Title: PSI 2018/2019 - String Theory Review - Lecture 3

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Collection: PSI 2018/2019 - String Theory Review (Gaiotto)

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$$S[X^\mu, \delta_{ab}] = \frac{T}{2}\iint \frac{\partial X^\mu}{\partial v^a}\frac{\partial X_\mu}{\partial v_a}\, dv^1 dv^2$$

$$h_{ab} = \delta_{ab}$$

$$\sigma \equiv \sigma + 2\pi L$$

$$X^0, X^1 \dots X^d$$

$$v^1 = \tau$$
$$v^2 = \sigma$$

$$X(\sigma,0) = \sum x_n e^{in\frac{\sigma}{L}}$$

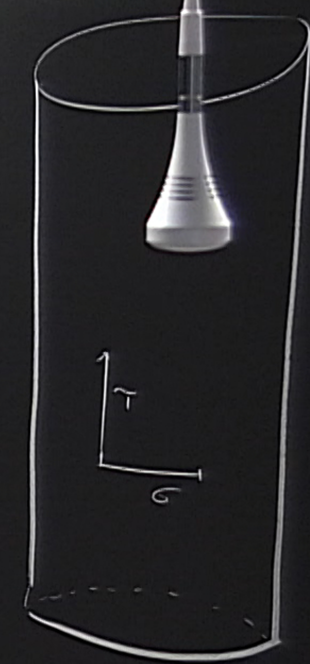
$$S[X] = T\int \partial_\zeta X\, \partial_{\bar\zeta} X$$

$$\Pi(\sigma,0) = \partial_\tau X(\sigma,0) = \sum p_n e^{in\frac{\sigma}{L}}$$

$$\tau = it$$

$$\zeta = \tau - i\sigma$$

$$g^{\mu\nu} = \eta^{\mu\nu}$$



Σ

$$X(\sigma,\tau) = x - \frac{2i}{L} p \tau + \sum_{n\neq 0} \left[\frac{i}{n} a_n e^{i\frac{n}{L}(\sigma+i\tau)} - \frac{i}{n} \bar{a}_n e^{-i\frac{n}{L}(\sigma-i\tau)} \right]$$

$$[a_n, a_m] = n\,\delta_{n+m}$$

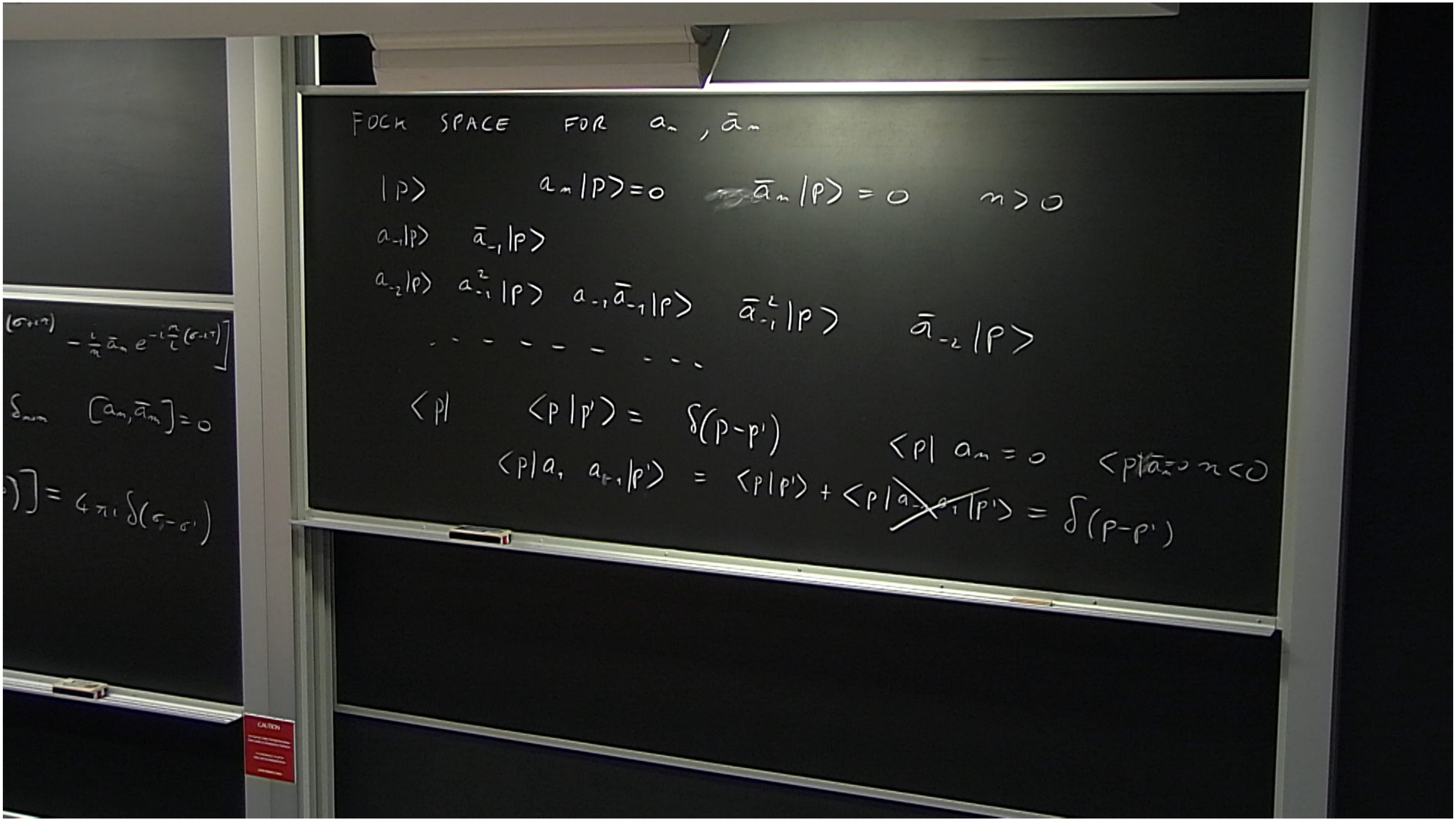
$$[\bar{a}_n, \bar{a}_m] = n\,\delta_{n+m}$$

