Title: Interdisciplinary Seminar: Holographic dualities and tensor renormalization group study of gauge theories

Speakers: Raghav Govind Jha

Series: Quantum Fields and Strings

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Abstract: In the first part of the talk, I will mention the ongoing numerical efforts using lattice calculations to understand the holographic dualities relating the super Yang-Mills (SYM) theories in various dimensions and their conjectured Type II supergravity theories in the decoupling limit. In the second part, I will discuss the tensor renormalization group study of the SU(2) gauge-Higgs model in two dimensions using the higher-order tensor renormalization group (HOTRG) algorithm and compare the results with the Monte Carlo simulations.

Holographic dualities and tensor renormalization group study of gauge theories.

Raghav Govind Jha

March 11, 2019

Overview

1. AdS/CFT and its generalisation for $p < 3$ *i.e.* non-conformal cases
2. Lattice formulation of $\mathcal{N} = 4$ super Yang-Mills (SYM) theory and its dimensional reductions.
3. Results for matrix models (BFSS and BMN) and 2d sixteen supercharge SYM theory
4. Tensor renormalization group (TRG) applied to 2d non-Abelian gauge/Higgs model
5. Some future directions.

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Gauge/Gravity duality

- 4d $\mathcal{N} = 4$ U(N) SYM theory associated to the world volume of N D3-branes is dual to Type IIB supergravity on $AdS_5 \times S_5$ in the decoupling limit (large N, λ) $\longrightarrow$ **AdS/CFT**
- Maximally supersymmetric (p+1)-dimensional YM theory dual to Dp-branes at low temperatures - dual description in terms of Type IIA/B (even/odd p) low-energy string theory. In this talk, we will not talk about $p > 3$

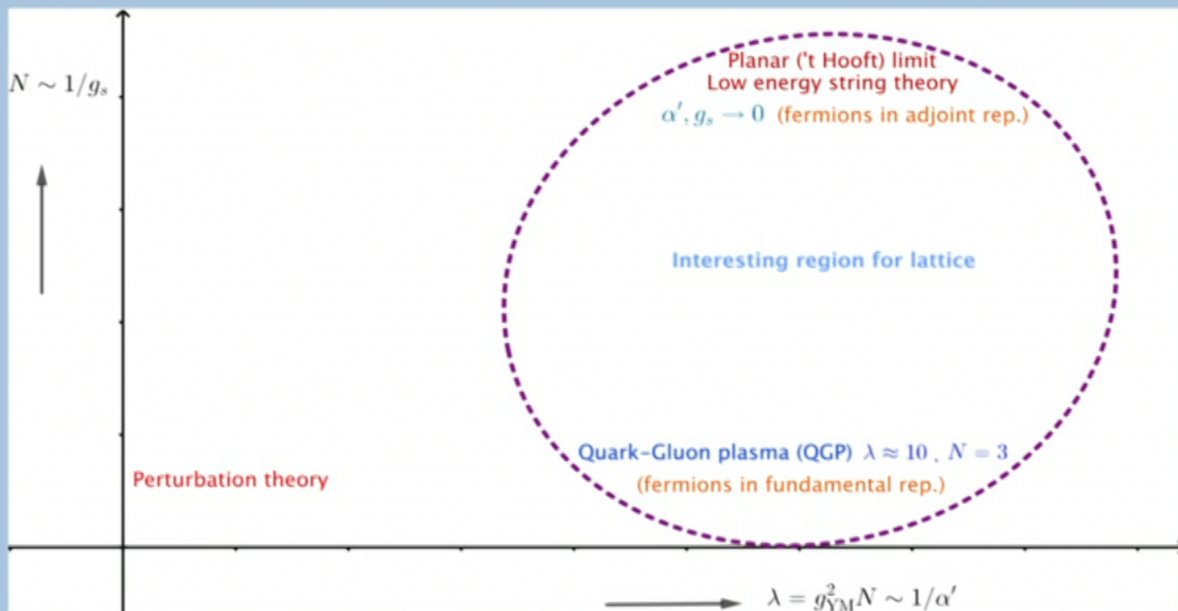
$\mathcal{N} = 4$ SYM action

$$S = \frac{N}{\lambda} \int \mathrm{d}^4x \ \mathrm{tr} \left\{ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2}(D_\mu X_i)^2 - \frac{1}{4}[X_i, X_j]^2 \right.$$
$$\left. + \Psi \ \partial\!\!\!/ \Psi + \frac{1}{2} \Psi_\alpha (\gamma_i)_{\alpha\beta} [X_i, \Psi_\beta] \right\}$$

