Title: Leggett-Garg Inequalities: Decisive Tests for Macrorealism and Protocols for Non-Invasive Measurements

Speakers: Jonathan Halliwell

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Abstract: The Leggett-Garg (LG) inequalities were introduced, as a temporal parallel of the Bell inequalities, to test macroscopic realism -- the view that a macroscopic system evolving in time possesses definite properties which can be determined without disturbing the future or past state. The talk will begin with a review of the LG framework. Unlike the Bell inequalities, the original LG inequalities are only a necessary condition for macrorealism, and are therefore not a decisive test. I argue, for the case of measurements of a single dichotomic variable Q, that when the original four three-time LG inequalities are augmented with a set of twelve two-time inequalities also of the LG form, Fine's theorem applies and these augmented conditions are then both necessary and sufficient [1]. A comparison is carried out with the alternative necessary and sufficient conditions for macrorealism based on no-signaling in time conditions which ensure that all probabilities for Q at one and two times are independent of whether earlier or intermediate measurements are made. I argue that the two tests differ in their implementation of the key requirement of non-invasive measurability so are testing different notions of macrorealism, and these notions are elucidated. I also describe some alternative protocols which achieve non-invasiveness, one involving continuous measurement of the velocity conjugate to Q [2], which was recently implemented in an experiment at IQC, the other involving a modification of the standard ideal negative measurement protocol [3].

[1] J.J.Halliwell, Phys Rev A96, 012121 (2017); A93, 022123 (2016); arxiv:1811.10408.

[2] J.J.Halliwell, Phys. Rev. A94, 052114 (2016).

[3] J.J.Halliwell, Phys. Rev. A99, 022119 (2019).

Leggett-Garg Inequalities: Decisive Tests for Macrorealism and Protocols for Non-Invasive Measurements

Jonathan Halliwell

Imperial College London

Perimeter Institute 6.3.19

Macroscropic Realism: Is the Moon Really There When No-one Looks?

Can a quantum system be thought of as following a definite trajectory?

Can macroscopic systems exist in superposition states?

- Relates to questions of classicality conditions in quantum theory – there is a hierarchy of such conditions.
- Macrorealism is, perhaps, the weakest notion of classicality. The Leggett-Garg inequalities were designed to test it.
- There exist other tests of macrorealism – no-signaling in time and coherence witness conditions – which are stronger.

This Talk

- Review of the Leggett-Garg Framework
- Extension of the LG Framework – a decisive and richer test of macrorealism. Clearer parallel to Bell experiments.
- Comparison with alternative, stronger conditions for macrorealism.
- Alternative methods for non-invasive measurements.

Based on: JJH, Phys Rev A93, 022123 (2016); A 94, 052131 (2016); A 94, 052114 (2016); A96, 012121 (2017); A99, 022119 (2019).

Current experiment: Shayan Majidy, Hement Katiyar and Raymond Laflamme

Outline

1. EPRB experiment, CHSH inequalities, Fine's theorem.
2. Macrorealism and Leggett-Garg tests.
3. Two-time measurements.
4. Conditions for macrorealism using extended Leggett-Garg inequalities.
5. Conditions for macrorealism using no-signaling in time.
6. Quantum-mechanical probabilities.
7. Non-invasive measurements

