Title: Counter-diabatic driving in many-body systems

Speakers: Dries Sels

Series: Tensor Network

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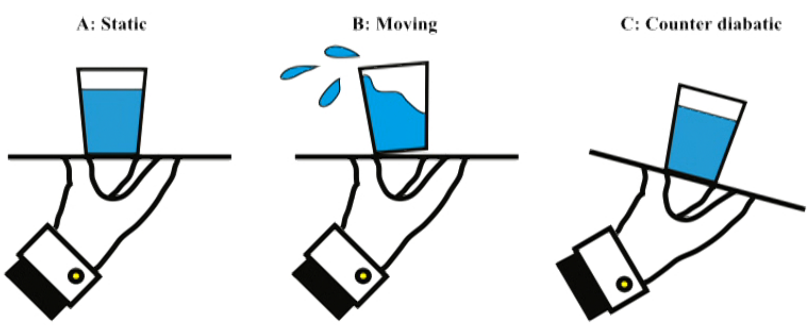
Abstract: Adiabatic evolution is a common strategy for manipulating quantum states. However, it is inherently slow and therefore susceptible to decoherence. Shortcuts to adiabaticity are methods of achieving faster adiabatic evolution, in order to maintain high fidelity in the presence of decoherence and noise. In this talk I will review recent progress on counter-diabatic (CD) driving for many-body systems. In particular, we will discuss a variational principle that allows to systematically compute approximate CD Hamiltonians. Two recent experiments will be discussed.

Counter-diabatic driving in quantum many body systems

Dries Sels & Anatoli Polkovnikov
Harvard-BU



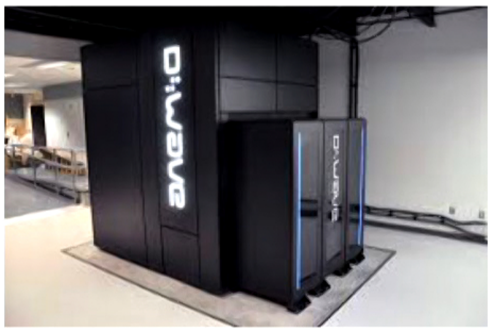
fwo Opening new horizons

Outline

- Non-adiabatic response and quantum geometry
- Counter-diabatic driving and quantum speed limit
- Variational adiabatic gauge fields for many body systems
- Some experimental implementations

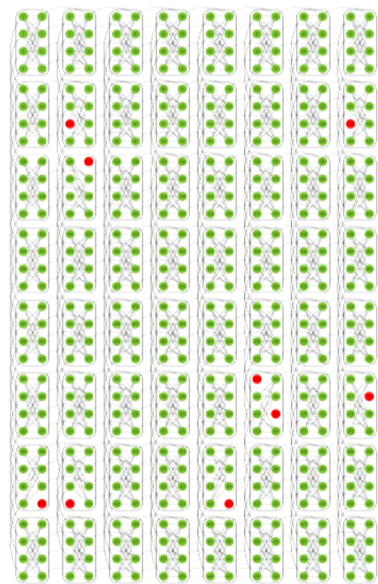
Motivation

- How to drive your system without exciting it?



Example : D-Wave machine

$$\sum_{ij} J_{ij} s_i^z s_j^z + \sum_i h_i(\tau) s_i^x$$



Non-adiabatic forces

- Consider a particle in an optical tweezer:

$$H = \frac{p^2}{2m} + V(x - \lambda(t))$$

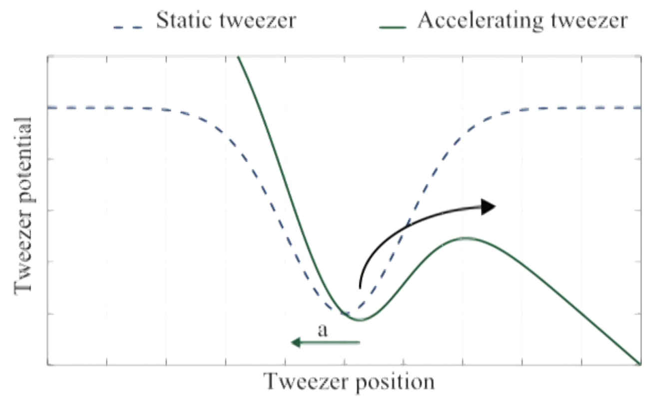
- Let's move with the atom: $|\psi\rangle = U(\lambda)\,|\phi\rangle$
  - Dynamics: $i\partial_t\,|\psi\rangle = H\,|\psi\rangle$

