Title: Assessment of the particle standard model: An alternative formulation.

Speakers: John Moffat

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Abstract: An assessment of the particle standard model and an alternative formulation of the model are presented. An ultraviolet complete particle model is constructed for the observed particles of the standard model. The quantum field theory associates infinite derivative entire functions with propagators and vertices, which make perturbative quantum loops finite and maintain PoincarÃ© invariance and unitarity of the model. The electroweak model SU(2) X U(1) group is treated as a broken symmetry group with non-vanishing experimentally determined boson and fermion masses. A spontaneous symmetry breaking of the vacuum by a scalar Higgs field is not invoked to restore boson and fermion masses to the initially massless SU(2) X U(1) Lagrangian of the standard model. The hierarchy naturalness problem of the Higgs boson mass is resolved and the model contains only experimentally observed masses and coupling constants. The renormalization group features of the scalar Higgs field are investigated and the model is shown to be asymptotically safe and free from the triviality problem. It is demonstrated that the finite nonlocal quantum field theory satisfies microcausality. The model can predict a stable vacuum evolution. Experimental tests to distinguish the standard model from the alternative model are proposed. The finite quantum field theory can be extended to quantum gravity.

Assessment of the Particle Standard Model: A Reformulation.

John Moffat
Perimeter Institute
Waterloo, Ontario, Canada

Talk given at Perimeter Institute, March 4, 2019

J. W. Moffat, arXiv:1812.01986

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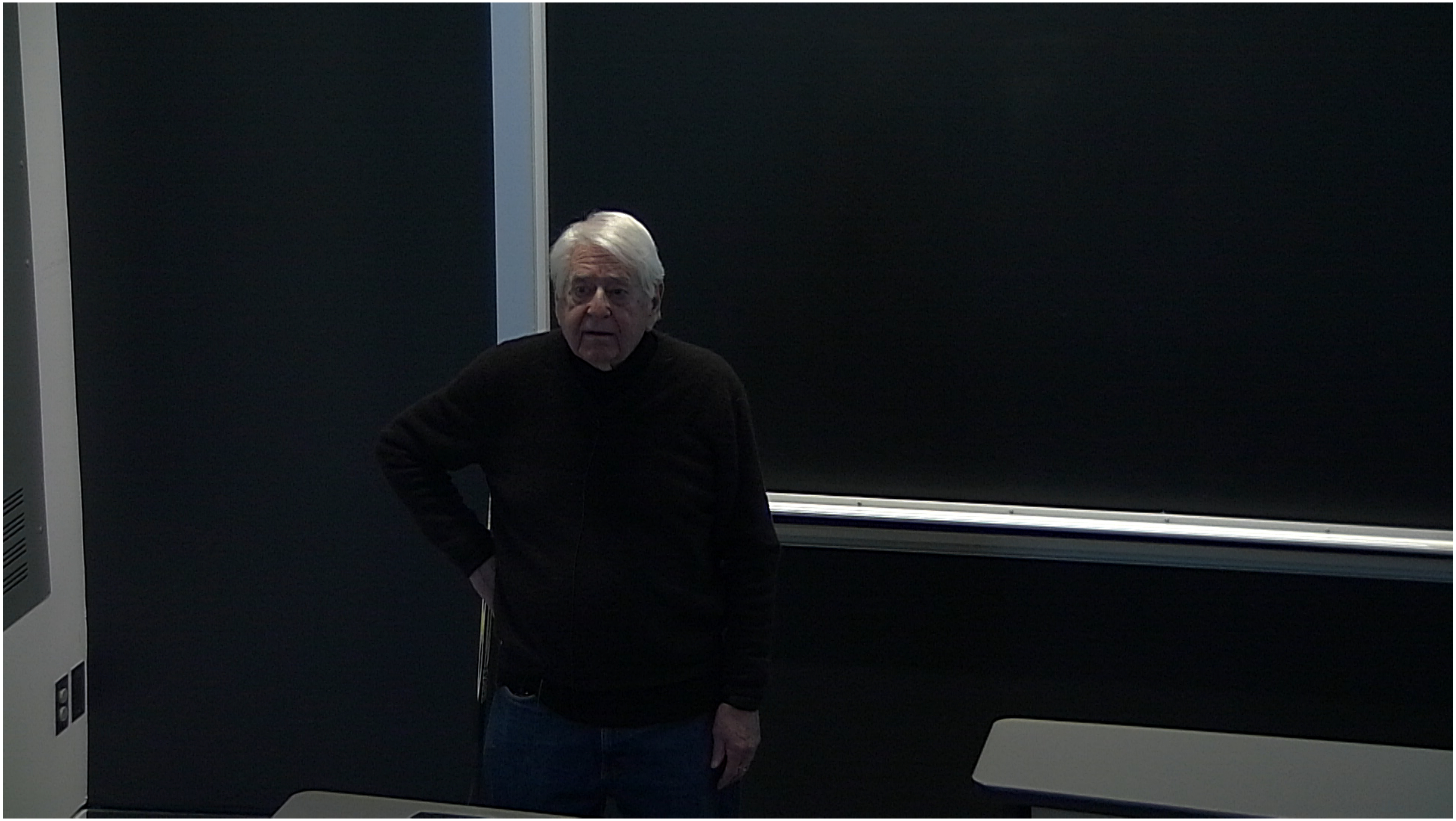

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“One of the amazing characteristics of nature is the variety of interpretational schemes. I always found that mysterious, and I do not know the reason why it is that the correct laws of physics are expressible in such a tremendous variety of ways. They seem to be able to get through several wickets at once.”

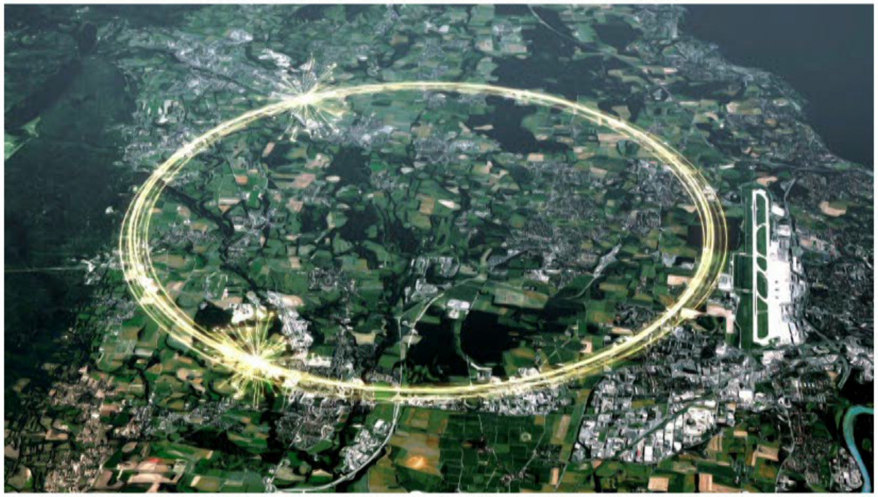
Richard Feynman, Cornell University lectures, 1964.

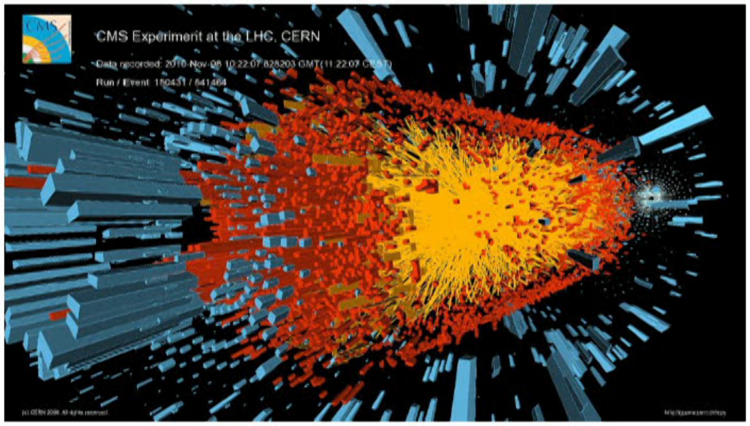
“It is not always so that theories which are equivalent are equally good, because one of them may be more suitable than the other for future developments.”