1.1 The EPRB Experiment

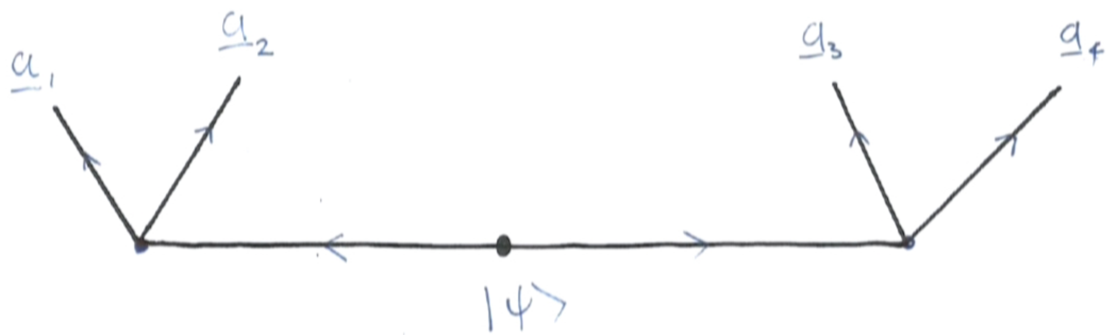

Figure: Testing Local Realism using the EPRB experiment.
Measurements are made of $p(s_1, s_3)$, $p(s_1, s_4)$, $p(s_2, s_3)$, $p(s_2, s_4)$, where $s_i = \pm 1$.

1.2 EPRB and the CHSH Inequalities

- $p(s_1, s_3), p(s_1, s_4), p(s_2, s_3), p(s_2, s_4)$ satisfy no signaling (NS):

$$\sum_{s_1} p(s_1, s_3) = p(s_3) = \sum_{s_2} p(s_2, s_3), \quad etc.$$

- Seek a probability $p(s_1, s_2, s_3, s_4)$ such that

$$p(s_1, s_3) = \sum_{s_2, s_4} p(s_1, s_2, s_3, s_4), \quad etc.$$

- If such a probability exists then the correlation functions

$$C_{ij} = \sum_{s_1, s_2, s_3, s_4} s_i s_j \ p(s_1, s_2, s_3, s_4),$$

satisfy the eight CHSH inequalities, e.g.

$$-2 \leq C_{13} + C_{14} + C_{23} - C_{24} \leq 2.$$

1.3 Fine's Theorem

Fine's theorem: The eight CHSH inequalities plus the NS conditions, are also a sufficient condition for the construction of $p(s_1, s_2, s_3, s_4)$

- CHSH inequalities together with the NS conditions are a necessary and sufficient condition for Local Realism.
- Similarly for three measurements and the Bell inequalities,

$$1 + C_{12} + C_{23} + C_{13} \geq 0$$

  (plus three more).
- The existence of the probability $p(s_1, s_2, s_3, s_4)$ is deduced indirectly from the partial snapshots $p(s_1, s_3)$, $p(s_1, s_4)$, $p(s_2, s_3)$, $p(s_2, s_4)$ – it is not explicitly measured.
- QM can sometimes supply candidate underlying probabilities (e.g. DH approach). CHSH then implies bounds on the interference.

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2.1 Macrorealism and Leggett-Garg Tests

The LG inequalities (Leggett and Garg, 1985) entail sequential measurements in time on a single system. They are designed to test Macrorealism (MR):

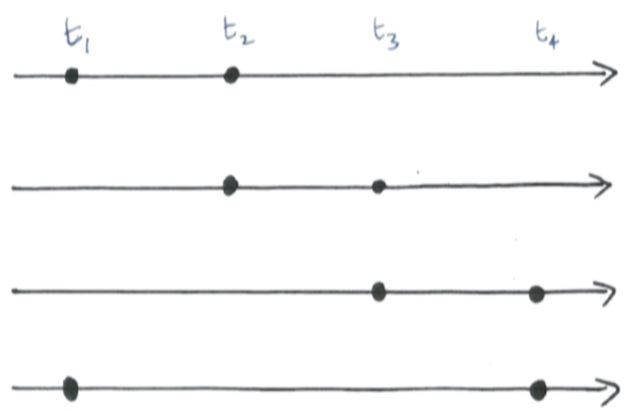
1. Macrorealism per se (MRps): the system is in a definite state at each moment of time;
2. Non-invasive measurability (NIM): the state can be measured without disturbing the subsequent dynamics;
3. Induction (Ind): future measurements do not affect the present state.

We write $\mathrm{MR} = \mathrm{MRps} \wedge \mathrm{NIM} \wedge \mathrm{Ind}$.

