Title: PSI 2018/2019 - String Theory Review - Lecture 2

Speakers: Davide Gaiotto

Collection: PSI 2018/2019 - String Theory Review (Gaiotto)

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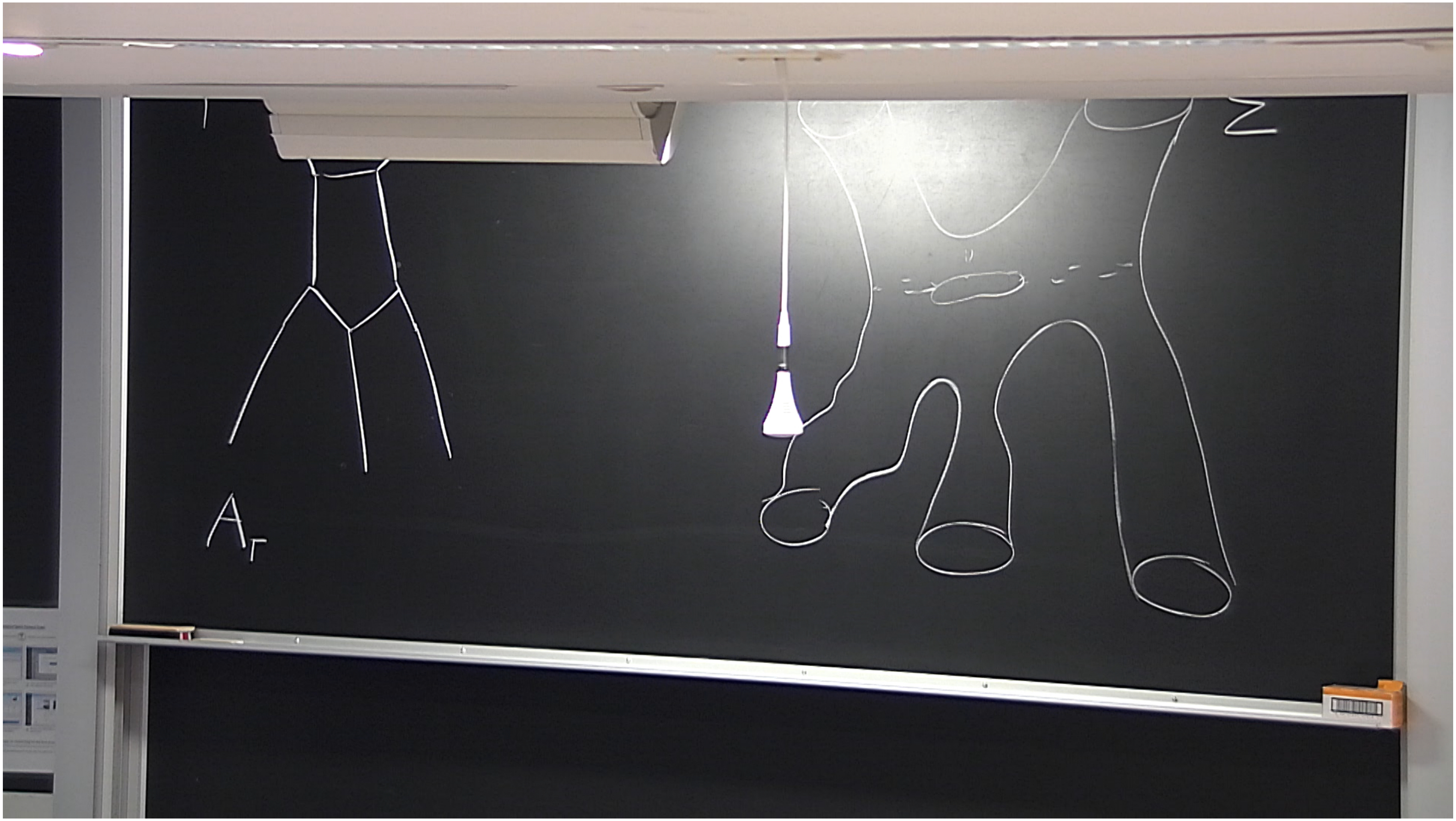
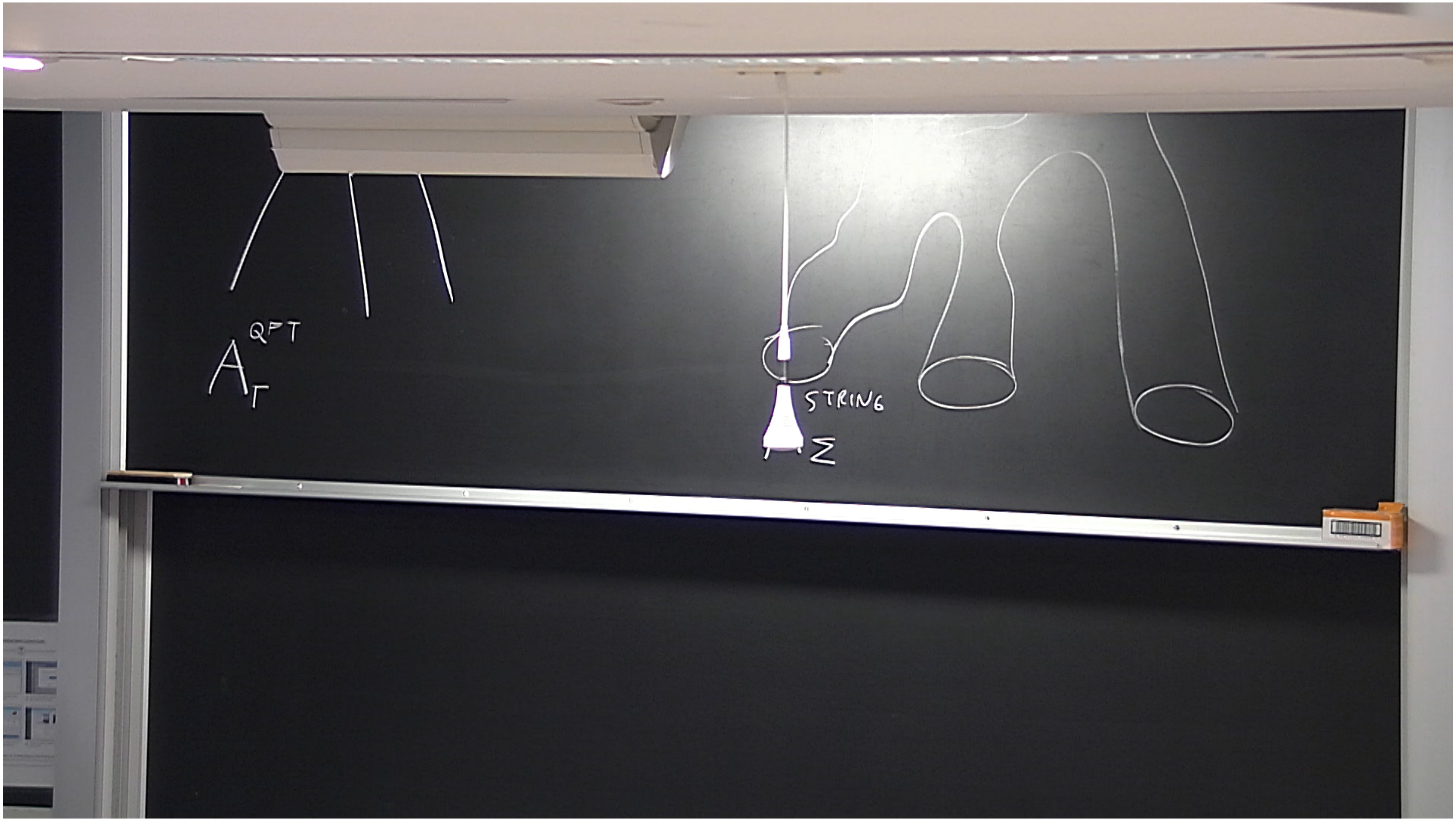


