Title: Topological Holography Course - Lecture 6

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Collection: Topological Holography Course (Costello)

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Boundary condition:

$A^{0,1}$ divisible by n

A sol^n to EOM which violates the BC is

$$A_{z=0,a} \sim \delta_{z=0} t^a + \frac{N n^2 d\bar{w}}{(1+\|w\|^2)^2} \frac{1}{z^2}$$

$t^a \in gl(K|K)$

A_{z_0, a_0}

A_{z_1, a_1}

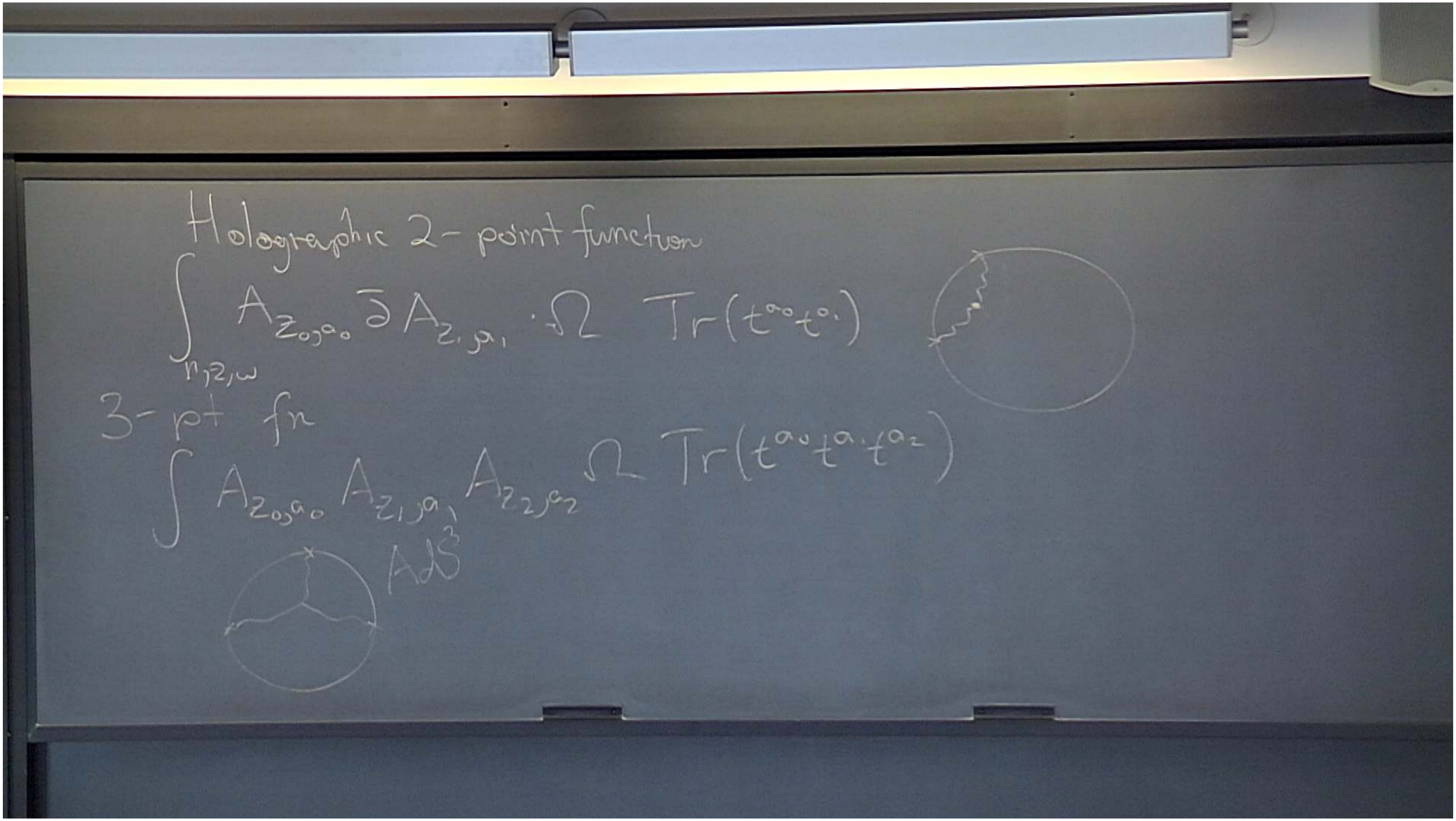
A_{z_2, a_2}

Holographic 2-point function

$$\int_{r,z,w} A_{z_0,a_0}\, \bar{\partial} A_{z_1,a_1}\, \Omega \quad \mathrm{Tr}(t^{a_0} t^{a_1})$$

3-pt fn