Review: Emary, Lambert and Nori (2014)
Critique: Maroney and Timpson (2014)

2.2 The Leggett-Garg Inequalities

- Measurements are made of a single variable $Q = \pm 1$ at pairs of times $t_1 < t_2 < t_3 < t_4$, to determine the four pairwise probabilities $p(s_i, s_j)$ and hence C_{ij} (for $ij = 12, 23, 34, 14$).



- MR $\Longrightarrow$ underlying probability exists $\Longrightarrow$ LG inequalities:

$$-2 \leq C_{12} + C_{23} + C_{34} - C_{14} \leq 2,$$

(plus six more).

2.3 LG Violations in a Simple Spin Model

- Take $\hat{Q} = \mathbf{a} \cdot \sigma$ and Hamiltonian $H = \frac{1}{2}\omega\sigma_x$. We find

$$\begin{aligned} C_{12} &= \frac{1}{2}\langle \hat{Q}(t_1)\hat{Q}(t_2) + \hat{Q}(t_2)\hat{Q}(t_1)\rangle = \mathbf{a}(t_1) \cdot \mathbf{a}(t_2) \\ &= \cos\omega(t_2 - t_1) \end{aligned}$$

  closely analogous to EPRB.

- LG inequalities with $t_1 = t$, $t_2 = 2t$, $t_3 = 3t$, $t_4 = 4t$ are

$$-2 \leq 3\cos\omega t - \cos 3\omega t \leq 2$$

  Maximal violation of $2\sqrt{2}$ at $\omega t = \pi/4$.

- Similar results for models with higher dimensional Hilbert spaces.

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- Similar results for models with higher dimensional Hilbert spaces.

2.4 Non-invasive Measurements

Ideal negative measurements: the detector is coupled to $Q = +1$ at the first time. A null result implies $Q = -1$.
(Knee et al, 2012; Robens et al. 2015; Katiyar et al 2016)

This eliminates alternative explanations by classical models with invasiveness. (Montina, 2012; Yearsley, 2013).

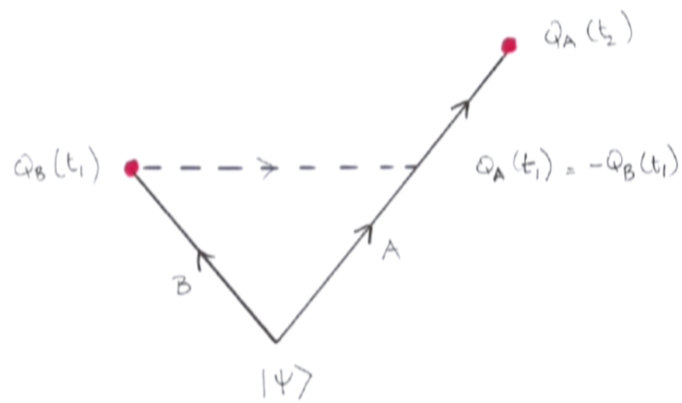
Ideal negative measurements still cause wave function collapse!

Measurement of $p(s_1, s_2)$ typically involves an ancilla whose state becomes entangled with Q at t_1, i.e. we have two systems each with 2 (or more) states, hence the similarity to EPRB.

Weak measurements are frequently used. Although the disturbance can be made very small the effect measured is (often) of the same order of magnitude. (Palacios-Laloy et al, 2010)

2.5 Aside: A Simple Direct Connection to the EPRB Case

- Suppose the LG system is particle A of an EPRB pair (with a magnetic field near A for $t > t_1$).



- Measure Q_B at time t_1. Deduce the value of Q_A, using $Q_A = -Q_B$ at time t_1, without disturbing A.
- Measure Q_A at time t_2.

2.6 Comments

- The LG framework seeks to rule out certain types of hidden variable theories. Most formulations rule out only HV theories of the GRW type.
- It does not rule out de Broglie-Bohm type theories, unless locality can be invoked (Maroney and Timpson, 2014; Bacciagalupi 2014).
- The NS relations

$$\sum_{s_1} p(s_1, s_2) = p(s_2) = \sum_{s_3} p(s_2, s_3)$$

do NOT hold in general in LG tests. Fine's theorem does not hold and LG inequalities are a necessary but not sufficient condition for MR.