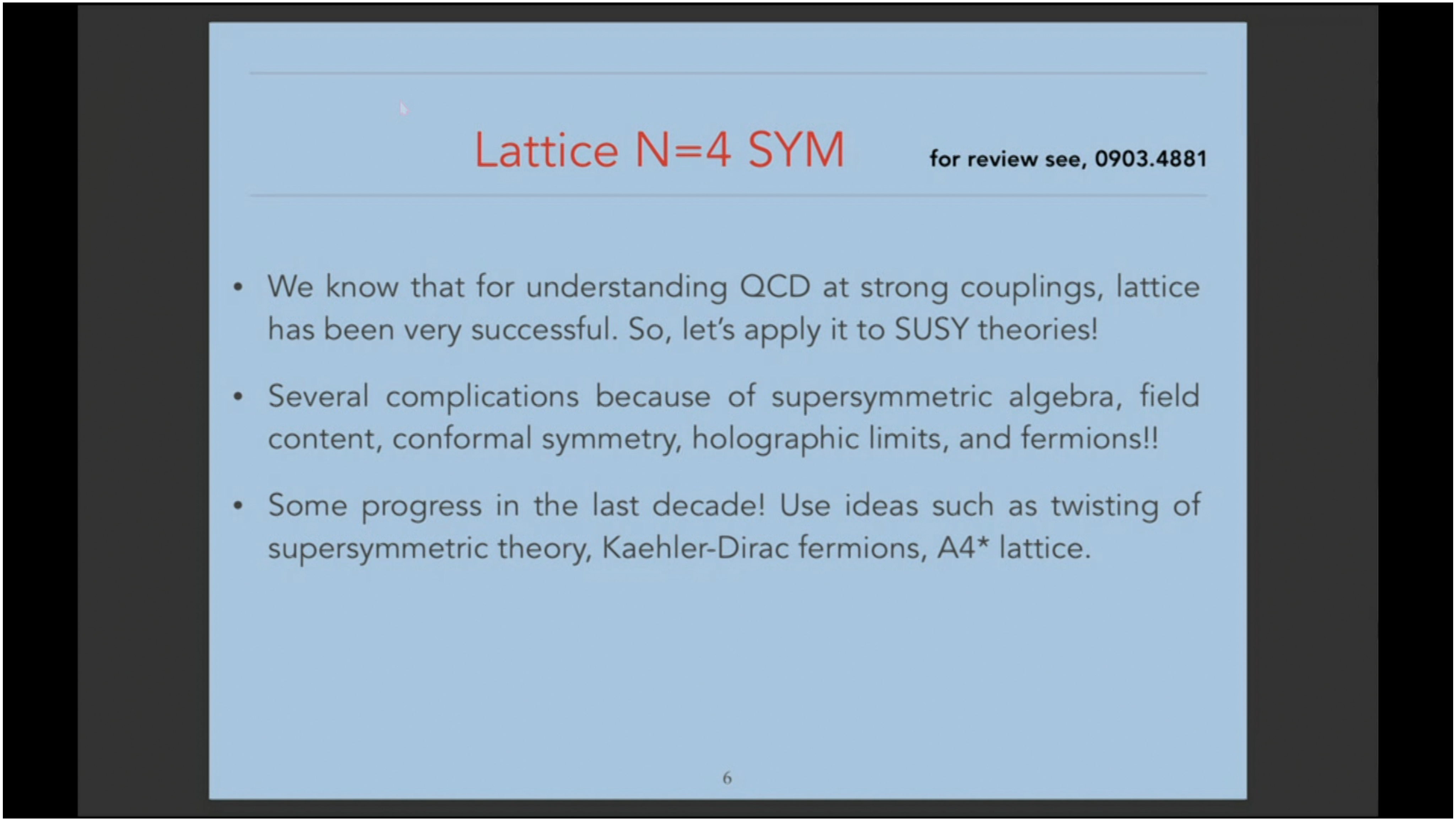
- SU(N) gauge theory with four fermions and six scalars, all massless and in adjoint rep.
- Supersymmetric: 16 supercharges
- Fields and Q's transform under global SO(6) R-symmetry
- Conformal: β function is zero for all 't Hooft coupling, $\lambda = g_{YM}^2 N$

For this theory to be dual to classical (super)gravity, we need planar limit and strong coupling.

(S)YM theory



$N \sim 1/g_s$
Planar ('t Hooft) limit
Low energy string theory
$\alpha', g_s \to 0$ (fermions in adjoint rep.)
Interesting region for lattice
Quark-Gluon plasma (QGP) $\lambda \approx 10$, $N = 3$
(fermions in fundamental rep.)
Perturbation theory
$\lambda = g_{YM}^2 N \sim 1/\alpha'$


Lattice N=4 SYM
for review see, 0903.4881
• We know that for understanding QCD at strong couplings, lattice has been very successful. So, let's apply it to SUSY theories!
• Several complications because of supersymmetric algebra, field content, conformal symmetry, holographic limits, and fermions!!
• Some progress in the last decade! Use ideas such as twisting of supersymmetric theory, Kaehler-Dirac fermions, A4* lattice.
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Topological twisting and the lattice

(Vafa, Witten, Yamron, Marcus)

- Maximal twisting procedure gives,

$$SO(4)_{\rm tw} \equiv {\rm diag}\,[SO(4)_{\rm Euc} \times SO(4)_R]\,; \quad SO(4)_R \subset SO(6)_R$$

- Q's transform with integer spin under this new rotation group.
- Results in one scalar supercharge (0-form), which is exactly preserved on the lattice. This construction similar to one by orbifolding (*Kaplan et al.*)
- Not all supersymmetric (SUSY) theories possible to study on the lattice. In fact, N=4 SYM is the only theory (in 4d) possible to study on lattice! No complains here!

Set of theories possible to study on the lattice

Theory	R-symmetry group	Twisted/orbifold construction
$d = 2, \mathcal{Q} = 4, \mathcal{N} = 2$	$SO(2) \times U(1)$	✓
$d = 2, \mathcal{Q} = 8, \mathcal{N} = 4$	$SO(4) \times SU(2)$	✓
$d = 2, \mathcal{Q} = 16, \mathcal{N} = 8$	$SO(8)$	✓
$d = 3, \mathcal{Q} = 4, \mathcal{N} = 1$	$U(1)$	X
$d = 3, \mathcal{Q} = 8, \mathcal{N} = 2$	$SO(3) \times SU(2)$	✓
$d = 3, \mathcal{Q} = 16, \mathcal{N} = 4$	$SO(7)$	✓
$d = 4, \mathcal{Q} = 4, \mathcal{N} = 1$	$U(1)$	X
$d = 4, \mathcal{Q} = 8, \mathcal{N} = 2$	$SO(2) \times SU(2)$	X
$d = 4, \mathcal{Q} = 16, \mathcal{N} = 4$	$SO(6)$	✓

Dimensionally reduced SYMs

- Reduce the four dimensional theory down to one dimension (time) to get BFSS (Banks-Fischler-Shenker-Susskind) model.
- Add mass deformation to this theory to study matrix model on pp-wave space-time, known as PWMM or BMN (Berenstein-Maldacena-Nastase) model
- Reduce the theory to (1+1)- dimensions to study SYM theory dual to Type IIB supergravity (SUGRA) having different black hole solutions (*uniform* black string and *localised* black hole) and transition between them.