  ➡ $\tilde{H} = U_\lambda^\dagger H U_\lambda - iU_\lambda^\dagger \partial_t U_\lambda$
  - Hence:

$$\tilde{H} = \frac{p^2}{2m} + V(x) - \dot{\lambda}p$$

  - Shift the momentum:

$$\tilde{H}' = \frac{p^2}{2m} + V(x) + m\ddot{\lambda}x$$



General

Time-dependent Hamiltonian:

$$H(\lambda_\mu(t))$$

Adiabatic gauge potential:

$$A_\mu = iU^\dagger \partial_{\lambda_\mu} U$$

Moving frame Hamiltonian:

$$\tilde{H} = U^\dagger H U - \dot{\lambda}_\mu A_\mu$$

Adiabatic gauge potential

- Berry connection is the expectation value of the gauge potential

$$\langle 0| A_\mu |0\rangle$$

- Berry curvature:

$$F_{\mu\nu} = -i \langle 0| [A_\mu, A_\nu] |0\rangle$$

- Metric tensor

$$g_{\mu\nu} = \frac{1}{2} \langle 0| A_\mu A_\nu + A_\nu A_\mu |0\rangle$$

  - Defines a metric on the ground state manifold. Can be used to detect and classify phase transitions.
    - Universal near second order QPT: $g \sim |\lambda - \lambda_c|^{d\nu - 2}$

M. Kolodrubetz, D. Sels, P. Mehta, A. Polkovnikov, Physics Reports 697 (2017)

Non-adiabatic response

- Let's look at the corrections to the ground state energy

General

$$\tilde{H} = U^\dagger H U - \dot{\lambda}_\mu A_\mu$$

First order:

$$\Delta E_1 = \dot{\lambda}_\mu \langle 0| A_\mu |0\rangle$$

Second order:

$$\Delta E_2 = \sum_{n=0} \frac{|\langle 0| \dot{\lambda}_\mu A_\mu |n\rangle|^2}{E_0 - E_n}$$

Translation

$$\tilde{H} = U^\dagger H U - \dot{\lambda} p$$

First order:

$$\Delta E_1 = \dot{\lambda} \langle 0| p |0\rangle = 0$$

Second order:

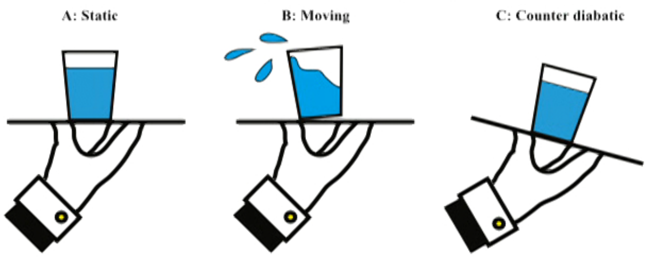
$$\Delta E_2 = \dot{\lambda}^2 \sum_{n\neq 0} \frac{|\langle 0| p |n\rangle|^2}{E_0 - E_n}$$

Effective mass of the classical parameter: $\Delta E_2 = -\frac{m\dot{\lambda}^2}{2}$

Counter-diabatic drive

- Can we keep the system adiabatic even if we change our parameters fast?
  - Just cancel out the gauge potential!