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3.1 No Signaling in Time (NSIT)

The analogue of NS in EPRB for temporal correlations is the NSIT condition (Brukner and Kofler, 2013):

$$\sum_{s_1} p_{12}(s_1, s_2) = p_2(s_2)$$

- It characterizes both NIM and MRps at two times. When it holds Fine's theorem applies.
- NSIT is not satisfied in general by QM for sequential measurements.
- Brukner and Kofler (2013) and Clemente and Kofler (2015) sought alternative definitions of MR using a set of NSIT conditions only, without the LG inequalities.

There is a different way of meeting Fine's theorem using a different implementation of NIM avoiding sequential measurements.

3.2 Key Issue – Probability Distributions at Two Times

- Since $[\hat{Q}(t_1), \hat{Q}(t_2)] \neq 0$, the existence of $p(s_1, s_2)$ at the two-time level in the LG framework is not guaranteed.
- I.e. MR may fail already at the two time level unlike EPRB.
- If a MR description exists, there could be a number of different ways of assigning probabilities to such pairs of observables.
- Different probability assignments correspond to different measurement protocols and potentially different implementations of NIM.
- Look for alternative ways to determine

$$p_{12}(s_1, s_2) = \frac{1}{4}\left(1 + \langle Q_1 \rangle s_1 + \langle Q_2^{(1)} \rangle s_2 + C_{12} s_1 s_2\right).$$

3.3 Checking MR at the Two-Time Level

- We proceed indirectly. Measure $\langle Q_1 \rangle$, $\langle Q_2 \rangle$ and C_{12} (respecting NIM) in three different experiments.
- Attempt to construct the probability from its moments:

$$q(s_1, s_2) = \frac{1}{4}\left(1 + \langle Q_1 \rangle s_1 + \langle Q_2 \rangle s_2 + C_{12} s_1 s_2\right)$$

- In a MR theory, we must have,

$$(1 + s_1 Q_1)(1 + s_2 Q_2) \geq 0,$$

and averaging we obtain the two-time LG inequalities:

$$q(s_1, s_2) \geq 0.$$

- Since NIM is assumed satisfied, $q(s_1, s_2)$ is a measure of MRps only (unlike the usual NSIT condition).
- If $q(s_1, s_2) < 0$, MRps fails at the two-time level.

3.4 NIM Comes in Two Types

There are TWO natural implementations of NIM. They correspond to different notions of MR.

- Piecewise NIM_{pw}: do numerous experiments to measure all the moments with NIM satisfied in each individual experiment. $q(s_1, s_2)$ is deduced indirectly.

- Sequential NIM_{seq}: $p(s_1, s_2)$ is determined by sequential measurements in a single experiment. NIM is imposed via the NSIT condition:

$$\sum_{s_1} p_{12}(s_1, s_2) = p_2(s_2)$$

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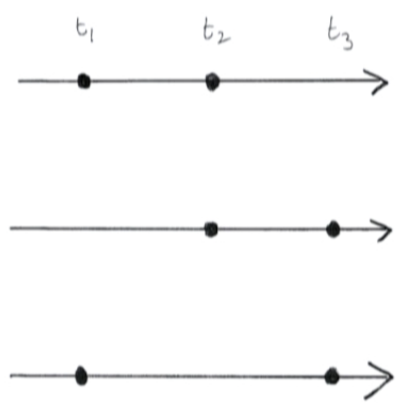
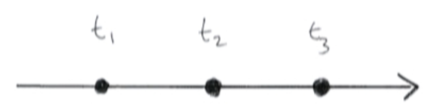
4.1 Two Routes to Defining Macrorealism at Three Times

- Piecewise NIM_{pw}: do numerous different non-invasive measurements of the moments. The existence of the probabilities $q(s_i, s_j)$, $p(s_1, s_2, s_3)$, is deduced indirectly.

- Sequential NIM_{seq}: measurements are made at all three times in a single experiment and the probability $p(s_1, s_2, s_3)$ is measured directly. NSIT conditions are imposed to ensure NIM.