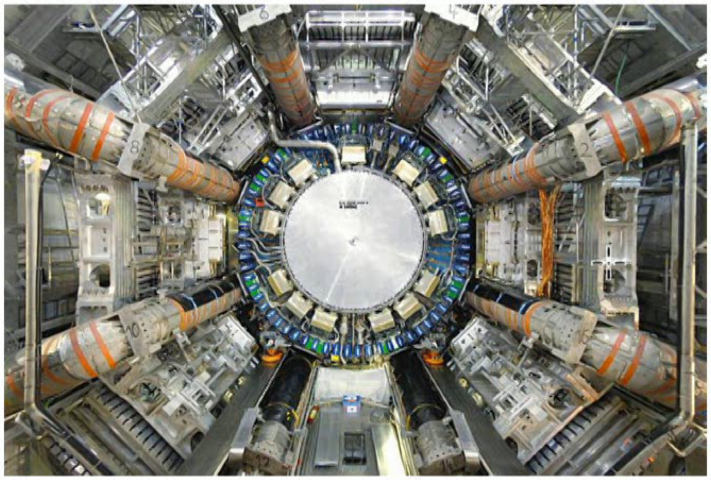
Paul Dirac.

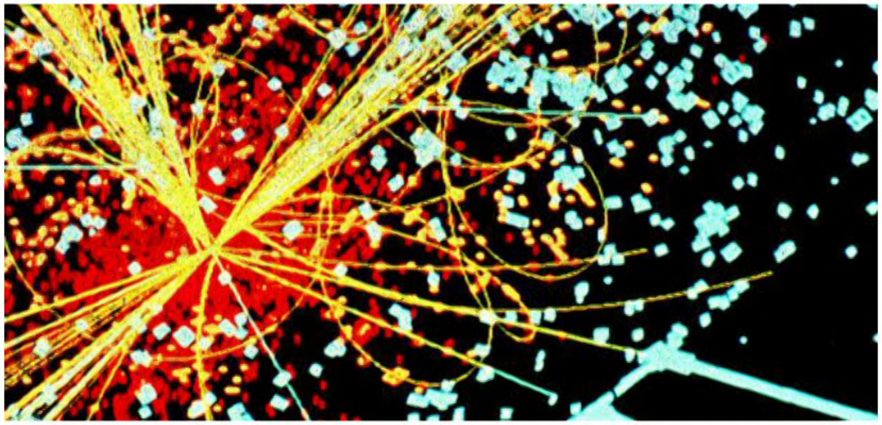
“Since the consideration of point interactions lead at once to infinities, it appears reasonable to regard it as a limit of some `smoothed out’ interaction with finite radius, as this radius decreases to zero.” ... “An approach of this kind means, essentially, the rejection of any unjustified consideration of point interaction by means of a δ -function. “

Lev Landau, 1955.









2019-03-03

The Large Hadron Collider at CERN

1. The Standard Model

- The particle content of the standard model consists of the 18 particles (group SU(3) X SU(2) X U(1)):

6 quarks: u,d,s,c,b,t   Quark flavor doublets: u c t   Three generations
d s b

6 leptons: e, μ, τ, ν_μ ν_e ν_τ Lepton flavor doublets: e μ τ Three generations
ν_e ν_μ ν_τ

6 bosons: γ, W, Z, H, g, G (?)

- As of the closing for an upgrade of the LHC, no new particles have been discovered. This includes the lack of discovery of supersymmetric particles and dark matter particles. Moreover, higher spatial dimensions greater than 3 have not been detected at the LHC.

- The lack of discovery of new particles has brought on a potential crisis in high energy physics. It has thwarted attempts to verify many beyond the standard models: supersymmetry, superstrings, Technicolor, Grand Unified Theories, Conformal invariant Higgs particle model. No experimental evidence supports a composite Higgs boson.

2. Why a Reformulation of the Standard Model? (arXiv: 1812.01986)

- In an early paper, I formulated a gauge theory model of the standard model (SM) with a spontaneous symmetry breaking of the vacuum and $SU(2) \times U(1) \rightarrow U(1)_{EM,}$ Including a Higgs boson field, based on a finite quantum field theory (QFT) (JWM, Phys. Rev. D 1990).

- We will address the following question: If at the beginning of the development of the standard model (SM), the QFT had been a finite theory free of divergencies, how would the SM have been formulated?

- The 12 quarks and leptons, the weak interaction $W^{\pm}, Z^0$ vector bosons and the scalar Higgs boson would be discovered and complete the detected particle content of the SM. However, guaranteeing the renormalizability of the electroweak (EW) sector would not have been a problem needing resolution, because a perturbatively finite QFT would be the foundation of the theory.

- Massive charged W bosons produce non-renormalizable loop contributions:

$$\text{Amplitude} = \int d^4p(\text{propagators})\cdots$$

- The massive W propagator is of the form:

$$iD_V^{\mu\nu}(p^2) = \frac{i\left(-\eta^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}\right)}{p^2 - M^2}$$

- For large momentum p it is

$$iD_V^{\mu\nu}(p^2) \sim \frac{ip^\mu p^\nu}{p^2 M^2}$$

- Power counting shows that the loop integral diverge for large loop momenta. Introducing a cutoff Λ_C violated Lorentz invariance, gauge invariance and unitarity. The finite QFT theory guarantees that the loop momentum integrals are finite, while maintaining Lorentz invariance and unitarity with a W boson mass.

- The motivation of the SM is through symmetry. The renormalizability of the model requires a gauge invariance symmetry. This succeeds for QCD (SU(3)) and QED (U(1)), for the eight colored gluons and the photon are massless guaranteeing gauge invariance.
- The SM is successful in describing experimental data. The discovery of the Higgs boson at m_H = 125 GeV has supported the standard scenario of the spontaneous symmetry breaking of the electroweak group $SU(2)_L \times U(1)_Y$ through a non-zero vacuum expectation value of the complex scalar field.
- The scalar field potential in the SM has the Ginsberg-Landau form:

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

- The Higgs boson mass is determined to be

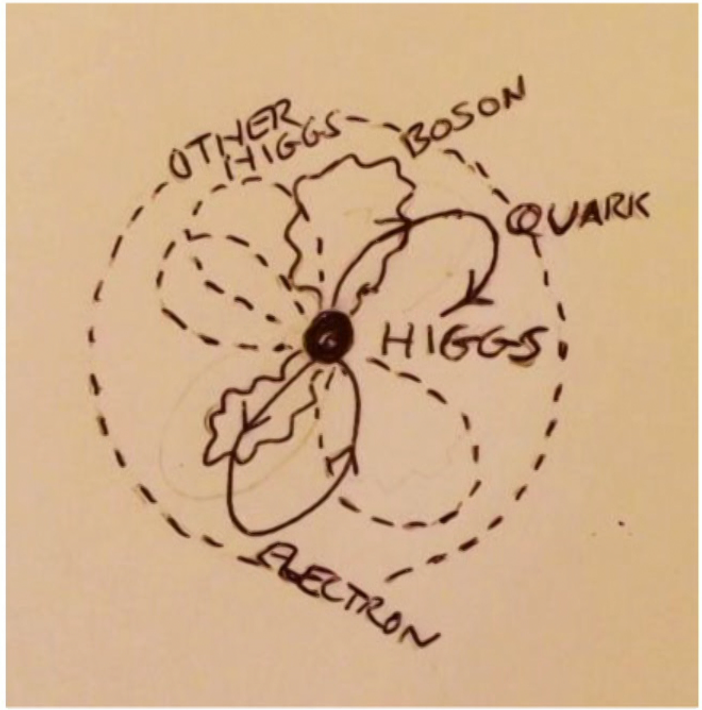
$$m_H^2 = m_{\text{bare}}^2 + \delta m_H^2$$