$$H_{CD} = H(\lambda_t) + \dot{\lambda}_\mu A_\mu$$



  - Rotate back into H if possible

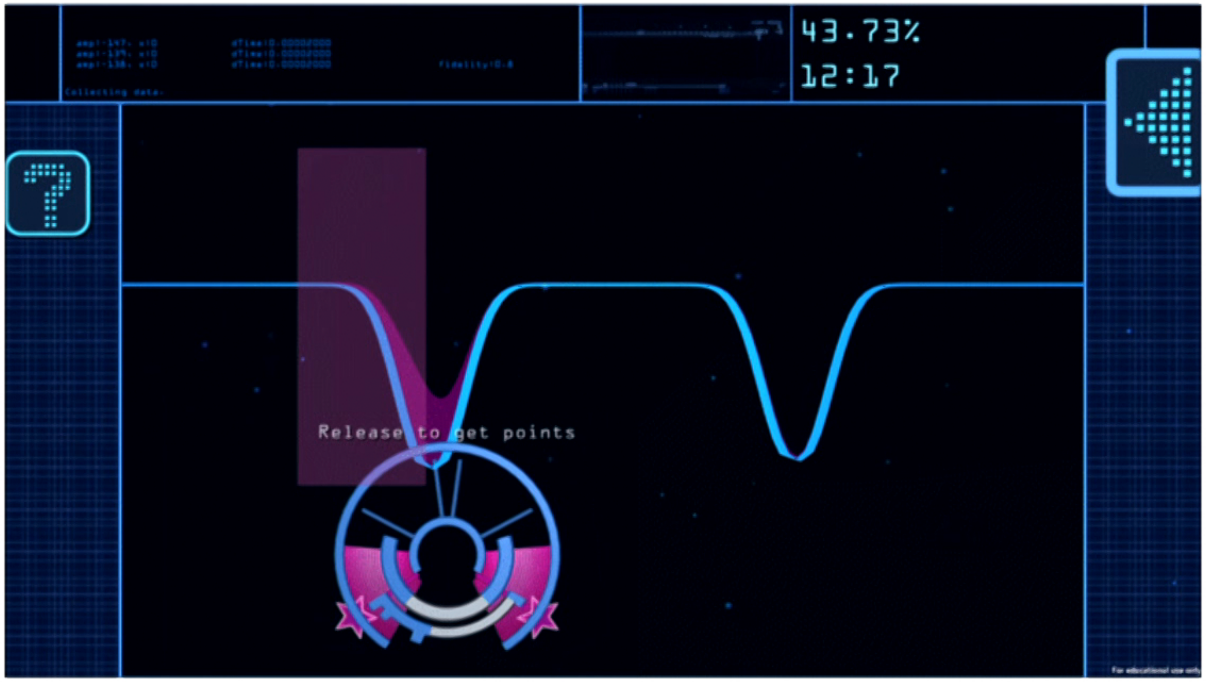
$$H_{FF} = R^\dagger H_{CD}(\lambda_t) R - i R^\dagger \partial_t R$$

M. Demirplak, S. A. Rice (2003), M. Berry (2009), S. Deffner, A. Del Campo, C. Jarzynski (2014+), …

Let's play a game

J. J. W. H. Sørensen et al., Nature 532, 210–213; 2016

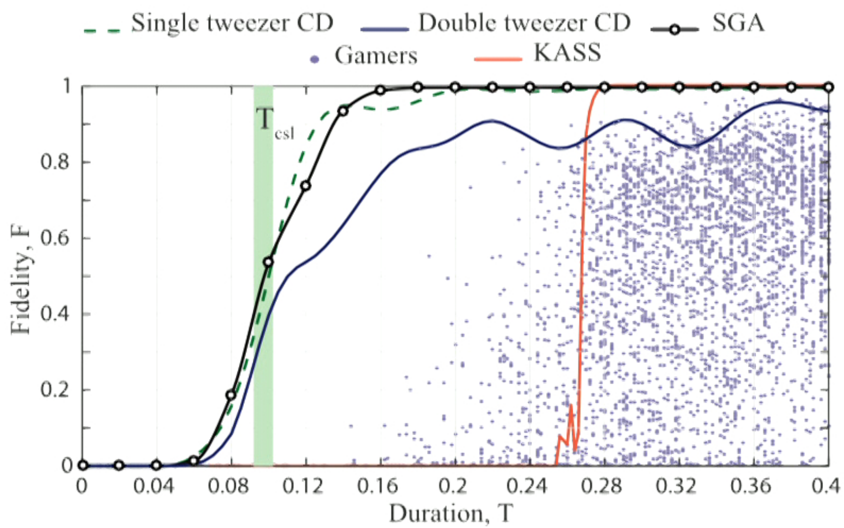
Let's play a game



J. J. W. H. Sørensen et al., Nature 532, 210–213; 2016

Effective translation

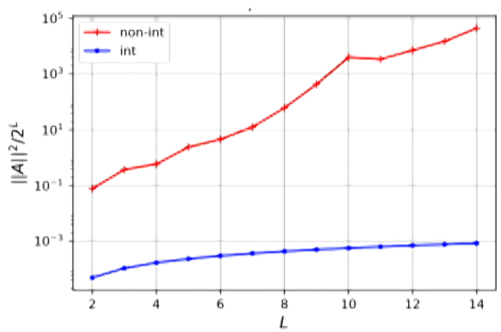
- System is described by: $H = \frac{p^2}{2m} + V_0(x) + V_1(x - \lambda)$
- Let's assume: $A_\lambda = v(\lambda)p$
- Transform to: $H_{FF} = H - \partial_t(v(\lambda)\dot{\lambda})x \approx \frac{p^2}{2m} + V_0(x) + V_1(x - \lambda_{FF}(t))$