4.2 Defining Macrorealism with LG Inequalities for Three Times

Use NIM_{pw}: Measure C_{12}, C_{23}, C_{13} non-invasively and $\langle Q_1 \rangle$, $\langle Q_2 \rangle$, $\langle Q_3 \rangle$ in six experiments.

- Look for an underlying probability $p(s_1, s_2, s_3)$ matching them, in 3 steps:

$$p(s_i) = \frac{1}{2}(1 + s_i \langle Q_i \rangle) \geq 0.$$
$$q(s_i, s_j) = \frac{1}{4}(1 + s_i \langle Q_i \rangle + s_j \langle Q_j \rangle + s_i s_j C_{ij})$$

- $q(s_i, s_j) \geq 0$ for a MR theory, but can be negative in QM.
- $q(s_i, s_j)$ formally satisfies "NSIT", by construction $\Longrightarrow$ conditions for Fine's theorem are met. E.g.,

$$\sum_{s_1} q(s_1, s_2) = p(s_2) = \sum_{s_3} q(s_2, s_3)$$

This says nothing about signaling.

4.3 MR from LGI continued

- We now have an exact mathematical parallel with Bell case: $p(s_1, s_2, s_3)$ exists if and only if the following 16 inequalities hold. The usual 4 three-time LG inequalities

$$
\begin{aligned}
1 + C_{12} + C_{23} + C_{13} &\geq 0, \\
1 - C_{12} - C_{23} + C_{13} &\geq 0, \\
1 + C_{12} - C_{23} - C_{13} &\geq 0, \\
1 - C_{12} + C_{23} - C_{13} &\geq 0,
\end{aligned}
$$

together with the 12 two-time LG inequalities,

$$1 + s_i \langle Q_i \rangle + s_j \langle Q_j \rangle + s_i s_j C_{ij} \geq 0.$$

- These test a particular version of MR:

$$\mathrm{MR}_{weak} = \mathrm{NIM}_{pw} \wedge \mathrm{LG}_{12} \wedge \mathrm{LG}_{23} \wedge \mathrm{LG}_{13} \wedge \mathrm{LG}_{123} \wedge \mathrm{Ind}$$

- Similarly for four-time LG inequalities.

4.4 MR from LGI: Comments

- The 12 two-time LG inequalities provide the extra restrictions that elevate the usual three-time inequalities from necessary to necessary and sufficient conditions for MR.
- Recent experiments could be readily adapted to carry out this decisive test of MR by including measurements of $\langle Q_1 \rangle, \langle Q_2 \rangle, \langle Q_3 \rangle$.
- See current experiment by Majidy, Katiyar and Laflamme.
- Many LG experiments to date already test certain two-time LG inequalities, using simplifications such as $Q_1 = 1$, since

$$1 + C_{12} + C_{23} + C_{13} \geq 0,$$

becomes

$$1 + \langle Q_2 \rangle + \langle Q_3 \rangle + C_{23} \geq 0.$$

4.5 MR from LGI: Generalizations

- Generalizations of the LG inequalities to n-times:

$$C_{12} + C_{23} + \cdots C_{n-2,n-1} - C_{n-1,n} \leq n - 2$$

plus all possible variants with an odd number of minus signs. Fine's theorem generalizes (JJH and Mawby, 2019).

- LG experiments which measure higher-order correlation functions, such as $D_{123} = \langle Q_1 Q_2 Q_3 \rangle$, have been carried out (Bechtold et al, 2016).

- The necessary and sufficient conditions for MR are then the eight conditions of the form (JJH, 2019):

$$1 + \langle Q_1 \rangle + \langle Q_2 \rangle + \langle Q_3 \rangle + C_{12} + C_{13} + C_{23} + D_{123} \geq 0.$$