$$\int A_{z_0,a_0} A_{z_1,a_1} A_{z_2,a_2} \Omega \quad \mathrm{Tr}(t^{a_0} t^{a_1} t^{a_2})$$


Holographic 2-point function
$\int_{r,z,w} A_{z_0,a_0} \bar\partial A_{z_1,a_1} \cdot \Omega \; \mathrm{Tr}(t^{a_0} t^{a_1})$
3-pt fn
$\int A_{z_0,a_0} A_{z_1,a_1} A_{z_2,a_2} \Omega \; \mathrm{Tr}(t^{a_0} t^{a_1} t^{a_2})$
AdS^3

A is really in $\Omega^{0,1}(\overline{SL_2(\mathbb{C})}, \mathcal{O}(-D))$

D is the boundary, $n=0$

If near boundary, we write A as a function (section of $\mathcal{O}$)

$$\bar{\partial}_{\mathcal{O}(-D)} = n \bar{\partial} n^{-1}$$

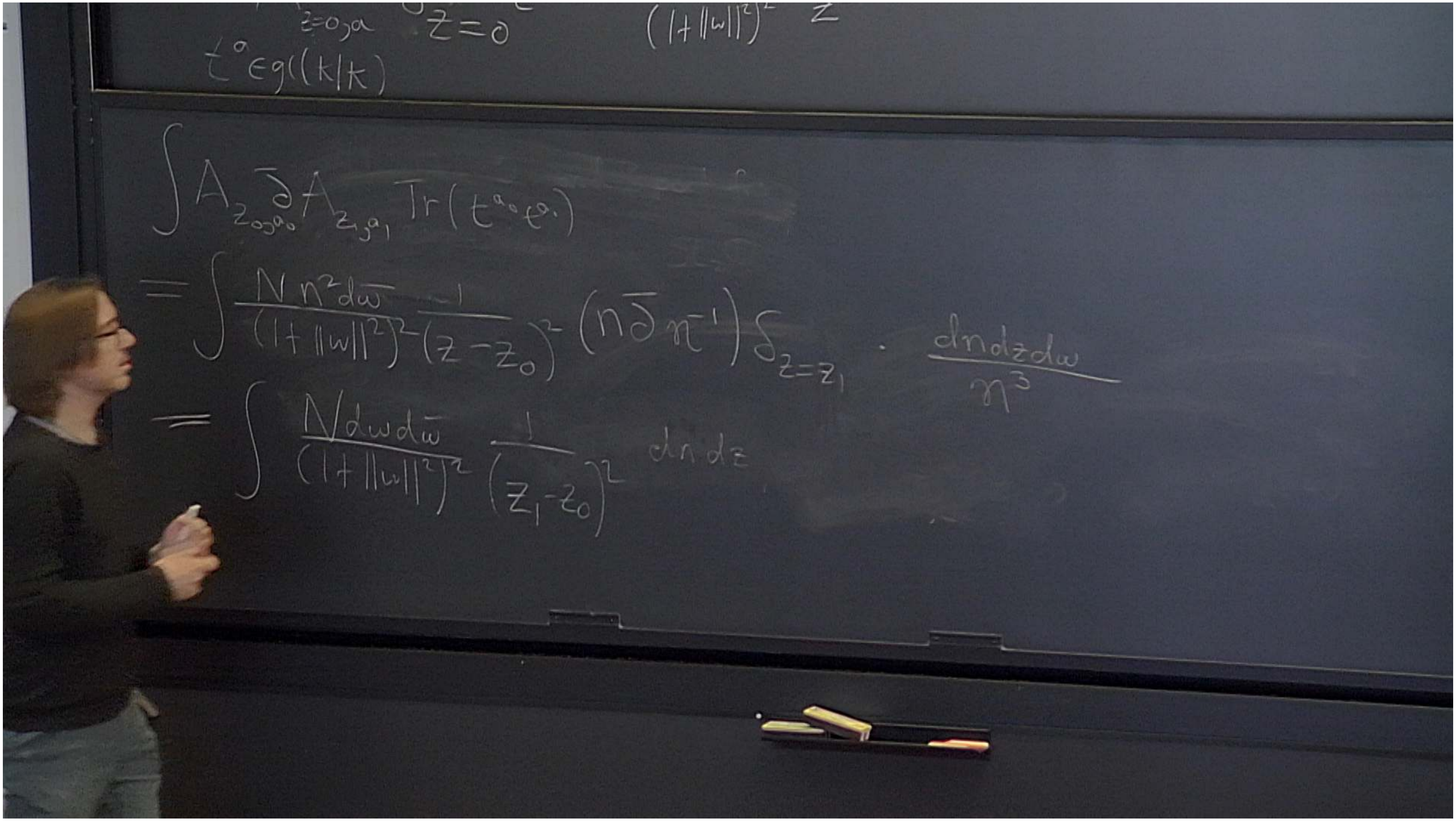
As, 1, viewed as a section of $\mathcal{O}(-D)$, has a first order pole

