Title: PSI 2018/2019 - String Theory Review - Lecture 4

Speakers: Davide Gaiotto

Collection: PSI 2018/2019 - String Theory Review (Gaiotto)

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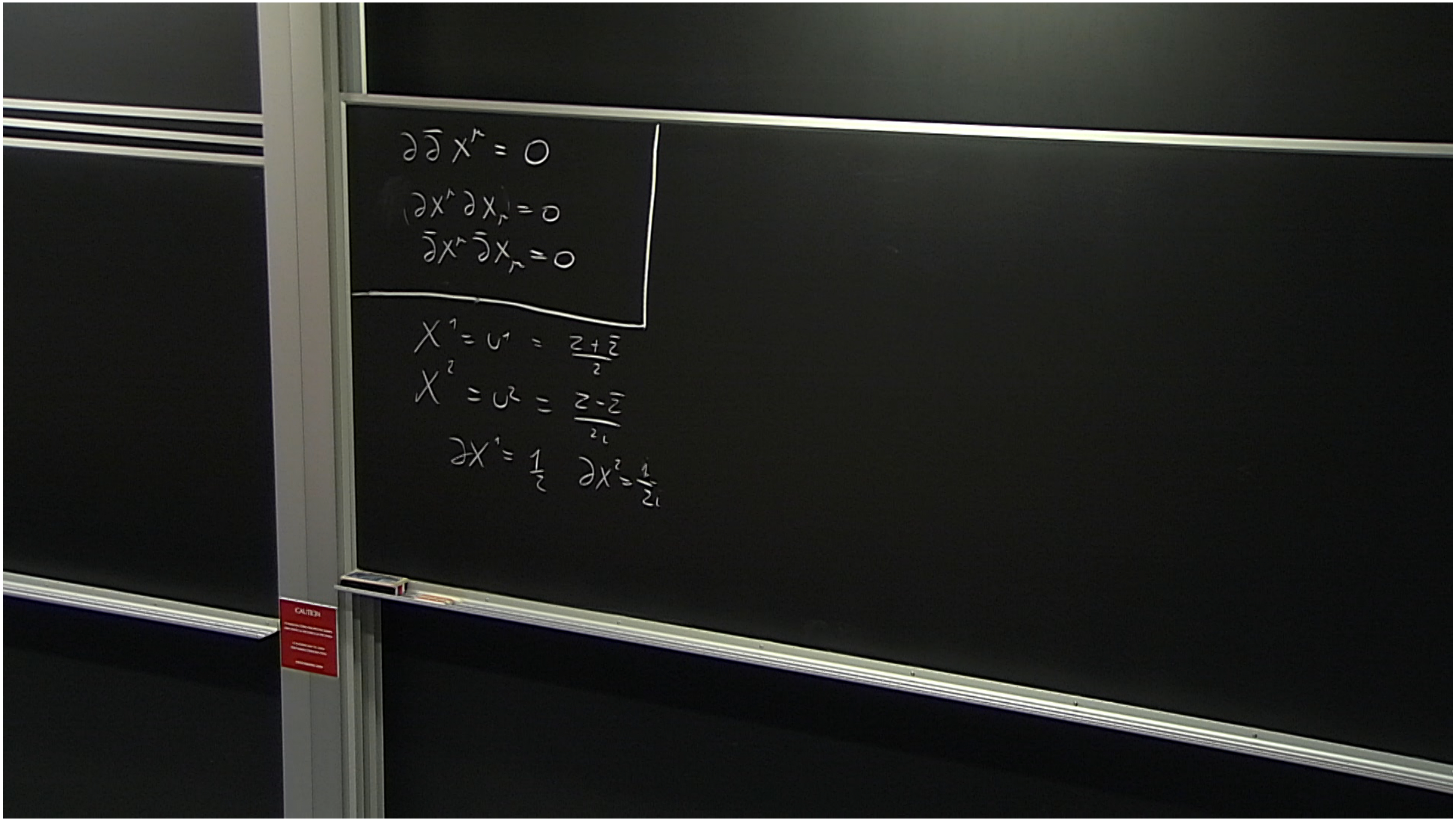


$\partial\bar{\partial}X^\mu = 0$
$\partial X^\mu \partial X_\mu = 0$
$\bar{\partial}X^\mu \bar{\partial}X_\mu = 0$
$X^1 = u^1 = \frac{z+\bar{z}}{2}$
$X^2 = u^2 = \frac{z-\bar{z}}{2i}$
$\partial X^1 = \frac{1}{2} \quad \partial X^2 = \frac{1}{2i}$

$$\langle \mathrm{PHYS} |\, T(s)\, | \mathrm{PHYS} \rangle = 0$$

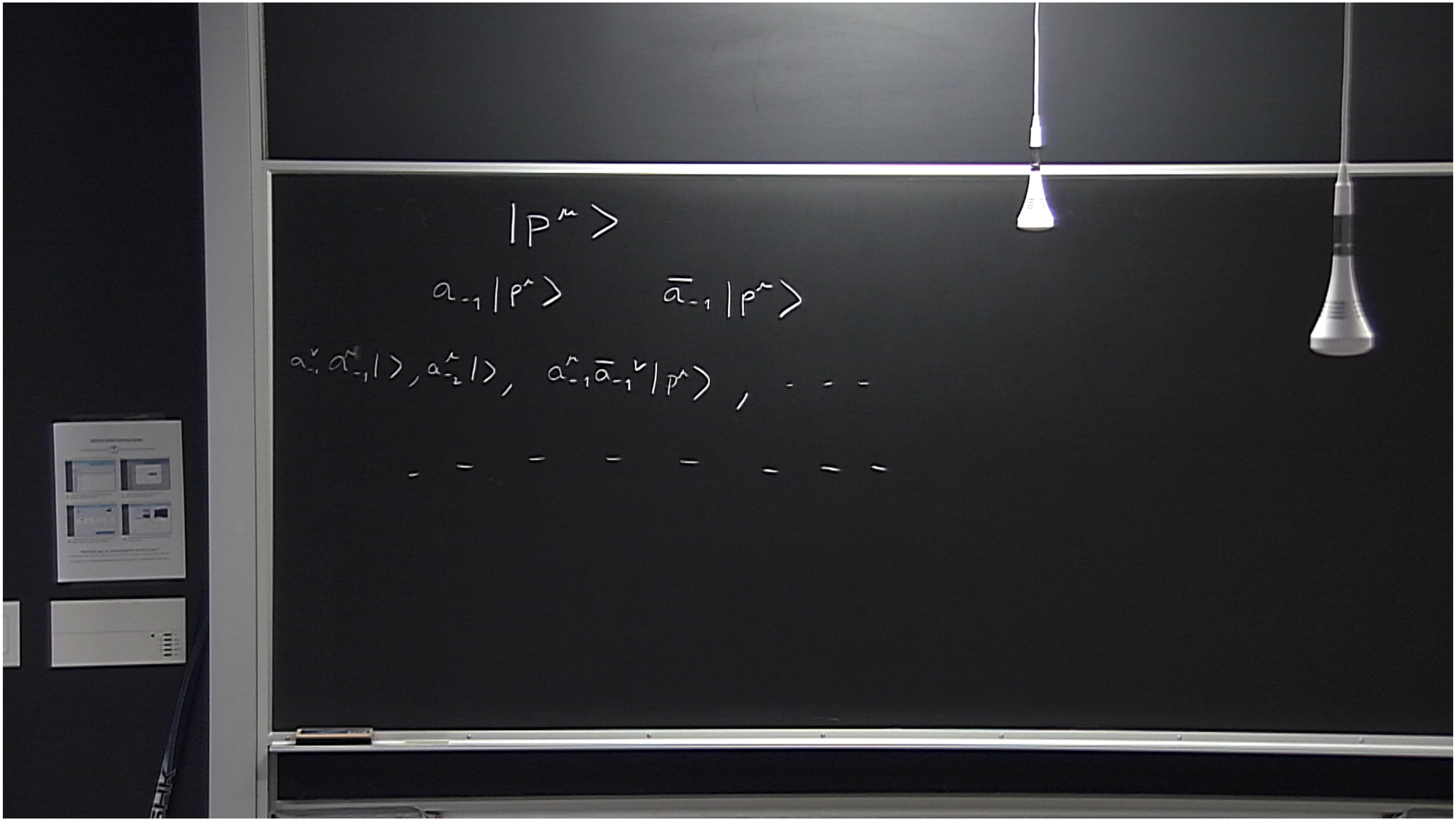
$d=26$

$$L_n | \mathrm{PHYS} \rangle = 0 \quad n>0 \qquad \langle \mathrm{PHYS} | L_n = 0 \quad n<0$$

$$(L_0 - 1) | \mathrm{PHYS} \rangle = 0$$

$$| \mathrm{NULL} \rangle = L_n | \cdots \rangle \quad n>0$$

$$\mathcal{H}_{\mathrm{STRING}} = \frac{\mathcal{H}_{\mathrm{PHYS}}}{\mathcal{H}_{\mathrm{NULL}}}$$


$|p^\mu\rangle$
$a_{-1}|p^\mu\rangle$ $\bar{a}_{-1}|p^\mu\rangle$
$a^\nu_{-1}a^\mu_{-1}|\rangle$, $a^\mu_{-2}|\rangle$, $a^\mu_{-1}\bar{a}^\nu_{-1}|p^\mu\rangle$, ...

$$\langle \mathrm{PHYS} | \, T(z) \, | \mathrm{PHYS} \rangle = 0$$

$d = 26$

$$L_n | \mathrm{PHYS} \rangle = 0 \qquad n > 0 \qquad\qquad \langle \mathrm{PHYS} | L_n = 0 \qquad n < 0$$

