

Title: TBA

Speakers: Alexander Braverman

Collection: Cohomological Hall Algebras in Mathematics and Physics

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Plan

Explain some generalization of
geometric Satake / examples of
local geometric Langlands
involving Lie super-algebra of $(n|m)$

→ Physics input (Gaiotto)

→ Finkelberg, Ginzburg & Travkin

Plan

Explain some generalization of
geometric Satake / examples of
local geometric Langlands
involving Lie super-algebra $gl(n|m)$

→ Physics input (Gaiotto)

→ Finkelberg, Ginzburg & Travkin

Specific plan

① Recall usual geometric Satake

② "Whittaker geometric Satake"

(Fundamental local equivalence)

Gartsgory-Lurie

③ Generalization for $GL(n|n)$

④ Reformulation in terms of
local geometric Langlands

⑤ Physics

(5-d)

$gl(n|n)$

⑤ Physics interpretation

(S-duality for NS5/D5
branes).

$$\textcircled{1} \quad K = \mathbb{C}(t)$$

$$G_R =$$

$$\mathbb{C} \\ \mathbb{C} = \mathbb{C}(t)$$

G - connected reductive group

$$G_R = G(K)/G(O)$$

$$\text{Perv}_{G(O)}(G_R) \simeq \text{Rep } G^V$$

ive group

$$\begin{array}{c} \parallel \\ \text{Perv}_{G(O)}(G(K)/G(O)) \\ \downarrow \\ \text{Geometric Satake equivalence} \end{array}$$

Langlands dual group.

What doesn't work here?

(a) Not true on derived level

$D_{G/O}(G \otimes_G)$ can be described algebraically

ands
group.

$$G_R = G(K)/G(O)$$

$$\text{Perv}_{G(O)}(G_R) \simeq \text{Rep } G^V$$

$$\begin{array}{c} \parallel \\ \text{Perv}(G(O) \backslash G(K)/G(O)) \\ \downarrow \\ \text{Geometric Satake equivalence} \end{array}$$

↑
Langlands
dual group.

ductive group

What doesn't work here?

(a) Not true on derived level

$D(G \otimes_{\mathbb{C}} \mathbb{C})$ can be described algebraically

(b) Doesn't generalize to quantum groups

Langlands
dual group.

Try:

LHS

RHS

||

$$U_q(\sigma^v) = \kappa p.$$

⑤ Physics interpretation

(S-duality for NS5/D5
branes).

GR_G come with a determinant
line bundle ($\Rightarrow$ central ext. of $G(K)$)

$\forall c \in \mathbb{C}$ can study $D\text{-mod}_{G(\mathbb{C})}^c (GR_G) \sim \mathcal{U}_q(\mathfrak{g}^v)$ - wrong
 $q = \exp(2\pi i/c)$

① $K = \mathbb{C}(t)$

$\mathcal{O} = \mathbb{C}[[t]]$

GR_G

G -connected reductive group

Try:

LHS

RHS

||

$$D\text{-mod}_{G(\mathbb{C})}^c(G_{\mathbb{R}_G}) \neq$$

$$U_q(\mathfrak{g}^v)\text{-rep.}$$

$$\text{If } c \notin \mathbb{Q} \quad \text{LHS} = \text{Vect.}$$

algebraically

turn groups

More notation

$$N \subset G$$

max. unipotent

$\chi_0: N \rightarrow G_a$ non-degenerate character

Extend this to $\chi: N(K) \rightarrow G_a$

$$\chi = \text{Res}_{t=0} \chi_t$$

$\text{Wh}^*(GR_G) = "$ $(N(K), \gamma)$ -equiv. D-modules
 on π_K "
 (S. Rastin)

Defined both as derived and as an abelian category
 (Factorization category on

character

G_a

$\text{Whit}(GR_G) = "$ $(N(K), \gamma)$ -equiv. D-modules
 on GR_G "
 (S. Rastin)

character

G_a

Defined both as derived and as an abelian category

(Factorization category on a curve)

Thm $\text{Whit}(GR_G) \cong \text{Rep } G^\vee$

True both for abelian & for derived cat.

Kazhdan-Lusztig equivalence

$$U_q(\mathfrak{g}^v)\text{-rep} \simeq KL_c(\hat{\mathfrak{g}}^v).$$

$$q = e^{2\pi i/c}$$

$$c \in \mathbb{R}_{\geq 0}$$

$\hat{\mathfrak{g}}^v$ - affine Kac-Moody

$KL_c(\hat{\mathfrak{g}}^v)$ = cat. of $\hat{\mathfrak{g}}^v$ -modules
of level c which are $G^v(\mathbb{O})$ -integrable.