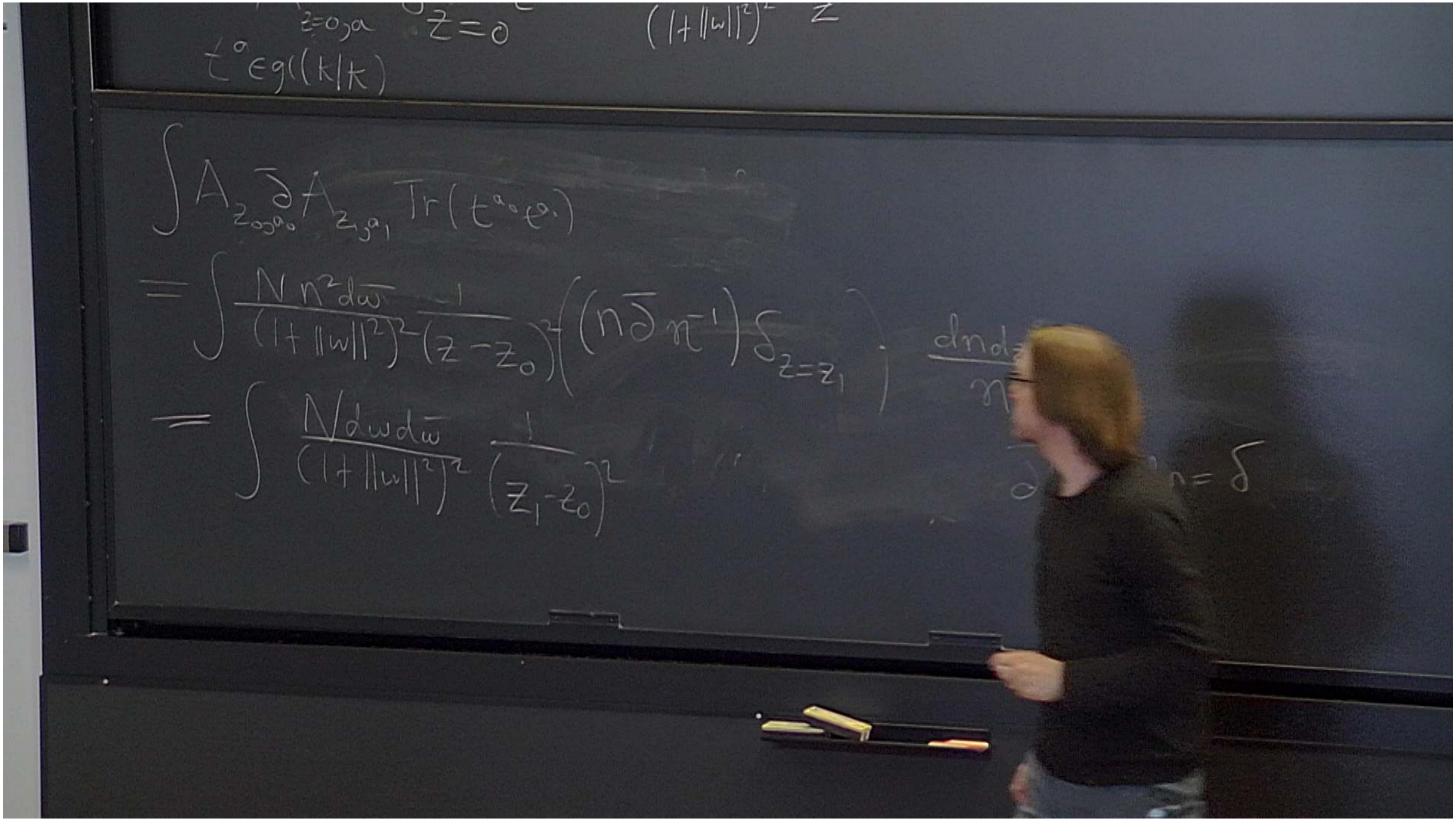
$z=0,a \qquad z=0 \qquad (1+\|w\|^2)^2 \quad z$