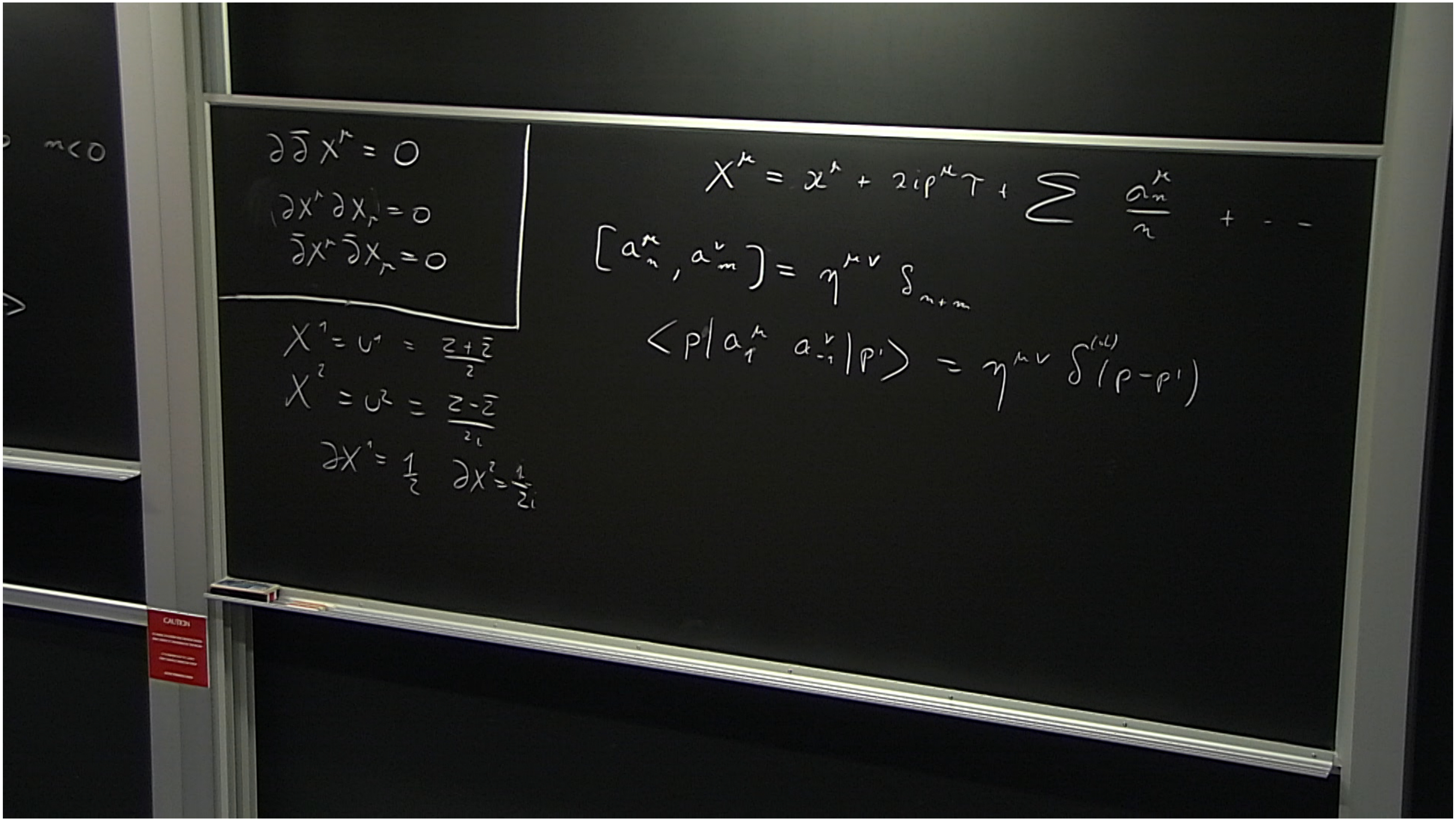
$$\bar{L}_n | \mathrm{PHYS} \rangle = 0$$

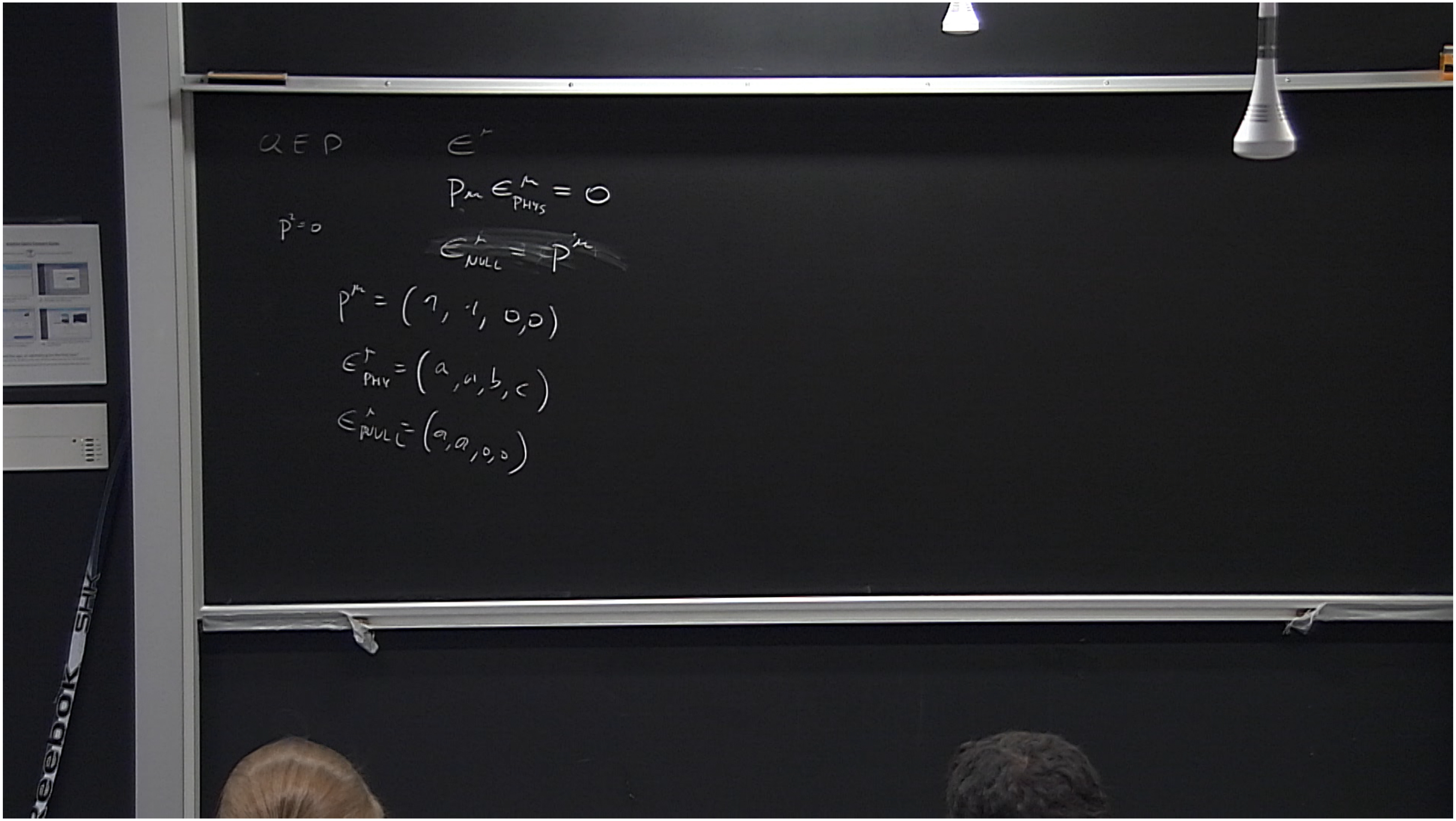
$$(L_0 - 1) | \mathrm{PHYS} \rangle = 0$$

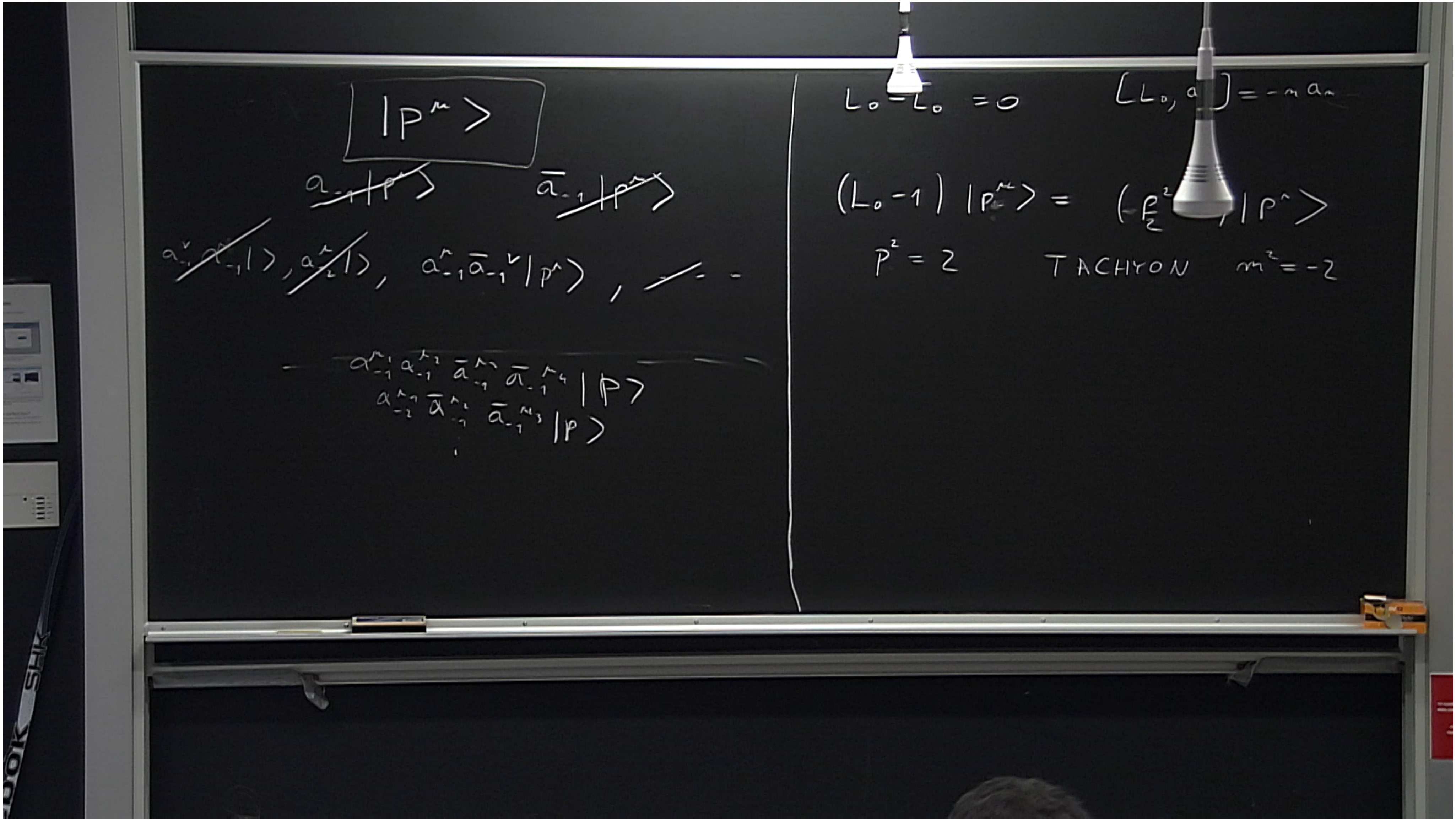
$$(\bar{L}_0 - 1) | \mathrm{PHYS} \rangle = 0$$

$$| \mathrm{NULL} \rangle = L_n | \cdots \rangle \quad n > 0 \qquad \text{OR} \qquad | \mathrm{NULL} \rangle = \bar{L}_n | \cdots \rangle$$