$$[a_n, \bar{a}_m] = 0$$

$$[x, p] = i$$

$$[X(\sigma,0), i\partial_\tau X(\sigma',0)] = 4\pi i\,\delta(\sigma-\sigma')$$

$$[a_1, a_{-1}] = 1$$


FOCK SPACE FOR a_n , ā_n
|p>   a_n|p>=0   ā_n|p> = 0   n>0
a_{-1}|p>   ā_{-1}|p>
a_{-2}|p>   a_{-1}^2|p>   a_{-1}ā_{-1}|p>   ā_{-1}^2|p>   ā_{-2}|p>
<p| <p|p'> = δ(p-p')   <p| a_n = 0   <p|ā_n = 0 n<0
<p|a_1 a_{-1}|p'> = <p|p'> + <p|a_{-1} a_1|p'> = δ(p-p')
[a_n, ā_m] = 0
= 4πi δ(σ-σ')

$$X(\sigma,\tau) = x - \frac{2i}{L} p \tau + \sum_{n\neq 0} \left[\frac{i}{n} a_n e \ldots - \frac{i}{n} \bar{a}_n e^{-i\frac{n}{L}(\sigma - i\tau)} \right]$$

$$[a_n, a_m] = n\,\delta_{n+m} \qquad [\bar{a}_n, \bar{a}_m] = n\,\delta_{n+m} \qquad [a_n, \bar{a}_m] = 0$$

$$[x, p] = i \qquad [X(\sigma,0), i\partial_\tau X(\sigma',0)] = 4\pi i\,\delta(\sigma - \sigma')$$

$$[a_1, a_{-1}] = 1$$

$$\partial_s \partial_{\bar{s}} X$$

$$\partial_s X = \frac{1}{L} p + \sum_{n\neq 0} \frac{1}{L} a_n e^{-\frac{n}{L} s}$$

$$a_0 \equiv p$$
$$\bar{a}_0 = -p$$

$$\partial_{\bar{s}} X = \frac{1}{L} p - \sum \frac{1}{L} \bar{a}_n e^{-\frac{n}{L} \bar{s}}$$

$$[x, p] = i$$

$$[X(\sigma,0), i\partial_\tau X(\sigma',0)] = 4\pi i\,\delta(\sigma-\sigma')$$

$$[a_1, a_{-1}] = 1$$

$$\partial_s \partial_{\bar{s}} X = 0$$

$$\partial_s X = \frac{1}{L} p + \sum_{n\neq 0} \frac{1}{L} a_n e^{-\frac{n}{L} s}$$

$$a_0 \equiv p$$

$$\bar{a}_0 = -p$$

$$\partial_{\bar{s}} X = \frac{1}{L} p - \sum \frac{1}{L} \bar{a}_n e^{-\frac{n}{L} s}$$

$$X \to X + f(z) + g(\bar{z})$$

$$\partial_s X \quad \partial_{\bar{s}} X \;)$$

$\partial_s \bar\partial_s X = 0$

$= -\frac{1}{\ell^2}\frac{e^{\frac{1}{\ell}(s+s')}}{\left(e^{\frac{s}{\ell}}-e^{\frac{s'}{\ell}}\right)^2}$

FOCK SPACE FOR a_n , $\bar a_n$

$|p\rangle$ $\quad a_n|p\rangle = 0 \quad \bar a_n|p\rangle = 0 \quad n>0 \quad \hat p|p\rangle = p|p\rangle$

$a_{-1}|p\rangle \quad \bar a_{-1}|p\rangle$

$a_{-2}|p\rangle \quad a_{-1}^2|p\rangle \quad a_{-1}\bar a_{-1}|p\rangle \quad \bar a_{-1}^2|p\rangle \quad \bar a_{-2}|p\rangle$

. . . .