A_Γ
Σ


QFT
A
Γ
STRING
Σ

STRING

Σ

$$A_\Gamma = \int d\ell \int_{x:\,\Gamma \to R^{d-1,1}} Dx \; e^{S(x,\ell)}$$

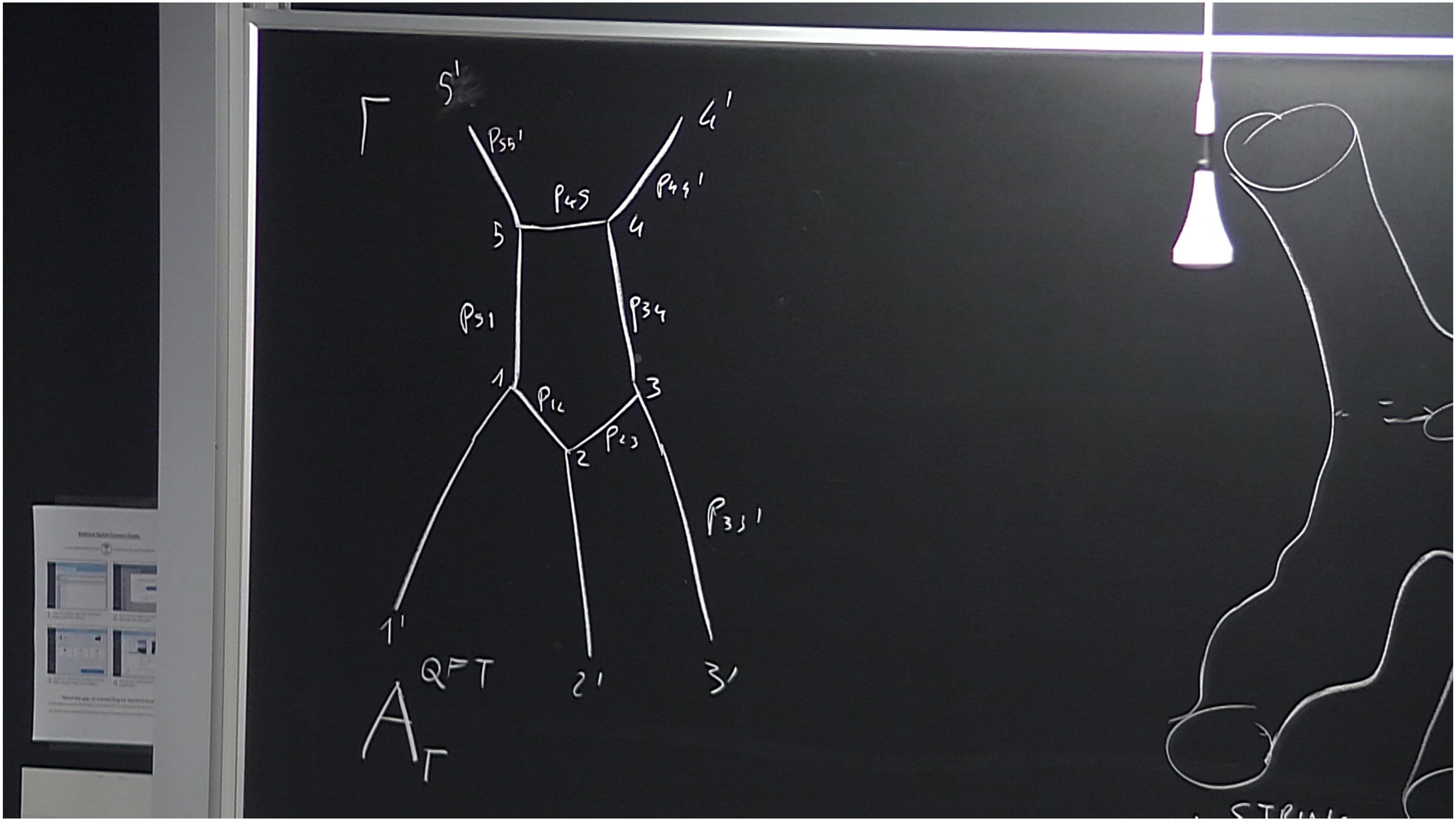
$$A_\Sigma = \int_{\mathcal{M}[\Sigma]} d\mu \int_{x:\,\Sigma \to R^{d-1,1}} Dx \; e^{-S(x,\mu)}$$