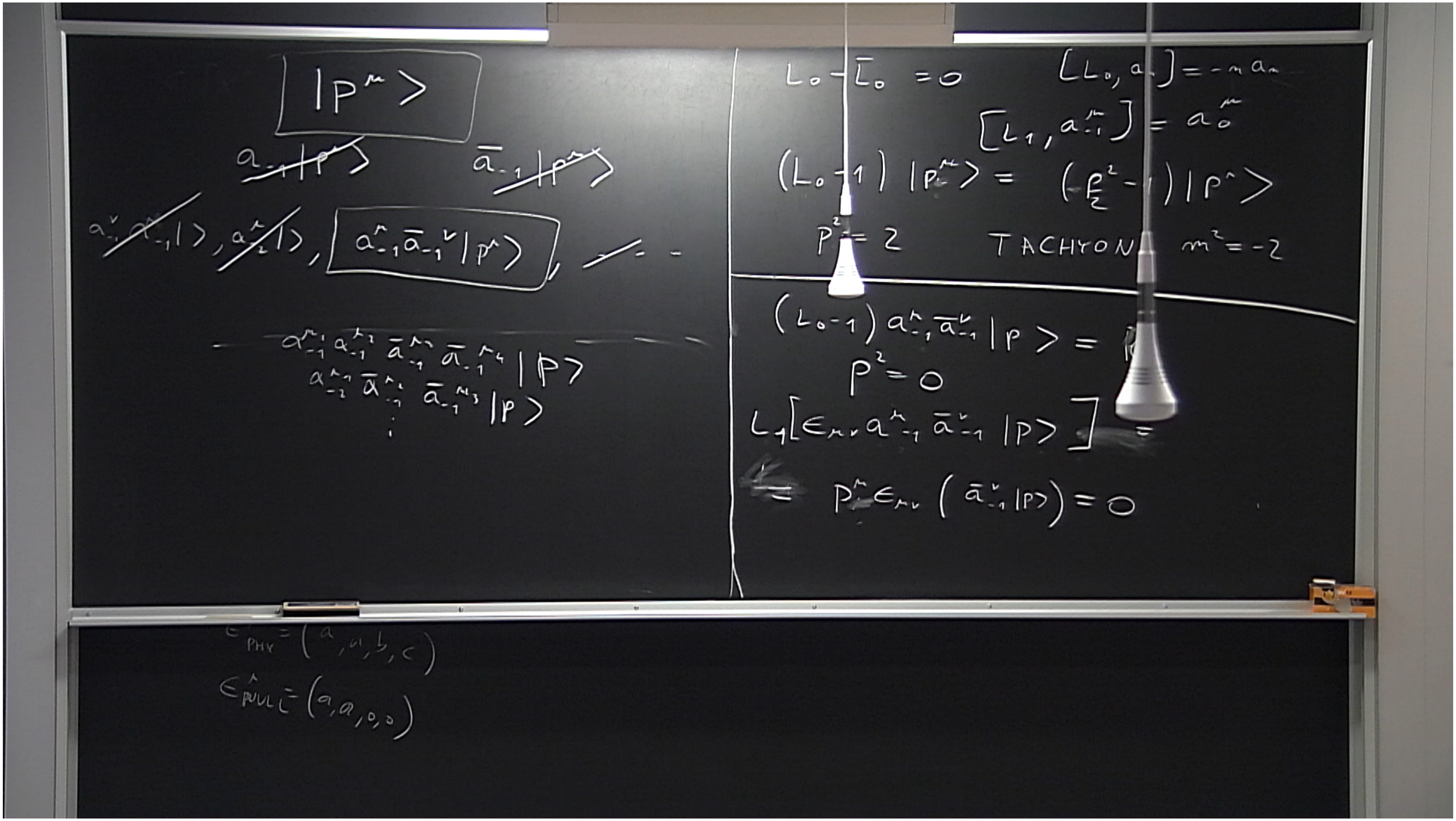
$$\mathcal{H}_{\mathrm{STRING}} = \frac{\mathcal{H}_{\mathrm{PHYS}}}{\mathcal{H}_{\mathrm{NULL}}}$$

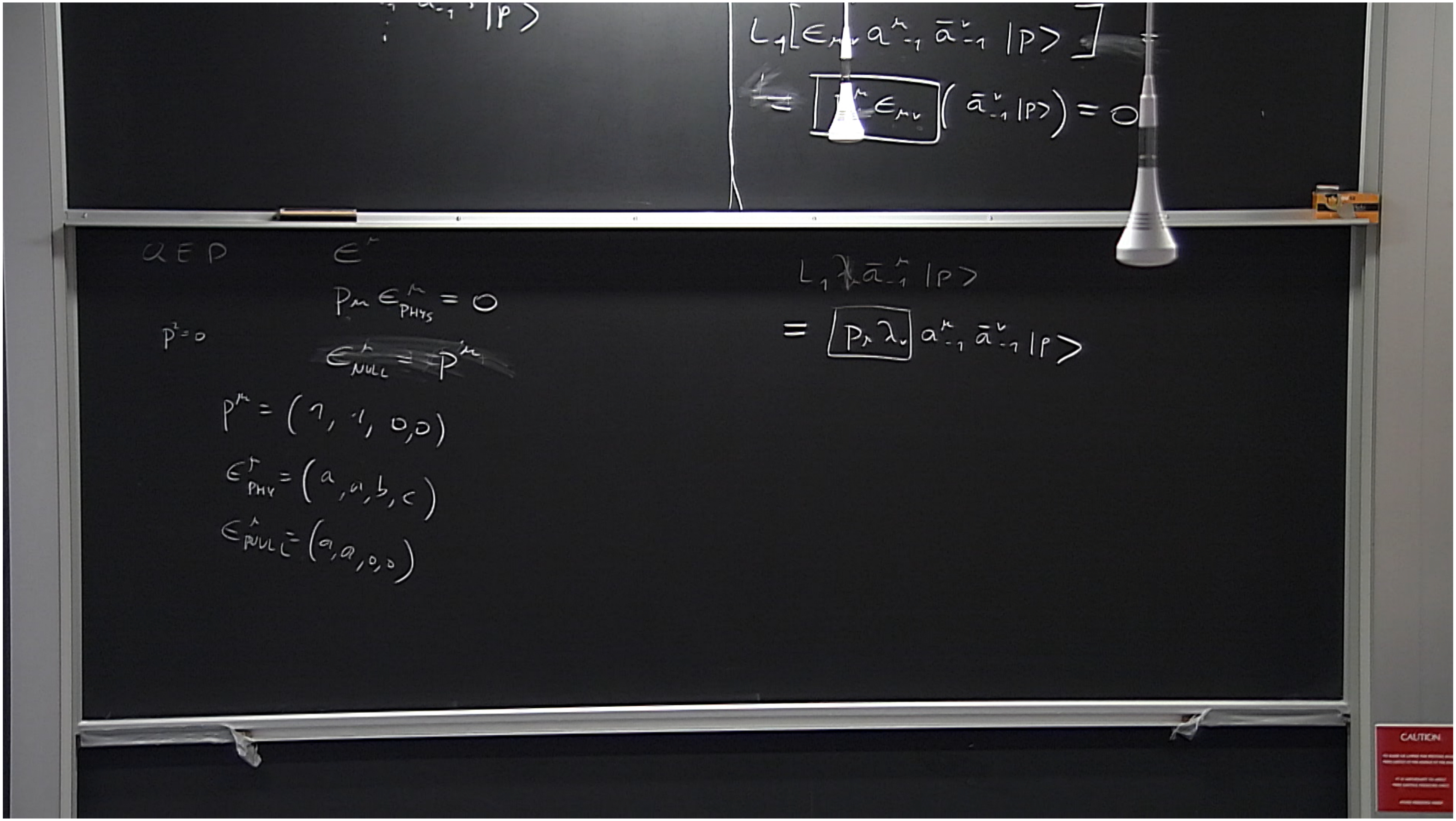

$m<0$
$\partial\bar\partial X^\mu = 0$
$\partial X^\mu \partial X_\mu = 0$
$\bar\partial X^\mu \bar\partial X_\mu = 0$
$X^1 = u^1 = \frac{z+\bar z}{2}$
$X^2 = u^2 = \frac{z-\bar z}{2i}$
$\partial X^1 = \frac{1}{2} \quad \partial X^2 = \frac{1}{2i}$
$X^\mu = x^\mu + 2ip^\mu\tau + \sum \frac{a_n^\mu}{n} + \dots$
$[a_n^\mu, a_m^\nu] = \eta^{\mu\nu}\delta_{n+m}$
$\langle p| a_1^\mu a_{-1}^\nu |p'\rangle = \eta^{\mu\nu}\delta^{(d)}(p-p')$

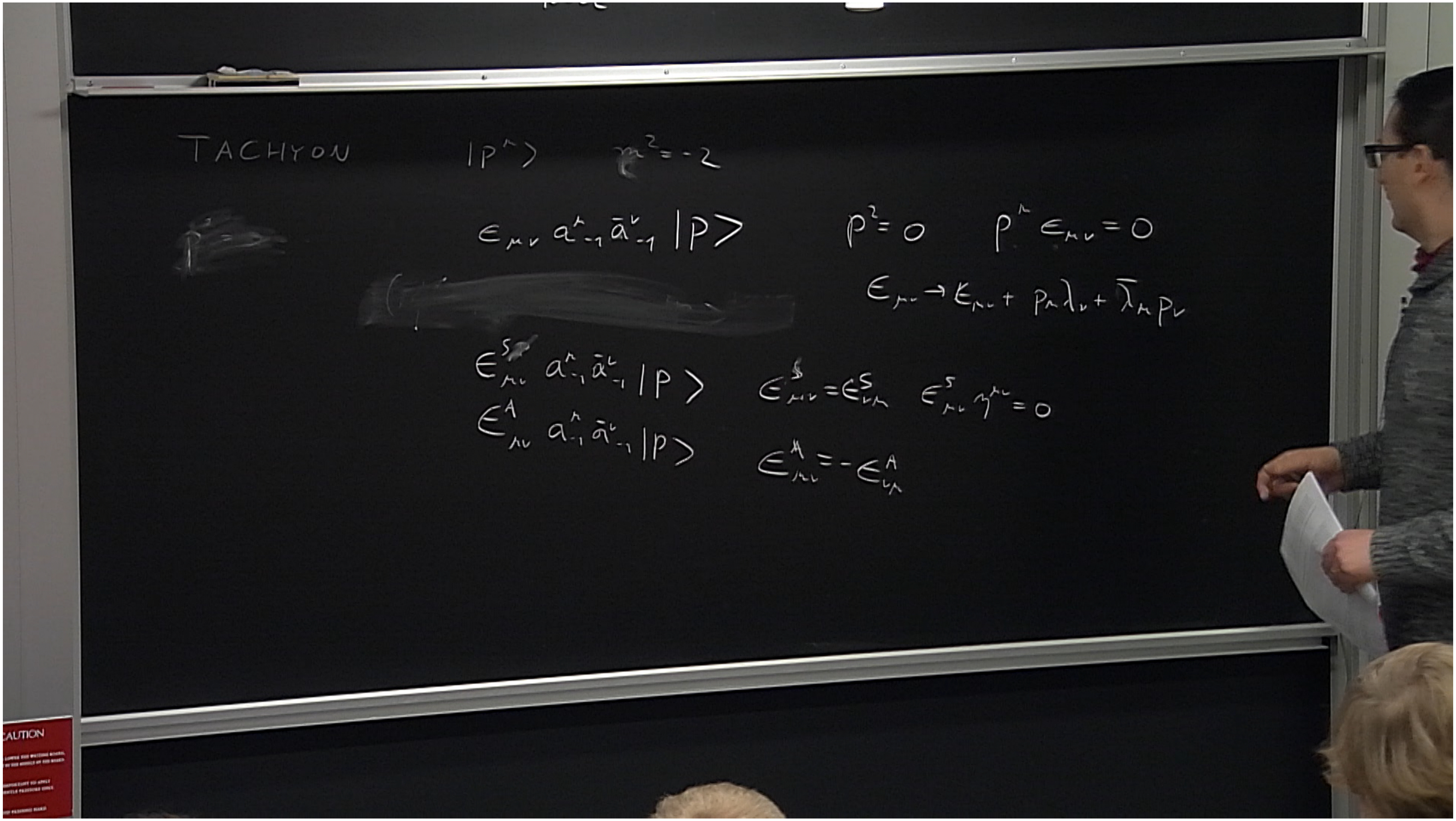

QED
ϵ^μ
$p_\mu \epsilon^\mu_{PHYS} = 0$
$p^2 = 0$
$\epsilon^\mu_{NULL} = p^\mu$
$p^\mu = (1, -1, 0, 0)$
$\epsilon^\mu_{PHY} = (a, a, b, c)$
$\epsilon^\mu_{NULL} = (a, a, 0, 0)$
Reebok
SHK