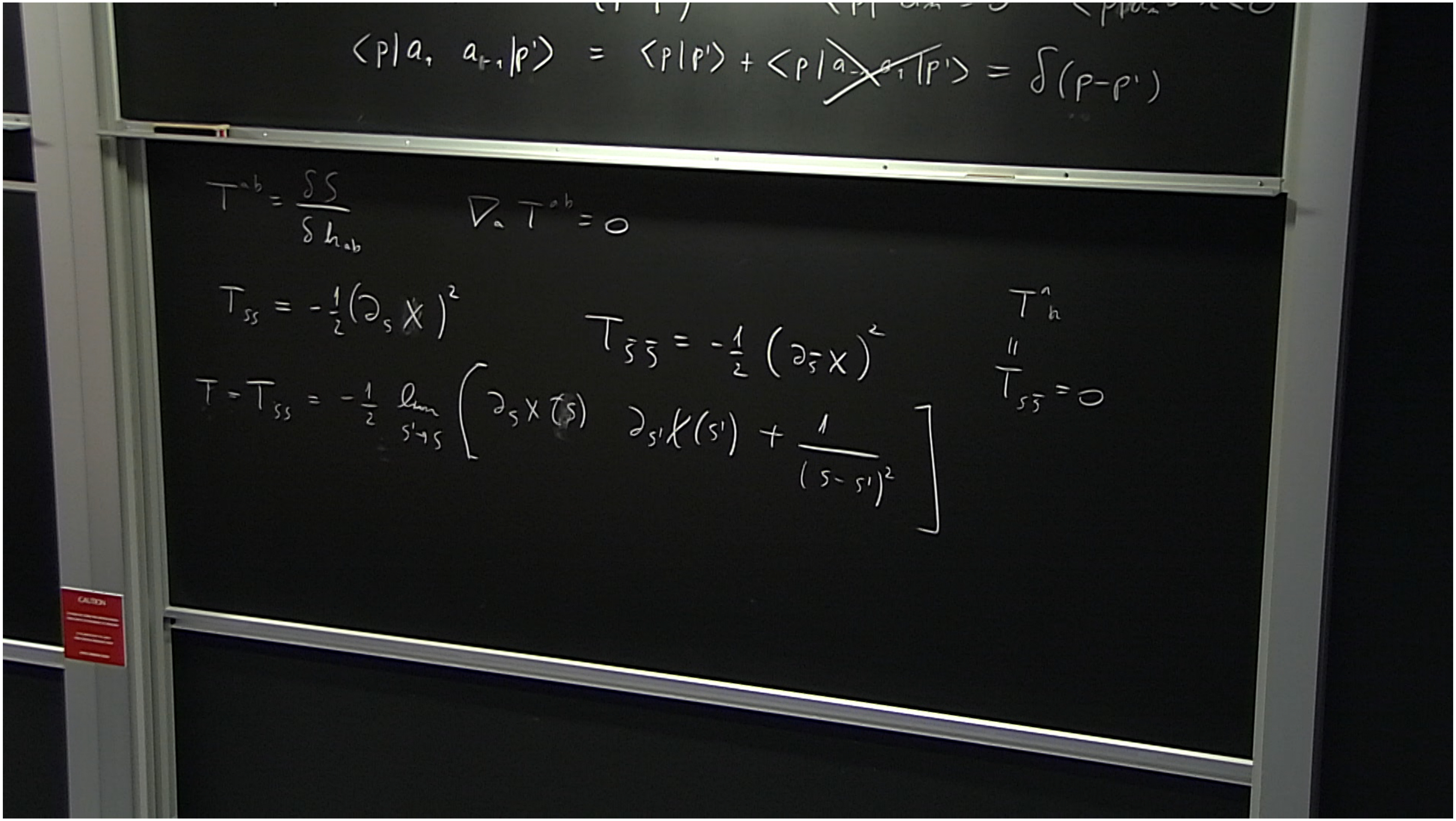
$\langle p| \quad \langle p|p'\rangle = \delta(p-p') \quad \langle p|\, a_n = 0 \quad \langle p|\bar a_n = 0 \quad n<0$

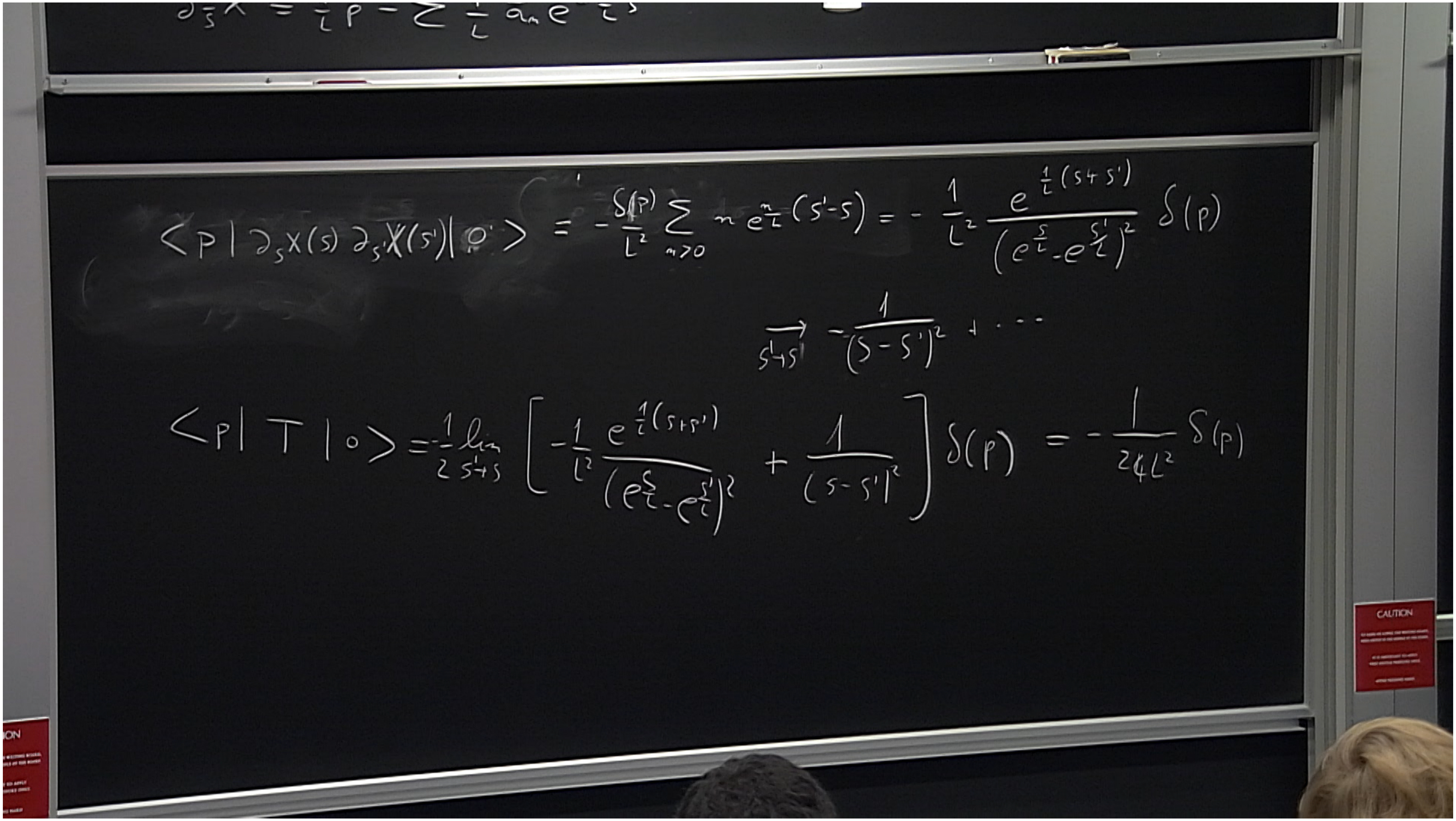
$\langle p|a_1\, a_{-1}|p'\rangle = \langle p|p'\rangle + \langle p|\cancel{a_{-1} a_1}|p'\rangle = \delta(p-p')$