Regime of supergravity validity (p<3)

To have a valid SUGRA limit, we need:

- Radius of curvature large in units of α' , which means $T \ll 1$ or $\lambda \gg 1$
- The string coupling, $g_s \ll 1$, which means that *N* should be large.

These two combined gives the following : $1 \ll \lambda_p \beta^{3-p} \ll N^{\frac{10-2p}{7-p}}$,
note that for *p*=3, this reduces to the familiar limit.

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BFSS matrix model

The action is,

$$S_{\text{BFSS}} = \frac{N}{4\lambda} \int dt \, \mathrm{Tr} \left[(D_t X^i)^2 - \frac{1}{2} \left[X^I, X^J \right]^2 + \Psi^T D_t \Psi + i \Psi^T \gamma^j [\Psi, X^j] \right]$$

This model can be obtained by the dimensional reduction of $\mathcal{N} = 1$ SUSY in ten dimensions.

I, J runs from 0 … 9, and we have total of sixteen Ψ , where all fields are N x N matrices and in the adjoint representation of the gauge group SU(N).

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State-of-the-art lattice results

- $a_0 = 7.41$ is known from SUGRA calculations.
- Finite-T corrections => α' corrections in Type II string theory

The coefficients a_1, a_2 unknown from supergravity. We only know that corrections start at the $(\alpha')^3$ …

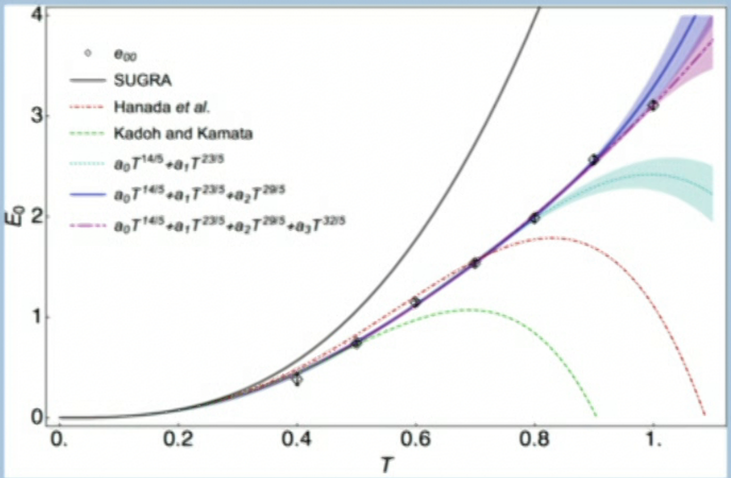


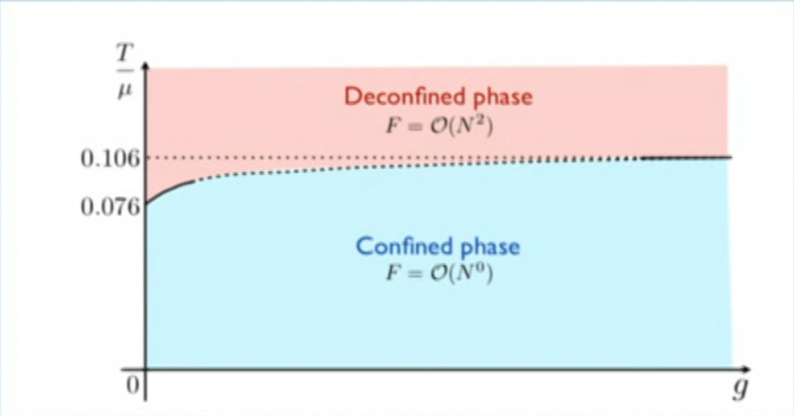
Figure from arXiv: **1606.04951**

(M. Hanada, G. Ishiki et al.)

(N=32, L=32)

Phase transition?

Can we find study some holographic model with some interesting phase structure? BFSS matrix model only has a deconfined phase. However, when we consider the massive deformation of this model (known as BMN or PWMM) model, there is an interesting phase structure.



arXiv: 1411.5541 (Costa, Penedones, Greenspan, Santos)

BMN matrix model

$$S_{BMN} = S_{BFSS} - \frac{N}{4\lambda}\int d\tau \, \mathrm{Tr}\left(\frac{\mu^2}{3^2}\left(X^I\right)^2 + \frac{\mu^2}{6^2}\left(X^i\right)^2 + \frac{2\mu}{3}\epsilon_{IJK}X^I X^J X^K + \frac{\mu}{4}\Psi^T\left(\gamma^{123}\right)\Psi \right)$$

The flat directions of the BFSS model are lifted by giving masses to SO(3) and SO(6) scalars. In addition, there is a cubic scalar term which is also known as 'Myers term' plus a fermion term. Unlike BFSS, this model is on pp-wave spacetime. Even after the addition of these mass terms, supersymmetry is intact.

Dual classical gravity solution valid when N is large and coupling is strong.

BMN phase diagram

(S. Catterall, RGJ, A. Joseph, D. Schaich, T.Wiseman, 19XX.XXXXX)

Use naive discretisation to study the model on the lattice. Since, these theories is super renormalizable in d < 4, such a discretization is good enough for 1d SUSY theories.

The order parameter for the thermal (deconfinement) transition is Polyakov loop. Finite volume phase transition in the large N limit.