Many body systems

- Whole other ballgame
  $$\langle n| A_\lambda |m\rangle = \langle n| U^\dagger i\partial_\lambda U |m\rangle = \langle n(\lambda)| i\partial_\lambda |m(\lambda)\rangle$$
- Hence
  $$\langle n| A_\lambda |m\rangle = -i\frac{\langle n| \partial_\lambda H |m\rangle}{E_n - E_m}$$
- If the system is ergodic we are in trouble:

$$|\langle n|A_\lambda|m\rangle = \frac{|\langle n|\partial_\lambda H|m\rangle}{|E_n - E_m|} \sim \frac{e^{-S/2}}{e^{-S}} = e^{S/2}$$



What now?

- We should be less stringent. Eigenstates of the same energy density are locally indistinguishable anyway.

- Here is our trick:

  - Observe: $\langle n | A_\lambda | m \rangle = -i \frac{\langle n | \partial_\lambda H | m \rangle}{E_n - E_m}$

$$\Rightarrow \quad \partial_\lambda H + i[A, H] = F_{BO}$$

$$\Rightarrow \quad [\partial_\lambda H + i[A, H], H] = 0$$

$$S(O) = \mathrm{Tr}(\partial_\lambda H + i[O, H])^2 \rightarrow A = \mathrm{argmin} S(O)$$

Variational method

$$S(O) = \mathrm{Tr}(\partial_\lambda H + i[O, H])^2 \to A = \mathrm{argmin} S(O)$$

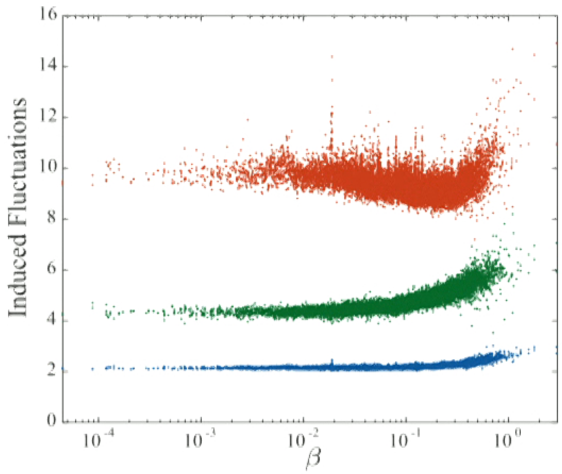
- Now we can just restrict to sensible, local, experimentally relevant operators.
- No local approximation can ever be close to the exact result, so is minimizing S sensible?
  - Yes! S(O) is a measure for the rate of increase of the energy fluctuations:

$$\frac{\partial \delta E_n^2}{\partial t} \propto \langle n| \left(\partial_\lambda H + i[A, H]\right)^2 |n\rangle$$

Flipping spins

- Consider $$H = \sum_i \left(\sigma^z_{i+1}\sigma^z_i + 0.8\sigma^z_j + 0.9\sigma^x_j + h_x(t)\sigma^x_0 \right)$$
- Then we can expand in terms of strings of spins

$$H = \sum_i \alpha_i \sigma^y_i + \sum_{ij} \left(\beta_{i,j}\sigma^x_i\sigma^y_j + \gamma_{i,j}\sigma^z_i\sigma^y_i \right)$$



15 Site chain

No CD drive

1 Body CD

2 body CD

Free lattice gas

- Let's see what happens for free particles

$$H = -\sum_i (c^\dagger_{i+1} c_i + h.c.) + \sum_i V(i,\lambda) c^\dagger_i c_i$$

- Let's take the most local Ansatz

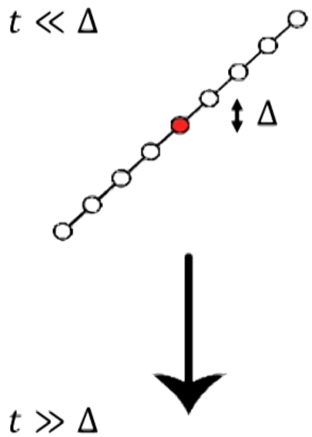
$$A = i \sum_i \alpha_i (c^\dagger_{i+1} c_i - h.c.)$$