$$\partial_{\bar{s}} X = \frac{\cdot}{L} p - \sum \frac{\cdot}{L} a_n e^{\cdots}$$

$$\langle p | \partial_s X(s) \, \partial_{s'} X(s') | 0 \rangle = -\frac{\delta(p)}{L^2} \sum_{n>0} n \, e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{\left(e^{\frac{s}{L}} - e^{\frac{s'}{L}}\right)^2} \delta(p)$$

$$\xrightarrow[s' \to s]{} -\frac{1}{(s-s')^2} + \cdots$$


$\langle p | a_1 \; a_{-1} | p' \rangle = \langle p | p' \rangle + \cancel{\langle p | a_{-1} a_1 | p' \rangle} = \delta(p - p')$
$T^{ab} = \frac{\delta S}{\delta h_{ab}}$
$\nabla_a T^{ab} = 0$
$T_{ss} = -\frac{1}{2} (\partial_s X)^2$
$T_{\bar{s}\bar{s}} = -\frac{1}{2} (\partial_{\bar{s}} X)^2$
$T^a{}_a = T_{s\bar{s}} = 0$
$T = T_{ss} = -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \; \partial_{s'} X(s') + \frac{1}{(s - s')^2} \right]$


$\partial_s X = \frac{1}{L} p - \sum \frac{1}{L} a_n e$
$\langle p | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{\delta(p)}{L^2} \sum_{n>0} n e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{\left(e^{\frac{s}{L}} - e^{\frac{s'}{L}}\right)^2} \delta(p)$
$\xrightarrow[s' \to s]{} -\frac{1}{(s-s')^2} + \cdots$
$\langle p | T | 0 \rangle = -\frac{1}{2} \lim_{s' \to s} \left[-\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{\left(e^{\frac{s}{L}} - e^{\frac{s'}{L}}\right)^2} + \frac{1}{(s-s')^2} \right] \delta(p) = -\frac{1}{24 L^2} \delta(p)$

$$\langle p | a_1 \, a_{-1} | p' \rangle = \langle p | p' \rangle + \cancel{\langle p | a_{-1} a_1 | p' \rangle} = \delta(p - p')$$

$$\sum \langle | a_n a_{-n} | \rangle = \sum_{n=1}^{\infty} n$$

$$T^{ab} = \frac{\delta S}{\delta h_{ab}} \qquad \nabla_a T^{ab} = 0 \qquad T^a_a \propto R[h_{ab}]$$

$$T_{ss} = -\frac{1}{2} (\partial_s X)^2 \qquad T_{\bar{s}\bar{s}} = -\frac{1}{2} (\partial_{\bar{s}} X)^2 \qquad T^a_a = T_{s\bar{s}} = 0$$

$$\underset{L=1}{T} = T_{ss} = -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \, \partial_{s'} X(s') + \frac{1}{(s - s')^2} \right]$$

$$T = -\frac{1}{24} - \sum_n L_n e^{-ns}$$

$$L_n = \frac{1}{2} \sum_m :a_{n-m} a_m:$$

$$T = T_{ss} = -\frac{1}{2}\lim_{s'\to s}\left[\partial_s X(s)\,\partial_{s'}X(s') + \frac{\quad}{(s-s')^2}\right]$$