- The radiative corrections are given by

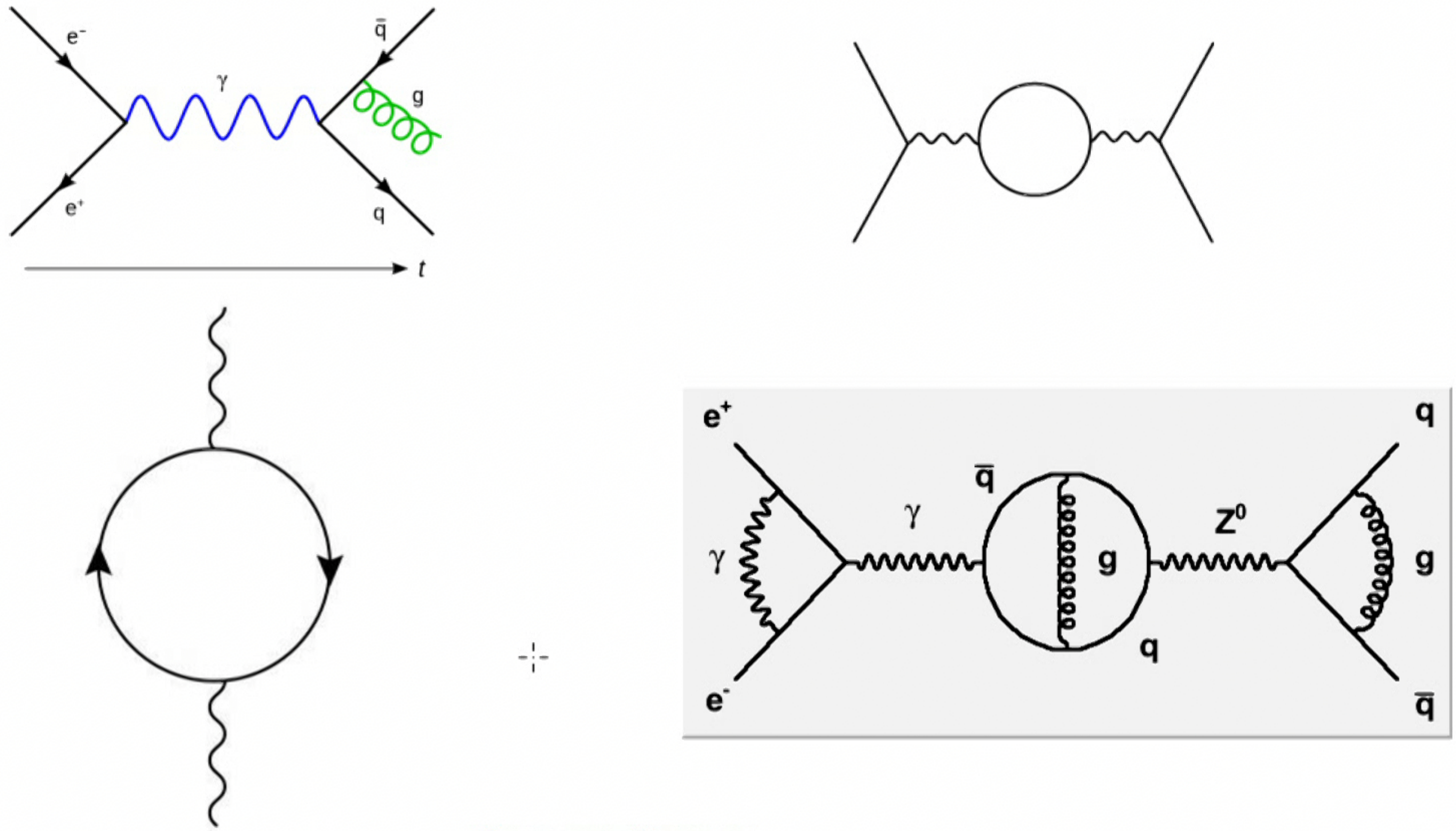
$$\delta m_H^2 = \frac{y_t^2}{16\pi^2}\Lambda_C^2 + \delta\mathcal{O}(m_{\text{weak}}^2)$$

where y_t is the top quark coupling constant or order 1, and the loop is quadratically divergent. The SM is expected to be valid up to the Planck energy $\Lambda_C \sim 10^{19}$ GeV and any new particle above 1 TeV would create a severe fine tuning problem (Weisskopf 1939, Wilson 1971, 't Hooft 1980).

- In the SM the absolute masses of the W and Z bosons, the quarks and leptons and the Higgs boson mass are not determined. The mass of the Higgs boson fails to be restricted to the light mass m_H = 125 GeV, because of the quadratic divergence of the quantum radiative Higgs loop graphs.



Credit: Mathew J. Strassler. The “bare” Higgs particle “dressed” with virtual particles.



Feynman diagrams

- The Higgs mass fine tuning problem could be eliminated if supersymmetric particles, or new particles such as the top quark partners were discovered that can cancel the quadratic radiative corrections of the Higgs loop graphs. The LHC at CERN has not discovered supersymmetric particles or new particles that can cancel the quadratically divergent contributions to the radiative corrections and eliminate the Higgs mass fine tuning problem.

- The SM is based on the assumption that the W and Z boson masses and the quark and lepton masses *are initially zero* allowing for the gauge invariance symmetry of the weak interaction group SU(2) X U(1). This assumption is driven by the need for a renormalizable perturbation theory.

- The *ad hoc* choice of the Scalar potential $V(\phi)$ and the imaginary mass produces spontaneous symmetry breaking of the vacuum and three Goldstone bosons, which are absorbed by the gauge bosons W and Z , leaving the photon massless.

- We present a model which can eliminate the Higgs mass fine tuning, hierarchy problem, and stabilize the vacuum evolution of the Universe. We can incorporate massive neutrinos. The model accepts from the beginning that SU(2) X U(1) is a broken symmetry group with non-zero experimentally measured W, Z , Higgs boson masses and quark and lepton masses. The problem of infinite renormalizability is resolved by regulated propagators for the Feynman loop diagrams of QED, QCD and weak interactions.

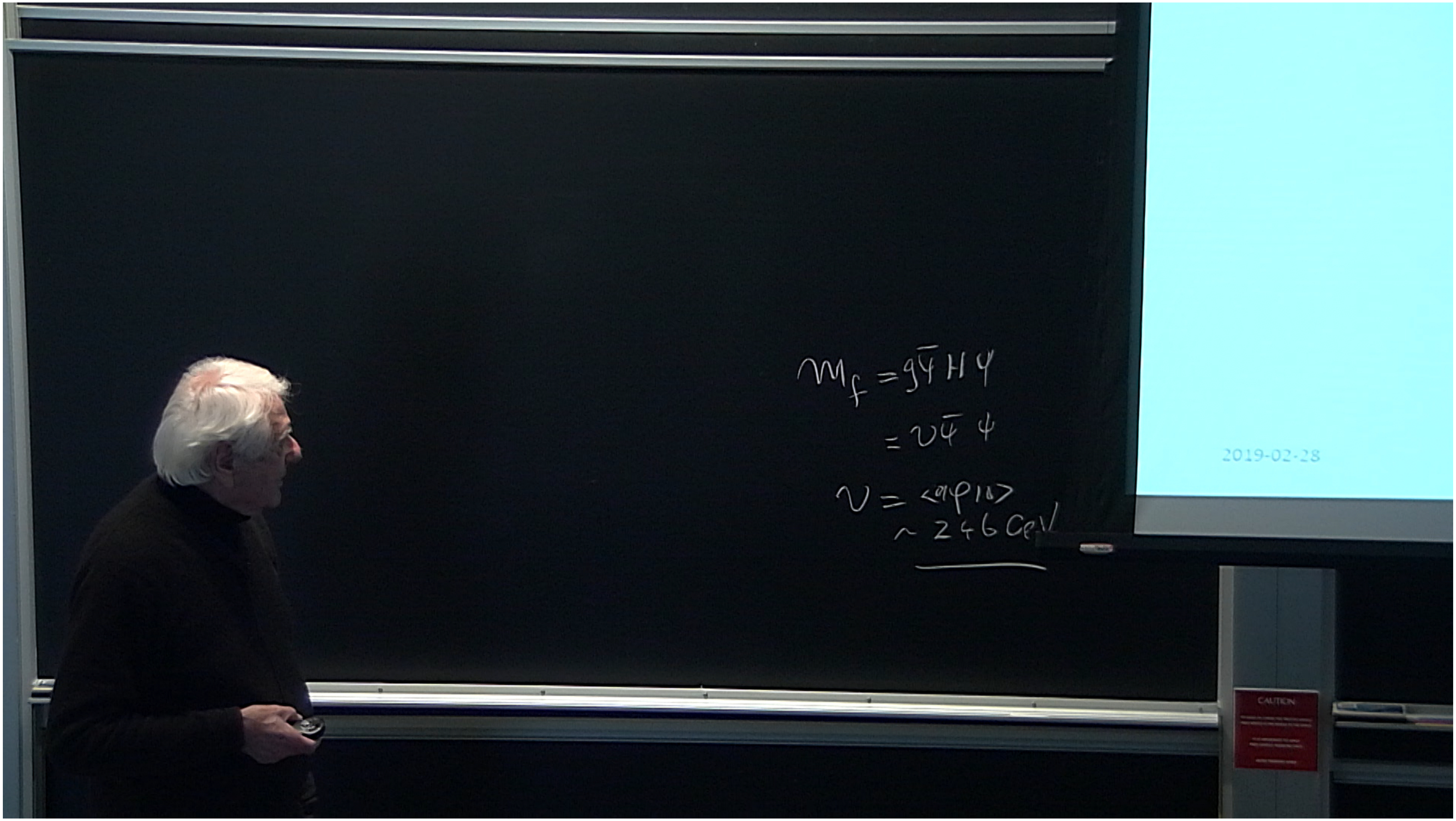
- The propagators are regulated by an infinite derivative entire function $\mathcal{E}(p^2)$, which is analytic and holomorphic in the complex p^2 plane with a pole or essential singularity at $p^2 \rightarrow \infty$. The QFT is finite, unitary and Poincaré invariant to all orders of perturbation theory.