$$\int Dx \; e^{S_{\text{PARTICLE}}(x,\ell)}$$

$$x : \Gamma \longrightarrow R^{d-1,1}$$

$$A_\Sigma = \int_{\mathcal{M}[\Sigma]} d\mu \int Dx \; e^{-S_{\text{STRING}}(x,\mu)}$$

$$x : \Sigma \rightarrow R^{d-1,1}$$


Γ
5'
4'
P55'
P45
P44'
5
4
P51
P34
1
3
P12
P23
2
P33'
1'
2'
3'
QFT
AΓ

$$A_\Gamma = \int \prod_{(a,b)\in \text{INTERNAL LINE}} dp_{ab} \prod_{a \in \text{INTERNAL VERTICES}} \delta\left(\sum_b p_{ab}\right) \prod_{(a,b)\in \text{INTERNAL LINE}} \frac{1}{p_{ab}^2+m^2}$$

$$A_\Gamma = \int \prod_{(a,b)\in \text{INTERNAL LINE}} dp_{ab} \prod_{a \in \text{INTERNAL VERTICES}} \delta\Big(\sum_b p_{ab}\Big) \prod_{(a,b)\in \text{INTERNAL LINE}} \frac{1}{p_{ab}^2 + m^2}$$

$$\int \prod_{(a,b)\in \text{INTERNAL LINE}} dp_{ab} \prod_{a\in \text{INTERNAL VERTICES}} \delta\left(\sum_b p_{ab}\right) \prod_{(a,b)\in \text{INTERNAL LINE}} \frac{1}{p_{ab}^2+m^2}$$

$$\frac{1}{p_{ab}^2+m^2} = \int_0^\infty d\ell_{ab}\, e^{-\ell_{ab} p_{ab}^2 - \ell_{ab} m^2}$$

$$\delta\left(\sum_b p_{ab}\right) = \int dx_a^d\, e^{i x_a \sum_b p_{ab}}$$

$$A_\Gamma = \prod_{a \in \text{INTERNAL}} \int dx_a^D \, e^{i x_a \sum_{b \in \text{EXTERNAL}} p_{ab}} \prod_{\substack{(a,b) \\ \in \\ \text{INTERNAL}}} \int_0^\infty d\ell_{ab} \, e^{-\frac{(x_a - x_b)^2}{4 \ell_{ab}} - \ell_{ab} m^2}$$