Theorem (Gaiitsgory-Lurie)
 twisted D-modules

$$\mathrm{Whit}^c(\mathrm{GR}_c) \cong \mathrm{KL}^{\frac{1}{c}}(\hat{\mathfrak{g}}^v)$$

True always for derived cat.

and if $c \in \mathbb{Q}_{\geq 0}$ also true

for abelian categories

$\mathfrak{g}^v$ -modules
 are $G^v(\mathcal{O})$ -integrable.

$$n = m \quad \text{LHS} \quad \mathcal{D}_{\text{mod}}^c_{GL(n, \mathcal{O})} (G_{R_{GL(m)}} \times K^n)$$

$$\text{RHS} \quad KL^{\frac{1}{c}}(\widehat{gl(n/m)}) \quad \frac{1}{c} \neq \infty$$

$$n > m$$

$$GL(n) \times GL(m) \text{-equivariant} \quad \frac{1}{c} = \infty$$

$$\text{modules over } \Lambda(\text{Hom}(\mathbb{C}^n, \mathbb{C}^m)) \otimes \Lambda(\text{Hom}(\mathbb{C}^m, \mathbb{C}^n))$$

$$\frac{1}{c} \neq \infty.$$

$$\frac{1}{c} = \infty$$

$$n = n-1$$

$$D^c\text{-mod}_{GL(n-1,0)}(G_{R_{GL(n)}}) \simeq KL^{\frac{1}{2}}(\widehat{gl(n|n-1)})$$

$$n = n-1$$

$$D^c\text{-mod}_{GL(n-1, \mathcal{O})} (G_{R_{GL(n)}}) \cong KL^{\frac{1}{2}}(\widehat{gl(n|n-1)})$$

$$\frac{1}{c} \neq \infty.$$

In general for $m < n \rightsquigarrow U_{n,m} \subset GL(n)$
unipotent subgroup

Normalized by $GL(m)$.

with $\chi_{n,m}: U_{n,m} \rightarrow \mathbb{G}_a$.

$$\frac{1}{c} = \infty$$

$$D\text{-mod}((U_{n,m}^{(K)} \times GL(m, \mathcal{O})) \backslash GL(n, K) / GL(n, \mathcal{O}))$$

generalization

fix numbers

$n \geq m \geq 0$

LHS

$n=m$

$$\mathcal{D}\text{-mod}_{GL(n,0)}^c (Gr_{GL(m)} \times K^n)$$

$n > m$

Conjecture, theorem

for $c=0$ (B-Finkelberg-Ginzburg
Travkin)

$GL(n) \times GL(m)$ -equivariant
modules over $\Lambda(\text{Hom}(\mathbb{C}^n, \mathbb{C}^m))$

$\otimes \Lambda(\text{Hom}(\mathbb{C}^m, \mathbb{C}^n))$

RHS

$$KL^{\frac{1}{c}}(\widehat{gl(n|m)})$$

$\frac{1}{c}$

Quantum local geometric Langlands.

A -alg group

$\mathcal{C}$ - "nice" category.

$\Rightarrow$ notion of triang ("D-module") action of A on $\mathcal{C}$.

Ex X - nice space with action of A
 $\mathcal{C} = \text{D-mod}(X)$

Weak action $\Leftrightarrow$ action of $\mathcal{U}(\text{wh}(A))$ on $\mathcal{C}$

Strong action $\Leftrightarrow$ action of $\text{D-mod}(A)$ on $\mathcal{C}$

Variant:

$$| \rightarrow G_m \rightarrow \tilde{A} \rightarrow A \rightarrow |$$

$\forall c \in \mathcal{C}$ Finiton of strong A -action of
level $c \Leftrightarrow \text{D-mod}^c(A)$ acts on $\mathcal{C}$.

Local geometric langlands

Categories with $G(K)$ -action of level c

$| \mathcal{C}$

Categories with $G^v(K)$ -action of level $-\frac{1}{c}$

$\mathcal{C} \longrightarrow \mathcal{C}^v$

s.t.

$$\mathcal{C}^{G(0)} \simeq (\mathcal{C}^v)^{G^v(0)}$$

$$D\text{-mod}^c (GR_G)^V$$

$$\parallel$$

$$D\text{-mod}^{-\frac{1}{c}} (GR_G^V)$$

$$v(\theta)$$

Gaiitsgory-Lurie comes from

$$\mathrm{Whit}^c(G(K))^\vee = \hat{\sigma}_G^{\vee \frac{1}{c}} - \mathrm{mod}.$$

$$\mathrm{D-mod}^c(G(K)/(N(K), \chi))$$

$$D\text{-mod}^c$$

$$n > m$$

$$GL(n, k) / (U_{n, m} \times X_{n, m}) \stackrel{V}{=} \widehat{OGL(n/m)}^{-\frac{1}{2}} \text{-mod}$$

$\uparrow$ $GL(n, k) \times GL(m, k)$

Similarly, $n = m$

$$D\text{-mod}^c \left(GL(n, k) \times k^n \right) \stackrel{V}{=} \widehat{OGL(n/n)}^{-\frac{1}{2}} \text{-mod}$$

$\uparrow$ $GL(n, k) \times GL(m, k)$