5.1 Defining Macrorealism with NSIT Conditions

Use NIM_{seq}: Measure Q_1, Q_2 and Q_3 in a single experiment. Require that $p_{123}(s_1, s_2, s_3)$ is a probability for three independent variables. Impose:

$$\begin{aligned}
\mathrm{NSIT}_{(1)23} : \sum_{s_1} p_{123}(s_1, s_2, s_3) &= p_{23}(s_2, s_3) \\
\mathrm{NSIT}_{1(2)3} : \sum_{s_2} p_{123}(s_1, s_2, s_3) &= p_{13}(s_1, s_3) \\
\mathrm{NSIT}_{(2)3} : \sum_{s_2} p_{23}(s_2, s_3) &= p_3(s_3)
\end{aligned}$$

These test a strong notion of MR:

$$\begin{aligned}
\mathrm{MR}_{strong} &= \mathrm{NIM}_{seq} \wedge \mathrm{MRps} \wedge \mathrm{Ind} \\
&= \mathrm{NSIT}_{(1)23} \wedge \mathrm{NSIT}_{1(2)3} \wedge \mathrm{NSIT}_{(2)3} \wedge \mathrm{Ind}
\end{aligned}$$

(Clemente and Kofler 2015, 2016; Maroney and Timpson, 2014)

5.2 NSIT Conditions vs LG Inequalities

- NSIT conditions define MR using equalities, whereas the LG framework defines MR using inequalities.
- NSIT conditions are checking both NIM_{seq} and MRps.
- In the LG framework NIM_{pw} holds by design, so tests MRps directly.
- This is why MR_{strong} seems to involve much more stringent conditions than MR_{weak}.
- Can also consider an intermediate version of MR:

$$\mathrm{MR}_{int} = \mathrm{NSIT}_{(1)2} \wedge \mathrm{NSIT}_{(2)3} \wedge \mathrm{NSIT}_{(1)3} \wedge \mathrm{LG}_{123} \wedge \mathrm{Ind}$$

Clearly

$$\mathrm{MR}_{strong} \implies \mathrm{MR}_{int} \implies \mathrm{MR}_{weak}$$

6.1 QM Two-Time Measurement Formulae

- NIM_{seq} involves requiring the standard measurement formula

$$p(s_1, s_2) = \mathrm{Tr}\left(P_{s_2}(t_2) P_{s_1}(t_1) \rho P_{s_1}(t_1)\right)$$

where $P_s = \frac{1}{2}(1 + s\hat{Q})$, to satisfy NSIT.

- In NIM_{pw}, the non-invasively measured $\langle Q_1 \rangle$, $\langle Q_2 \rangle$ and C_{12} determine the quasi-probability

$$q(s_1, s_2) = \mathrm{Re}\ \mathrm{Tr}\left(P_{s_2}(t_2) P_{s_1}(t_1) \rho\right).$$

It formally satisfies NSIT but can be negative.

6.2 Sequential Measurements vs Quasi-probability

The sequential measurement probability p and quasi-probability q are related by

$$p(s_1, s_2) = q(s_1, s_2) + \frac{1}{8}\langle[\hat{Q}_1, \hat{Q}_2]\hat{Q}_1\rangle_{s_2}$$

- NSIT for $p(s_1, s_2) \implies$ zero interference $\implies q(s_1, s_2) \geq 0$.
- However, MRps may hold, $q(s_1, s_2) \geq 0$, but NSIT fails.
- $q(s_1, s_2) \geq 0$ requires only that the interferences are bounded.
- Similarly at three times: MR_{strong} requires zero interference but MR_{weak} allows non-zero interferences.

Summary so Far

There exist two natural notions of NIM, piecewise and sequential, and two corresponding sets of necessary and sufficient conditions for MR.

- NIM_{pw}: moments are measured in a number of non-invasive experiments and the results of partial snapshots combined.

  $$\mathrm{MR}_{weak} = \mathrm{NIM}_{pw} \wedge \mathrm{LG}_{12} \wedge \mathrm{LG}_{23} \wedge \mathrm{LG}_{13} \wedge \mathrm{LG}_{123} \wedge \mathrm{Ind}$$

  Requires bounded interference. Direct tests of MRps. Elevates LG to a decisive test for MR.