$|p^\mu\rangle$
$\cancel{a_{-1}|p^\mu\rangle}$
$\cancel{\bar{a}_{-1}|p^\mu\rangle}$
$\cancel{a^\nu_{-1}a^\mu_{-1}|\rangle}$, $\cancel{a^\mu_{-2}|\rangle}$, $a^\mu_{-1}\bar{a}^\nu_{-1}|p^\mu\rangle$, ...
$a^{\mu_1}_{-1}a^{\mu_2}_{-1}\bar{a}^{\mu_3}_{-1}\bar{a}^{\mu_4}_{-1}|p\rangle$
$a^{\mu_1}_{-2}\bar{a}^{\mu_2}_{-1}\bar{a}^{\mu_3}_{-1}|p\rangle$
$L_0-\bar{L}_0=0$
$[L_0, a_m]=-m\,a_m$
$(L_0-1)\,|p^\mu\rangle = (-\frac{p^2}{2}$
$|p^\mu\rangle$
$p^2=2$
TACHYON
$m^2=-2$


|p^μ>
a_{-1}|p^μ>   ā_{-1}|p^μ>
α^ν_{-1} α^μ_{-1}|>, α^μ_{-2}|>, α^μ_{-1} ᾱ^ν_{-1}|p^μ>, ...
α^{μ1}_{-1} α^{μ2}_{-1} ᾱ^{μ3}_{-1} ᾱ^{μ4}_{-1}|p>
α^{μ1}_{-2} ᾱ^{μ2}_{-1} ᾱ^{μ3}_{-1}|p>
L_0 − L̄_0 = 0
[L_0, a_n] = −n a_n
[L_1, a^μ_{-1}] = a^μ_0
(L_0 − 1)|p^μ> = (p²/2 − 1)|p^μ>
p² = 2   TACHYON   m² = −2
(L_0 − 1) a^μ_{-1} ā^ν_{-1}|p> =
p² = 0
L_1[ε_{μν} a^μ_{-1} ā^ν_{-1}|p>] =
= p^μ ε_{μν} (ā^ν_{-1}|p>) = 0
ε_PHY = (a, a, b, c)
ε_NULL = (a, a, 0, 0)


$L_1[\epsilon_{\mu\nu} a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle] =$
$= \boxed{p^\mu \epsilon_{\mu\nu}} (\bar{a}^\nu_{-1} |p\rangle) = 0$
QED
ϵ^μ
$p_\mu \epsilon^\mu_{PHYS} = 0$
$p^2 = 0$
$\epsilon^\mu_{NULL} = p^\mu$
$p^\mu = (1, 1, 0, 0)$
$\epsilon^\mu_{PHY} = (a, a, b, c)$
$\epsilon^\mu_{NULL} = (a, a, 0, 0)$
$L_1 \lambda_\mu \bar{a}^\mu_{-1} |p\rangle$
$= \boxed{p_\nu \lambda_\nu} a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle$
CAUTION


TACHYON
$|p^\mu\rangle$ $m^2 = -2$
$\epsilon_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle$ $p^2 = 0$ $p^\mu \epsilon_{\mu\nu} = 0$
$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + p_\mu \lambda_\nu + \bar{\lambda}_\mu p_\nu$
$\epsilon^S_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle$ $\epsilon^S_{\mu\nu} = \epsilon^S_{\nu\mu}$ $\epsilon^S_{\mu\nu} \eta^{\mu\nu} = 0$
$\epsilon^A_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle$ $\epsilon^A_{\mu\nu} = -\epsilon^A_{\nu\mu}$
CAUTION

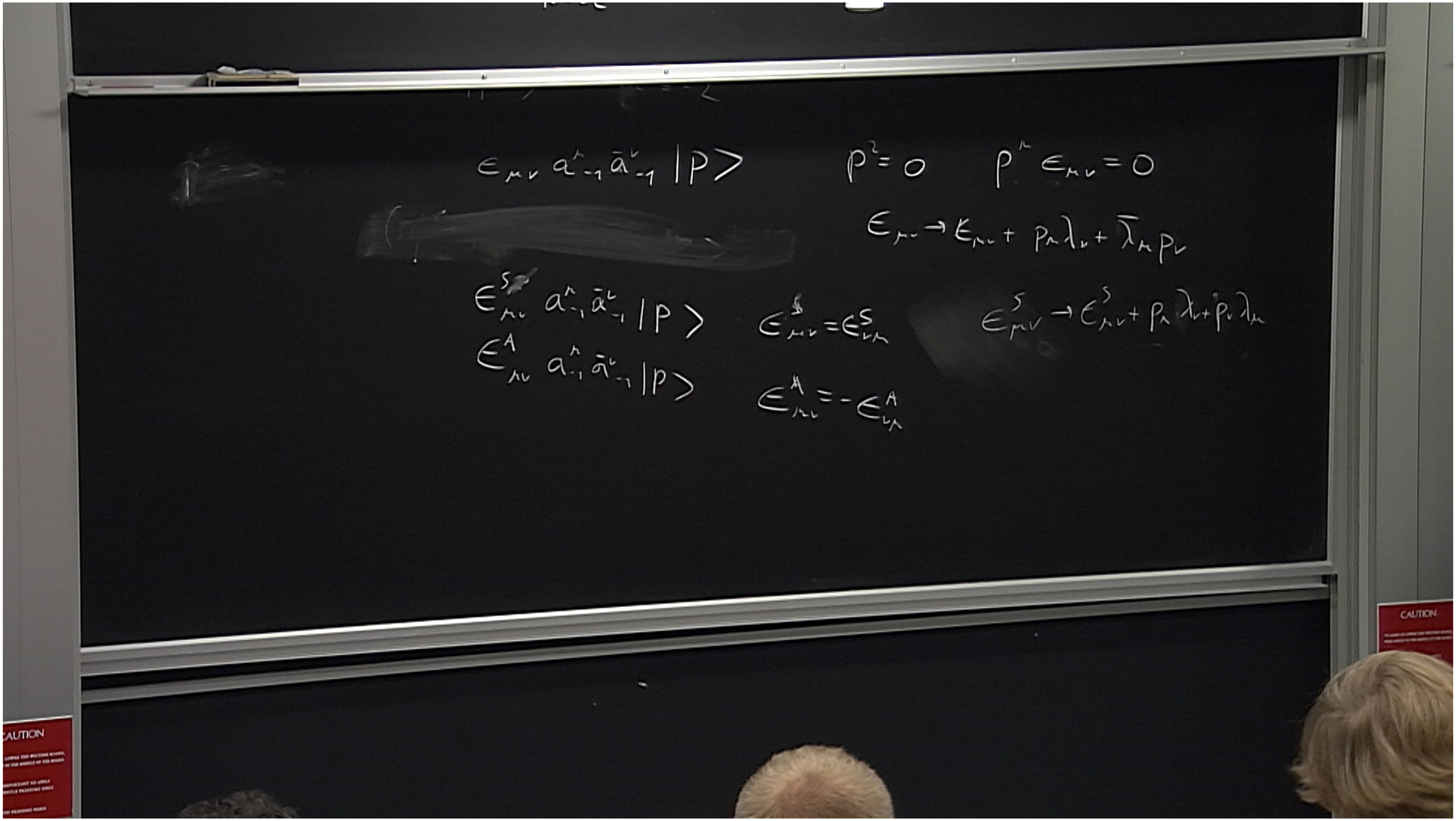
TACHYON $|p^\mu\rangle$ $m^2 = -2$

$$\epsilon_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle \qquad p^2 = 0 \qquad p^\mu \epsilon_{\mu\nu} = 0$$

$$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + p_\mu \lambda_\nu + \bar{\lambda}_\mu p_\nu$$

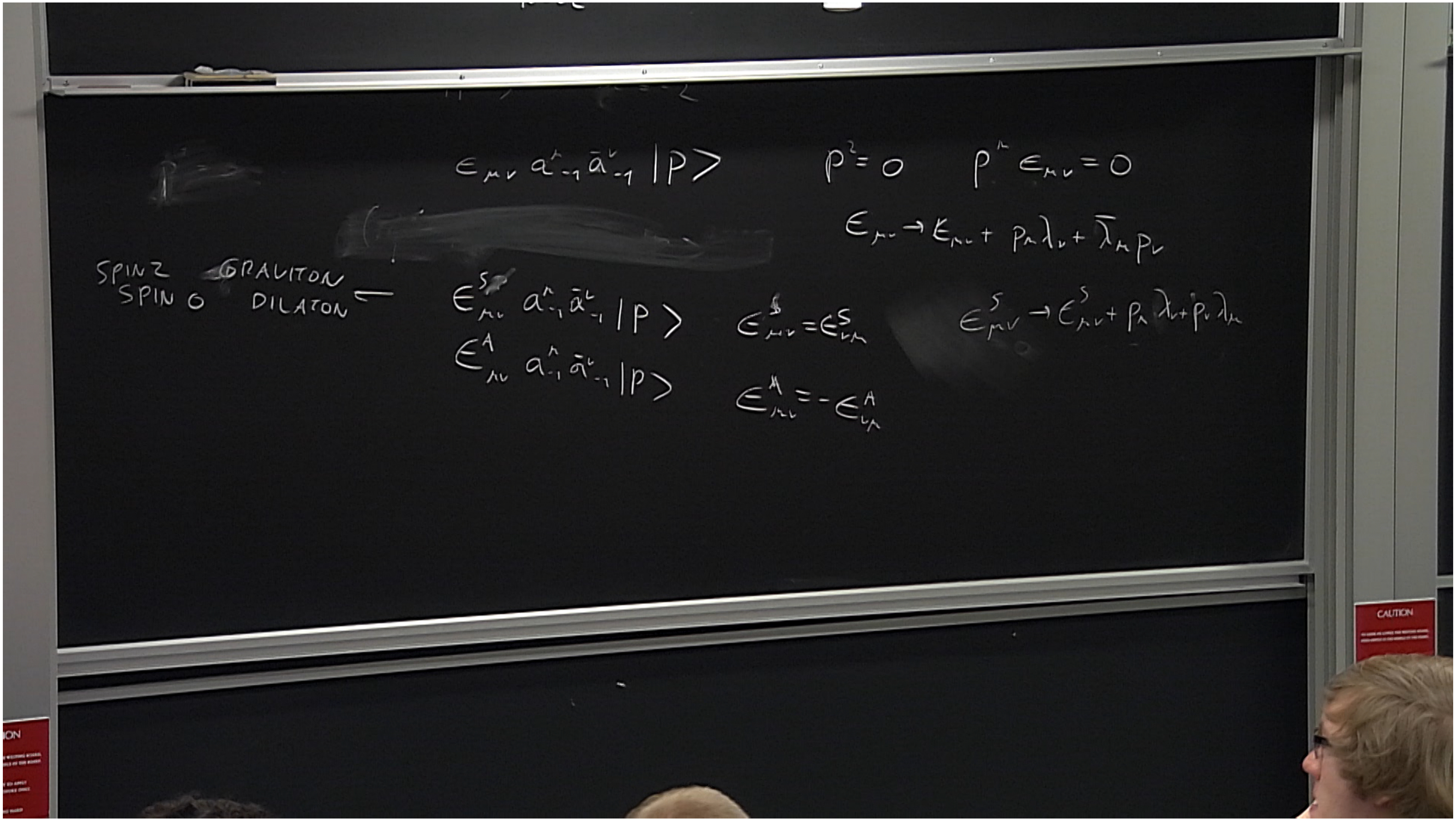
$$\epsilon^S_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle \qquad \epsilon^S_{\mu\nu} = \epsilon^S_{\nu\mu}$$

$$\epsilon^A_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle \qquad \epsilon^A_{\mu\nu} = -\epsilon^A_{\nu\mu}$$


