Title: A new method of constructing binary black hole initial data

Speakers: Istvan Racz

Series: Strong Gravity

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Abstract: By applying a parabolic-hyperbolic formulation of the constraints and superposing Kerr-Schild black holes, a simple method is introduced to initialize time evolution of binary systems. As the input parameters are essentially the same as those used in the post-Newtonian (PN) setup the proposed method interrelates various physical expressions applied in PN and in fully relativistic formulations. The global ADM charges are also determined by the input parameters, and no use of boundary conditions in the strong field regime is made.

A new method of constructing binary black hole initial data

István Rácz

istvan.racz@fuw.edu.pl & racz.istvan@wigner.mta.hu

Faculty of Physics, University of Warsaw, Warsaw, Poland

Wigner Research Center for Physics, Budapest, Hungary

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Waterloo, Ontario, Canada, 6 March 2019

Outline:

Motivations:

GW observations:

- inspiral and merger of binary black holes is of distinguished importance for the emerging field of gravitational wave astronomy
- non-linearities necessitate the use of accurate numerical approaches in determining the emitted waveforms
- precision of these simulations—in particular, their initializations—is of critical importance in enhancing the detection of gravitational wave signals

Based on:

- I. Rácz: *Constraints as evolutionary systems*, Class. Quantum Grav. **33** 015014 (2016); [arXiv:1508.01810]
- I. Rácz: *A simple method of constructing binary black hole initial data*, Astronomy Reports **62** 953-958 (2018); [arXiv:1605.01669]
- I. Rácz: *Supplemental Material* (2016) : http://www.kfki.hu/~iracz/ SM-BH-data.pdf
- I. Rácz: *On the ADM charges of multiple black holes* , [arXiv:1608.02283]

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Intitialization:

The constraints:

- vacuum initial data: (h_{ij}, K_{ij}) on a 3-dimensional manifold Σ
  - evolution equations $\mathscr{L}_n h_{ij} = ...$ & $\mathscr{L}_n K_{ij} = ...$ (in analogy $\dot{\mathbf{x}} = \mathbf{v}$ & $\dot{\mathbf{v}} = \mathbf{f}$)

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\begin{aligned}
{}^{(3)}R + (K^e{}_e)^2 - K_{ef}K^{ef} &= 0 \\
D_e K^e{}_a - D_a K^e{}_e &= 0
\end{aligned}
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where D_a denotes the covariant derivative operator associated with h_{ab}

- it is an underdetermined system: 4 equations for 12 variables

The conformal (elliptic) method:

Lichnerowicz A (1944) and York J W (1972):

- replace

$$h_{ij} = \phi^4\, \widetilde{h}_{ij} \quad \text{and} \quad K_{ij} - \tfrac{1}{3}\, h_{ij}\, K^l{}_l = \phi^{-2}\, \widetilde{K}_{ij}$$

- using these variables the constraints are put into the **semilinear elliptic system**

$$\widetilde{D}^l \widetilde{D}_l \phi - \tfrac{1}{8}\, \widetilde{R}\, \phi + \tfrac{1}{8}\, \widetilde{K}_{ij} \widetilde{K}^{ij}\, \phi^{-7} - \tfrac{1}{12}\, (K^l{}_l)^2\, \phi^5 = 0$$

  where $\widetilde{D}_l$, $\widetilde{R}$, ... $\widetilde{h}_{ij}$

- $\widetilde{K}_{ij} = \widetilde{K}^{[L]}_{ij} + \widetilde{K}^{[TT]}_{ij}$, where $\widetilde{K}^{[L]}_{ij} = \widetilde{D}_i X_j + \widetilde{D}_j X_i - \tfrac{2}{3}\, \widetilde{h}_{ij} \widetilde{D}^l X_l$

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Impressive mathematical developments since 1944 but ...

- either "constancy" of $K^l{}_l$ or "smallness" of the TT part of $\widetilde{K}_{ij}$ is required
- it is highly implicit due to its elliptic character and the replacements $h_{ij} = \phi^4\,\widetilde{h}_{ij}$ and $K_{ij} = \frac{1}{3}\,\phi^4\,\widetilde{h}_{ij}\,K^l{}_l + \phi^{-2}\,\widetilde{K}_{ij}$ $\Longrightarrow$
  - no direct control on the physical parameters of the initial data specifications
- **boundary conditions:**
  - are known to influence solutions everywhere in their domains
  - the inner boundary conditions—they are applied with excision in the black hole interior—cannot simply be supported by intuition (**trumpet data ...**)
  - Bowen-York type initial data: $\widetilde{h}_{ij}$ is flat $\widetilde{h}_{ij} = \delta_{ij}$ and $K^l{}_l = 0$

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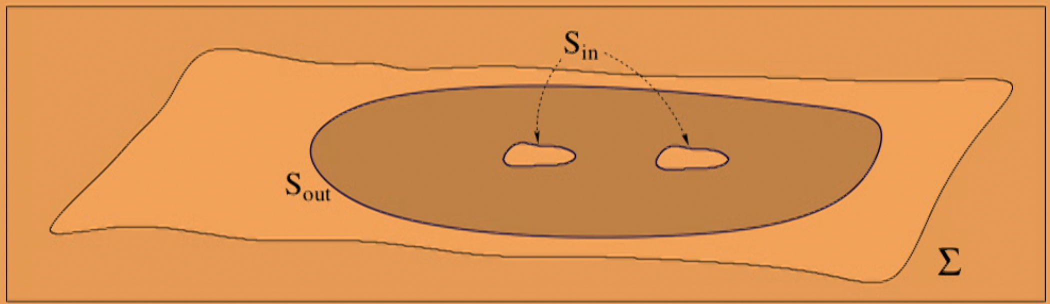