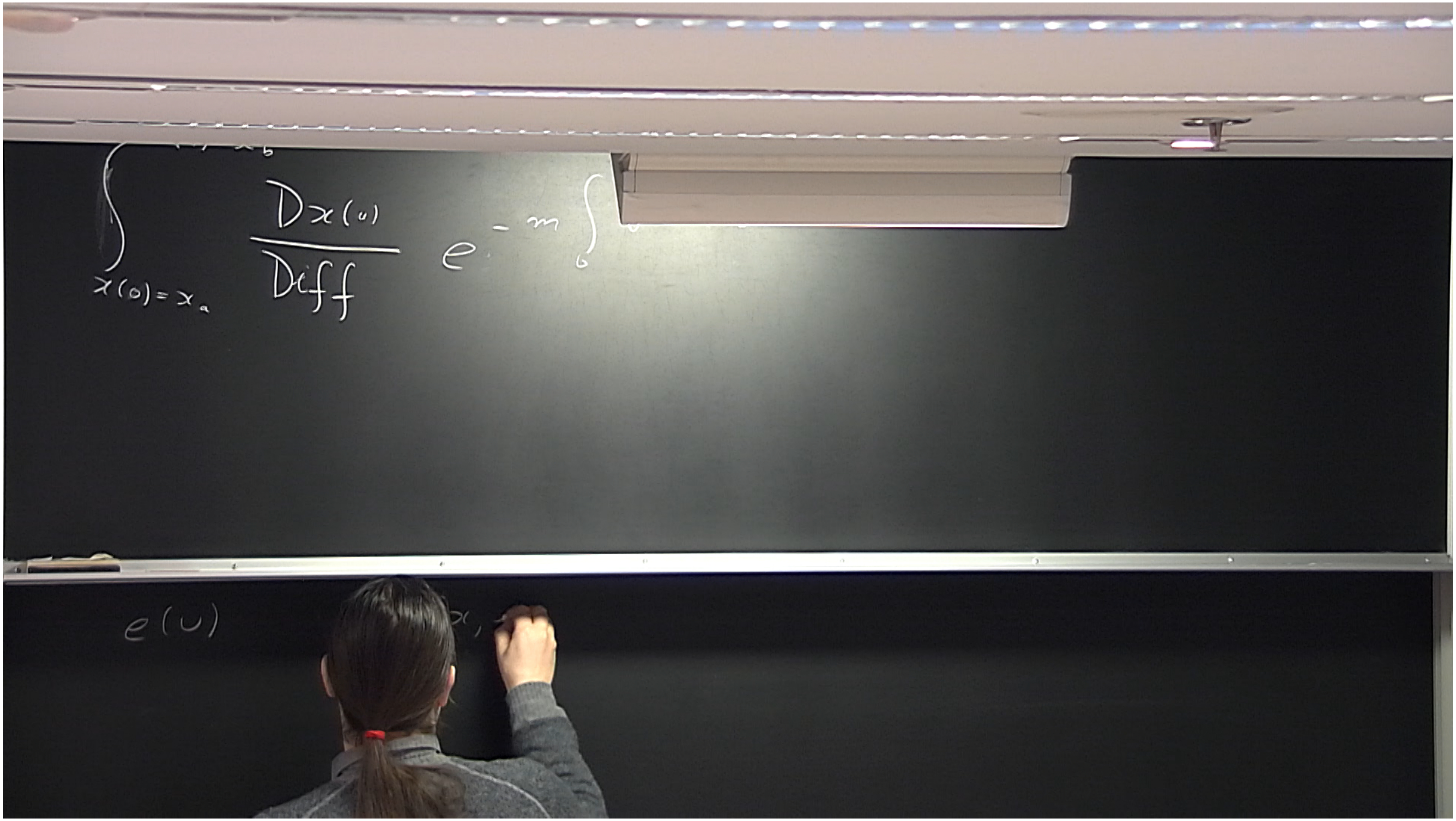
$$A_\Gamma = \prod_{a \in \text{INTERNAL}} \int dx_a^D \, e^{i x_a \sum_{b \in \text{EXTERNAL}} p_{ab}} \prod_{(a,b) \in \text{INTERNAL}} \int_0^\infty d\ell_{ab} \, e^{-\frac{(x_a - x_b)^2}{4\ell_{ab}} - \ell_{ab} m^2}$$

$$G(x_{a,b}) = \int_0^\infty d\ell_{ab} \, e^{-\frac{(x_a - x_b)^2}{4\ell_{ab}} - \ell_{ab} m^2}$$

$$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^\mu}{du}\frac{dx_\mu}{du}}\, du$$

$$x(u) : [0,1] \to R^{d-1,1}$$

$$u = f(u')$$


Dx(u)
Diff
x(0) = x_a
e(u)

$e(u)$

$$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1}\dot{x}^2 + m^2 e\right] du$$

$$e(u)\,du = e'(u')\,du'$$

$$\boxed{\begin{aligned} x(u) &\to x\big(f(u)\big) \\ e(u) &\to e\big(f(u)\big)\frac{df}{du} \end{aligned}}$$

$$\int_0^1 e\,du = \ell$$

GAUGE-FIX

$e = \frac{1}{\ell}$ CONSTANT

$e(u)$

$$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1}\dot{x}^2 + m^2 e\right] du$$