$L=1$

$$T = -\frac{1}{24} - \sum_n L_n e^{-ns}$$

$$\bar{T} = -\frac{1}{24} + \sum \bar{L}\, e^{-n\bar{s}}$$

$$L_n = \frac{1}{2}\sum_m :a_{n-m}\,a_m:$$

$$L_0|p\rangle = \frac{1}{2}p^2\,|p\rangle$$

$$L_0\, a_{-1}|p\rangle = \left(\frac{1}{2}p^2 + 1\right)|p\rangle$$

$$E = L_0 + \bar{L}_0 - \frac{1}{12} \qquad P = L_0 - \bar{L}_0$$

FOCK SPACE FOR a_n, $\bar{a}_n$

$|p\rangle$ $\quad a_n|p\rangle = 0 \quad \bar{a}_n|p\rangle = 0 \quad n>0 \quad \hat{p}|p\rangle = p|p\rangle$

$a_{-1}|p\rangle \quad \bar{a}_{-1}|p\rangle$

$a_{-2}|p\rangle \quad a_{-1}^2|p\rangle \quad a_{-1}\bar{a}_{-1}|p\rangle \quad \bar{a}_{-1}^2|p\rangle \quad \bar{a}_{-2}|p\rangle$

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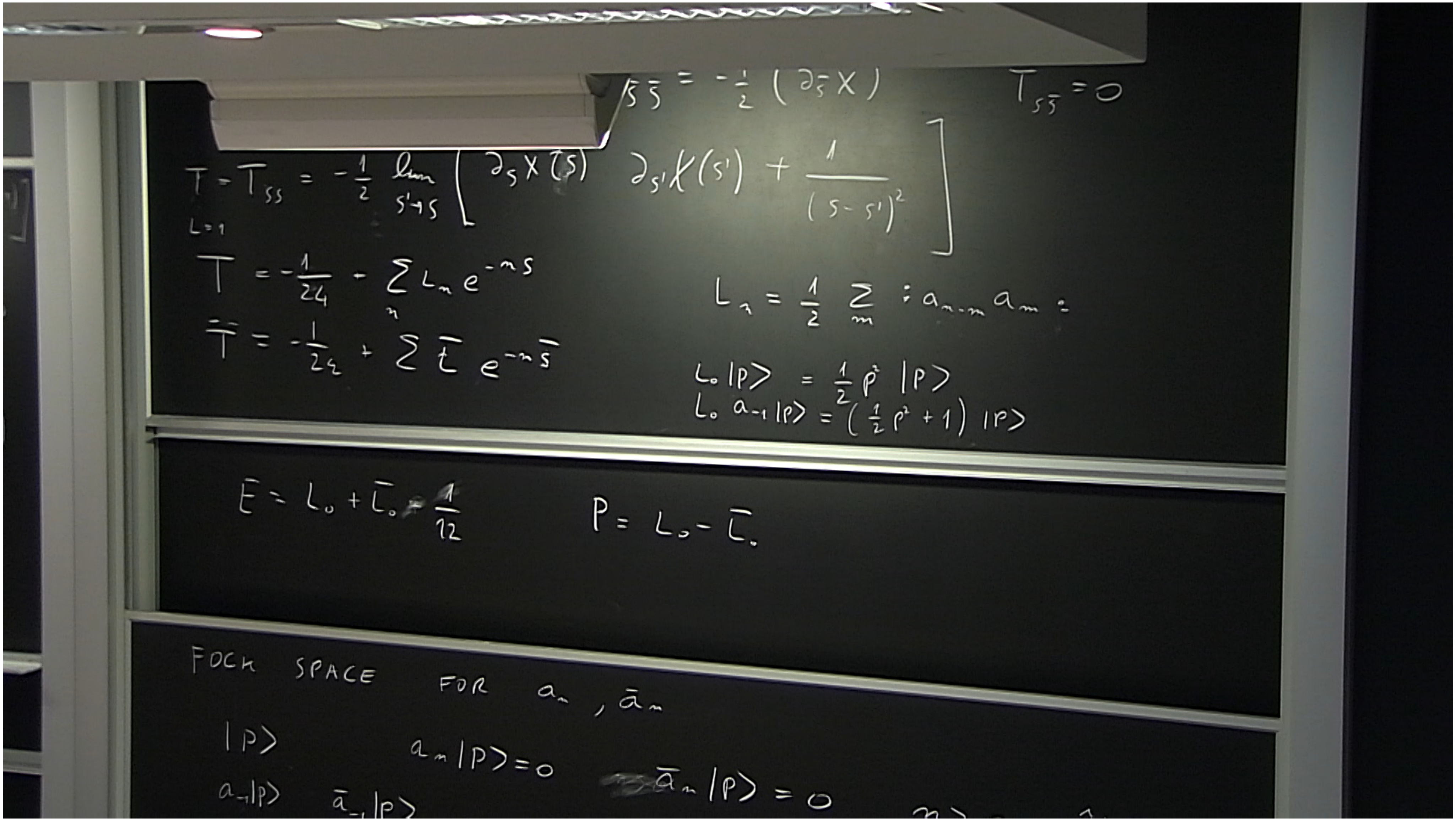
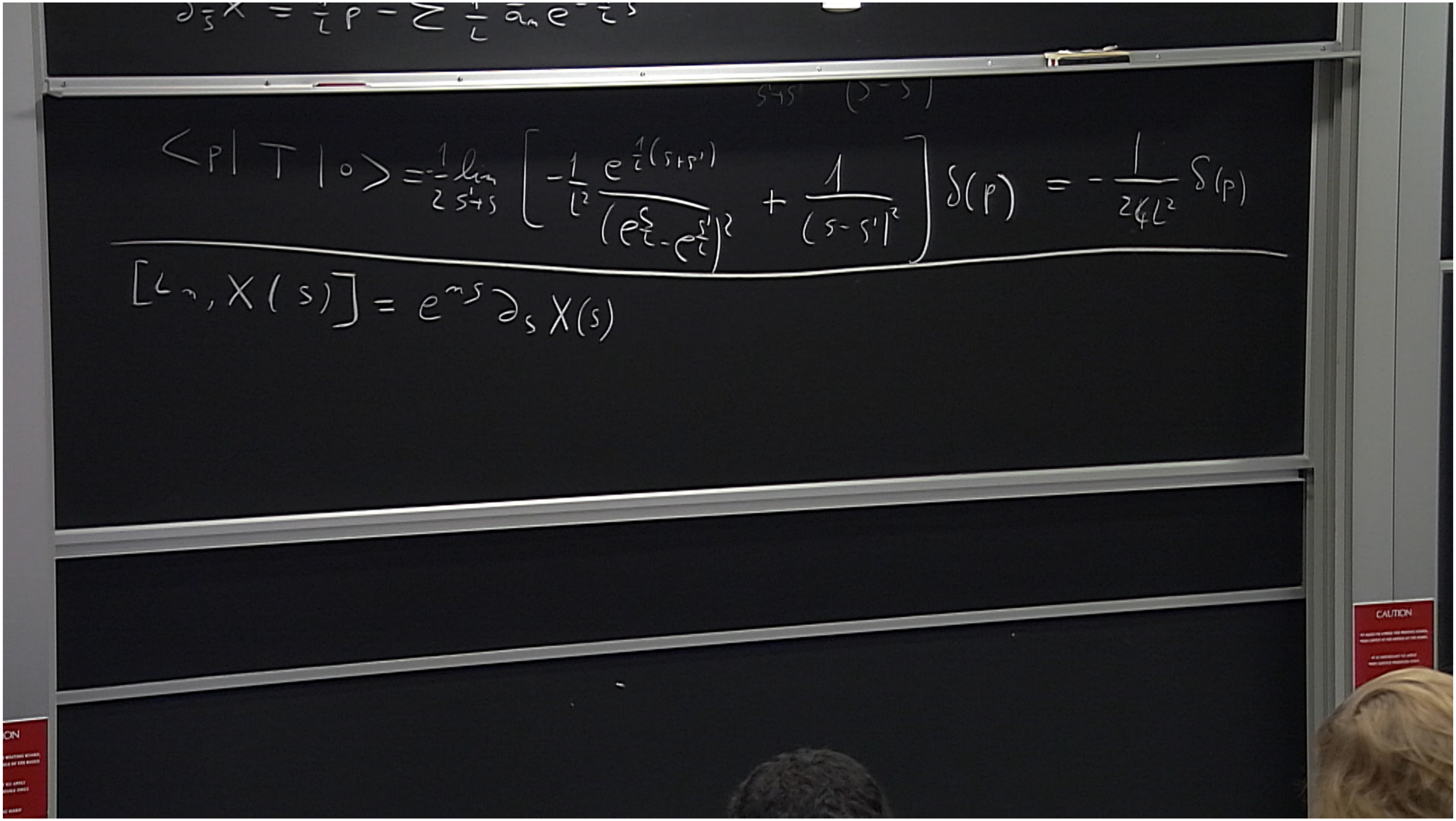
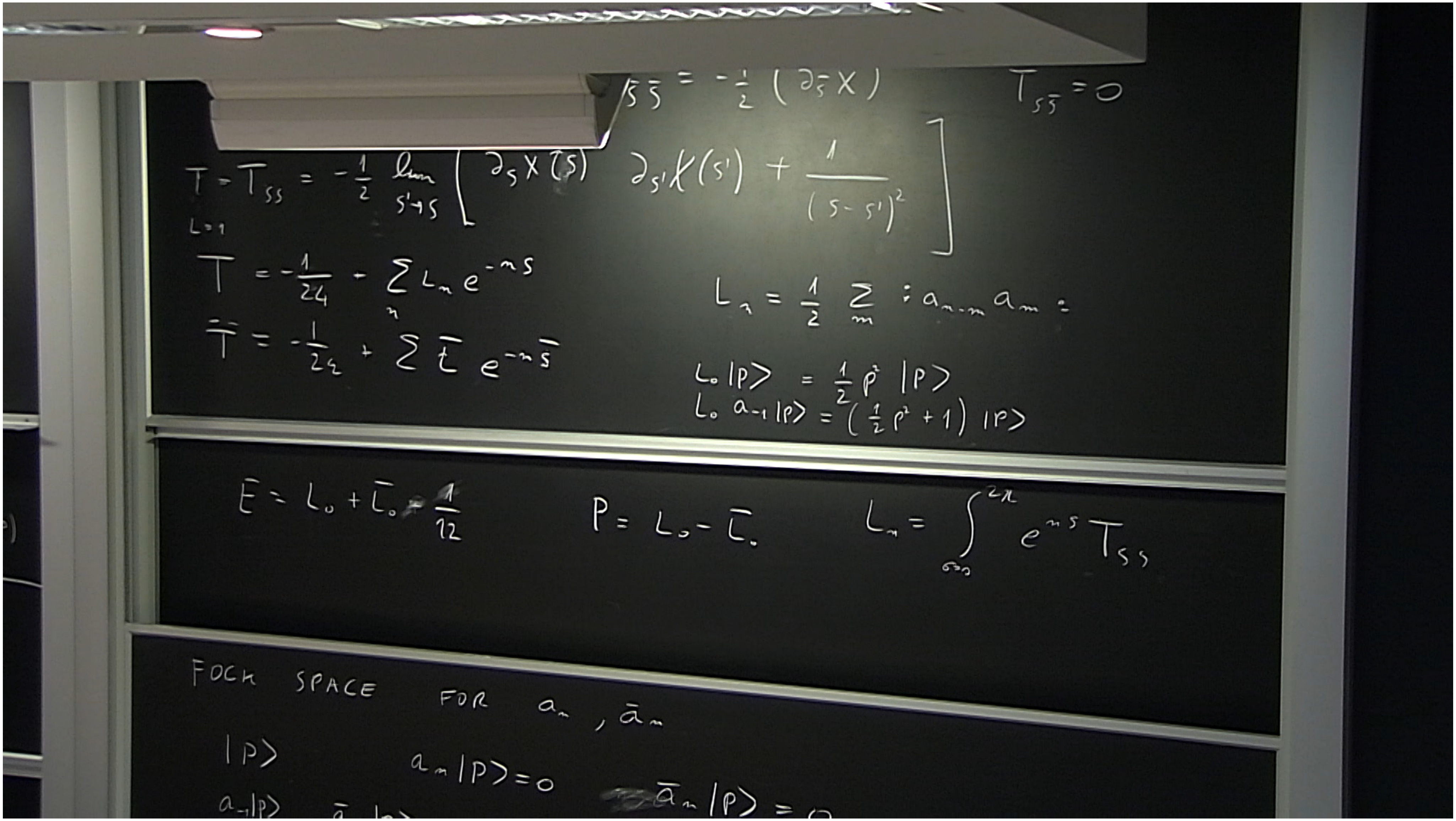