- Compute some traces

$$-3\partial_i^2 \alpha + (\partial_i V)^2 \alpha = \partial_i \partial_\lambda V$$

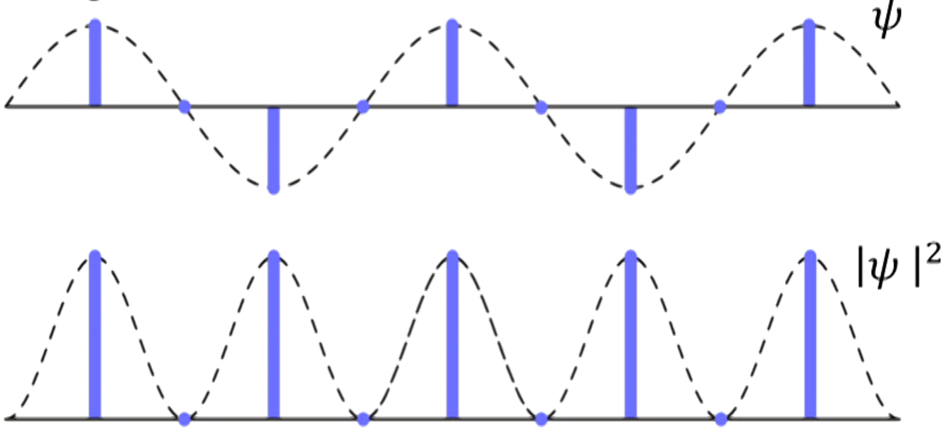
  - Exact solution for linear potential

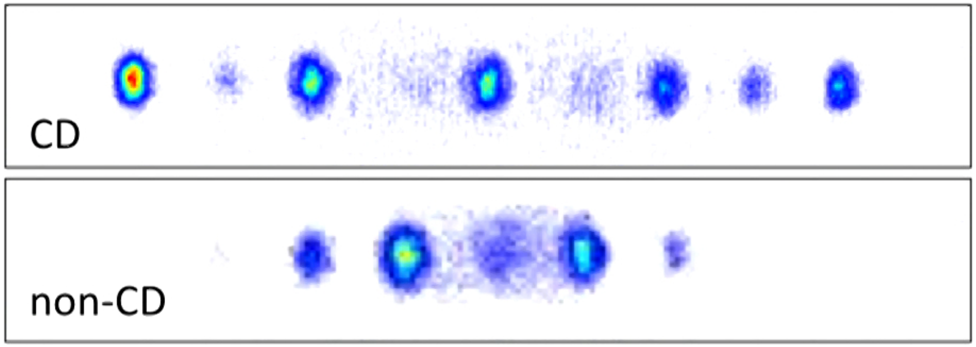
preparation by counterdiabatic driving