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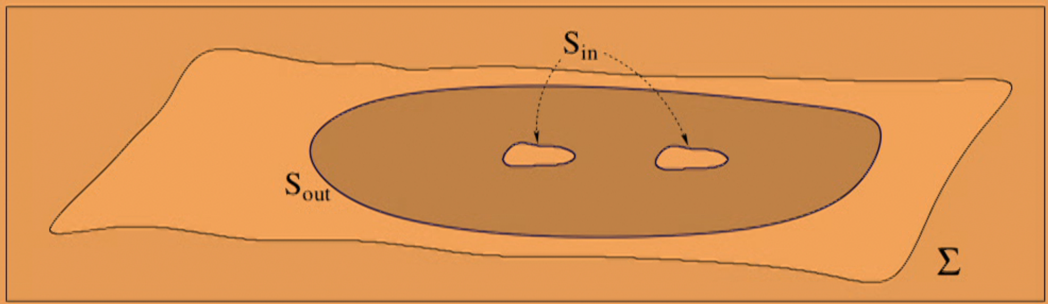

New variables by applying $2+1$ decompositions:

Splitting of the metric h_{ij}:

assume: $\Sigma \approx \mathbb{R} \times \mathscr{S}$

Σ is smoothly foliated by a one-parameter family of two-surfaces $\mathscr{S}_\rho$:
$\rho = const$ level surfaces of a smooth real function $\rho : \Sigma \to \mathbb{R}$ with $\partial_i \rho \neq 0$

$$\Longrightarrow \quad \widehat{n}_i = \widehat{N}\, \partial_i \rho \;\ldots\; \& \ldots\; h^{ij} \longrightarrow \widehat{n}^i = h^{ij}\widehat{n}_j \longrightarrow \widehat{\gamma}^i{}_j = \delta^i{}_j - \widehat{n}^i \widehat{n}_j$$

- choose ρ^i to be a vector field on Σ: the integral curves... & $\rho^i \partial_i \rho = 1$
- 'lapse' and 'shift' of ρ^i

$$\rho^i = \widehat{N}\,\widehat{n}^i + \widehat{N}^i\,, \quad \text{where} \quad \widehat{N} = \rho^j \widehat{n}_j \quad \text{and} \quad \widehat{N}^i = \widehat{\gamma}^i{}_j\,\rho^j$$

- induced metric, extrinsic curvature and acceleration of the $\mathscr{S}_\rho$ level surfaces:

$$\widehat{\gamma}_{ij} = \widehat{\gamma}^k{}_i\,\widehat{\gamma}^l{}_j\,h_{kl} \qquad \widehat{K}_{ij} = \tfrac{1}{2}\mathscr{L}_{\widehat{n}}\widehat{\gamma}_{ij} \qquad \dot{\widehat{n}}_i := \widehat{n}^e \nabla_e \widehat{n}_i = -\widehat{D}_i \ln \widehat{N}$$

- the metric h_{ij} can then be given as

$$h_{ij} = \widehat{\gamma}_{ij} + \widehat{n}_i \widehat{n}_j \quad \Longleftrightarrow \quad \{\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}\}$$

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$2+1$ decompositions:

Splitting of the symmetric tensor field K_{ij}:

- $$K_{ij} = \boldsymbol{\kappa}\, \widehat{n}_i \widehat{n}_j + [\widehat{n}_i\, \mathbf{k}_j + \widehat{n}_j\, \mathbf{k}_i] + \mathbf{K}_{ij}$$

  where

  $$\boldsymbol{\kappa} = \widehat{n}^k \widehat{n}^l\, K_{kl}\,, \quad \mathbf{k}_i = \widehat{\gamma}^k{}_i \widehat{n}^l\, K_{kl} \quad \text{and} \quad \mathbf{K}_{ij} = \widehat{\gamma}^k{}_i \widehat{\gamma}^l{}_j\, K_{kl}$$

- the **trace** and **trace free** parts of $\mathbf{K}_{ij}$

  $$\mathbf{K}^l{}_l = \widehat{\gamma}^{kl}\, \mathbf{K}_{kl} \quad \text{and} \quad \mathring{\mathbf{K}}_{ij} = \mathbf{K}_{ij} - \tfrac{1}{2}\, \widehat{\gamma}_{ij} \mathbf{K}^l{}_l$$

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The new variables:

- $$(h_{ij}, K_{ij}) \quad \Longleftrightarrow \quad (\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$$

- these variables retain the physically distinguished nature of h_{ij} and K_{ij}

The parabolic-hyperbolic form of the constraints:

An evolutionary system for the constrained fields $\widehat{N}, \mathbf{k}_i$ and $\mathbf{K}^l{}_l$:

$$\overset{\star}{K}\,[\,(\partial_\rho \widehat{N}) - \widehat{N}^l(\widehat{D}_l\widehat{N})\,] - \widehat{N}^2(\widehat{D}^l\widehat{D}_l\widehat{N}) - \mathcal{A}\,\widehat{N} - \mathcal{B}\,\widehat{N}^3 = 0$$

$$\mathscr{L}_{\widehat{n}}\mathbf{k}_i - \tfrac{1}{2}\,\widehat{D}_i(\mathbf{K}^l{}_l) - \widehat{D}_i\boldsymbol{\kappa} + \widehat{D}^l\overset{\circ}{\mathbf{K}}_{li} + \widehat{N}\overset{\star}{K}\,\mathbf{k}_i + [\,\boldsymbol{\kappa} - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,]\,\dot{\widehat{n}}_i - \dot{\widehat{n}}^l\,\overset{\circ}{\mathbf{K}}_{li} = 0$$

$$\mathscr{L}_{\widehat{n}}(\mathbf{K}^l{}_l) - \widehat{D}^l\mathbf{k}_l - \widehat{N}\overset{\star}{K}\,[\,\boldsymbol{\kappa} - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,] + \widehat{N}\,\overset{\circ}{\mathbf{K}}_{kl}\overset{\star}{K}{}^{kl} + 2\,\dot{\widehat{n}}^l\,\mathbf{k}_l = 0\,,$$

where $\widehat{D}_i$ denotes the covariant derivative operator associated with $\widehat{\gamma}_{ij}$

$$\overset{\star}{K} = \tfrac{1}{2}\,\widehat{\gamma}^{ij}\,\mathscr{L}_\rho\widehat{\gamma}_{ij} - \widehat{D}_j\widehat{N}^j$$