$T_{\bar{s}\bar{s}} = -\frac{1}{2}(\partial_{\bar{s}} X)$
$T_{s\bar{s}} = 0$
$T = T_{ss} = -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \, \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$
$L = 1$
$T = -\frac{1}{24} + \sum_n L_n e^{-ns}$
$\bar{T} = -\frac{1}{24} + \sum \bar{L} e^{-n\bar{s}}$
$L_n = \frac{1}{2} \sum_m :a_{n-m} a_m:$
$L_0 |p\rangle = \frac{1}{2} p^2 |p\rangle$
$L_0 a_{-1} |p\rangle = (\frac{1}{2} p^2 + 1) |p\rangle$
$E = L_0 + \bar{L}_0 - \frac{1}{12}$
$P = L_0 - \bar{L}_0$
FOCK SPACE FOR a_n, $\bar{a}_n$
$|p\rangle$
$a_n |p\rangle = 0$
$\bar{a}_n |p\rangle = 0$
$a_{-1}|p\rangle$
$\bar{a}_{-1}|p\rangle$


$\langle p | T | 0 \rangle = -\frac{1}{2} \lim_{s' \to s} \left[-\frac{1}{l^2} \frac{e^{\frac{1}{l}(s+s')}}{(e^{\frac{s}{l}} - e^{\frac{s'}{l}})^2} + \frac{1}{(s-s')^2} \right] \delta(p) = -\frac{1}{24 l^2} \delta(p)$
$[L_n, X(s)] = e^{ns} \partial_s X(s)$
CAUTION


$T_{s\bar s} = 0$
$T = T_{ss} = -\frac{1}{2}\lim_{s'\to s}\left[\partial_s X(s)\,\partial_{s'}X(s') + \frac{1}{(s-s')^2}\right]$
$L=1$
$T = -\frac{1}{24} + \sum_n L_n e^{-ns}$
$\bar T = -\frac{1}{24} + \sum \bar L e^{-n\bar s}$
$L_n = \frac{1}{2}\sum_m :a_{n-m}a_m:$
$L_0|p\rangle = \frac{1}{2}p^2|p\rangle$
$L_0 a_{-1}|p\rangle = (\frac{1}{2}p^2+1)|p\rangle$
$E = L_0 + \bar L_0 - \frac{1}{12}$
$P = L_0 - \bar L_0$
$L_n = \int_{s=0}^{2\pi} e^{ns}T_{ss}$
FOCK SPACE FOR a_n, $\bar a_n$
$|p\rangle$
$a_n|p\rangle = 0$
$\bar a_n|p\rangle =$
$a_{-1}|p\rangle$

$$\partial_s X = \frac{1}{L}p + \sum_{n=0} \frac{1}{L} a_n e^{\cdots}$$

$a_0 = p$

$\bar{a}_0 = -p$

$$\partial_{\bar{s}} X = \frac{1}{L} p - \sum \frac{1}{L} \bar{a}_n e^{-\frac{n}{L} s}$$

$$\langle p | T | 0 \rangle = -\frac{1}{2} \lim_{s' \to s} \left[-\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{\left(e^{\frac{s}{L}} - e^{\frac{s'}{L}}\right)^2} + \frac{1}{(s-s')^2} \right] \delta(p) = -\frac{1}{24 L^2} \delta(p)$$

$$[L_n, X(s)] = e^{ns} \partial_s X(s)$$

$s \to s + \epsilon e^{ns}$

$X(s) \to X(s + \epsilon e^{ns})$

$$[L_n, \partial_s X(s) \partial_{s'} X(s')] = \cdots$$

$$T_{s's'}(s') = \left(\frac{\partial s}{\partial s'}\right)^2 T_{ss}(s(s'))$$

$$[L_n, T_{ss}(s)] = e^{ns}(2n + \partial_s)\left(T(s) + \frac{1}{24}\right) + \frac{1}{12} n (n^2 - 1) e^{ns}$$