- A violation of perturbative unitarity of scattering amplitudes is avoided by the exchange of the scalar Higgs boson between SM particles. The Higgs mass hierarchy and naturalness problem is resolved by having the Higgs coupling Λ_H energy scale controlling Higgs-fermion and Higgs self-interaction loops satisfy $\Lambda_H \lesssim 1\,\mathrm{TeV}$. This energy condition can be checked by future experiments determining the strength of the radiative couplings of the Higgs particle to the SM particles in loop diagrams. The other particle loop diagrams will be controlled by the energy scale $\Lambda_M > 1\,\mathrm{TeV}$.

3. Finite Quantum Field Theory

The regularized propagator is the Green's function $G(x,x')$ for the Klein-Gordon operator:

$$(\Box_x + m^2)\tilde{\Delta}(x-x') \equiv \mathcal{E}(x-x') = -\frac{1}{4\pi^2\Lambda_x^4}\exp\Big(-(x-x')^2/2\Lambda_x^2\Big)$$


m_f = gψ̄Hψ
= vψ̄ψ
v = <0|φ|0>
~ 246 GeV
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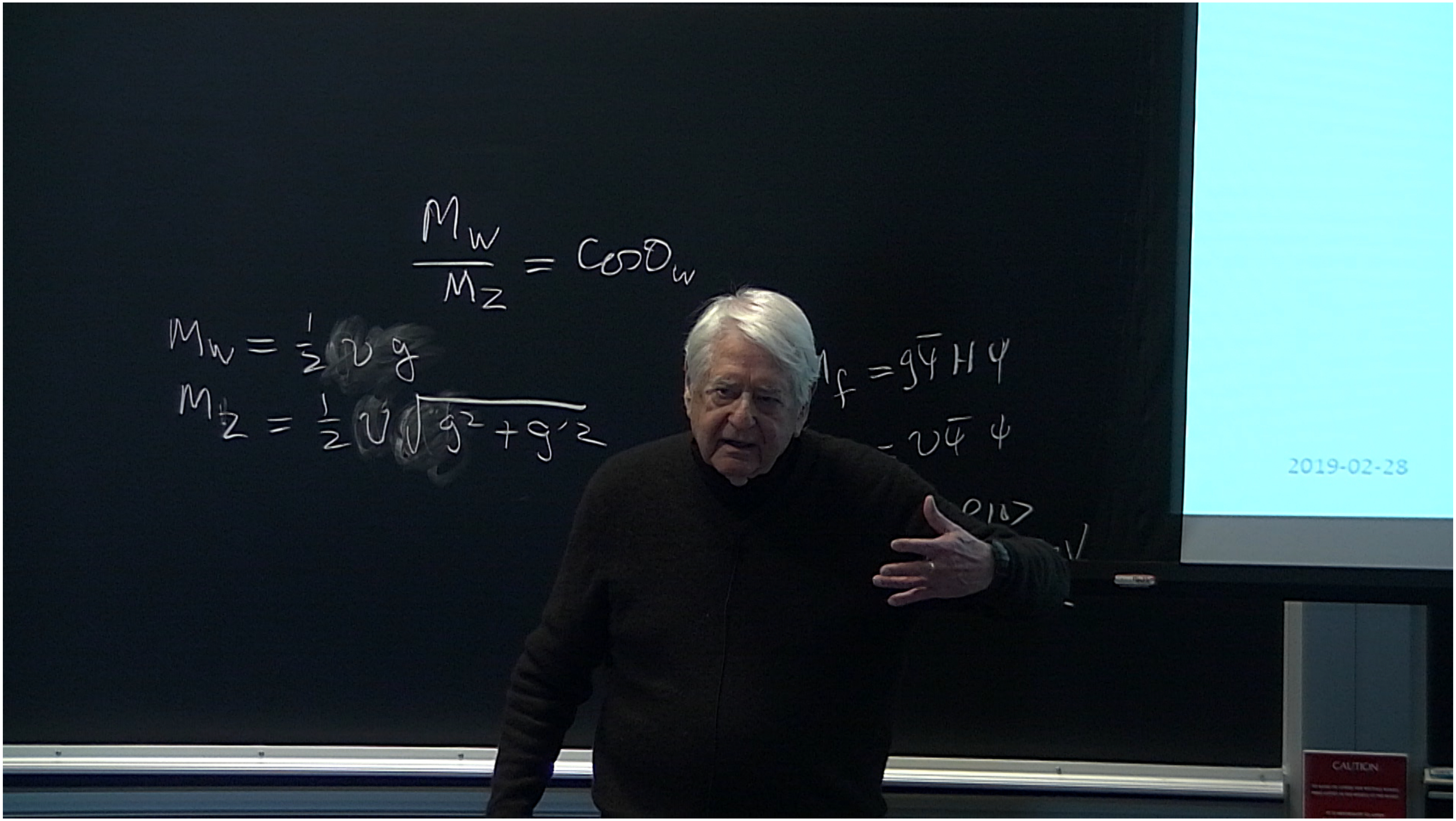

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- In the limit $\Lambda_x \to 0$, we obtain the Local Klein-Gordon equation:

$$(\Box_x + m^2)\Delta(x - x') = -\delta^4(x - x')$$

A Fourier transform yields

$$\mathcal{E}(P) = \exp\left(-P^2/2\Lambda_P^2\right)$$

- The probability distribution yields $\mathcal{P}(p) = |\mathcal{E}(p)|^2$ and for $\Lambda_P \to 0$:

$$\int_{-\infty}^{\infty} d^4P\mathcal{E}(P) \to \int_{-\infty}^{\infty} d^4P\delta^4(P) = 1.$$

- We have for a free particle $p_\mu \to -i\hbar\frac{\partial}{\partial x_\mu}$ and

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$$\exp(\Box/\Lambda_x^2) \to \exp(-p^2/2\Lambda_P^2)$$

$$\frac{M_W}{M_Z} = \cos\theta_W$$

$$M_W = \frac{1}{2} v g$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$m_f = g \bar{\psi} H \psi$$

$$= v \bar{\psi} \psi$$

$$v = \langle 0|\varphi|0\rangle$$

$$\sim 246 \text{ GeV}$$

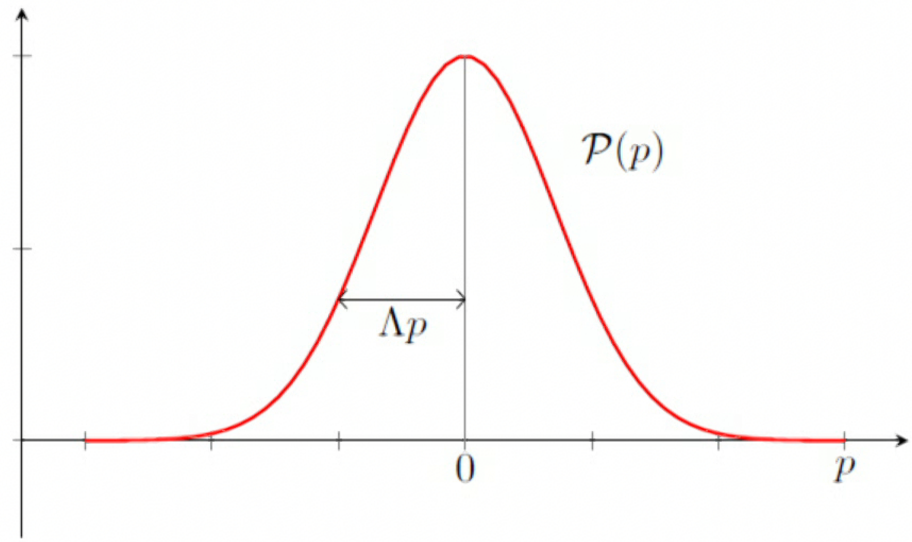


Figure 1: The probability distribution corresponding to the entire function distribution $\mathcal{E}(p)$. The value of p is almost certain to be found within the p range Λ_p. In the limit $\Lambda_p \to 0$ the peak at the origin becomes infinitely high and narrow and the distribution $\mathcal{E}(p)$ becomes a delta function $\delta(p)$.