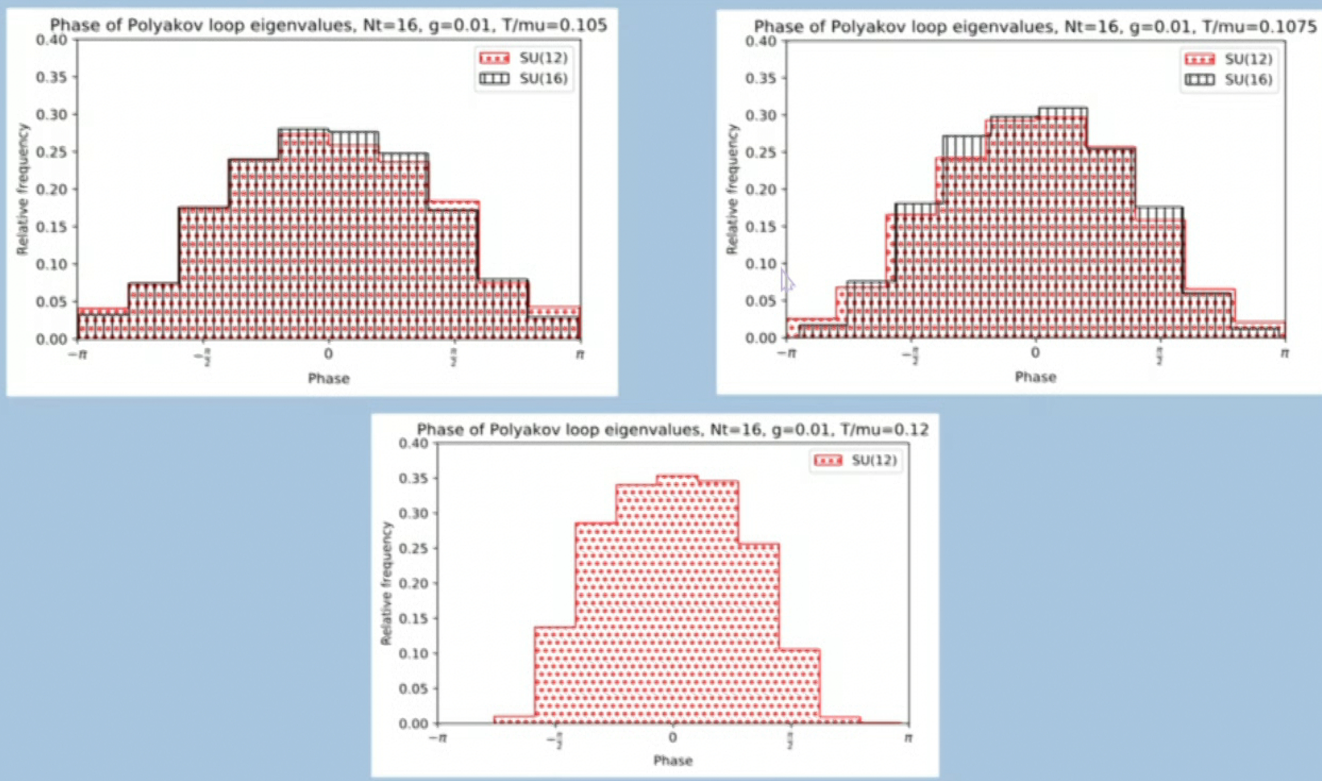
In fact, it might also be interesting to measure Entanglement Entropy (EE) for these finite-T transitions (Ryu and Takayanagi have argued that it plays a role similar to an "order parameter) !

$\langle P \rangle \neq 0$ Deconfined $\qquad$ $\langle P \rangle = 0$ Confined

First explore, $g \to 0$, and see if the perturbative results (large N) are reproduced.

Understanding BMN phase diagram .. continued

(S. Catterall, RGJ, A. Joseph, D. Schaich, T.Wiseman, 19XX.XXXXX)



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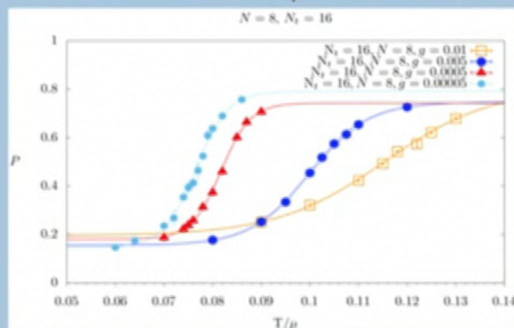
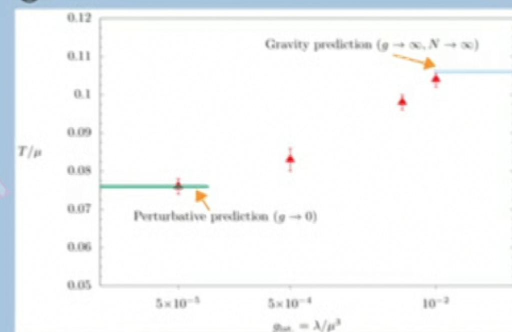
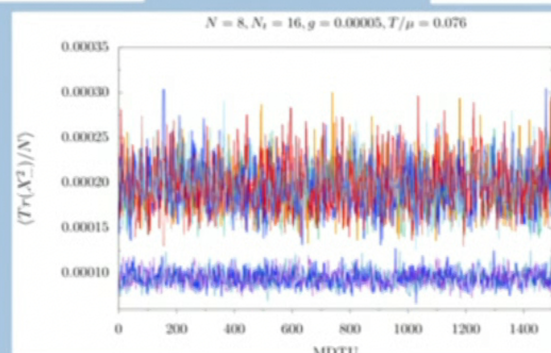
Understanding BMN phase diagram .. continued

(S. Catterall, RGJ, A. Joseph, D. Schaich, T.Wiseman, 19XX.XXXXX)

Lattice results interpolate between the two limits and we see the dependence of the critical temperature on coupling! (T/μ vs. g) !!