- NIM_{seq}: sequential measurements are made at three times in a single experiment.

  $$\mathrm{MR}_{strong} = \mathrm{NSIT}_{(1)23} \wedge \mathrm{NSIT}_{1(2)3} \wedge \mathrm{NSIT}_{(2)3} \wedge \mathrm{Ind}$$

  Requires zero interference. Tests a combination or MRps and NIM.

7.1 Modified Ideal Negative Measurements

- Ideal negative measurements: the detector is coupled to $Q = +1$ at the first time. A null result implies $Q = -1$.
- INMs still collapses the wave function so NSIT fails:

$$\sum_{s_1} p_{12}(s_1, s_2) \neq p_2(s_2)$$

How do we check experimentally that INMs are non-invasive?

- The value of C_{12} is insensitive to diagonalization at t_1.
- Introduce a briefly acting decoherence mechanism at t_1. (E.g. use the ancilla to perform a blind measurement). NSIT becomes satisfied – fails due to clumsiness. (JJH, 2019).

7.2 Continuous in Time Velocity Measurement (CTVM)

- First note that

$$C_{12} = \langle Q_1 Q_2 \rangle = 1 - \frac{1}{2}\langle [Q_2 - Q_1]^2 \rangle$$

- Assume there exists a velocity $v(t) = \dot{Q}(t)$.

$$Q_2 - Q_1 = \int_{t_1}^{t_2} dt \; v(t).$$

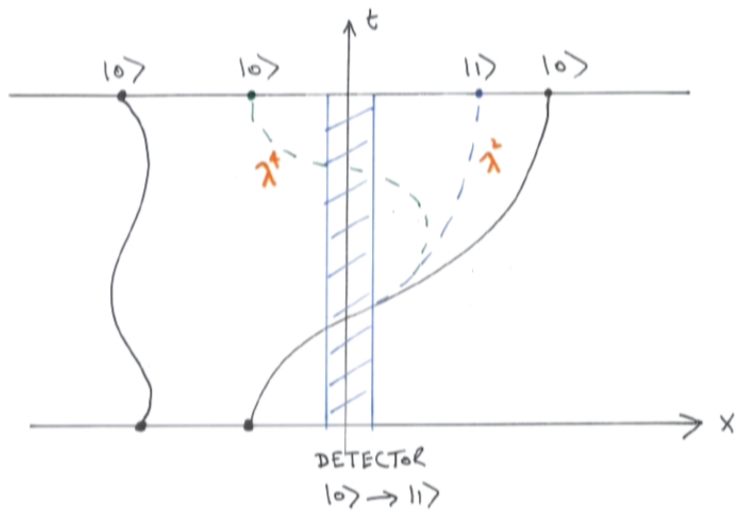
RHS can be measured using a weak coupling λ to a detector continuous in time.

- Assume that $Q(t)$ changes sign at most once during $[t_1, t_2]$. This is reasonable in some models and includes regimes in which there is substantial LG violation.

JJH, Phys Rev A 94, 052114 (2016). Measured by Majidy, Katiyar and Laflamme.

7.3 CTVM/Waiting Detector

For illustrative purposes suppose $Q = \mathrm{sign}(X)$.



The effect of interest, $p(|1\rangle)$, is of order λ^2 but the back-action disturbance is order λ^4, so is approximately non-invasive for $\lambda \ll 1$.

Summary

- Necessary and sufficient conditions for MR using augmented LG inequalities.
- Alternative methods for non-invasive measurements.
- Generalizations of LG to more times and higher order correlators.

All of the above are of interest to test experimentally. Some progress has been made (Majidy, Katiyar, Laflamme).

2.6 Comments

- The LG framework seeks to rule out certain types of hidden variable theories. Most formulations rule out only HV theories of the GRW type.
- It does not rule out de Broglie-Bohm type theories, unless locality can be invoked (Maroney and Timpson, 2014; Bacciagalupi 2014).
- The NS relations

$$\sum_{s_1} p(s_1, s_2) = p(s_2) = \sum_{s_3} p(s_2, s_3)$$

do NOT hold in general in LG tests. Fine's theorem does not hold and LG inequalities are a necessary but not sufficient condition for MR.