- The uncertainty Λ_x in position space x, and the uncertainty Λ_p in momentum space can be related by Heisenberg's uncertainty principle:

$$\Lambda_x \Lambda_p \geq \hbar$$

- Let us consider causality in our finite QFT. We define the nonlocal field operator:

$$\tilde{\phi}(x) = \int d^4x' \mathcal{F}(x - x')\phi(x') = \mathcal{F}(\mathcal{E}(x))\phi(x)$$

$\mathcal{F}(x) = \mathcal{F}(\mathcal{E})$ is an entire function distribution operator. We have

$$\langle 0|\tilde{\phi}(x)\tilde{\phi}(x')|0\rangle = \tilde{\Delta}(x - x') = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\exp(-ip\cdot(x - x'))\mathcal{F}(x - x')$$

$$[\tilde{\phi}(x), \tilde{\phi}(x')] = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}[\exp(-ip\cdot(x-x')) - \exp(ip\cdot(x-x'))]\mathcal{F}(x-x')$$

$$= \tilde{\Delta}(x-x') - \tilde{\Delta}(x'-x)$$

When $(x - x')^2 < 0$, we can perform a Lorentz transformation on the second term yielding $(x - x') \to -(x - x')$ giving:

$$[\tilde{\phi}(x), \tilde{\phi}(x')] = 0$$

- We conclude that causality is preserved and that no measurement in the nonlocal QFT can affect another measurement outside the light cone. It is accepted that the phenomenon of quantum entanglement is a generic nonlocal behavior of quantum mechanics but that there is no violation of causality. No information is exchanged at the speed of light, so that causality is not violated. We have demonstrated that microcausality is preserved in nonlocal finite QFT. This has the important consequence that time-ordered products of nonlocal field operators can be defined.

- The action is written as $S_\phi = S_F(\phi) + S_I(\phi)$

where $S_F(\phi) = \int d^4x \left(\frac{1}{2}\phi(x)\mathcal{K}\phi(x)\right)$ $\qquad \mathcal{E} = \exp\left(\frac{\mathcal{K}}{2\Lambda^2}\right)$

The regulated propagator is

$$i\tilde{\Delta} = \frac{i\mathcal{E}^2}{\mathcal{K}} = i\int \frac{d\tau}{\Lambda^2}\exp\left(\tau\frac{\mathcal{K}}{\Lambda^2}\right)$$

- The bare regulated Feynman propagators in Euclidean momentum space are

$$i\tilde{\Delta}_F(p) = i\frac{\exp(-(p^2+m^2)/\Lambda_p^2)}{p^2+m^2} \quad i\tilde{S}_F = i\frac{(\not{p}+m)\exp(-(p^2+m^2)/\Lambda_p^2)}{p^2+m^2}$$

- The anomalous results for triangle fermion graphs and the decay rate $\pi^0 \to 2\gamma$ are correctly produced in finite QFT.

- The Callan-Symanzik equations are satisfied for our (length) energy scales Λ_i :

$$\left[\Lambda_i \frac{\partial}{\partial \Lambda_i} + \beta(\tilde{g}_i)\frac{\partial}{\partial \tilde{g}_i} - 2\gamma(\tilde{g})\right]\Gamma^{(n)} = 0.$$

where $\tilde{g}_i$ are running coupling constants associated with entire function vertices.

- The vertices of string theory contain nonlocal entire function factors $\exp(-\alpha' p^2)$, where α' is the string tension, which cause loops to converge in Euclidean space (Gross-Jevichi (1987)). Our finite nonlocal QFT is formulated in D=4-dimensional spacetime.

4. Electroweak Sector

- Let us consider the electroweak (EW) sector. We assume that the $SU(2)_L$ X $U(1)_Y$ group is broken by the masses of the bosons and the fermion particles. The EW model Lagrangian is given by

$$\mathcal{L}_{\rm EW} = \sum_{\psi_L} \tilde{\psi}_L \left[\gamma^\mu \left(i\partial_\mu - \frac{1}{2}\tilde{g}\tau^a W^a_\mu - \tilde{g}'\frac{Y}{2}B_\mu\right)\right]\psi_L$$

$$+\sum_{\psi_R} \tilde{\psi}_R \left[\gamma^\mu \left(i\partial_\mu - \tilde{g}'\frac{Y}{2}B_\mu\right)\right]\psi_R - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \mathcal{L}_M + \mathcal{L}_{m_f}$$

$$\mathcal{L}_{\rm Higgs} = \left|\left(i\partial_\mu - \frac{1}{2} + \mathcal{L}_M + \mathcal{L}_{m_f}\frac{Y}{2}B_\mu\right)\phi\right|^2 + \frac{1}{2}m_H^2\phi^2$$

$$L_{\rm QED} = \sum_{\psi_L} \tilde{\psi}_L \left[\gamma^\mu \left(i\partial_\mu - \frac{1}{2}\tilde{e}\right)A_\mu\right]\psi_L + \sum_{\psi_R} \tilde{\psi}_L \left[\gamma^\mu \left(i\partial_\mu - \frac{1}{2}\tilde{e}\right)A_\mu\right]\psi_R$$

$$- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{m_f}$$

$$\tilde{g}(p^2) = g\mathcal{E}(p^2/\Lambda_M^2) \text{ and } \tilde{g}'(p^2) = g'\mathcal{E}(p^2/\Lambda_M^2)$$

$$\tilde{g}_H(p^2) = g_H\mathcal{E}(p^2/\Lambda_H^2) \text{ and } \tilde{g}'_H(p^2) = g'_H\mathcal{E}(p^2/\Lambda_H^2) \quad \tilde{e}(p^2) = e\mathcal{E}(p^2/\Lambda_M^2)$$

- The dynamically broken symmetry group SU(2) X U(1) has a mass Lagrangian density:

$$\mathcal{L}_M = \frac{1}{8}b^2 g^2[(W^1_\mu)^2 + (W^2_\mu)^2] + \frac{1}{8}b^2[g^2(W^3_\mu)^2 - 2gg'W^3_\mu B^\mu + g^{'2}B^2_\mu]$$
$$= \frac{1}{4}g^2 b^2 W^+_\mu W^{-\mu} + \frac{1}{8}b^2(W_{3\mu}, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g^{'2} \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix},$$

where b is the broken symmetry scale ~ 246 GeV not identified with the SM value $v = \langle\phi\rangle_0$ from spontaneous symmetry breaking of the vacuum. The broken symmetry mass matrix leads to the masses (Moffat, Eur. Phys. J Plus 2011):

$$M_W = \frac{1}{2}bg, \quad M_Z = \frac{1}{2}b(g^2 + g'^2)^{1/2}, \quad M_A = 0.$$

$$\mathcal{L}_M = M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$

- The boson and fermion mass Lagrangians are

$$\mathcal{L}_M = \frac{1}{2}M_W^2 W^{a\mu}W_\mu^a + \frac{1}{2}M_B^2 B^\mu B_\mu \qquad \mathcal{L}_{m_f} = -\sum_{\psi_L^i,\psi_R^j} m_{ij}^f(\tilde{\psi}_L^i\psi_R^j + \tilde{\psi}_R^i\psi_L^j)$$

- The $\mathcal{L}_{m_f}$ includes massive neutrinos.

We write $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$ and

$$Z_\mu = \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu \quad \text{and} \quad A_\mu = \cos\theta_w B_\mu + \sin\theta_w W_\mu^3$$

$$\sin^2\theta_w = \frac{g'^2}{g^2+g'^2} \quad \text{and} \quad \cos^2\theta_w = \frac{g^2}{g^2+g'^2} \qquad e = g\sin\theta_w = g'\cos\theta_w$$

$$L_I = -\frac{g}{\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}) - \tilde{g}\sin\theta_w J_{\text{em}}^\mu A_\mu - \frac{g}{\cos\theta_w}J_{\text{NC}}^\mu Z_\mu$$

- Quantization of the Proca massive vector bosons W and Z is physically consistent even though the SU(2) X U(1) gauge group is a dynamically broken symmetry group (Moffat, Eur. Phys. J. Plus 2011).