$\epsilon_{\mu\nu} a^{\mu}_{-1} \bar{a}^{\nu}_{-1} |p\rangle$
$p^2 = 0$
$p^{\mu} \epsilon_{\mu\nu} = 0$
$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + p_{\mu}\lambda_{\nu} + \bar{\lambda}_{\mu} p_{\nu}$
$\epsilon^{S}_{\mu\nu} a^{\mu}_{-1} \bar{a}^{\nu}_{-1} |p\rangle$
$\epsilon^{S}_{\mu\nu} = \epsilon^{S}_{\nu\mu}$
$\epsilon^{S}_{\mu\nu} \to \epsilon^{S}_{\mu\nu} + p_{\mu}\lambda_{\nu} + p_{\nu}\lambda_{\mu}$
$\epsilon^{A}_{\mu\nu} a^{\mu}_{-1} \bar{a}^{\nu}_{-1} |p\rangle$
$\epsilon^{A}_{\mu\nu} = -\epsilon^{A}_{\nu\mu}$

$$\Box h_{\mu\nu} = 0 \qquad \partial^\mu h_{\mu\nu} = 0 \qquad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu v_\nu + \partial_\nu v_\mu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$


$\epsilon_{\mu\nu}\, a^{\mu}_{-1} \bar{a}^{\nu}_{-1} |p\rangle$
$p^2 = 0$
$p^{\mu} \epsilon_{\mu\nu} = 0$
$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + p_{\mu}\lambda_{\nu} + \bar{\lambda}_{\mu} p_{\nu}$
SPIN 2 GRAVITON
SPIN 0 DILATON
$\epsilon^{S}_{\mu\nu}\, a^{\mu}_{-1} \bar{a}^{\nu}_{-1} |p\rangle$
$\epsilon^{S}_{\mu\nu} = \epsilon^{S}_{\nu\mu}$
$\epsilon^{S}_{\mu\nu} \to \epsilon^{S}_{\mu\nu} + p_{\mu}\lambda_{\nu} + p_{\nu}\lambda_{\mu}$
$\epsilon^{A}_{\mu\nu}\, a^{\mu}_{-1} \bar{a}^{\nu}_{-1} |p\rangle$
$\epsilon^{A}_{\mu\nu} = -\epsilon^{A}_{\nu\mu}$
CAUTION

$$\epsilon_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle \qquad p^2 = 0 \qquad p^\mu \epsilon_{\mu\nu} = 0$$

$$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + p_\mu \lambda_\nu + \bar{\lambda}_\mu p_\nu$$

SPIN 2 GRAVITON
SPIN 0 DILATON $\leftarrow$ $\epsilon^S_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle \qquad \epsilon^S_{\mu\nu} = \epsilon^S_{\nu\mu} \qquad \epsilon^S_{\mu\nu} \to \epsilon^S_{\mu\nu} + p_\mu \lambda_\nu + p_\nu \lambda_\mu$

$\leftarrow$ $\epsilon^A_{\mu\nu}\, \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |p\rangle \qquad \epsilon^A_{\mu\nu} = -\epsilon^A_{\nu\mu} \qquad \epsilon^A_{\mu\nu} \to \epsilon^A_{\mu\nu} + p_\mu \lambda_\nu - p_\nu \lambda_\mu$

$$\Box h_{\mu\nu} = 0 \qquad \partial^\mu h_{\mu\nu} = 0 \qquad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu v_\nu + \partial_\nu v_\mu$$

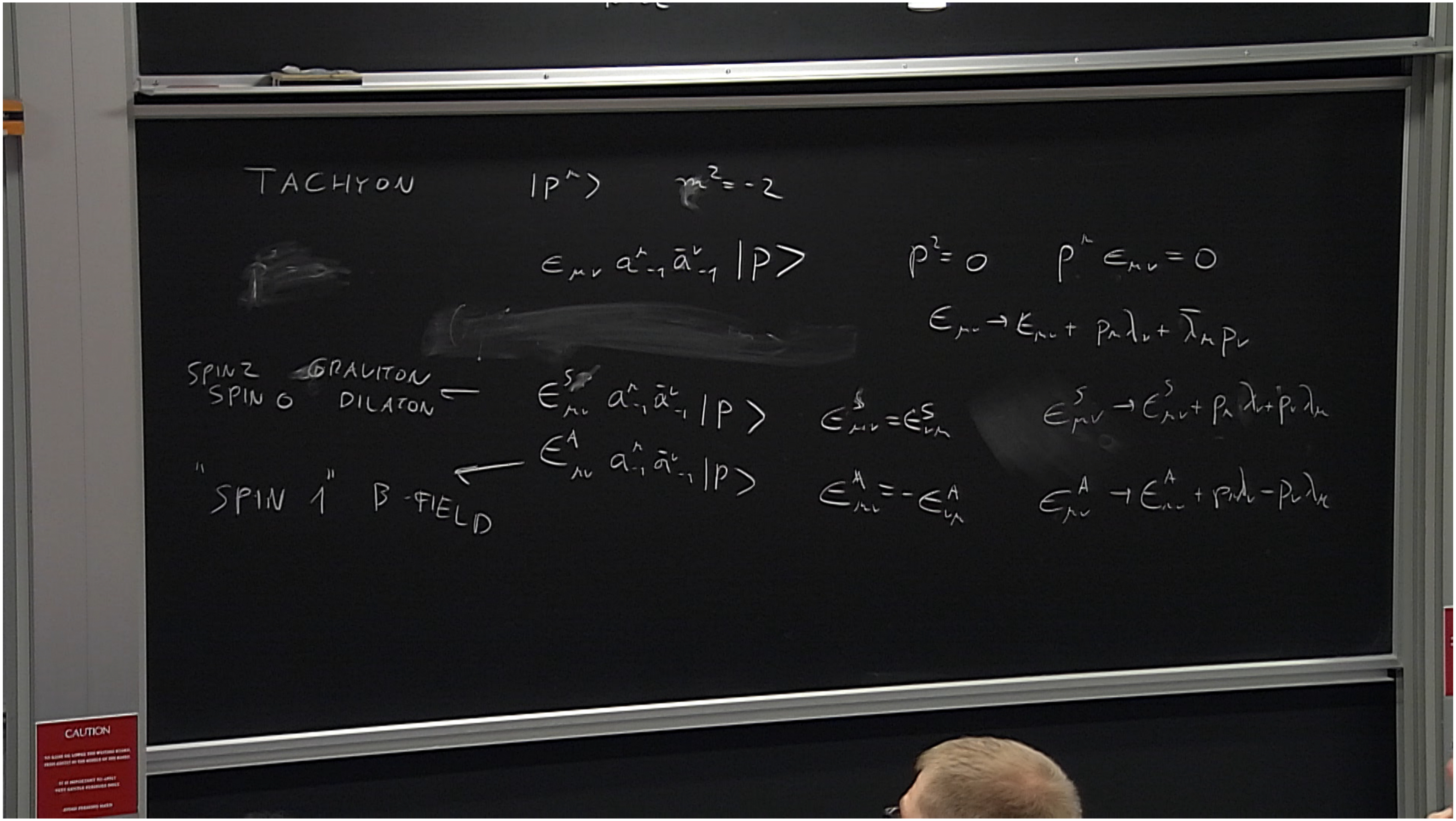
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$B = B_{\mu\nu}\, dx^\mu \wedge dx^\nu \qquad B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu$$