$$e(u)\,du = e'(u')\,du'$$

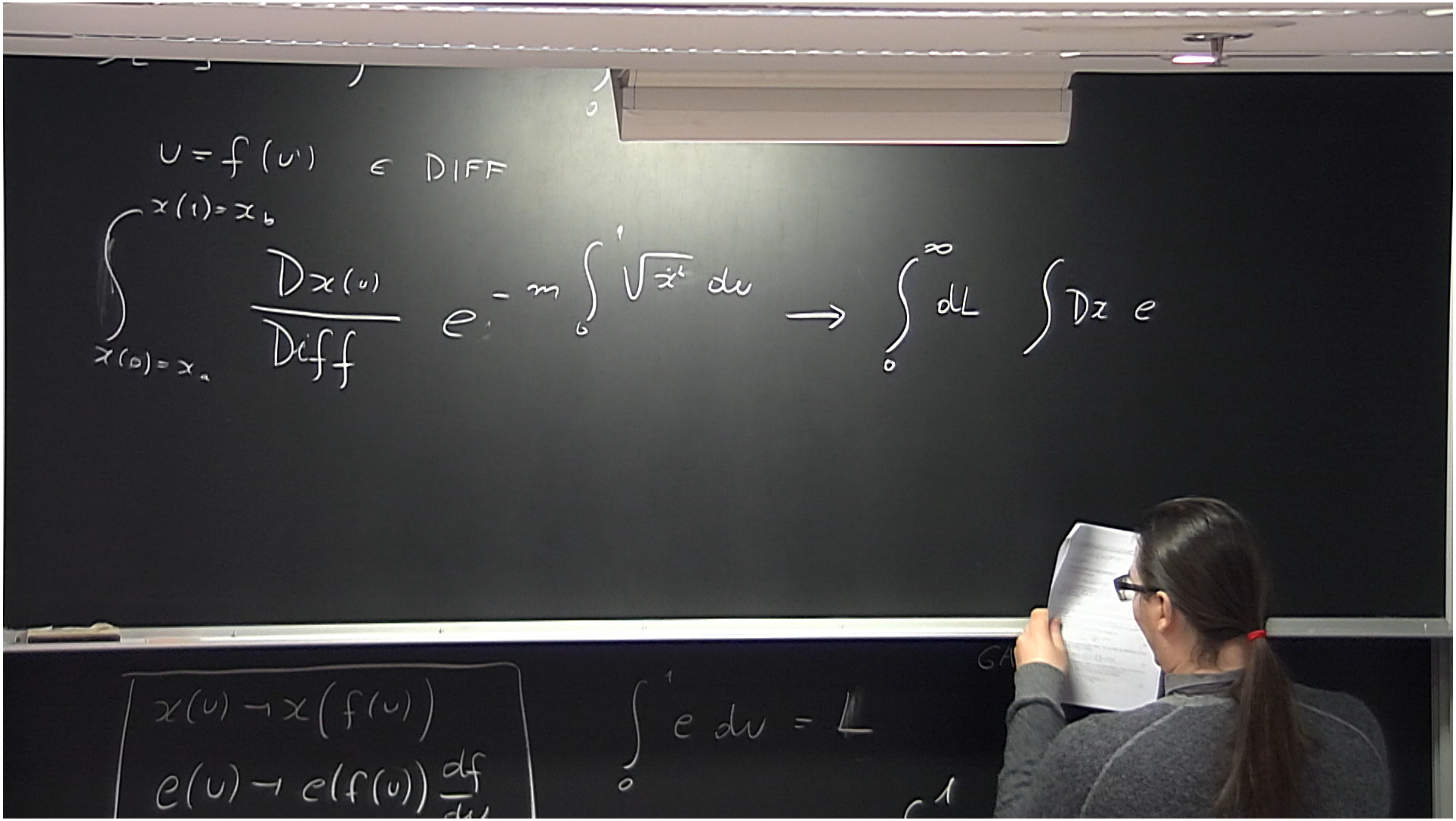
$$\boxed{\begin{aligned} x(u) &\to x\big(f(u)\big) \\ e(u) &\to e\big(f(u)\big)\frac{df}{du} \end{aligned}}$$