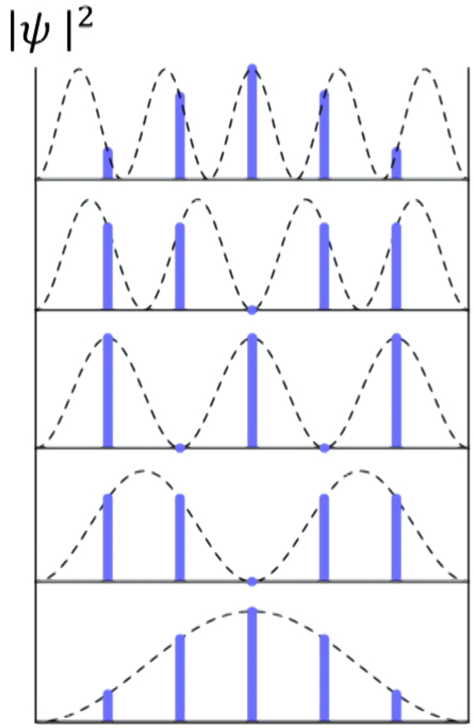
Bryce Gadway

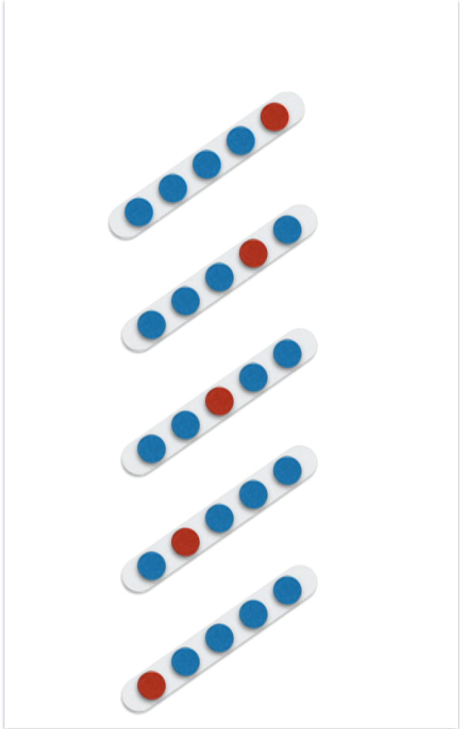
5th eigenstate of a 9-site "box"

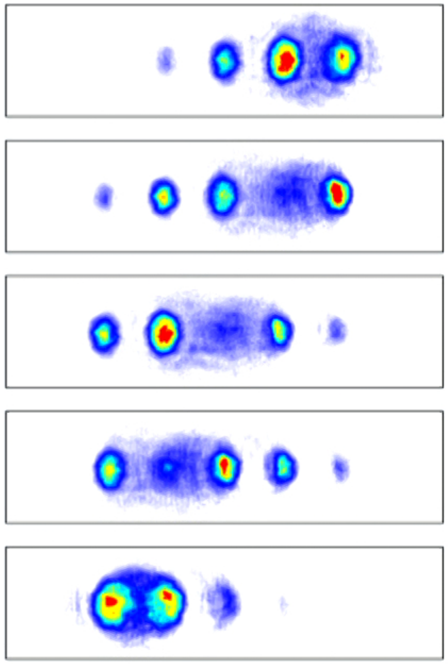




eigenstates of a 5-site "box"







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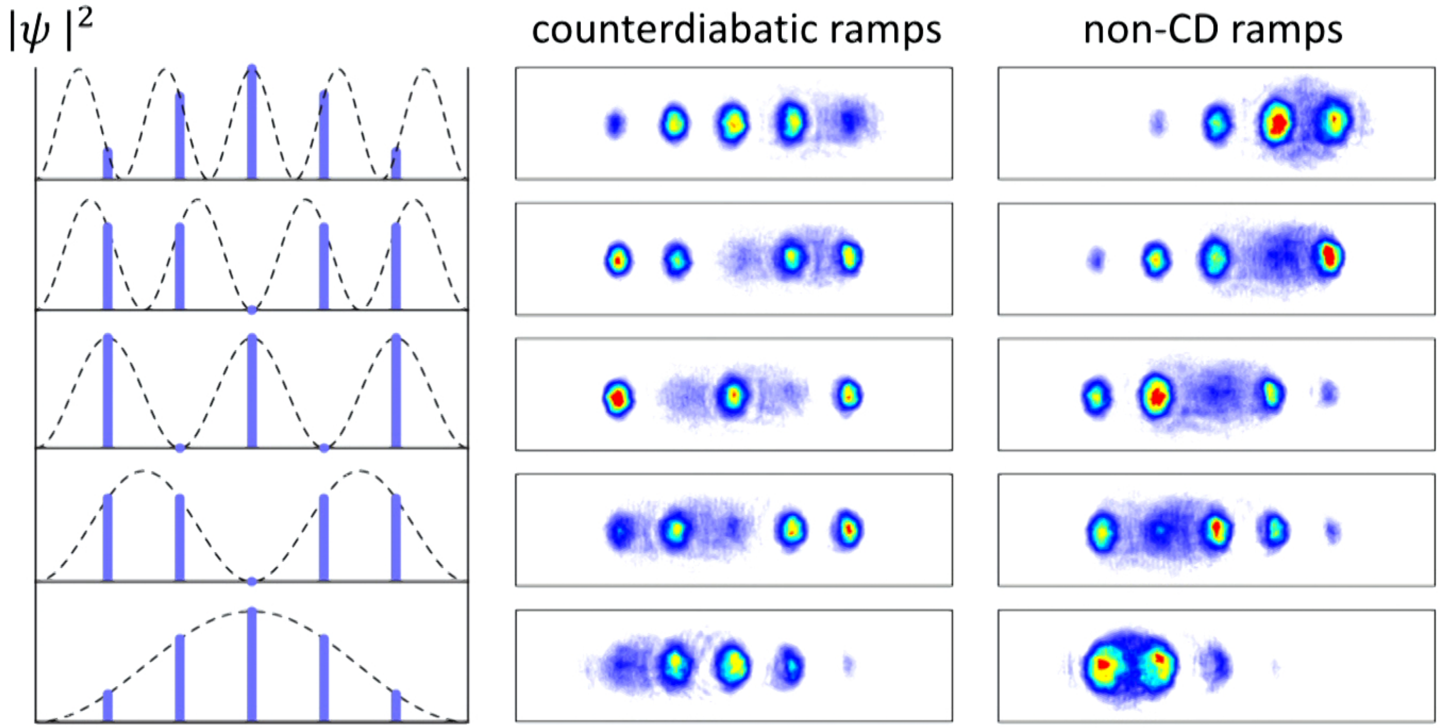