$(1 + \epsilon n e^{ns})^2 T(s + \epsilon e^{ns})$

$$\partial_{\bar s} X = \frac{1}{l} p - \sum \frac{1}{l} a_n e^{\dots}$$

$$[L_n, \partial_s X(s)\, \partial_{s'} X(s')] = \dots$$

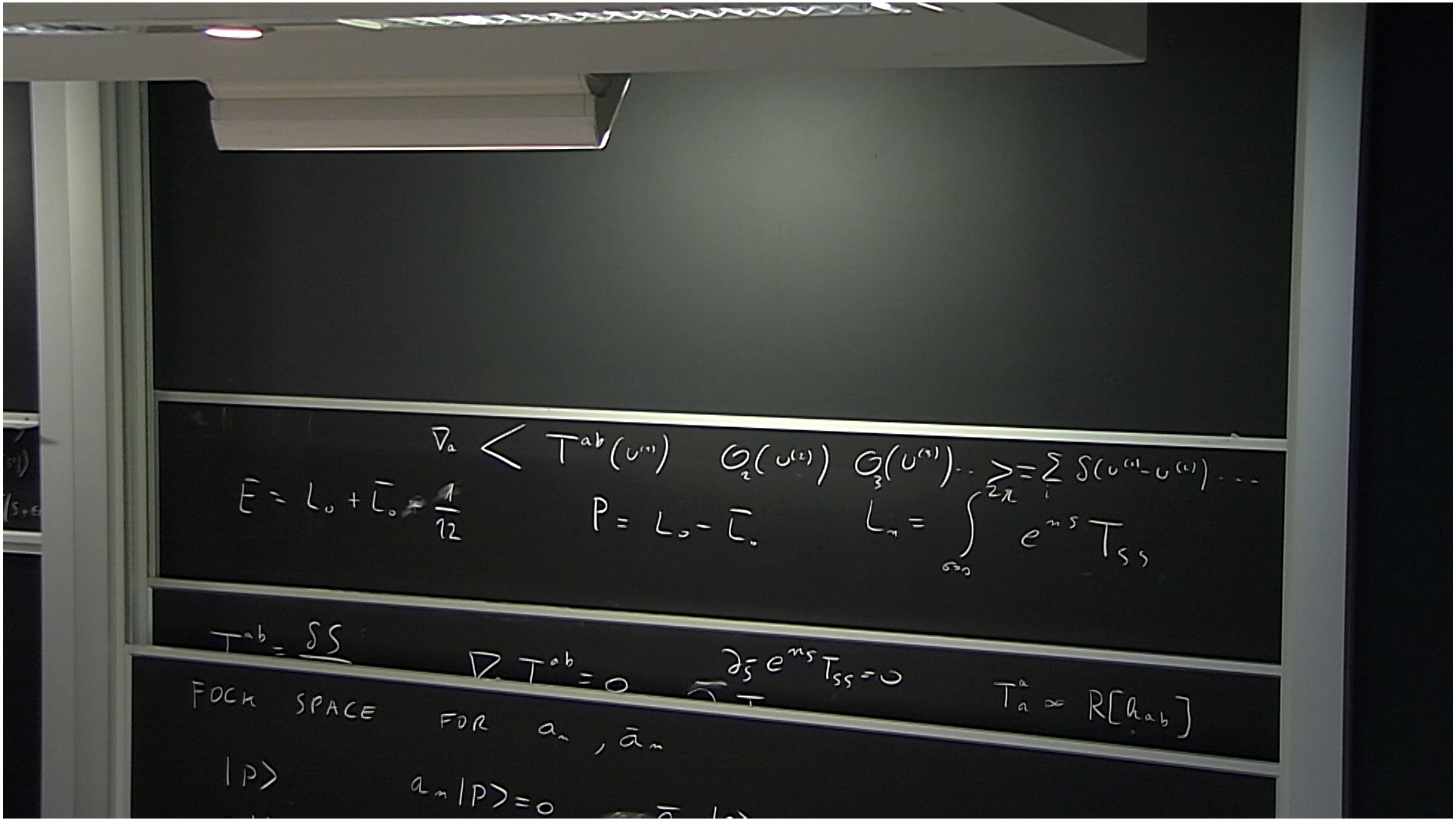
$$[L_n, T_{ss}(s)] = e^{ns}(2n + \partial_s)\left(T(s) + \frac{1}{24}\right) + \frac{1}{12} n(n^2-1) e^{ns}$$

$$T_{s's'}(s') = \left(\frac{\partial s}{\partial s'}\right)^2 T_{ss}(s(s'))$$

$$(1 + \epsilon n e^{ns})^2 T(s + \epsilon e^{ns})$$

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12} n(n^2-1)\delta_{n+m,0}$$

$$\{e^{ns}\partial_s, e^{ms}\partial_s\} = (n-m) e^{(n+m)s}\partial_s$$


FOCK SPACE FOR a, ā
|p⟩
a|p⟩=0