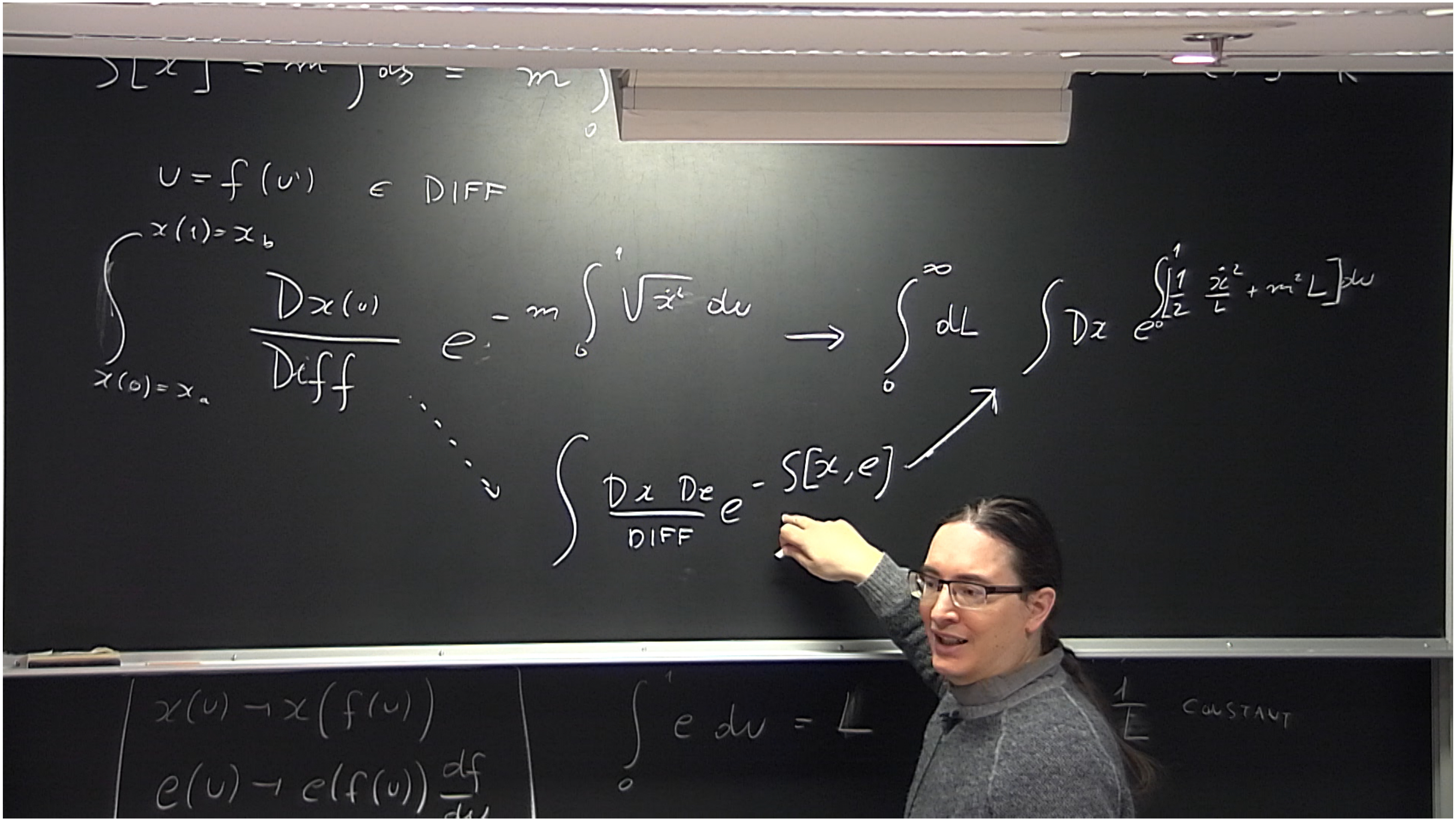
$$\int_0^1 e\,du = L$$

GAUGE-FIX

$e = \frac{1}{L}$ CONSTANT

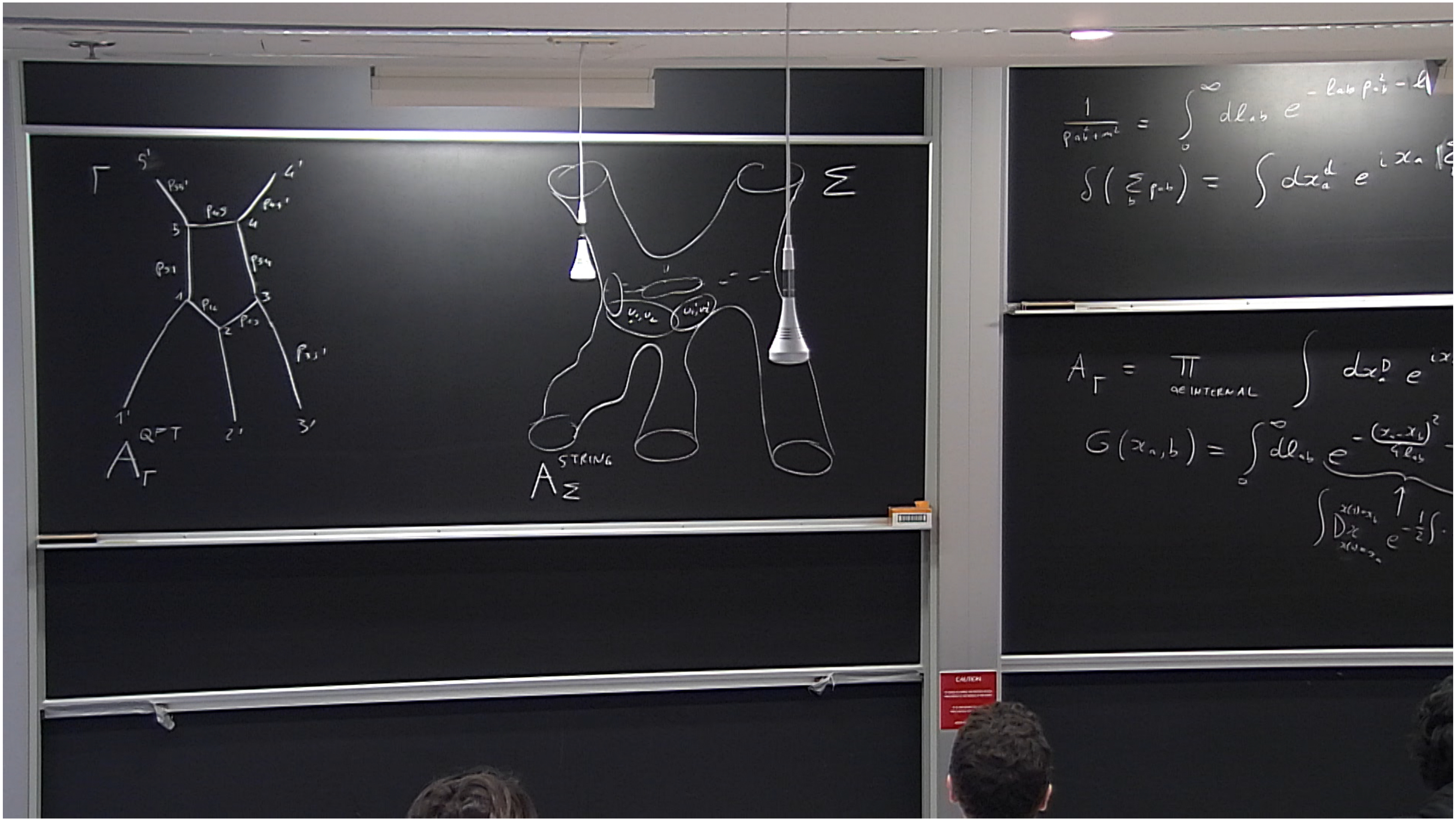
$$S^{\text{GAUGE-FIXED}} = \frac{1}{2}\int_0^1 \left[\frac{\dot{x}^2}{L} + m^2 L\right] du$$