5. Perturbative Unitarity and the Higgs Boson Sector

The scattering of two W_L results in a divergent term proportional to s:

$$W_L^+ + W_L^- \to W_L^+ + W_L^- \qquad i\mathcal{M}_{W_L} = ig^2\left[\frac{\cos\theta + 1}{8M_W^2}s + \mathcal{O}(1)\right]$$

- This behavior is corrected by the addition of the s-channel Higgs boson exchange in the high energy limit and for $\Lambda_M < 1 - 14\ \mathrm{TeV}$.:

$$i\mathcal{M}_H = -ig^2\left[\frac{\cos\theta + 1}{8M_W^2}s + \mathcal{O}(1)\right]$$

- We identify the EW energy scale:

$$b = E_{\rm EW} = \left(\frac{1}{\sqrt{2}}G_F\right)^{1/2} = 246\,\mathrm{GeV}$$

- In our finite QFT EW model, the potential $V(\phi)$ has only one minimum $\langle 0|\phi_H|0\rangle = 0$ corresponding to the true vacuum. With our tree graph rules, we obtain the decay of the Higgs boson:

$$\Gamma(H \to f\tilde{f}) = \frac{N_C}{8\pi E_{\rm EW}^2} m_f^2 m_H \beta_f^3 \qquad \beta_f = \sqrt{1 - 4m_f^2/m_H^2}$$

At the tree graph level given $\cos^2\theta_w$ we have

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_w} = 1 \quad \rho = \rho_{(0)} + \Delta\rho_{(1)} \quad \Delta\rho_{(1)} \sim \frac{3G_F M_W^2}{8\pi^2\sqrt{2}}\left(\frac{m_t^2}{M_W^2} + \frac{5}{6}\right)$$

$\rho_{(0)} = 1$ and $\Delta\rho_{(1)}$ denotes the one-loop top quark correction for $p^2 < \Lambda_M^2$ and for $\Lambda_M > 1\ \mathrm{TeV}$. From the Higgs particle m_H=125 GeV:

$$\Delta\rho_{H(1)} \sim -\frac{3G_F M_W^2}{8\pi^2\sqrt{2}}\left[\left(\frac{M_Z^2}{M_W^2} - 1\right)\ln\left(\frac{m_H^2}{M_W^2}\right)\right] \qquad \Delta\rho_{H(1)} \sim 10^{-4}$$

$$\Delta\rho_{(1)} \sim 0.01$$

6. Higgs Mass Hierarchy Problem and Fine-Tuning Problem

- The real scalar Higgs boson Lagrangian is

$$\mathcal{L}_H = \frac{1}{2}\phi\Box\phi - m_0^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

- The perturbative finite renormalization gives

$$Z = 1 + \delta Z(\lambda, m_H^2, \Lambda_H),$$

$$Zm_0^2 = Zm^2 + \delta m_H^2(\lambda, m_H^2, \Lambda_H^2)$$
$$Z^2\lambda_0 = \lambda + \delta Z(\lambda, m_H^2, \Lambda_H^2).$$

- The propagator in Euclidean momentum space is

$$i\tilde{\Delta}_H(p) \equiv \frac{i\mathcal{E}^2}{p^2 + m_H^2} = i\int \frac{d\tau}{\Lambda_H^2}\exp\left[-\tau\left(\frac{p^2 + m_H^2}{\Lambda_H^2}\right)\right]$$

- The one-loop Higgs boson self-energy graph is given by (Kleppe-Woodard 1993):

$$-i\Sigma = \frac{-iZ^{-2}\lambda}{2^5\pi^2} m_H^2 \Gamma\left(1/2, \frac{m_H^2}{\Lambda_H^2}\right)$$

$$\delta m_H^2 = \frac{\lambda}{32\pi^2}\left[\Lambda_H^2 - m_H^2 \ln\left(\frac{\Lambda_H^2}{m_H^2}\right) - m_H^2(1-\gamma) + \mathcal{O}\left(\frac{m_H^2}{\Lambda_H^2}\right)\right] + \mathcal{O}(\lambda^2)$$ 3):

- We now choose $\Lambda_H \sim 200\,\mathrm{GeV} - 1\,\mathrm{TeV}$ which yields $\delta m_H^2/m_H^2 \sim \mathcal{O}(1)$.

- The dominant Higgs particle radiative correction is the top quark correction:

$$\delta m_H^2 = \frac{y_t^2}{16\pi^2}\Lambda_H^2 + \delta\mathcal{O}(m_{\mathrm{weak}}^2)$$

- With the choic$\Lambda_H \sim 200\,\mathrm{GeV} - 1\,\mathrm{TeV}$, we get $\delta m_H^2/m_H^2 \sim \mathcal{O}(1)$, which resolves the Higgs boson mass hierarchy fine tuning problem.

7. The Finite Renormalization Group and Triviality Problem (Green-Moffat, in preparation)

- In SM for the Higgs scalar field, we obtain the coupling constant vertex correction $\delta\lambda$ and the β function at lowest in order in λ:

$$\delta\lambda \equiv \beta(\Lambda) = \frac{3\lambda^2}{16\pi^2}\ln\left(\frac{\Lambda_C}{m_H}\right)$$

- From the RG equation :

$$\left[\frac{\partial\lambda}{\partial\lambda} + \beta(\Lambda_H, \lambda)\frac{\partial}{\partial\lambda}\right]\Gamma^H = 0.$$

we obtain

$$\frac{1}{\lambda_{\rm ren}} = \frac{1}{\lambda_0} + \frac{3}{16\pi^2}\ln\left(\frac{\Lambda_C}{\mu}\right)$$

This gives the result for the renormalized coupling constant:

$$\frac{1}{\lambda_{\rm ren}} = \frac{1}{\lambda_0} + \frac{3}{16\pi^2}\ln\left(\frac{\Lambda_C}{\mu}\right) \qquad \lambda_{\rm ren} = \frac{\lambda_0}{1 + \frac{3\lambda_0}{16\pi^2}\ln\left(\frac{\Lambda_C}{\mu}\right)}$$

- The $\lambda_0\phi^4$ model is renormalizable producing finite scattering amplitudes and cross sections. However, by the definition of renormalizability, we must set the cutoff $\Lambda_C \to \infty$. This means that $\lambda_{\rm ren} = 0$ and the theory becomes a free field theory and trivial (Landau, Khalatnikov, Abrikosov (1954), Landau (1995)). This result holds even for $\lambda_0 \to \infty$. For $\Lambda_H = 1/\ell_H$, we cannot take the limit $\ell_H \to 0$ corresponding to the Dirac δ-function limit.

- In the finite QFT, the one loop, coupling constant vertex correction is given by

$$\delta\lambda = \frac{3\lambda^2}{16\pi^2}\int_0^{1/2} dx\Gamma\left(0, \frac{1}{1-x}, \frac{m_H^2}{\Lambda_H^2}\right) + \mathcal{O}(\lambda^2)$$

For $\mu < \Lambda$ this is expanded to give

$$\delta\lambda = \frac{3\lambda^2}{16\pi^2}\left[\ln\left(\frac{\Lambda_H}{m_H}\right) + \frac{1}{2}(\ln(2) - 1 - \gamma) + \mathrm{O}\left(\frac{m_H}{\Lambda_H}\right)\right] + \mathrm{O}(\lambda^2)$$

- We now obtain the β function:

$$\beta(\Lambda_H, \lambda) = \frac{3\Lambda_H^2}{16\pi^2} I(\Lambda_H, \lambda) \qquad I(\Lambda_H, \lambda) = \int_0^{1/2} dx \Gamma\left(0, \frac{1}{1-x}\frac{\mu^2}{\Lambda_H^2}\right)$$

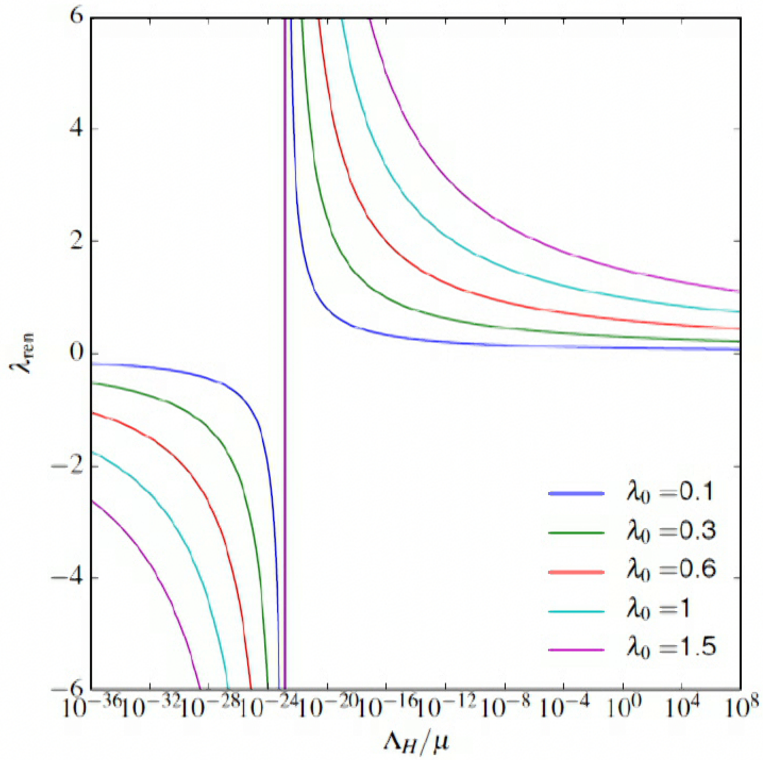
- The renormalized coupling constant is now given by

$$\frac{1}{\lambda_{\rm ren}} = \frac{1}{\lambda_0} + J(\Lambda_H, \mu) \qquad \lambda_{\rm ren} = \frac{\lambda_0}{1 + \lambda_0 J(\Lambda_H, \mu)}$$