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$$\mathcal{A} = (\partial_\rho\overset{\star}{K}) - \widehat{N}^l(\widehat{D}_l\overset{\star}{K}) + \tfrac{1}{2}[\,\overset{\star}{K}{}^2 + \overset{\star}{K}_{kl}\overset{\star}{K}{}^{kl}\,]$$

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- no restriction on $\widehat{N}^i, \widehat{\gamma}_{ij}, \boldsymbol{\kappa}$ and $\mathring{\mathbf{K}}_{ij}$ $\Longrightarrow$ they are freely specifiable on Σ
- the parabolic equation is uniformly parabolic in those subregions of Σ, where $\overset{\star}{K}$ is either positive or negative
- $\overset{\star}{K}$ depends exclusively on the freely specifiable fields $\widehat{\gamma}_{ij}$ and $\widehat{N}^i$ $\Longrightarrow$ its sign can be tailored according to the desire of the investigated problem
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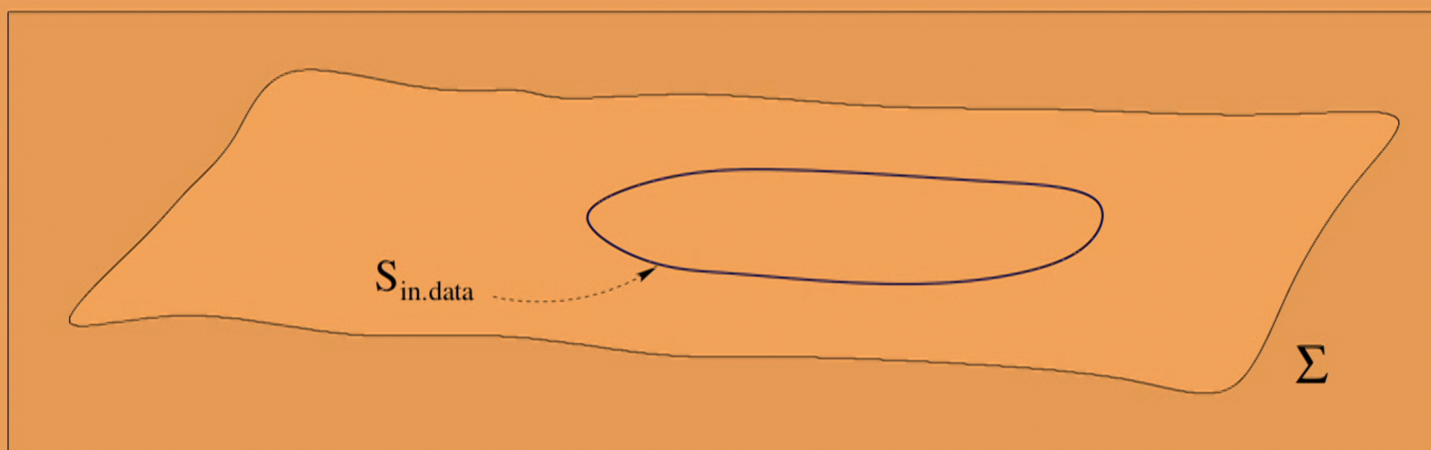
- the combined evolutionary system is (locally) well-posed
  - if suitable initial values for the constrained fields $\boxed{\widehat{N}, \mathbf{k}_i \text{ and } \mathbf{K}^l{}_l}$ are given, on some level surface $\mathscr{S}_0$ in Σ, then, in the domain of dependence of $\mathscr{S}_0$, unique solution exists to the evolutionary system
  - the fields $\boxed{h_{ij} \text{ and } K_{ij}}$ that can be reconstructed from the freely specifiable and constrained variables do satisfy the Hamiltonian and momentum constraints

Solving the constraints:

- (h_{ij}, K_{ij}) represented by the variables $(\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$
- the constraints comprise a **parabolic-hyperbolic** system for $(\widehat{N}, \mathbf{k}_i, \mathbf{K}^l{}_l)$
  - with freely specifiable variables on Σ and on $\mathscr{S}_{\rm in.data}$:

$$(\widehat{N}|_{\mathscr{S}_{\rm in.data}}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i|_{\mathscr{S}_{\rm in.data}}, \mathbf{K}^l{}_l|_{\mathscr{S}_{\rm in.data}}, \mathring{\mathbf{K}}_{ij})$$

  - a fixed $(+/-)$ sign of $\overset{\star}{K} = \frac{1}{2}\,\widehat{\gamma}^{ij}\,\mathscr{L}_\rho \widehat{\gamma}_{ij} - \widehat{D}_j \widehat{N}^j$ can be guaranteed



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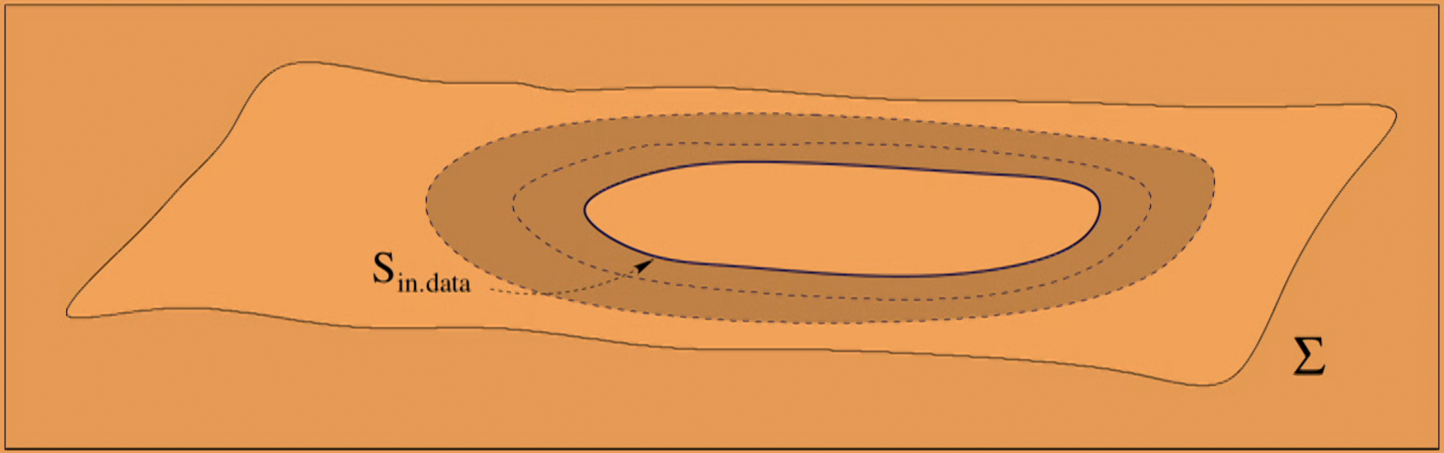