$u = f(u') \in \mathrm{DIFF}$
$\int_{x(0)=x_a}^{x(1)=x_b} \frac{Dx(u)}{\mathrm{Diff}} e^{-m\int_0^1 \sqrt{\dot{x}^2}\, du} \longrightarrow \int_0^\infty dL \int Dx\, e$
$x(u) \to x(f(u))$
$e(u) \to e(f(u)) \frac{df}{du}$
$\int_0^1 e\, du = L$


$U = f(U') \in \mathrm{DIFF}$
$\int_{x(0)=x_a}^{x(1)=x_b} \frac{Dx(u)}{\mathrm{Diff}} e^{-m\int_0^1 \sqrt{\dot{x}^2}\,du} \rightarrow \int_0^\infty dL \int Dx\, e^{\int_0^1 \left[\frac{1}{2}\frac{\dot{x}^2}{L} + m^2 L\right] du}$
$\int \frac{Dx\,De}{\mathrm{DIFF}} e^{-S[x,e]}$
$x(u) \rightarrow x(f(u))$
$e(u) \rightarrow e(f(u))\frac{df}{du}$
$\int_0^1 e\,du = L$
CONSTANT

$$A_\Gamma = \prod_{a \in \text{INTERNAL}} \int dx_a^D \, e^{i x_a \sum_{b \in \text{EXTERNAL}} p_{ab}} \prod_{\substack{(a,b) \\ \in \\ \text{INTERNAL}}} \int_0^\infty dl_{ab}\, e^{-\frac{(x_a - x_b)^2}{4 l_{ab}} - l_{ab} m^2}$$

$$G(x_a, b) = \int_0^\infty dl_{ab}\, \underbrace{e^{-\frac{(x_a - x_b)^2}{4 l_{ab}} - l_{ab} m^2}}_{\int_{x(0)=x_a}^{x(1)=x_b} Dx\, e^{-\frac{1}{2}\int \cdots}}$$


Γ
Σ
QFT
STRING
a∈INTERNAL

$$S[x] = T \iint_{\Sigma} \sqrt{\det_{(a,b)} \frac{dx^\mu}{du^a} \frac{dx_\mu}{du^b}} \, du^1 du^2$$

$$x(u^1, u^2)$$

$$u^1 \to f^1(u^1, u^2)$$

$$u^2 \to f^2(u^1, u^2)$$

$$S[X] = T \iint_{\Sigma} \sqrt{\det_{(a,b)} \frac{dX^{\mu}}{du^{a}} \frac{dX_{\mu}}{du^{b}}}\; du^1 du^2$$