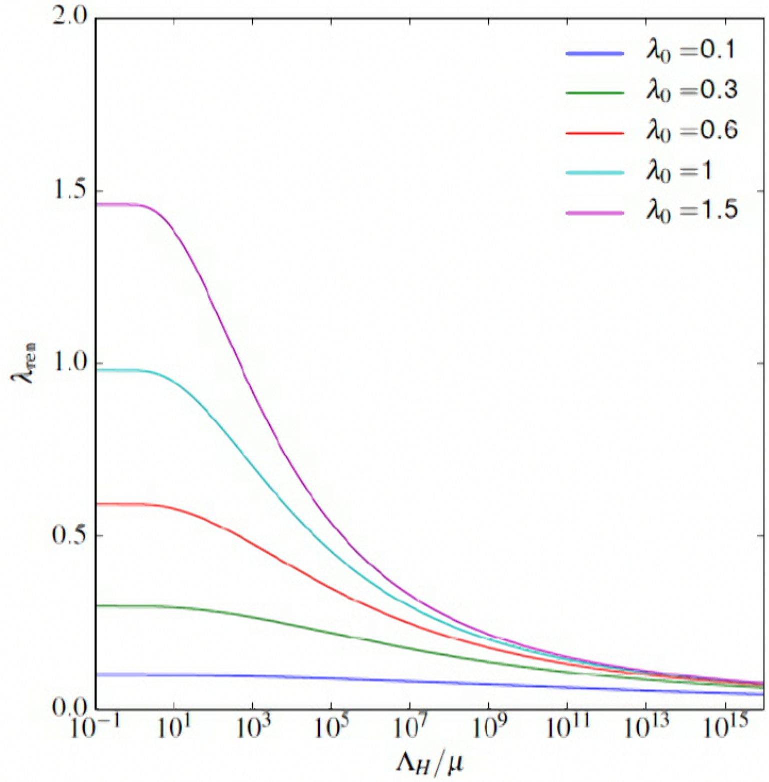
where

$$J(\Lambda_H, \mu) = \frac{3}{16\pi^2} \int \frac{d\Lambda_H}{\Lambda_H} I(\Lambda_H, \mu)$$

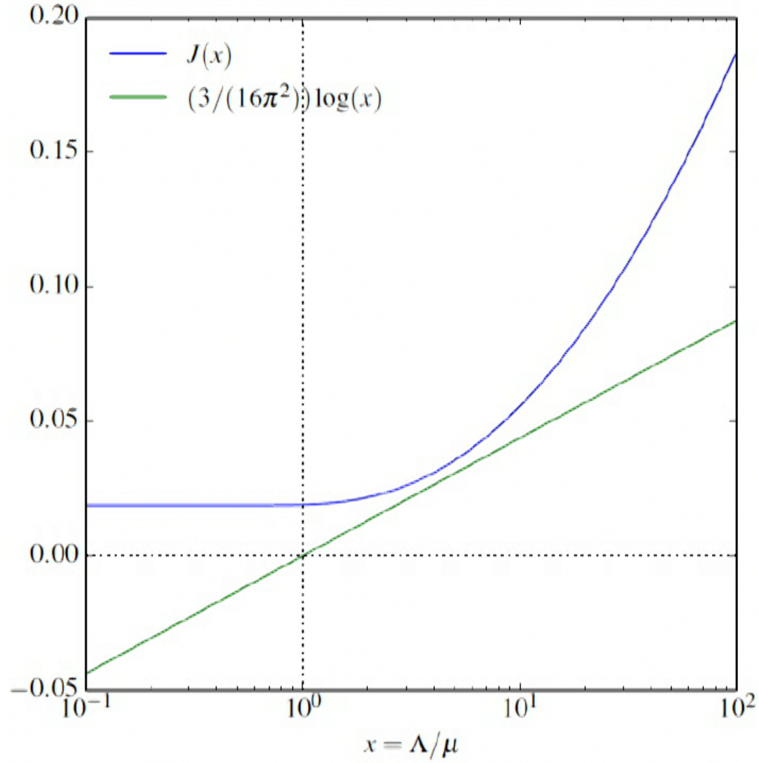
- We resolve the triviality problem for all values of Λ/μ for the scalar Higgs field. There is no Landau pole.



The SM occurrence of a Landau pole in $\lambda_{\rm ren}$.



$\lambda_{\rm ren}$ versus Λ_H/μ for various values of λ_0 showing that there is no Landau pole. As μ increases for fixed Λ_H, $\lambda_{\rm ren} \rightarrow \lambda_0$. $\lambda_{\rm ren}$ decreases as $\mu \sim p \sim 0$.



J(x) versus SM log(x) for x = Λ/μ.

- The results obtained for the running of $\lambda_{\rm ren}$ are for a single Higgs boson interacting with another Higgs boson. This cannot describe a realistic situation, for the Higgs coupling to other particles such as a top quark (the top quark-Higgs particle coupling $\lambda_t \sim \mathcal{O}(1)$) will play an important role making the pure Higgs coupling result for $\lambda_{\rm ren}$ not valid.

- We note that the energy scale Λ_M will not appear in the calculations of scattering amplitudes, decay amplitudes of particles and vacuum polarization for $p^2 \rightarrow 0$ (Evans, Moffat, Kleppe, Woodard, Phys. Rev. D 1991, Moffat 1104.5706, Moffat, Eur. Phys. J. Plus 2011).

- The proof of triviality by Landau et. al (Landau, Khalatnikov, Abrikosov 1954, 1955) was for QED and demonstrated that for $p^2 \sim m_e^2$ and $\Lambda_C \rightarrow$ infinity $e^2 \rightarrow 0$ and the QED has no interactions (free fields).

8. Stability of the Vacuum

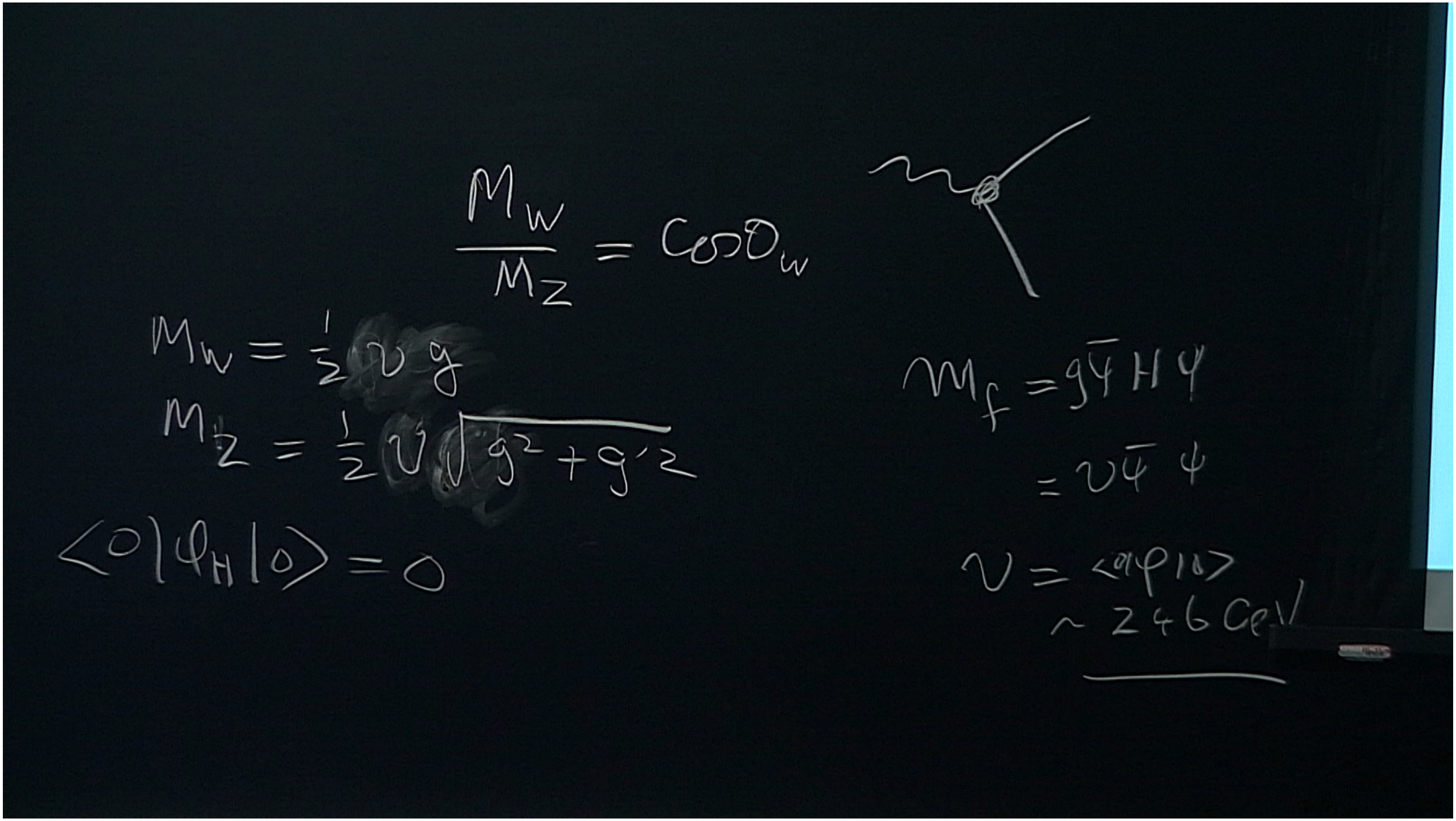
- A stable, true vacuum is determined by the global minimum of the scalar potential, which depends on the particle model's scalar fields.
- The spontaneous symmetry breaking of the vacuum in the SM produces a ``false'' vacuum, instead of a local minimum. If only the deepest minimum of the scalar field potential is occupied by the Universe, then its future is not threatened by an unstable vacuum state. A local minimum in the current Universe can become a deeper minimum in the potential, and only a finite barrier separates the potential from a bottomless pit, and the Universe can tunnel out into another state in which we cannot support life as we know it.

