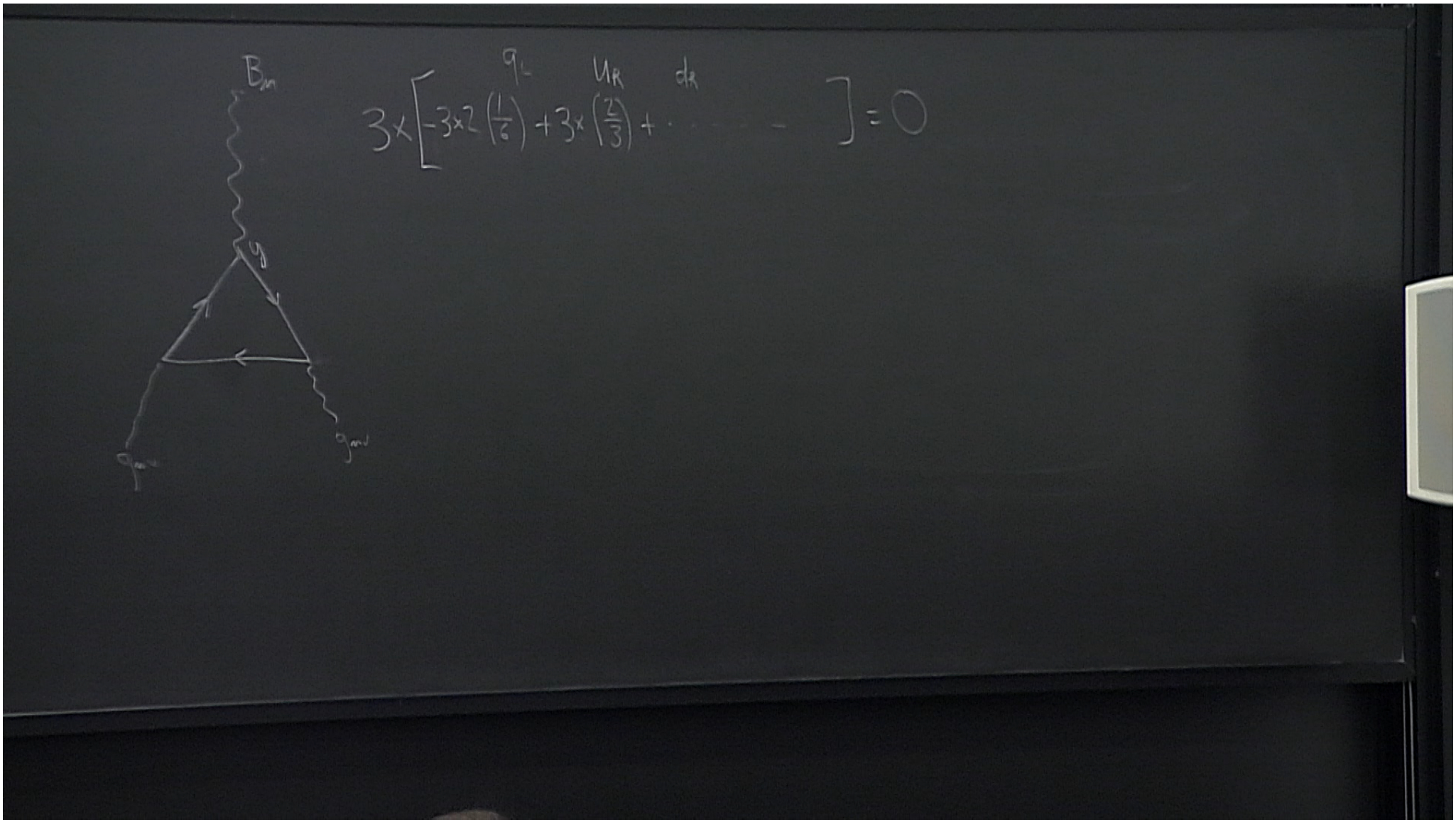
e_R

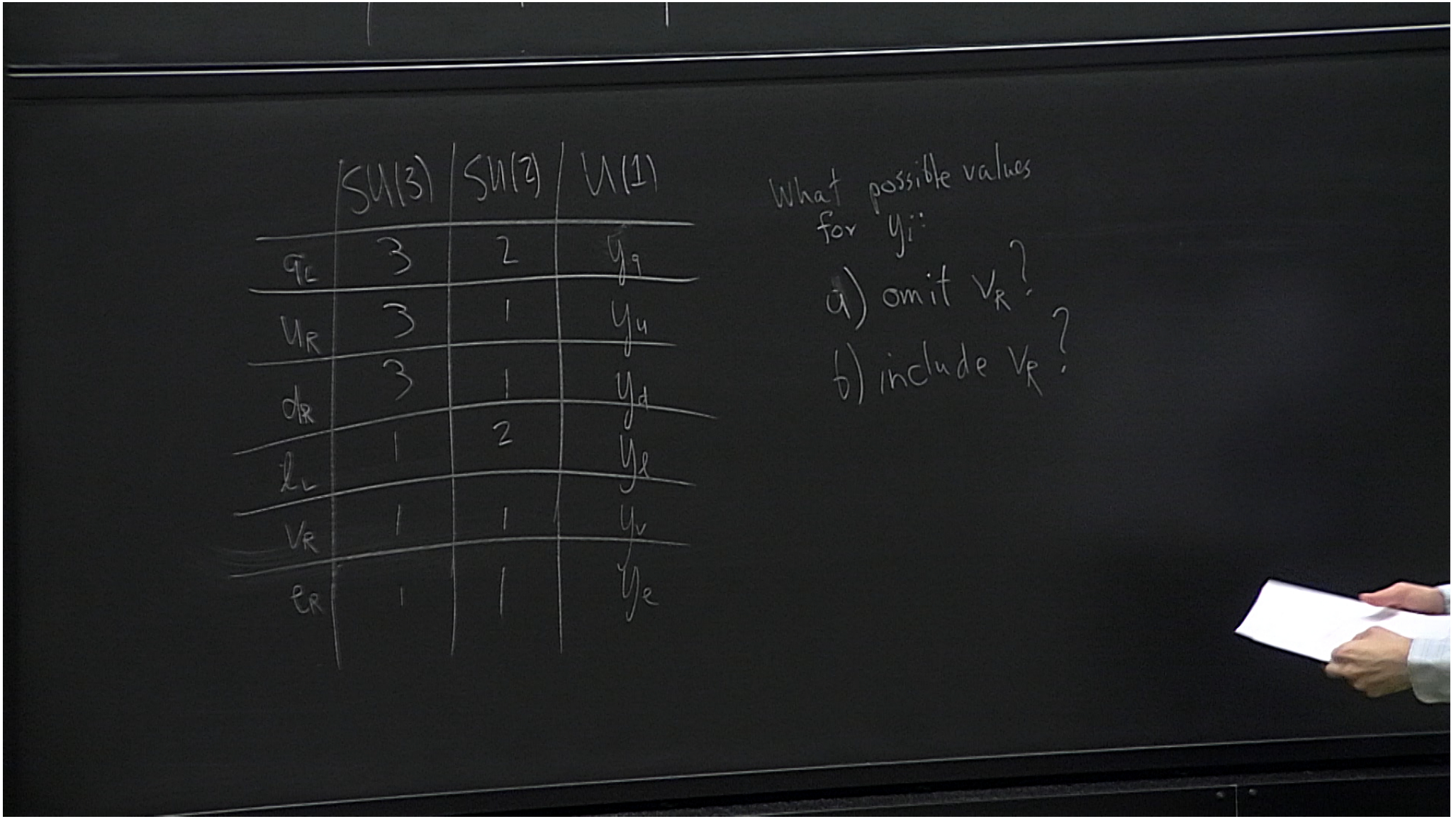
$U(1)_Y^3$

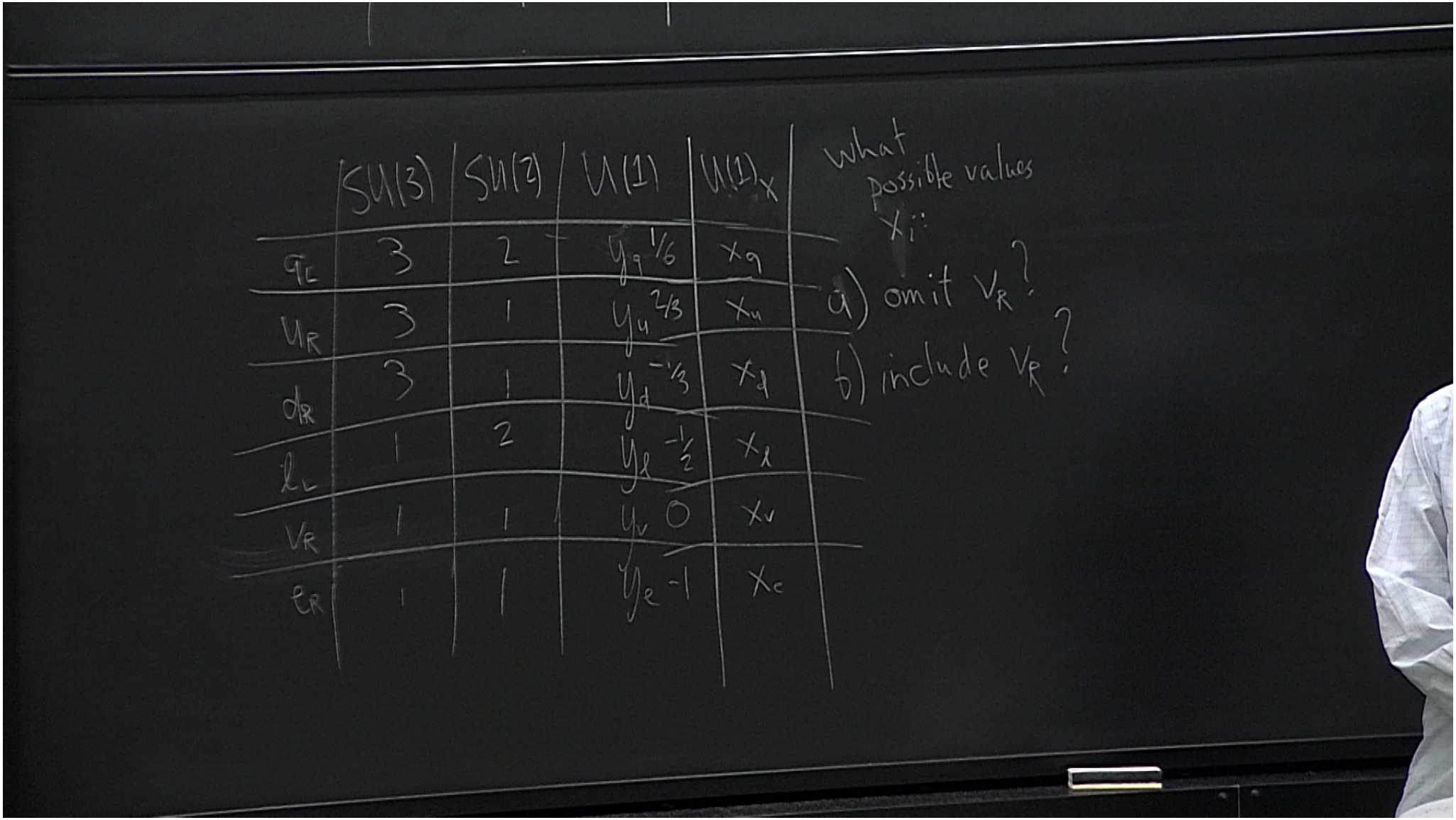


$$3\times\left[-3\times 2\left(\tfrac{1}{6}\right)^3 + 3\left(\tfrac{2}{3}\right)^3 + 3\left(-\tfrac{1}{3}\right)^3 - 2\left(-\tfrac{1}{2}\right)^3 + 1(-1)^3\right] = 0$$

(q_L, u_R, d_R, l_L, e_R)


B_μ
q_L
u_R
d_R
$3\times\left[-3\times 2\left(\frac{1}{6}\right)+3\times\left(\frac{2}{3}\right)+\cdots-\right]=0$
q_{anti}
q_{anti}


SU(3) SU(2) U(1)
q_L 3 2 y_q
u_R 3 1 y_u
d_R 3 1 y_d
l_L 1 2 y_l
ν_R 1 1 y_ν
e_R 1 1 y_e
What possible values for y_i:
a) omit ν_R?
b) include ν_R?


SU(3) SU(2) U(1) U(1)_X
q_L 3 2 y_q 1/6 x_q
u_R 3 1 y_u 2/3 x_u
d_R 3 1 y_d −1/3 x_d
l_L 1 2 y_l −1/2 x_l
ν_R 1 1 y_ν 0 x_ν
e_R 1 1 y_e −1 x_e
what possible values x_i:
a) omit ν_R?
b) include ν_R?