The Kerr black hole:

In Kerr-Schild form:

- $$g_{\alpha\beta} = \eta_{\alpha\beta} + 2H\ell_\alpha\ell_\beta$$
- inertial coordinates (t, x, y, z) adapted to the Minkowski background $\eta_{\alpha\beta}$

$$H = \frac{r^3 M}{r^4 + a^2 z^2}$$

$$\ell_\alpha = \left(1, \frac{r\,x + a\,y}{r^2 + a^2}, \frac{r\,y - a\,x}{r^2 + a^2}, \frac{z}{r}\right)$$

- the Boyer-Lindquist radial coordinate r is related to the spatial part of the inertial coordinates as

$$r^4 - (x^2 + y^2 + z^2 - a^2)\,r^2 - a^2\,z^2 = 0 \qquad \frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1$$

- the $r = const$ surfaces are "ellipsoids"
  - degenerate to a disk $x^2 + y^2 \leq a^2$ & $z = 0$ possessing the "ring singularity" (given as $x^2 + y^2 = a^2$ & $z = 0$) at its edge

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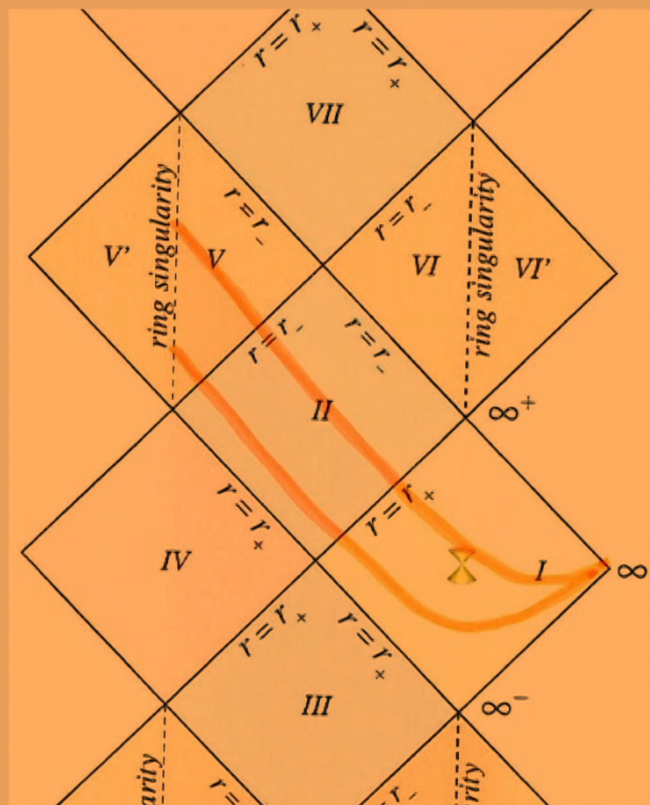
$t = const$ slices in Kerr spacetime:



the $\theta = \frac{\pi}{2}$ section

$$\Sigma \approx \mathbb{R}^3 \setminus \{\text{“ring singularity”}\}$$

Generic Kerr-Schild black holes:

- the Kerr-Schild metrics are form-invariant under Lorentz transformations
  - if a Lorentz transformation $x'^{\alpha} = \Lambda^{\alpha}{}_{\beta}\, x^{\beta}$ is performed
  - the metric retains its distinguished Kerr-Schild form

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Superposed Kerr-Schild black holes:

We are looking for suitable free(ly specifiable) data:

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  - $H^{[n]}$ and $\ell_\alpha{}^{[n]}$ correspond to the Kerr-Schild data for individual black holes
- (*) does not satisfy Einstein's equations
- good approximation close to the individual black holes
- Einstein tensor falls off at the rate $\mathcal{O}(|\vec{x}|^{-4})$, where $|\vec{x}| = \sqrt{x^2 + y^2 + z^2}$

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Choice for the free data: Take a foliation of the $t_{Mink} = 0$ time-slice of (*)

- $\widehat{N}^i, \widehat{\gamma}_{ij}, \boldsymbol{\kappa}$ and $\mathring{\mathbf{K}}_{ij}$ as if (*) solved the Einstein equations
- $\widehat{N}, \mathbf{K}^l{}_l$ and $\mathbf{k}_i$ on some level surface $\mathscr{S}_0$ in Σ deduced from (*) [only on $\mathscr{S}_0$!]
- **initial data surface**: the complementer of the two "ring" singularities in $t_{Mink} = 0$ hypersurface: $\Sigma \approx \mathbb{R}^3 \setminus \{ring_1 \text{ and } ring_2\}$

The initial-boundary value problem:

the $\mathscr{S}_\rho$ surfaces have tacitly been assumed to be compact without boundary:

- in numerical approaches Σ is chosen to be a large but bounded subset of $\mathbb{R}^3$
- the product structure $\Sigma \approx \mathbb{R} \times \mathscr{S}$ can be guaranteed by choosing the $\mathscr{S}_\rho$ leaves to be diffeomorphic to a closed disk in $\mathbb{R}^2$