1+1- SYM theory

Reduce the 4d theory down to two dimensions. Two additional scalars corresponding to the reduced directions. Only some region of parameter space in SYM theory has valid supergravity description which given by, $1 \ll \cdots \ll N^{2/3}$

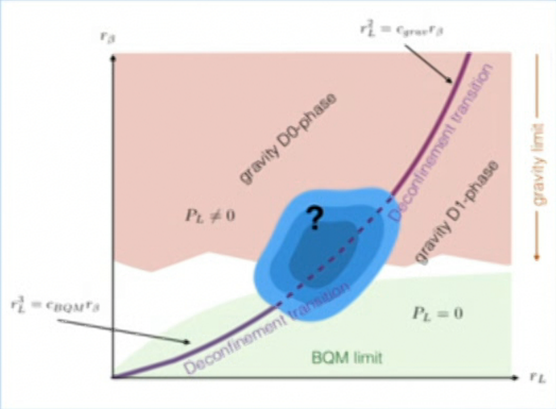
We measure Wilson line around the two cycles. The gravity solutions are static black holes, their Euclidean time circle is contractible so we expect a deconfined Polyakov line, $P_\beta \neq 0$. The homogeneity of the horizon is taken to indicate that the eigenvalues of Wilson line are uniformly distributed at large N (for which we will see the results later).

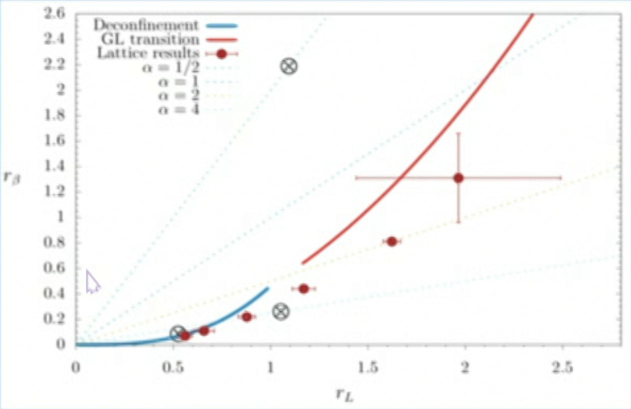
1+1- SYM details

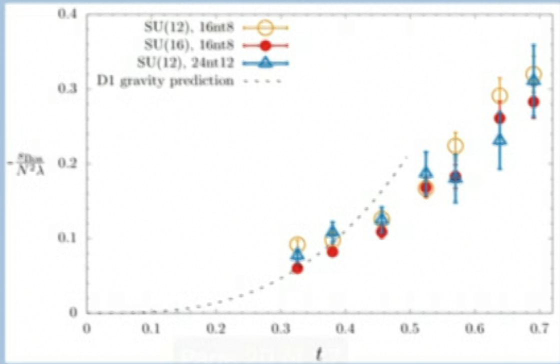
- Dimensionless coupling, $r_\tau = \sqrt{\lambda}\beta = 1/t$, $r_x = \sqrt{\lambda}L$, $\alpha = L/\beta = r_x/r_\tau$
- Santos et al. calculation from gravity side matches old work for square lattice. Gregory-Laflamme transition between localized black hole and homogeneous black string at $\alpha^2 r_\tau \sim 2.45$
- This is a topological transition on the gravity side, dual to *deconfinement* transition on the gauge theory side. In the large N limit and weak coupling, there is well-known Gross-Witten-Wadia transition.
- We study this theory on a reduced A4* —> A2* (triangular lattice)

1+1- SYM theory - results

(PRD 97, 086020, arXiv:1709.07025)







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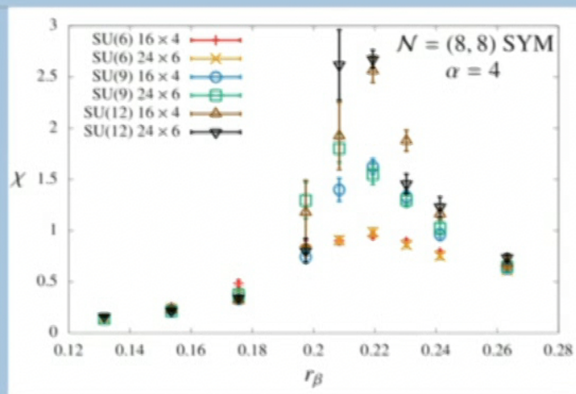

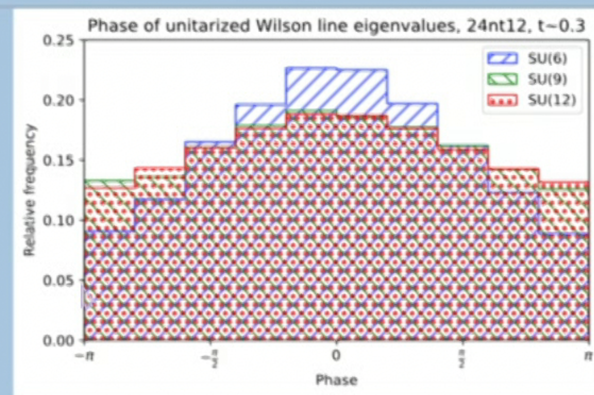

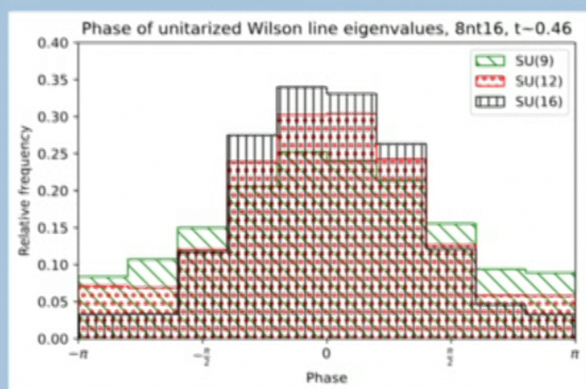

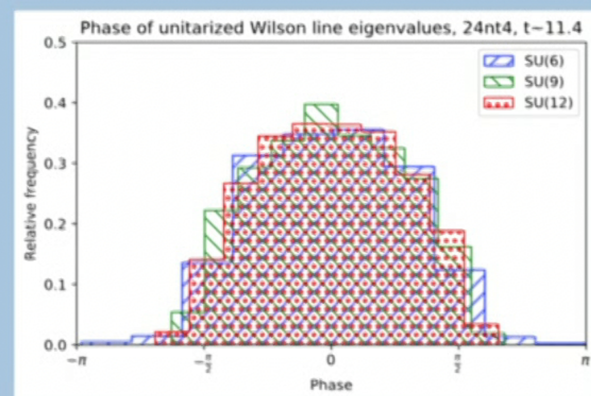

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Ongoing & future work in 3d SYM and N=4 SYM