Floquet fast forward

- Consider Landau-Zener problem:

$$H = \Delta\sigma_z + h(t)\sigma_x$$

- CD drive becomes

$$H_{CD} = \Delta\sigma_z + h(t)\sigma_x + \frac{1}{2}\frac{\Delta\dot{\lambda}}{\Delta^2 + \lambda^2}\sigma_y$$

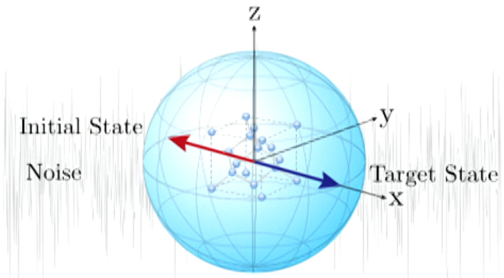
- How to generate y-field?

$$H_{FE} = \cos(\omega t)\sigma_z + C\omega \sin \omega t \sigma_x$$

High frequency expansion

$$\rightarrow \tilde{H}_F = J_1(C)\sigma_y + O(1/\omega)$$

- Slowly varying protocol in every period
- Higher harmonics for complex systems



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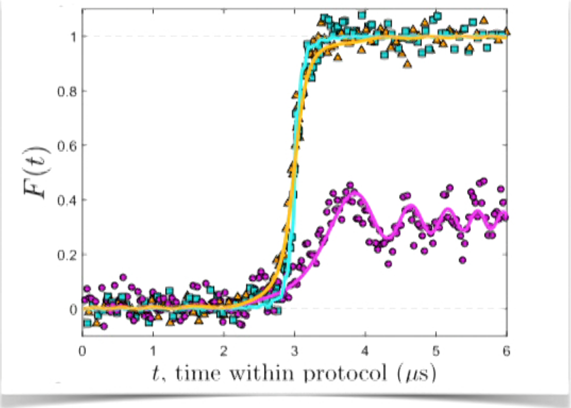