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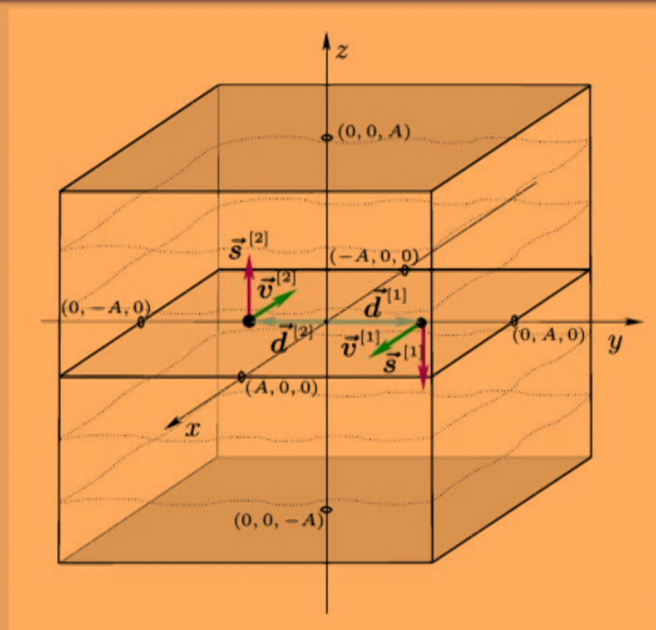

- for large enough value of A ...
- boundary of Σ:
  six squares each with edges of size $2A$
- the black holes are assumed to be located on the $z = 0$ plane
- speeds are parallel, spins are orthogonal to the $z = 0$ plane
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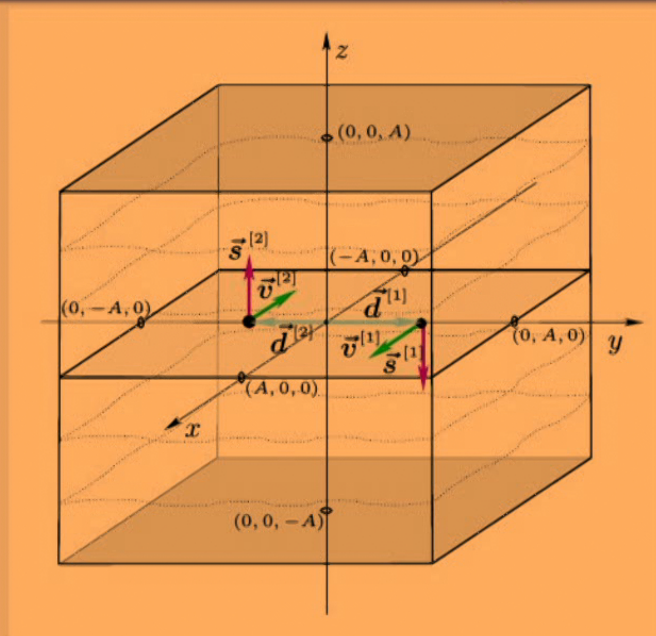

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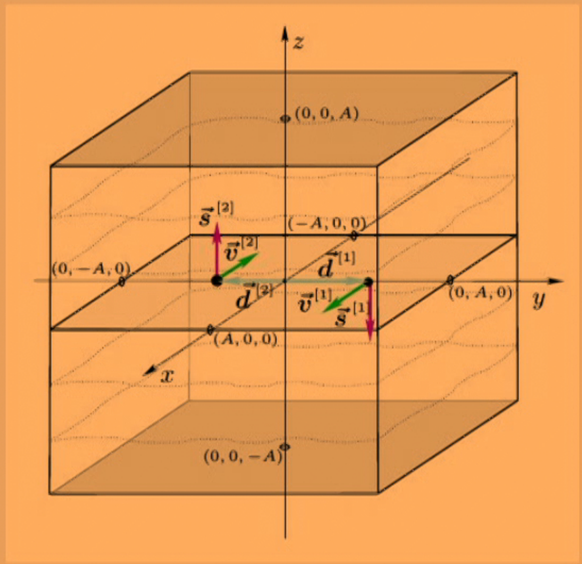
The critical coefficient $\overset{\star}{K}$:



- the sign of $\overset{\star}{K}$ decides whether the parabolic-hyperbolic system evolves in the positive or negative ρ-direction

$$\overset{\star}{K}\,[\,(\partial_\rho \widehat{N}) - \widehat{N}^l(\widehat{D}_l \widehat{N})\,] = \widehat{N}^2(\widehat{D}^l \widehat{D}_l \widehat{N}) + \mathcal{A}\,\widehat{N} + \mathcal{B}\,\widehat{N}^3$$

  - it propagates aligned ρ^i for positive $\overset{\star}{K}$, while anti-aligned for negative $\overset{\star}{K}$

- restrict considerations to a binary BH system arranged as indicated on the figure

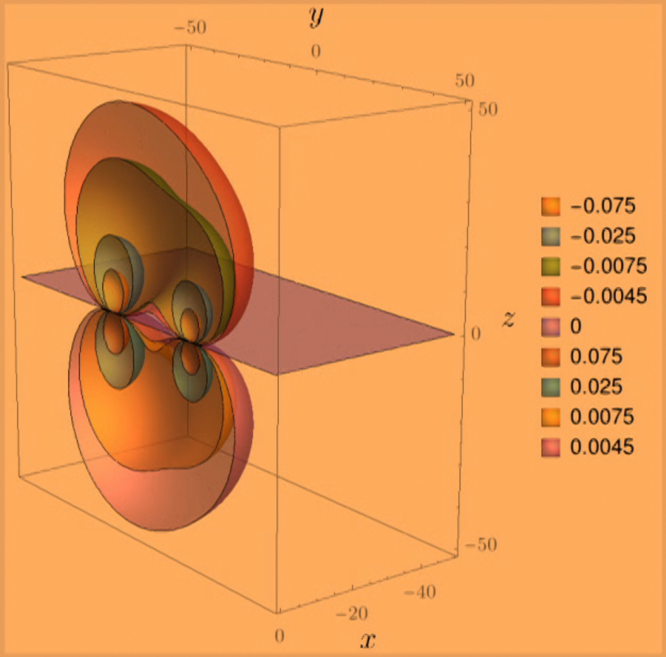
Splitting the boundary:

The princical coefficient $\overset{\star}{K}$:

- $\overset{\star}{K}$ can be given as the product of a strictly negative function and the z-coordinate
- $\overset{\star}{K}$ is positive below the $z = 0$ plane, while it is negative above that plane

$\overset{\star}{K} = const$ level surfaces:

$x < 0, \ 2A = 100$
$M^{[1]} = 1, \ \vec{d}^{[1]} = 20\,\vec{e}_y$
$\vec{v}^{[1]} = 0.5\,\vec{e}_x, \ a^{[1]} = 0.6$
$M^{[2]} = 2, \ \vec{d}^{[2]} = -10\,\vec{e}_y$
$\vec{v}^{[2]} = -0.25\,\vec{e}_x, \ a^{[2]} = -0.8$



Solving the initial-boundary value problem:

The parabolic-hyperbolic system:

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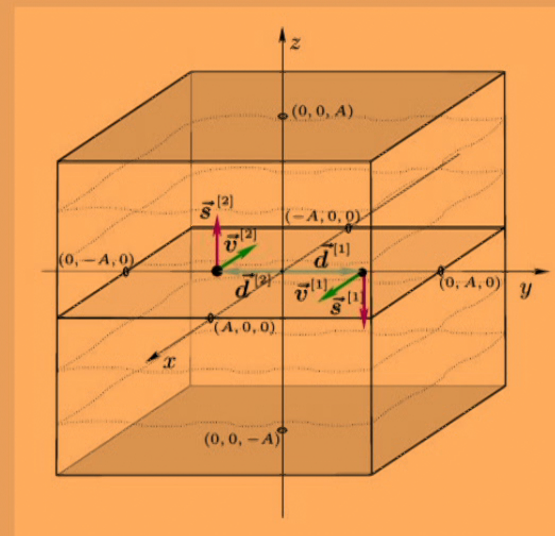
- solved by propagating, along the z-streamlines, initial values specified on the horizontal $z = \pm A$ squares
- boundary values are to be given on the four vertical sides of the cube



- $\widehat{N}, \mathbf{K}^l{}_l$ and $\mathbf{k}_i$ are developed on Σ^+ and Σ^- separately
  - global existence and matching of these solutions at their common Cauchy horizon—at the $z = 0$ plane—is of fundamental importance
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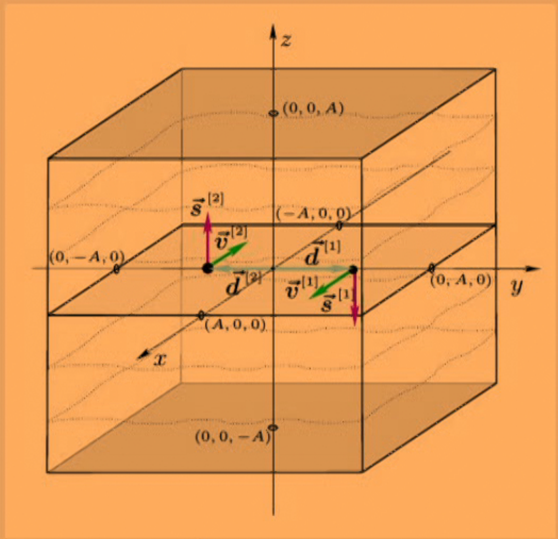

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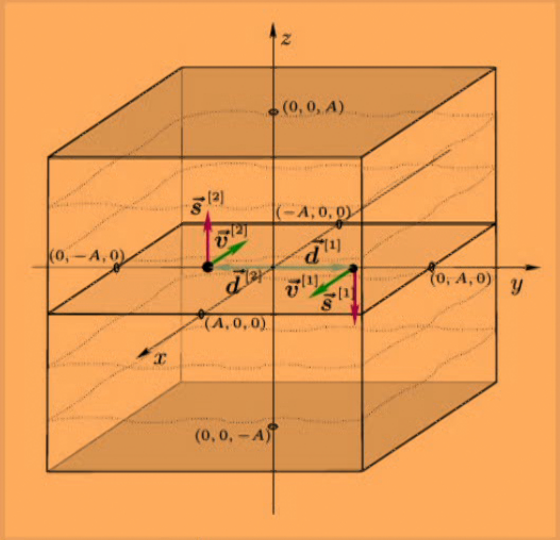

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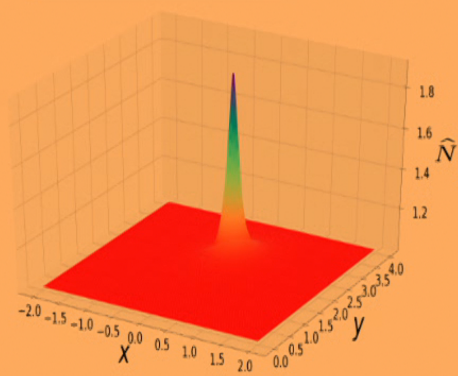
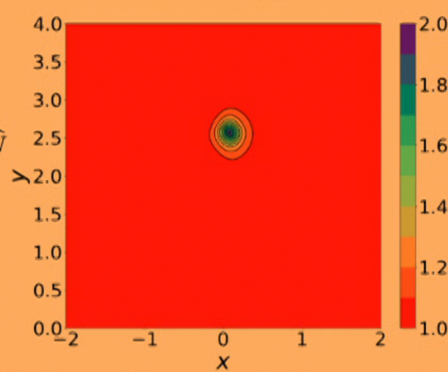
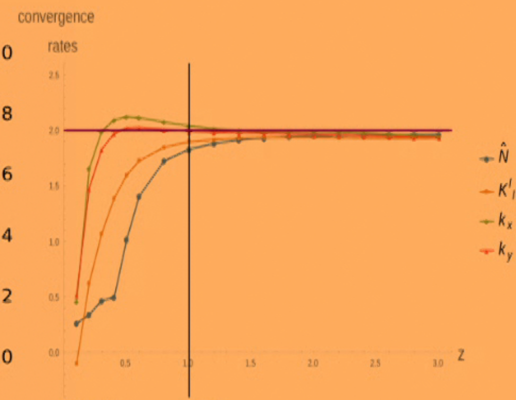
Exact Kerr: with input parameters $M = 0.5, a = 0.3M, v = 0.6, d = 5M$