Trying to understand the thermodynamics of dual uniform D2-branes from 3d SYM at strong coupling (in preparation, **19XX.XXXX**)

Calculate the static quark potential by computing Maldacena-Wilson loop and check predictions (**hep-th/9803002**). Well-known dependence in 4d : $V \sim \sqrt{\lambda}$

Generally, for a (p+1)-dim SYM theory in regimes where supergravity (SUGRA) description is valid, it is expected to follow $V \sim \lambda^{\frac{1}{5-p}}$

Speculative work: Can EE can be calculated on the lattice? For SUSY theories in lower dimensions?

Part II - Tensor networks

Several motivations to study tensor networks

1. Possibility of formulating gauge theories in terms of tensors can enable us to study theories where usual Monte Carlo methods fail (sign problem!).
2. Provides an arena for studying lower-dimensional critical systems and gapped systems faster and more efficiently than any other numerical method!
3. Expected to play important role in putting AdS/CFT on a firm footing via understanding the properties (geometry) of the bulk physics from entangled quantum state at the boundary.

2d non-Abelian gauge+Higgs model

(A Bazavov, S Catterall, RGJ, J Unmuth-Yockey, 1901.11443)

$S = \left(\frac{-\beta}{2} \mathrm{Tr}\Box - \frac{\kappa}{2} \mathrm{Tr} U \right)$, where β is the gauge coupling and κ is the matter coupling in the unitary gauge. The first term is the standard pure gauge Wilson action featuring a plaquette.

We expand the Boltzmann weights in terms of characters (called *character expansion*).

$$e^{-S_g} = \prod_x \sum_r F_r(\beta) \chi^r(UUU^\dagger U^\dagger)$$

$$e^{-S_\Phi} = \prod_{x,\mu} \sum_r F_r(\kappa) \chi^r(U_{x,\mu})$$

$F_r(\cdots)$ is expressed in terms of modified Bessel functions of first kind.

Link (A) and plaquette (B) tensors

$$A^{(\tau)}_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})}(\kappa) = \frac{1}{d_{r_r}} \sum_{\sigma=|r_r - r_l|}^{r_r + r_l} F_\sigma(\kappa) C^{r_r m_{rb}}_{r_l m_{lb} \sigma (m_{rb} - m_{lb})} \times C^{r_r m_{ra}}_{r_l m_{la} \sigma (m_{rb} - m_{lb})} .$$

$$B_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})(r_a m_{al} m_{ar})(r_b m_{bl} m_{br})} = \begin{cases} F_r(\beta) \;\; \delta_{m_{la}, m_{al}} \delta_{m_{ar}, m_{ra}} \delta_{m_{rb}, m_{br}} \delta_{m_{bl}, m_{lb}} & \text{if } r_l = r_r = r_a = r_b = r \\ 0 \quad \text{else.} \end{cases}$$

In case of Abelian-Higgs model, the A tensor only has factors of Bessel's function. With the knowledge of these two tensors, one can construct the fundamental tensor.

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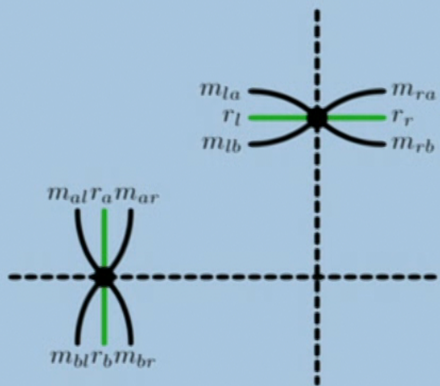
Link (A) and plaquette (B) tensors

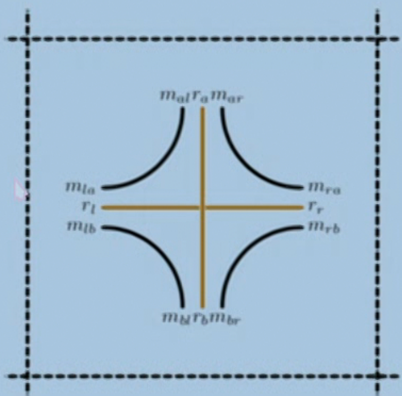
$$A^{(\tau)}_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})}(\kappa) = \frac{1}{d_{r_r}} \sum_{\sigma=|r_r-r_l|}^{r_r+r_l} F_\sigma(\kappa) C^{r_r m_{rb}}_{r_l m_{lb} \sigma (m_{rb}-m_{lb})} \times C^{r_r m_{ra}}_{r_l m_{la} \sigma (m_{rb}-m_{lb})} .$$

$$B_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})(r_a m_{al} m_{ar})(r_b m_{bl} m_{br})} = \begin{cases} F_r(\beta) \;\; \delta_{m_{la}, m_{al}} \delta_{m_{ar}, m_{ra}} \delta_{m_{rb}, m_{br}} \delta_{m_{bl}, m_{lb}} & \text{if } r_l = r_r = r_a = r_b = r \\ 0 \quad \text{else.} \end{cases}$$

In case of Abelian-Higgs model, the *A* tensor only has factors of Bessel's function. With the knowledge of these two tensors, one can construct the fundamental tensor.