$t^a \in gl(k|k)$

$$\int A_{z_0,a_0} \bar\partial A_{z_1,a_1} \operatorname{Tr}(t^{a_0} t^{a_1})$$

$$= \int \frac{N n^2 d\bar{w}}{(1+\|w\|^2)^2} \frac{-1}{(z-z_0)^2}$$


$t^a \in gl(K|K)$
$\int A_{z_0,a_0} \bar{\partial} A_{z_1,a_1} \mathrm{Tr}(t^{a_0} t^{a_1})$
$= \int \frac{N n^2 d\bar{w}}{(1+\|w\|^2)^2} \frac{-1}{(z-z_0)^2} (n \bar{\partial} n^{-1}) \delta_{z=z_1} \cdot \frac{dn\, dz\, dw}{n^3}$
$= \int \frac{N dw\, d\bar{w}}{(1+\|w\|^2)^2} \frac{1}{(z_1-z_0)^2} dn\, dz$


$t^a \in gl(K|K)$
$\int A_{z_0,a_0} \bar\partial A_{z_1,a_1} \mathrm{Tr}(t^{a_0} t^{a_1})$
$= \int \frac{N n^2 d\bar{w}}{(1+\|w\|^2)^2} \frac{1}{(z-z_0)^2} \left((n \bar\partial n^{-1}) \delta_{z=z_1} \right)$
$= \int \frac{N dw d\bar{w}}{(1+\|w\|^2)^2} \frac{1}{(z_1-z_0)^2}$

$$z=0,a \qquad z=0 \qquad (1+\|w\|^2)^2 \qquad z$$

$$t^a \in \mathfrak{gl}(k|k)$$

$$\int A_{z_0,a_0}\,\bar\partial A_{z_1,a_1}\,\mathrm{Tr}(t^{a_0}t^{a_1})$$

$$= \int \frac{N\,n^2\,d\bar w}{(1+\|w\|^2)^2}\,\frac{1}{(z-z_0)^2}\left((n\bar\partial n^{-1})\,\delta_{z=z_1}\right) \quad \frac{dn\,dz\,dw}{n^3}$$