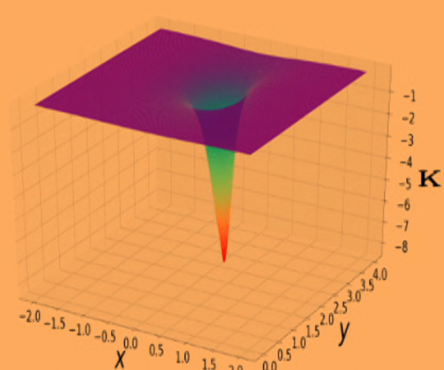
Exact Kerr

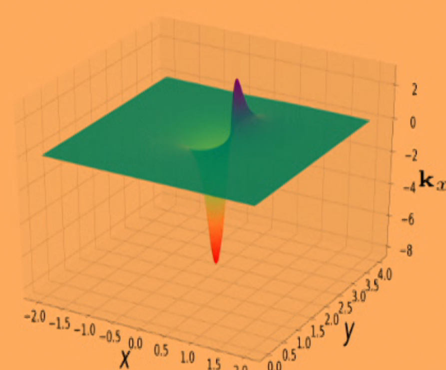


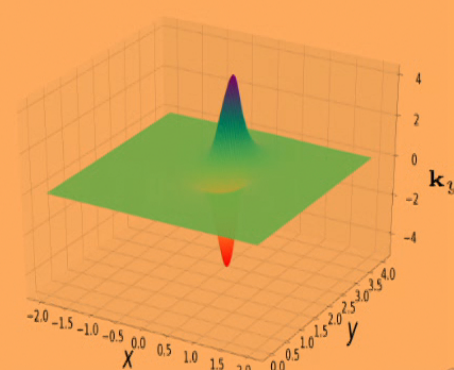




$$h_{ij} = \widehat{\gamma}_{ij} + \widehat{n}_i \widehat{n}_j, \quad K_{ij} = \boldsymbol{\kappa}\, \widehat{n}_i \widehat{n}_j + \widehat{n}_i \mathbf{k}_j + \widehat{n}_j \mathbf{k}_i + \mathbf{K}_{ij}$$