Diagrammatic representation of A and B tensors

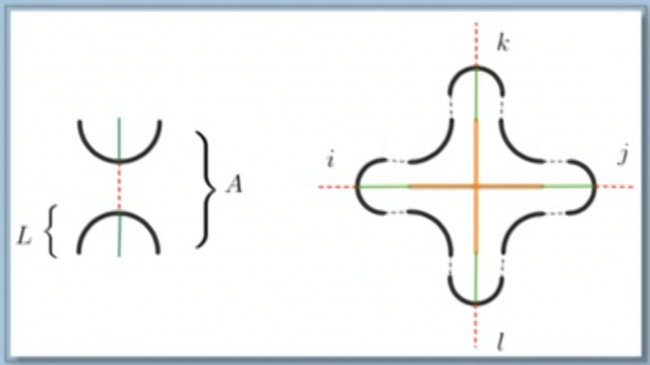




The fundamental tensor (T)

Using A and B tensor, we construct T tensor. And then several copies of T tensor we can make up the entire lattice. Note that we can just use A by exploiting the translational invariance of the lattice.

$$T_{ijkl}(\beta,\kappa) = \sum_{\alpha,\beta,\gamma,\delta} B_{\alpha\beta\gamma\delta}(\beta) L_{\alpha i} L_{\beta j} L_{\gamma k} L_{\delta l}(\kappa), \quad \text{where} \quad A = LL^T$$



Coarse-graining the tensor network

The partition function can then be written as, $Z(\beta, \kappa) = \mathrm{Tr}\left[\prod_x T^{(x)}(\beta, \kappa)\right]$

We use HOTRG (higher order tensor renormalisation group) to implement coarse-graining by truncating the local space to $D_{\text{bond}} = 50$. We choose $r_{max} = 1$, which corresponds to size of initial T to be 14^4. Using $r_{max} = 1/2$ would imply T of size 5^4. The choice of r_max depends on the gauge theory one wants to study, coupling regime, running/walking. In some cases, there are arguments that using an action other than Wilson action can improved truncation over "r", ex: heat kernel action.

Polyakov line in tensor formulation

Defined as,

$$\langle P \rangle \equiv \mathrm{Tr}\left[\prod_{\tau=0}^{N_\tau-1} D^{\frac{1}{2}}(U_{x,\tau})\right]$$

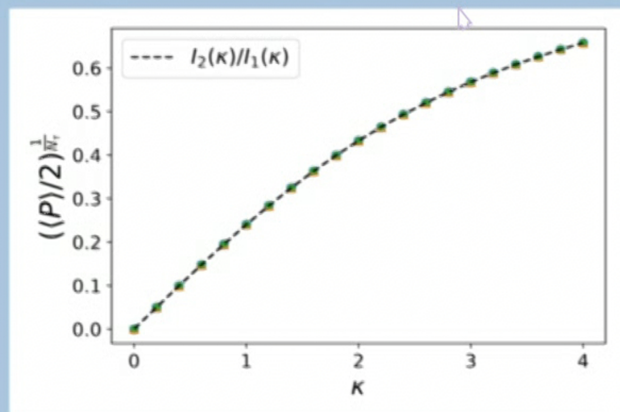
Impure tensor $\tilde{A}$ which contains the Polyakov line in the 1/2-rep can be constructed as,

$$\tilde{A}^{(\tau)}_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb}) ij}(\kappa) = \frac{1}{d_{r_r}} \sum_{r'=|\frac{1}{2}-r_l|}^{\frac{1}{2}+r_l} \sum_{\sigma=|r_r-r'|}^{r_r+r'} F_\sigma(\kappa) C^{r_r m_{rb}}_{r'(m_{lb}+i)\sigma(m_{rb}-m_{lb}-i)} C^{r_r m_{ra}}_{r'(m_{la}+j)\sigma(m_{rb}-m_{lb}-i)} C^{r'(m_{lb}+i)}_{r_l m_{lb} \frac{1}{2} i} C^{r'(m_{la}+j)}_{r_l m_{la} \frac{1}{2} j} .$$

One does the coarse-graining on a given time-slice and then inserts this tensor at the edge of the lattice (actually, can insert anywhere due to *translational* symmetry)

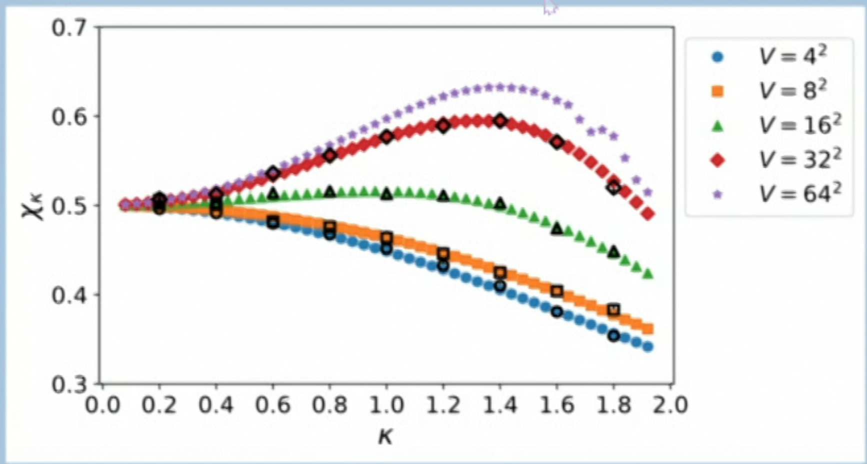
Exact result for $\beta = 0$

For $\beta = 0$, we can write the expectation value of Polyakov loop as, $\langle P \rangle = 2\left(\frac{I_2(\kappa)}{I_1(\kappa)}\right)^{N_\tau}$ and provides a simple check of the tensor formulation.



Other observables..