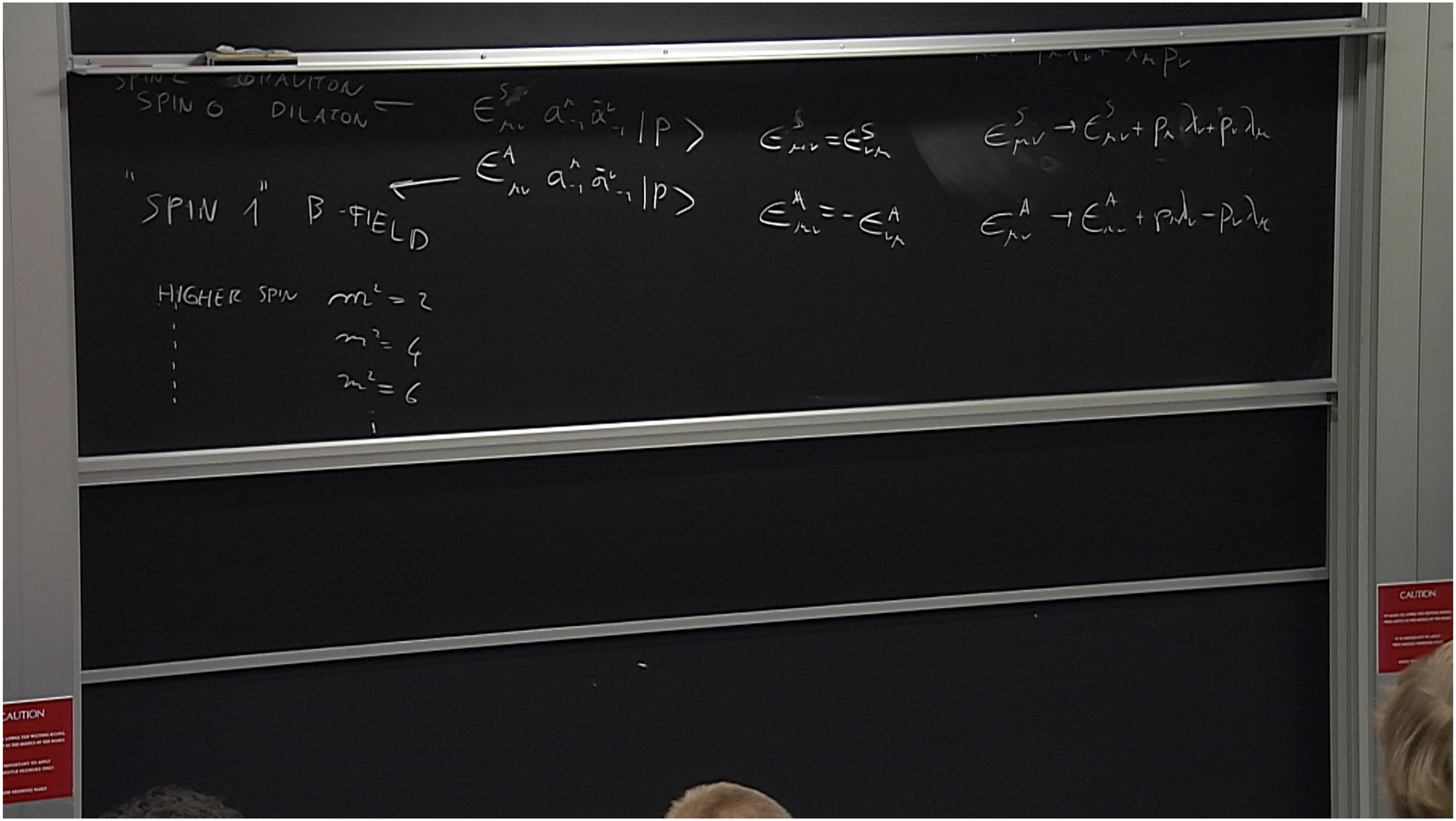
$$B \to B + d\lambda$$

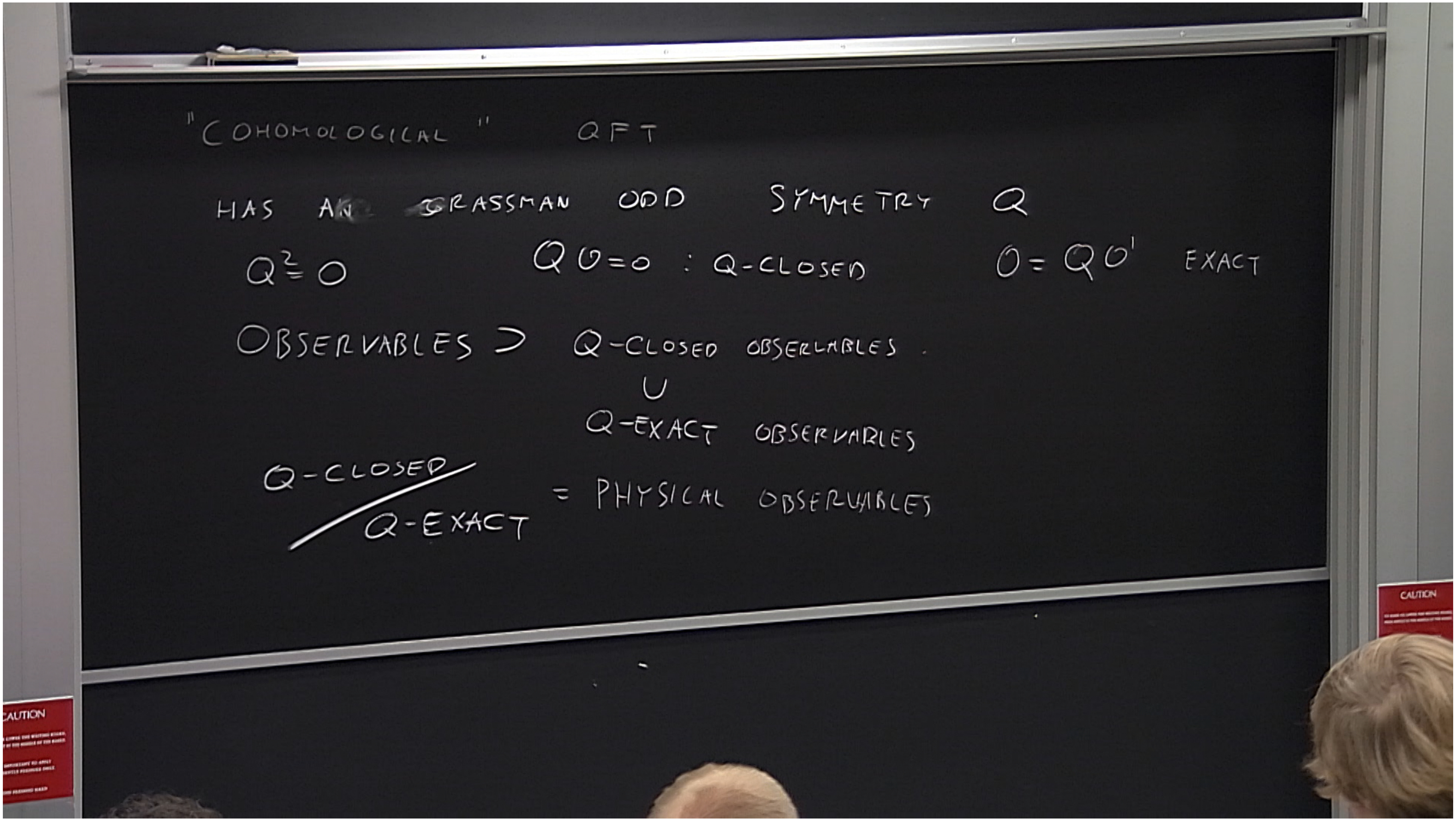
$$\int_\ell A = \int A_\mu \dot{x}^\mu$$

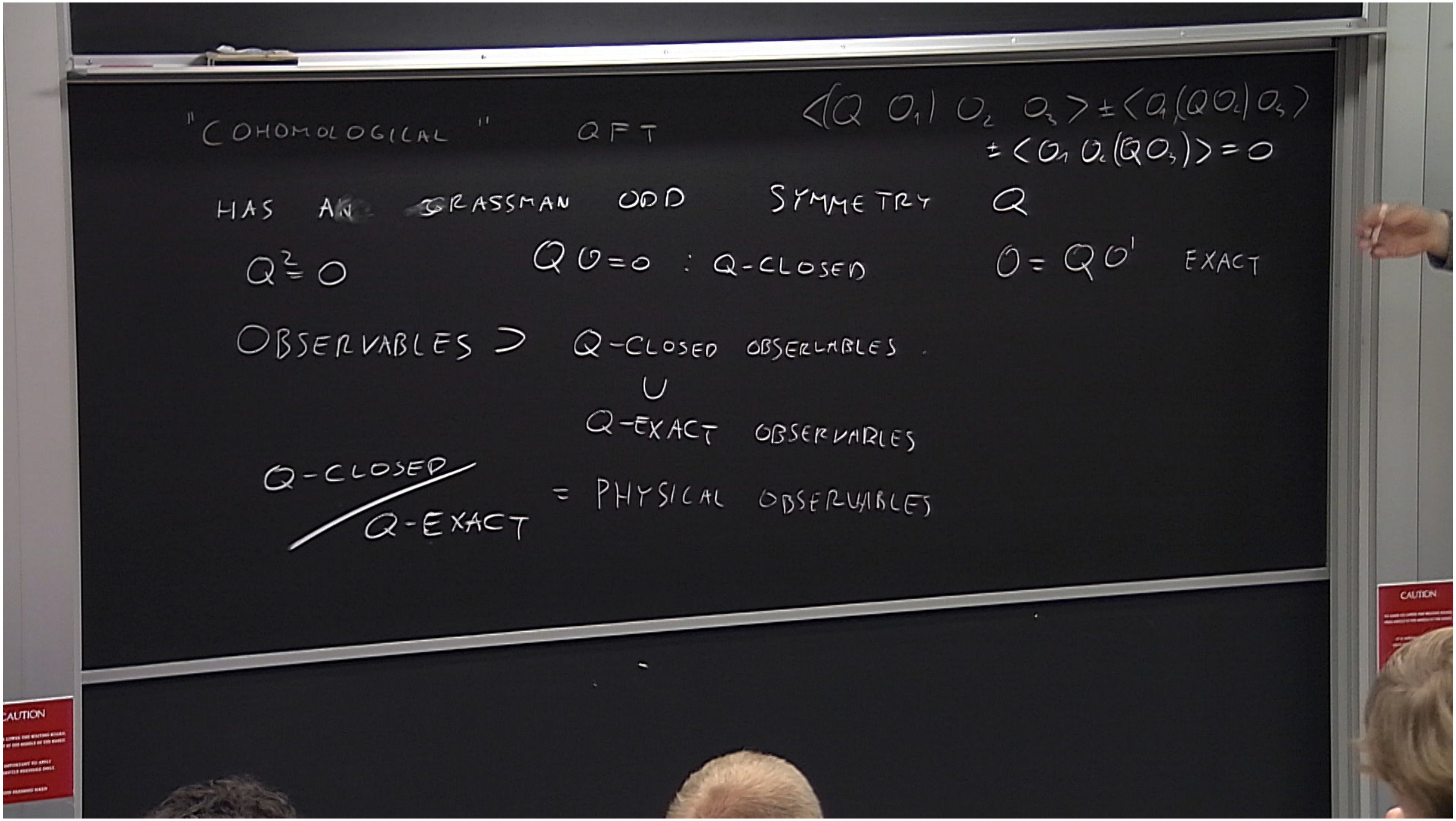
$$\int_\Sigma B = \int B_{\mu\nu} \frac{\partial x^\mu}{\partial v^1} \frac{\partial x^\nu}{\partial v^2}$$