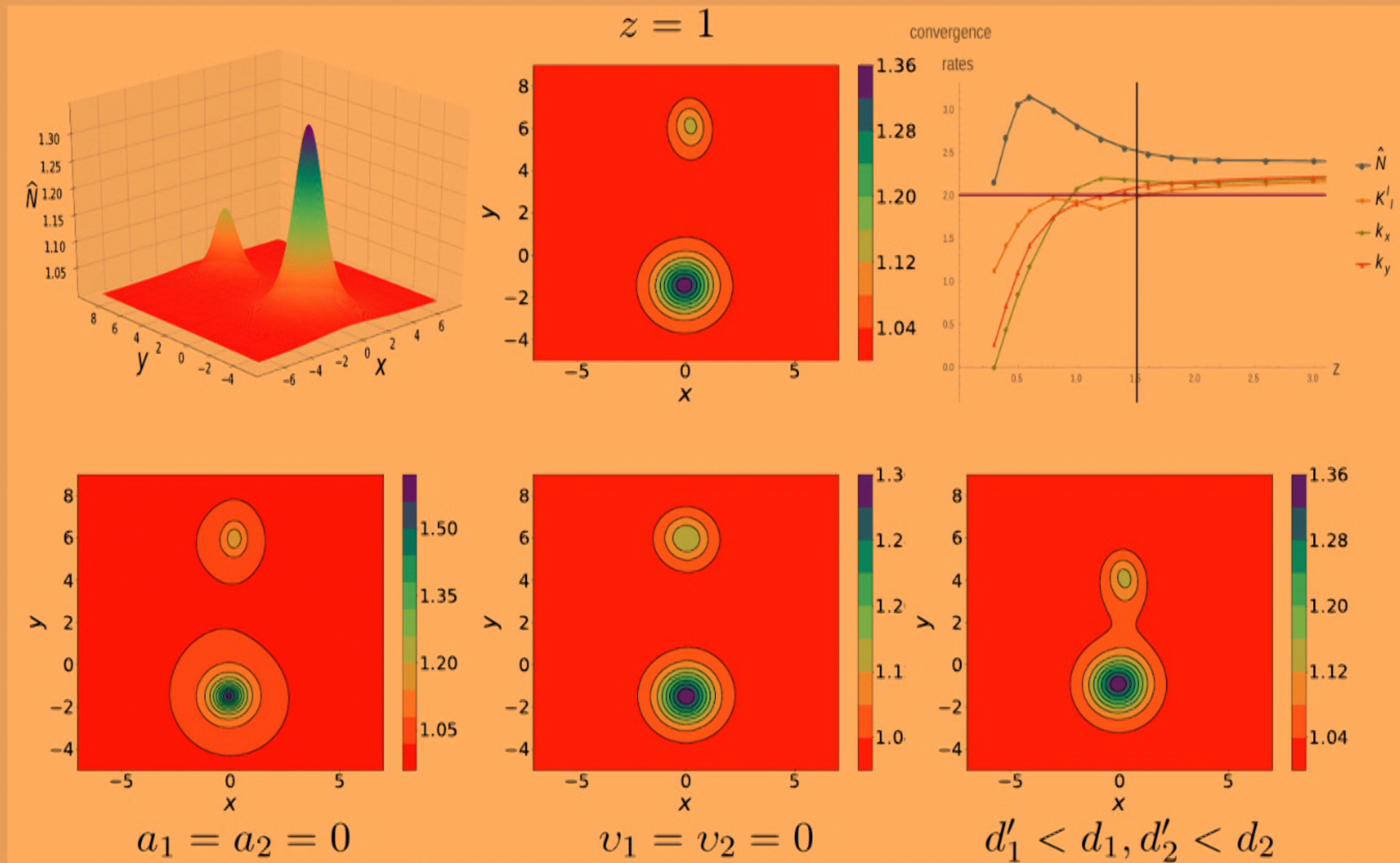




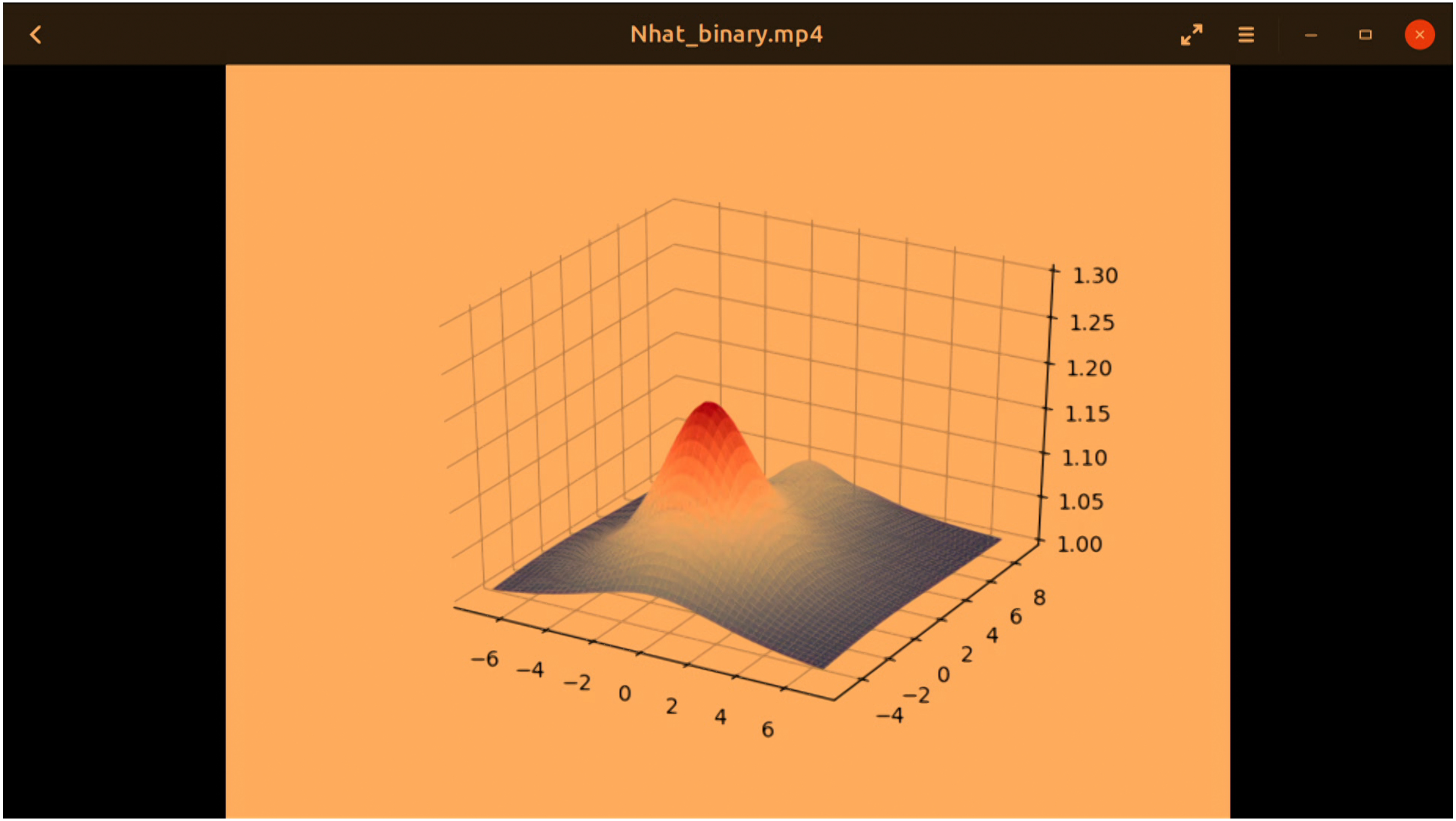


A binary system: $M_1 = 0.25, a_1 = 0.7, v_1 = 0.68, d_1 = 6$

$M_2 = 3M_1, a_2 = -0.9, v_2 = v_1/4, d_2 = d_1/4$



video: the evolution of $\widehat{N}$


Nhat_binary.mp4
1.30
1.25
1.20
1.15
1.10
1.05
1.00
−6 −4 −2 0 2 4 6
−4 −2 0 2 4 6 8

Input parameters and global ADM charges:

- **Input parameters:** the rest masses $M^{[n]}$, displacements $\vec{d}^{\,[n]}$, speeds $\vec{v}^{\,[n]}$ and spins $M^{[n]}a^{[n]}\vec{s}_\circ^{[n]}$ of the involved black holes
  - essentially the same as used in post-Newtonian description of binaries **!!!**
- **Global ADM charges:** in terms of the input parameters
  - though (*) does not satisfy Einstein's equations it is asymptotically flat
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  - the ADM quantities are linear in deviation from flat Euclidean space at infinity

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   - superposing Kerr-Schild black holes
2. the parabolic-hyperbolic equations solved as an initial-boundary value problem
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