Matter susceptibility defined as, $\chi_\kappa = \frac{1}{N_s N_\tau} \frac{\partial^2 \ln Z}{\partial \kappa^2}$ compared with the Monte Carlo results! Note that their computation time differ by factor of 1000! (cross-over behaviour around $\kappa \sim 1.5$)



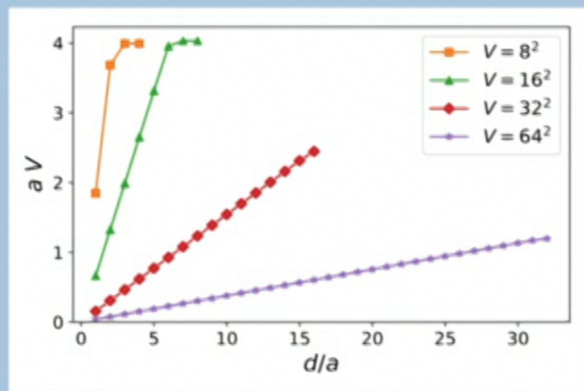
Polyakov loop correlation - Confining & Higgs Phase

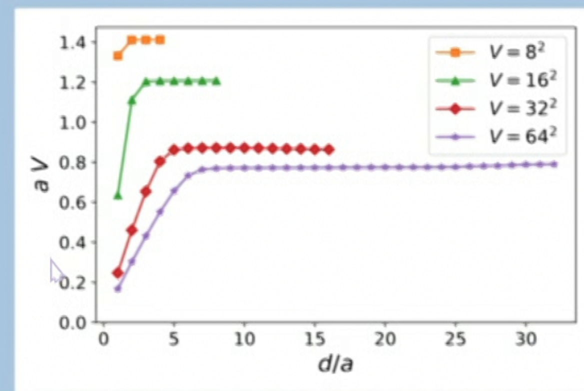
In the limit of zero temperature, the correlator is given as,

$C_{PP^{\dagger}}(R) = \exp\left[-\beta V(R)\right]$. This also provides a measure of monitoring confinement when $V \propto R$ (the slope gives σ, string tension). On the other hand, in a conformal theory, the potential is Coulomb like such as in N=4 SYM. In a Higgs-like region, it is constant and independent of R.

Two distinct phases: 1) Confining phase and, 2) Higgs-phase.

Results - Static potential

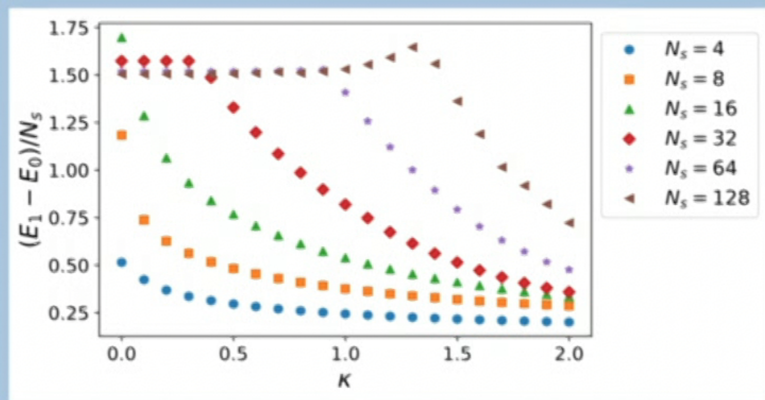




On the left, we have $\kappa = 0.50$, which is in the confining regime and on the right, we have $\kappa = 2.0$ which is in the Higgs like region. There is a crossover behaviour around $\kappa \sim 1.5$

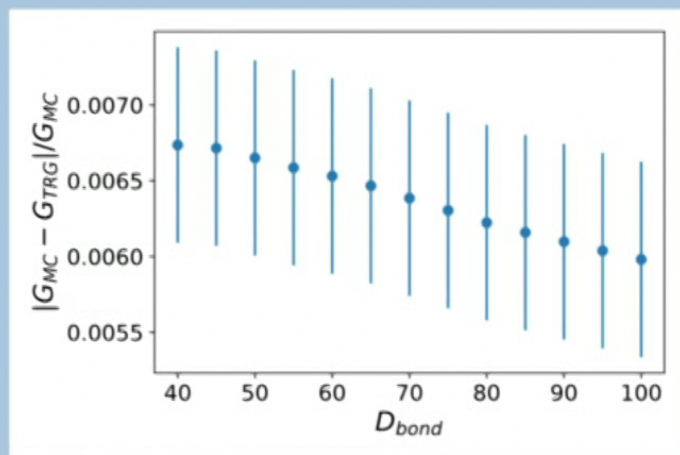
Mass gap and continuum limit

We calculate the mass gap by evaluating the energy difference between the first excited and the ground state. One finds that, $\frac{a}{\xi} \propto \ln\left(\frac{\lambda_1}{\lambda_0}\right)$ and we see that correlation length increases as continuum limit is taken $\beta = cN_s^2 \to \infty$ with $c = O(1)$



Difference in results between HOTRG and Monte Carlo

Though we find good agreement between tensor results and Monte Carlo, there seems to be small disagreement at $\kappa \geq 2$. We guess that this might be related to breaking of the HOTRG algorithm. We are exploring possible alternatives.



Future work!

- For precision holographic checks and understanding the nature of bulk geometry, we need to access larger N by parallelising the Monte Carlo over both the lattice volume and colours. Extract static potential in 3d SYM and look at the possibility of measuring the entanglement entropy as order parameter for black hole transitions (in fact, the EE is known for 3d N=8 SCFT where it goes as $N^{5/3}$ between IR fixed point dependence of $N^{3/2}$ and free field result of N^2, Ryu & Takayanagi, **arXiv:0605073**)
- One interesting possibility is to measure the Renyi entropy in this non-Abelian model. Also plan to revisit classical XY model with these more efficient tensor algorithms.