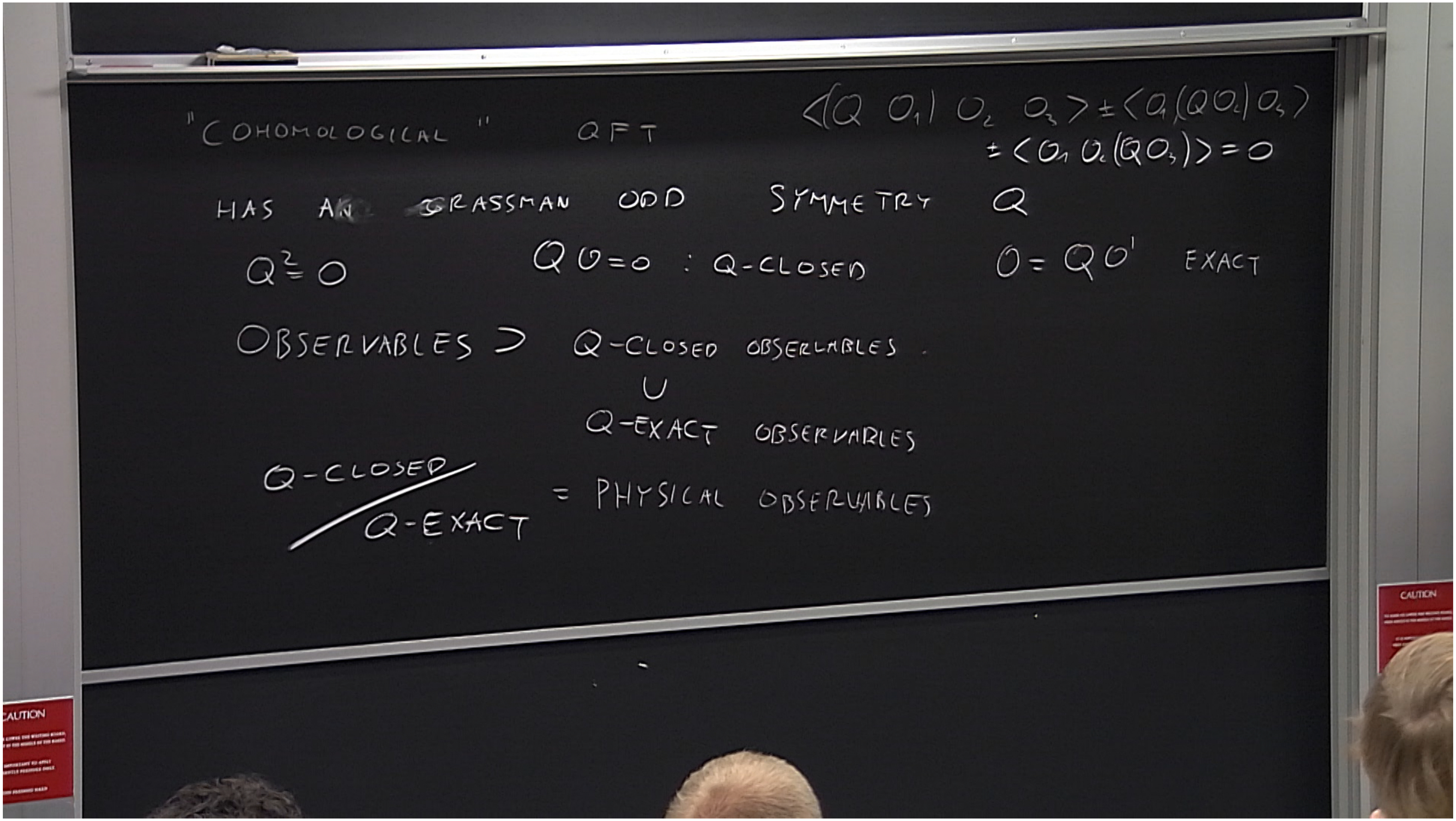


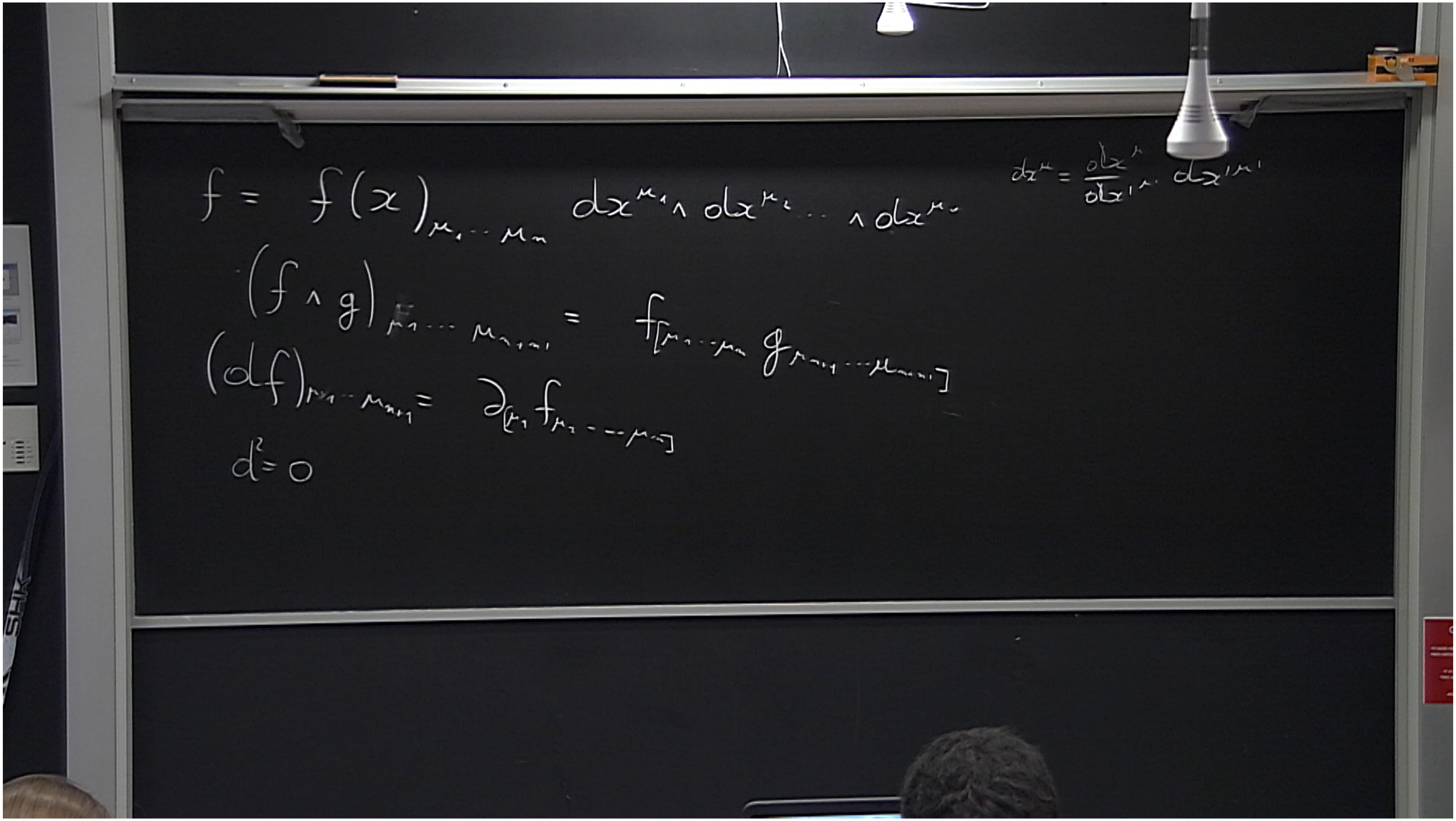
TACHYON $|p^\mu\rangle$ $m^2 = -2$
$\epsilon_{\mu\nu}\, a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle$ $p^2 = 0$ $p^\mu \epsilon_{\mu\nu} = 0$
$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + p_\mu \lambda_\nu + \bar{\lambda}_\mu p_\nu$
SPIN 2 GRAVITON
SPIN 0 DILATON
$\epsilon^S_{\mu\nu}\, a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle$ $\epsilon^S_{\mu\nu} = \epsilon^S_{\nu\mu}$ $\epsilon^S_{\mu\nu} \to \epsilon^S_{\mu\nu} + p_\mu \lambda_\nu + p_\nu \lambda_\mu$
$\epsilon^A_{\mu\nu}\, a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle$ $\epsilon^A_{\mu\nu} = -\epsilon^A_{\nu\mu}$ $\epsilon^A_{\mu\nu} \to \epsilon^A_{\mu\nu} + p_\mu \lambda_\nu - p_\nu \lambda_\mu$
"SPIN 1" B-FIELD

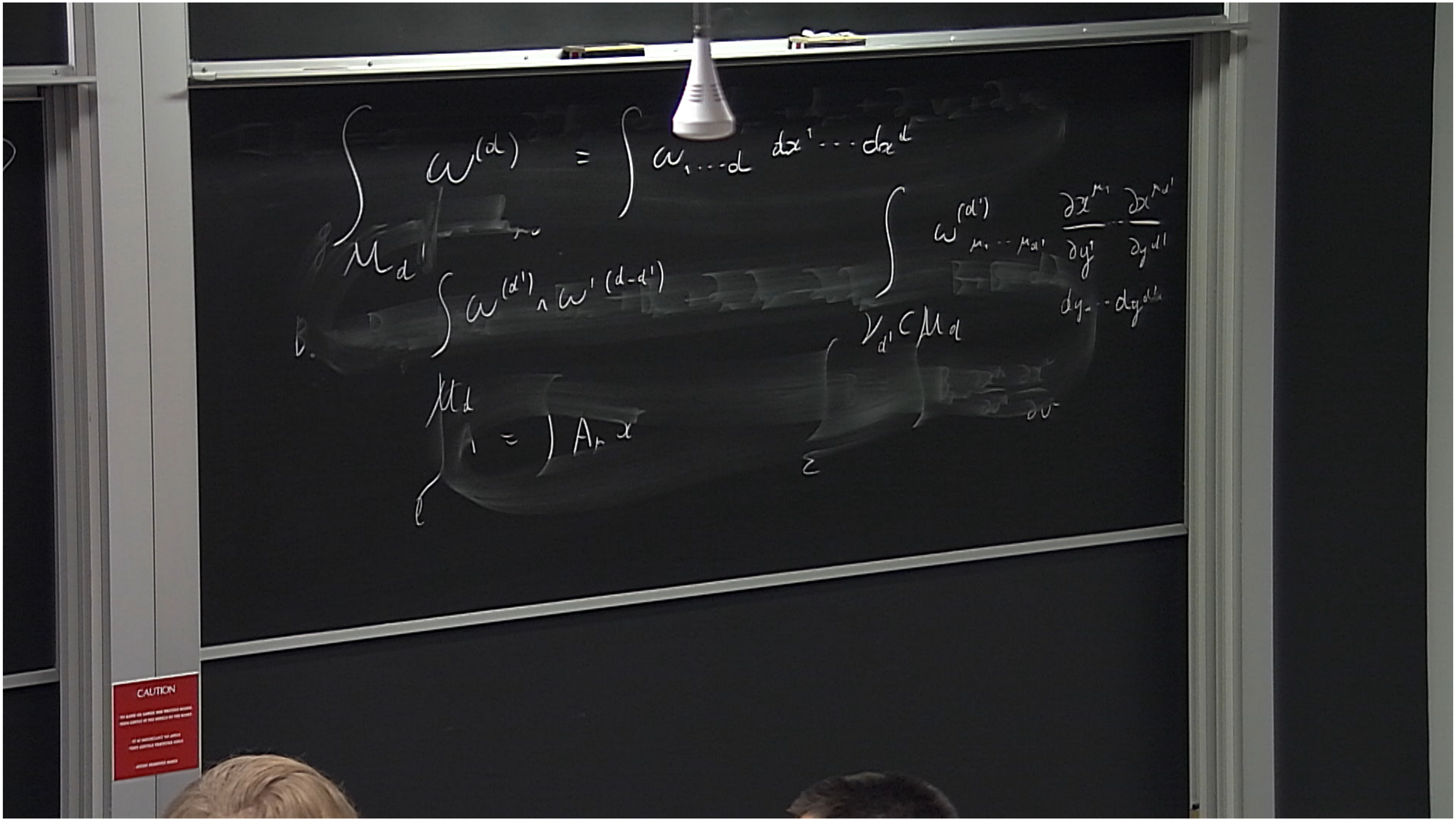

SPIN 2 GRAVITON
SPIN 0 DILATON ← $\epsilon^S_{\mu\nu} a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle$
$\epsilon^S_{\mu\nu} = \epsilon^S_{\nu\mu}$
$\epsilon^S_{\mu\nu} \to \epsilon^S_{\mu\nu} + p_\mu \lambda_\nu + p_\nu \lambda_\mu$
"SPIN 1" B-FIELD ← $\epsilon^A_{\mu\nu} a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle$
$\epsilon^A_{\mu\nu} = -\epsilon^A_{\nu\mu}$
$\epsilon^A_{\mu\nu} \to \epsilon^A_{\mu\nu} + p_\mu \lambda_\nu - p_\nu \lambda_\mu$
HIGHER SPIN $m^2 = 2$
$m^2 = 4$
$m^2 = 6$
CAUTION