$$= \int_w \frac{N\,dw\,d\bar w}{(1+\|w\|^2)^2}\,\frac{1}{(z_1-z_0)^2} \sim \frac{N}{(z_1-z_0)^2}$$

$$\bar\partial(n^{-1})\,dn = \delta$$

In the chiral algebra

$$I \in \mathrm{Hom}(\mathbb{C}^{k|k},\mathbb{C}^N) \quad J \in \mathrm{Hom}(\mathbb{C}^N,\mathbb{C}^{k|k})$$

A_{z_0} b.c. at z_0

$$A_{z_0} \cdot A_{z_1} = \sum_{i>0} \tilde{A}^{(i)}_{z_0} \frac{1}{(z_0-z_1)^i}$$

If $\tilde{\tilde{A}}_{z_2}$ is another field we must have

$$\int A_{z_0} \wedge A_{z_1} \wedge \tilde{\tilde{A}}_{z_2} = \sum_{i>0} \frac{1}{(z_0-z_1)^i} \int \tilde{A}^{(i)}_{z_0} \wedge \tilde{\tilde{A}}_{z_2}$$

$+$ reg. terms in $z_0 - z_1$

$$A_{z_0} \cdot A_{z_1} = \sum_{i>0} \tilde{A}^{(i)}_{z_0} \frac{1}{(z_0-z_1)^i}$$

A_{z_0} b.c. at z_0

If $\tilde{\tilde{A}}_{z_2}$ is another field we must have

$$\int A_{z_0} \wedge A_{z_1} \wedge \tilde{\tilde{A}}_{z_2} = \sum_{i>0} \frac{1}{(z_0-z_1)^i} \int \tilde{A}^{(i)}_{z_0} \wedge \tilde{\tilde{A}}_{z_2}$$

$+$ reg. terms in $z_0 - z_1$

So,

$$\sum \frac{1}{(z_0-z_1)^i} A^{(i)}_{z_0} = \bar{\partial}^{-1}\left(A_{z_0} \wedge A_{z_1}\right) + \text{reg.}$$

$\in \mathfrak{gl}(k|k)$

$$A_{z_0} = \delta_{z=z_0} + \frac{N\hbar^2\, d\bar{w}}{(1+\|w\|^2)^2}\,\frac{1}{(z-z_0)^2}$$

$$A_{z_1} = \quad '' \qquad ''$$

$$A_{z_0} = \bar\partial\left(\frac{1}{z-z_0}\right)$$

Therefore $$\frac{1}{z-z_0}A_{z_1} = \bar\partial^{-1}\left(A_{z_0}\wedge A_{z_1}\right)$$

$$= \frac{1}{z_1-z_0}\left(\delta_{z=z_1} + \frac{N\hbar^2\, d\bar{w}}{(1+\|w\|^2)^2}\,\frac{1}{(z-z_1)^2}\right) + \text{reg}$$

$$A_{z_0,a_0}\cdot A_{z_1,a_1}$$

$$\sim f^{a_2}_{a_0 a_1}\,\frac{1}{z_0-z_1}\,A_{z_0,a_2}$$

$$+\frac{1}{(z_0-z_1)^2}\,N\,\mathrm{Tr}\left(t^{a_0}t^{a_1}\right)$$

$\Omega^{0,2}(X)$ "gerbe"

Better

β, γ define a map $X \longrightarrow (\mathbb{C}^\times)^2$

and we're coupling $\alpha \in PV^{1,1}$ to hol. Rozansky-Witten theory on $\mathbb{C}^\times \times \mathbb{C}^\times$

This has $SL_2\mathbb{Z}$ symmetry S-duality