Alex Sushkov

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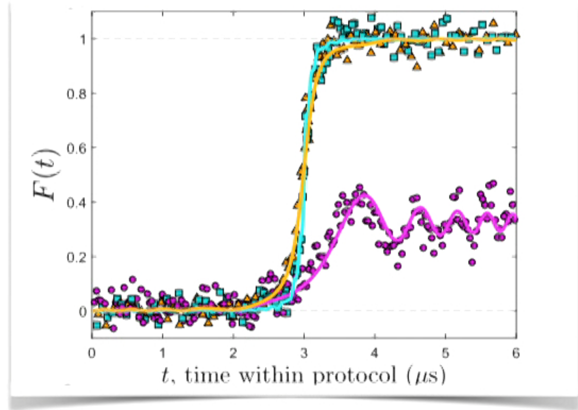
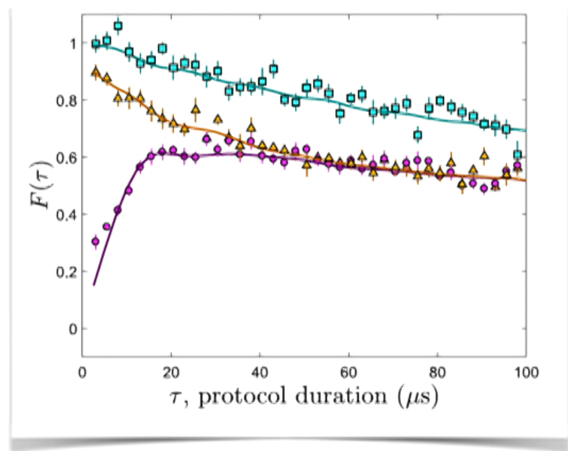
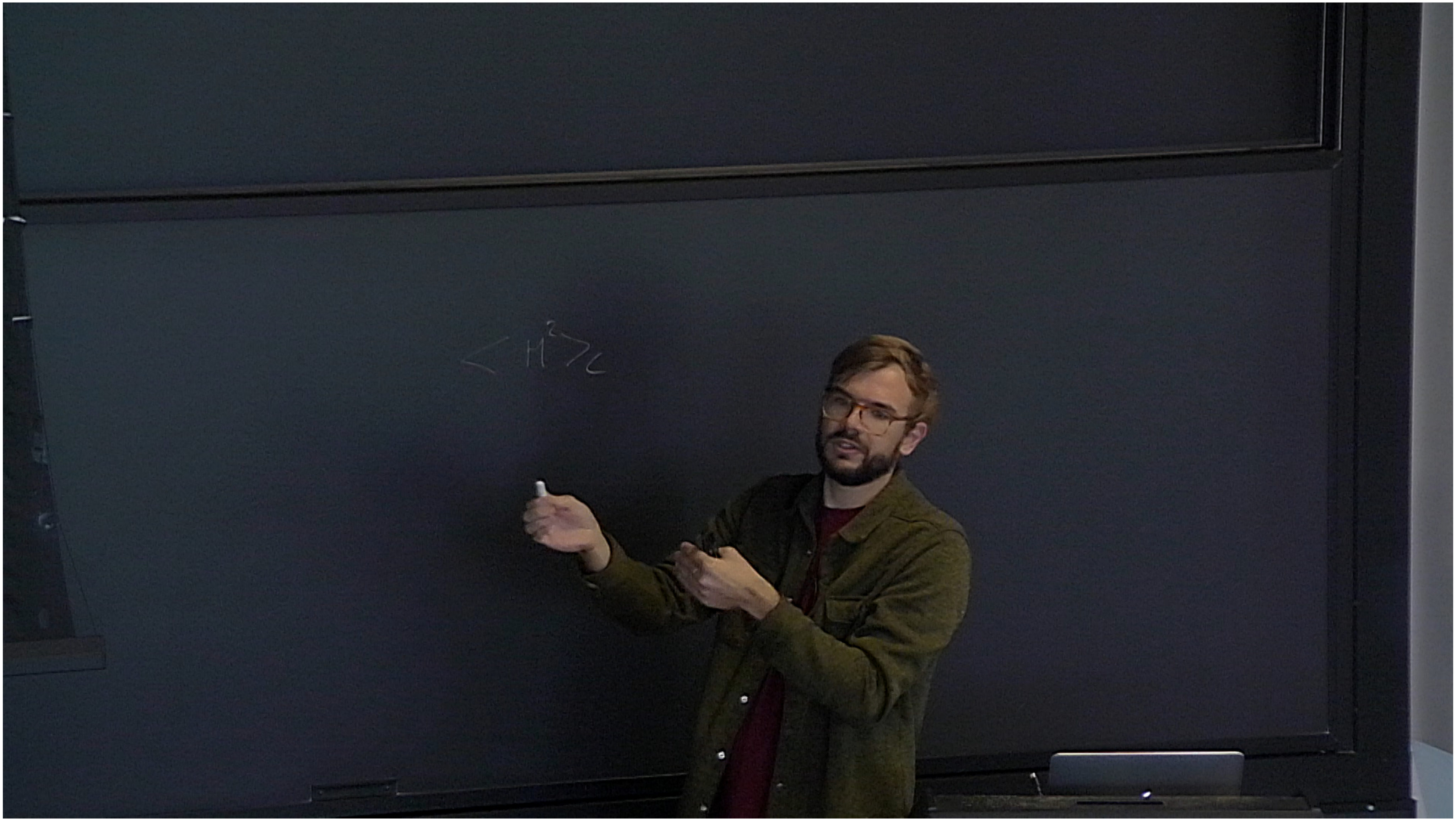




Conclusion

- Deep connection between non-adiabatic response and geometry
- Counter-diabatic driving can be used to suppress dissipation in many-body systems
- Variational method to compute gauge field
- Floquet implementation
- Many applications!

DS and AP, PNAS, Volume 114, 20 (2017)
DS, PRA 97, 040302(R), (2018)
Boyers et. al., arXiv:1811.09762, (2018)



< H² >c