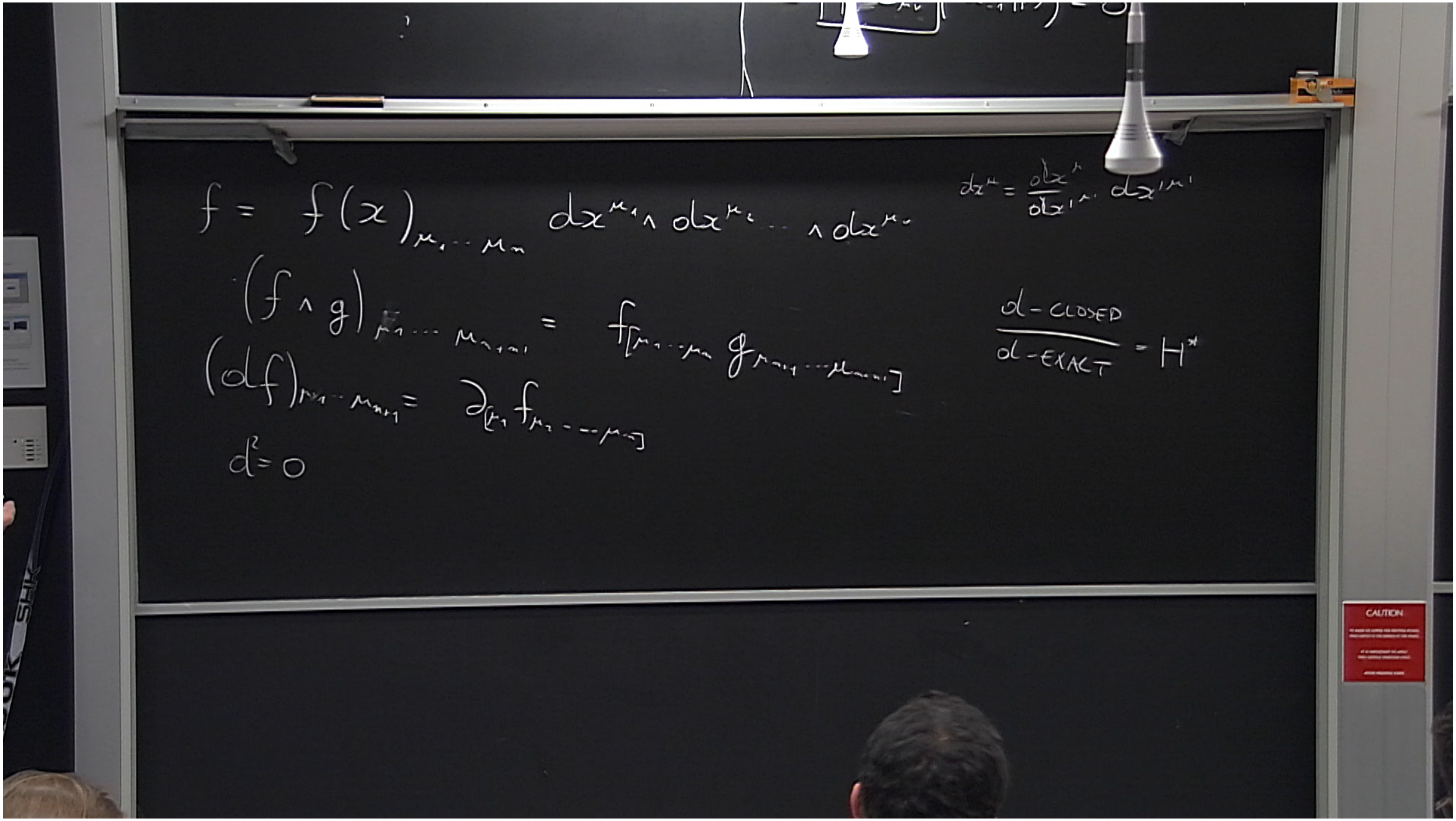

"COHOMOLOGICAL" QFT
HAS AN GRASSMAN ODD SYMMETRY Q
Q² = 0
QO = 0 : Q-CLOSED
O = QO' EXACT
OBSERVABLES ⊃ Q-CLOSED OBSERVABLES
∪
Q-EXACT OBSERVABLES
Q-CLOSED / Q-EXACT = PHYSICAL OBSERVABLES

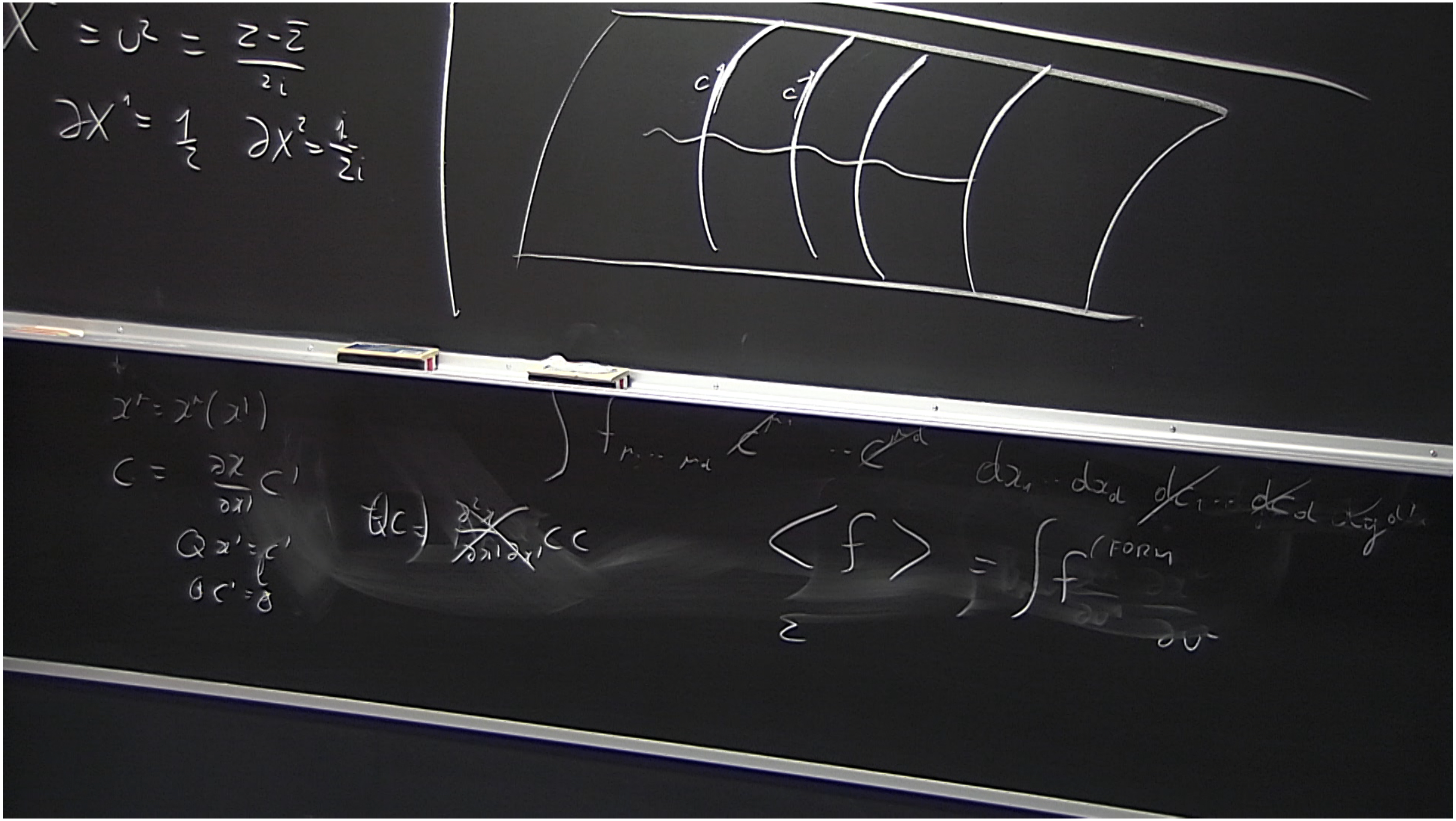

"COHOMOLOGICAL" QFT
$\langle (Q\,O_1)\, O_2\, O_3 \rangle \pm \langle O_1 (Q O_2) O_3 \rangle$
$\pm \langle O_1 O_2 (Q O_3) \rangle = 0$
HAS AN GRASSMAN ODD SYMMETRY Q
$Q^2 = 0$
$Q\,O = 0$: Q-CLOSED
$O = Q\,O'$ EXACT
OBSERVABLES ⊃ Q-CLOSED OBSERVABLES
∪
Q-EXACT OBSERVABLES
Q-CLOSED / Q-EXACT = PHYSICAL OBSERVABLES


"COHOMOLOGICAL" QFT
$\langle (Q O_1)\, O_2\, O_3 \rangle \pm \langle O_1 (Q O_2)\, O_3 \rangle$
$\pm \langle O_1 O_2 (Q O_3) \rangle = 0$
HAS AN GRASSMAN ODD SYMMETRY Q
$Q^2 = 0$
$Q\mathcal{O} = 0$: Q-CLOSED
$\mathcal{O} = Q\mathcal{O}'$ EXACT
OBSERVABLES $\supset$ Q-CLOSED OBSERVABLES
$\cup$
Q-EXACT OBSERVABLES
Q-CLOSED / Q-EXACT = PHYSICAL OBSERVABLES


$f = f(x)_{\mu_1 \cdots \mu_n} dx^{\mu_1} \wedge dx^{\mu_2} \cdots \wedge dx^{\mu_n}$
$dx^{\mu} = \frac{\partial x^{\mu}}{\partial x'^{\mu'}} dx'^{\mu'}$
$(f \wedge g)_{\mu_1 \cdots \mu_{n+m}} = f_{[\mu_1 \cdots \mu_n} g_{\mu_{n+1} \cdots \mu_{n+m}]}$
$(df)_{\mu_1 \cdots \mu_{n+1}} = \partial_{[\mu_1} f_{\mu_2 \cdots \mu_{n+1}]}$
$d^2 = 0$


$\int_{M_d} \omega^{(d)} = \int \omega_{1\cdots d}\, dx^1 \cdots dx^d$
$\int_{M_d} \omega^{(d')} \wedge \omega'^{(d-d')}$
$\int_{V_{d'} \subset M_d} \omega^{(d')}_{\mu_1 \cdots \mu_{d'}} \frac{\partial x^{\mu_1}}{\partial y^1} \cdots \frac{\partial x^{\mu_{d'}}}{\partial y^{d'}} dy_1 \cdots dy^{d'}$
CAUTION


$f = f(x)_{\mu_1 \cdots \mu_n} dx^{\mu_1} \wedge dx^{\mu_2} \cdots \wedge dx^{\mu_n}$
$(f \wedge g)_{\mu_1 \cdots \mu_{n+m}} = f_{[\mu_1 \cdots \mu_n} g_{\mu_{n+1} \cdots \mu_{n+m}]}$
$(df)_{\mu_1 \cdots \mu_{n+1}} = \partial_{[\mu_1} f_{\mu_2 \cdots \mu_{n+1}]}$
$d^2 = 0$
$dx^\mu = \frac{\partial x^\mu}{\partial x'^\nu} dx'^\nu$
$\frac{d\text{-CLOSED}}{d\text{-EXACT}} = H^*$
CAUTION


= u² = (z − z̄)/2i
∂x¹ = 1/2   ∂x² = 1/2i
c
c
x^μ = x^μ(x')
c = ∂x/∂x' c'
Q x' = c'
Q c' = 0
⟨f⟩ = ∫ f (FORM
dx₁ .. dx_d