- In our model we do not invoke a spontaneous symmetry breaking of the vacuum state. Consequently, we can expect the scalar field potential for the Universe lies in the deepest minimum of the potential with $\langle 0|\phi_H|0\rangle = 0$ and there is no threat to the stability of the vacuum and the future evolution of the Universe.

9. Experimental Tests of the Model

- Future Higgs boson experimental data, which will determine the sizes of the Higgs loop radiative corrections can distinguish our finite QFT model from the SM. This can check whether radiative Higgs –particle loops can experimentally satisfy the restriction $\Lambda_H \lesssim 1\,\mathrm{TeV}$ on their magnitudes. Future accelerator experiments at energies greater than $\Lambda_M > 1 - 14\,\mathrm{TeV}$ can test whether amplitudes and cross sections are damped by the entire function operator $\mathcal{E}(p^2)$ compared to the predicted amplitudes and radiative corrections in the SM.

- If no new particles are detected by the LHC and future high energy accelerators, what explains the lack of new fundamental particles up to the Planck energy, $E \sim 10^{19}\,\mathrm{GeV}$? Because amplitudes are damped exponentially fast in Euclidean momentum space for $E > \Lambda_M$ the ``desert" energy hierarchy problem can be explained in the model.


Mw / MZ = Cos θw
Mw = 1/2 v g
MZ = 1/2 v √(g² + g'²)
<0|φH|0> = 0
mf = gψ̄Hψ
= vψ̄ψ
v = <0|φ|0>
~ 246 GeV

- All the data for the Higgs boson interactions obtained from the LHC scattering of protons are determined by the Higgs boson decay products. The W, Z and Higgs particle lifetimes are of order $\tau \sim 10^{-22}$ sec and $c\tau \sim 10^{-12}$ cm, so they cannot be used as collision projectiles and targets.

- The range of the LEP accelerator had a electron-positron maximum collision energy of 200 Gev. No damping of the cross sections for W, ZW and quark and lepton cross sections was detected, except for the top quark due to its high energy threshold. The Higgs boson self-energies and radiative loops remain unresolved due to their energy thresholds $2m_t \sim 344$ GeV. However, precision measurements at a future International Linear Collider (ILC) or at a future Circular Electron-Positron Collider (CEPC) and the projected CERN FCC could measure the Higgs boson self-energy and radiative loop contributions and determine the magnitude of Λ_M.

- If it is found that the Higgs boson couplings and self-energies making up the radiative loop corrections are significantly damped at energies above > 300 GeV, then we are justified to choose $\Lambda_H < 1$ TeV, thereby, resolving the Higgs mass hierarchy problem.

10. Quantum Gravity

- The finite QFT has been extended to quantum gravity. The perturbative quantum gravity is formulated as an expansion around a fixed background metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

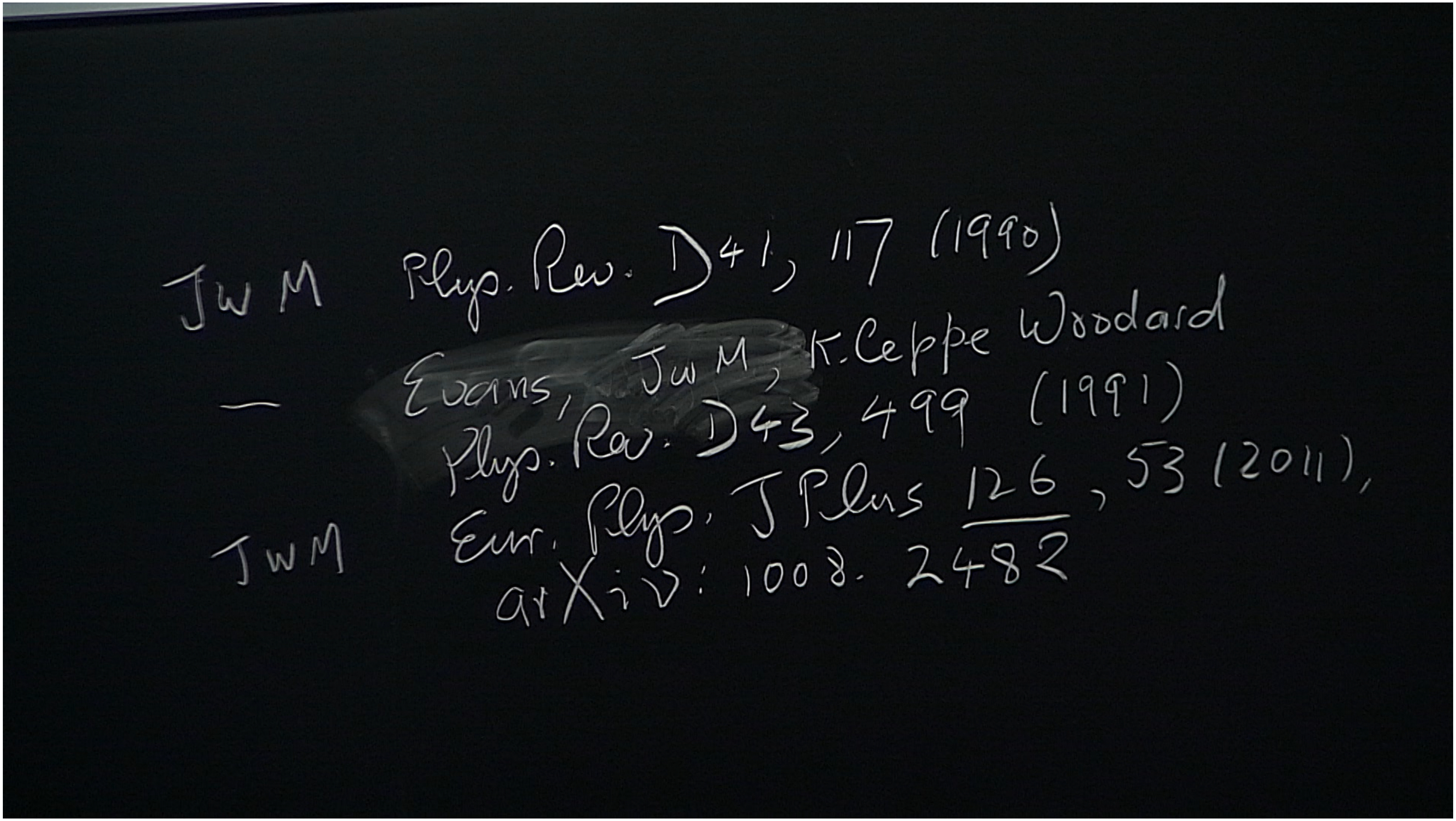
where $h_{\mu\nu} < 1$ and $\bar{g}_{\mu\nu}$ can be chosen to be the Minkowski metric $\eta_{\mu\nu}$.

- The quantum gravity theory is finite and unitary to all orders of perturbation theory and is based on the Einstein-Hilbert Lagrangian linear in the curvature.

- A consequence of the finite quantum gravity theory is that graviton scattering amplitudes, including gravitons coupled to gravitons and matter, are damped in Euclidean momentum space and quantum gravity is damped to zero at the Planck energy $\sim 10^{19}$ GeV.

- A solution of the cosmological constant problem can be achieved using the finite quantum gravity theory.

END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