$$\int \frac{DX}{\mathrm{DIFF}}\, e^{-S[X]}$$

$X(u^1, u^2)$

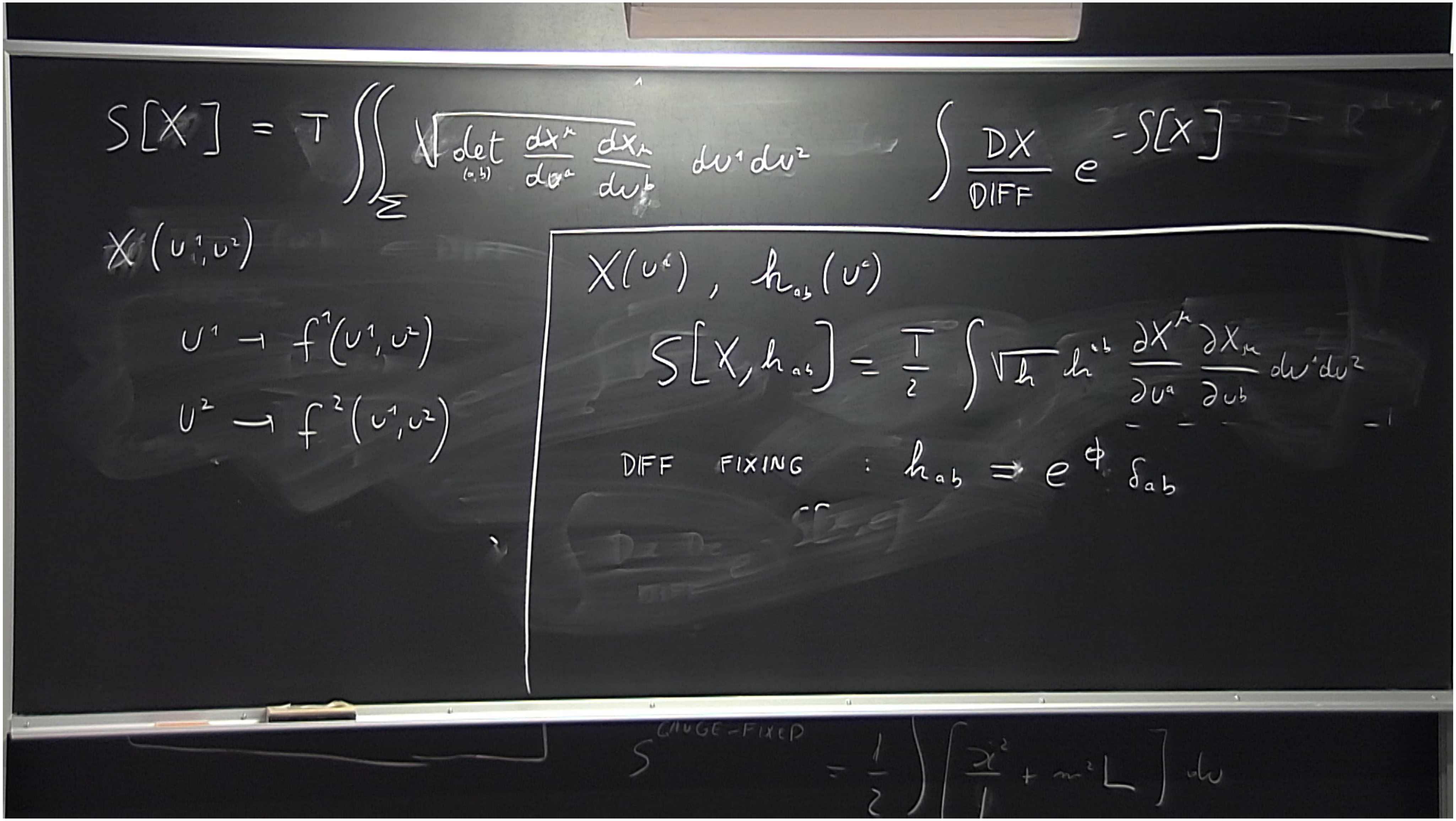
$$u^1 \to f^1(u^1, u^2)$$

$$u^2 \to f^2(u^1, u^2)$$

$X(u^a)\,,\ h_{ab}(u^c)$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^{\mu}}{\partial u^{a}} \frac{\partial X_{\mu}}{\partial u^{b}}\, du^1 du^2$$

$$S^{\text{GAUGE-FIXED}} = \frac{1}{2} \int \left[\frac{\dot{x}^2}{l} + m^2 l \right] dw$$


$S[X] = T \iint_\Sigma \sqrt{\det_{(a,b)} \frac{dX^\mu}{du^a} \frac{dX_\mu}{du^b}}\, du^1 du^2$
$\int \frac{DX}{\mathrm{DIFF}} e^{-S[X]}$
$X(u^1, u^2)$
$u^1 \to f^1(u^1, u^2)$
$u^2 \to f^2(u^1, u^2)$
$X(u^c),\ h_{ab}(u^c)$
$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u^b} du^1 du^2$
DIFF FIXING : $h_{ab} \Rightarrow e^{\phi} \delta_{ab}$

$$S[X] = T \iint_\Sigma \sqrt{\det_{(a,b)} \frac{dX^\mu}{du^a} \frac{dX_\mu}{du^b}}\; du^1 du^2$$

$$\int \frac{DX}{\text{DIFF}}\, e^{-S[X]}$$

$X(u^1, u^2)$

$$u^1 \to f^1(u^1, u^2)$$

$$u^2 \to f^2(u^1, u^2)$$

$X(u^c)\,,\; h_{ab}(u^c)$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u^b}\, du^1 du^2$$

DIFF FIXING : $h_{ab} \Rightarrow e^\phi \delta_{ab}$

$$S[X, e^\phi \delta_{ab}] = \frac{T}{2} \int \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u_a}\, du^1 du^2$$

$$S^{\text{GAUGE-FIXED}} = \frac{1}{2} \int \left[\frac{\dot{x}^2}{l} + m^2 L \right] dw$$

DIFF

$U^2 \to f^2(U^1, U^2)$

$$\int \frac{DX\, Dh}{\text{DIFF} \times \text{WEYL}} e^{-S[X,h]}$$

DIFF FIXING : $h_{ab} \Rightarrow \delta_{ab}$

WEYL

$$S[X, e^{\phi}\delta_{ab}] = \frac{T}{2}\int \frac{\partial X^\mu}{\partial U^a}\frac{\partial X_\mu}{\partial U_a} dU^1 dU^2$$

WEYL : $h_{ab} \to e^{\lambda} h_{ab}$

$e(u)$

$$S[x, e] = \frac{1}{2}\int_0^1 \left[e^{-1}\dot{x}^2 + m^2 e\right] du$$

$e(u)du = e'(u')du'$

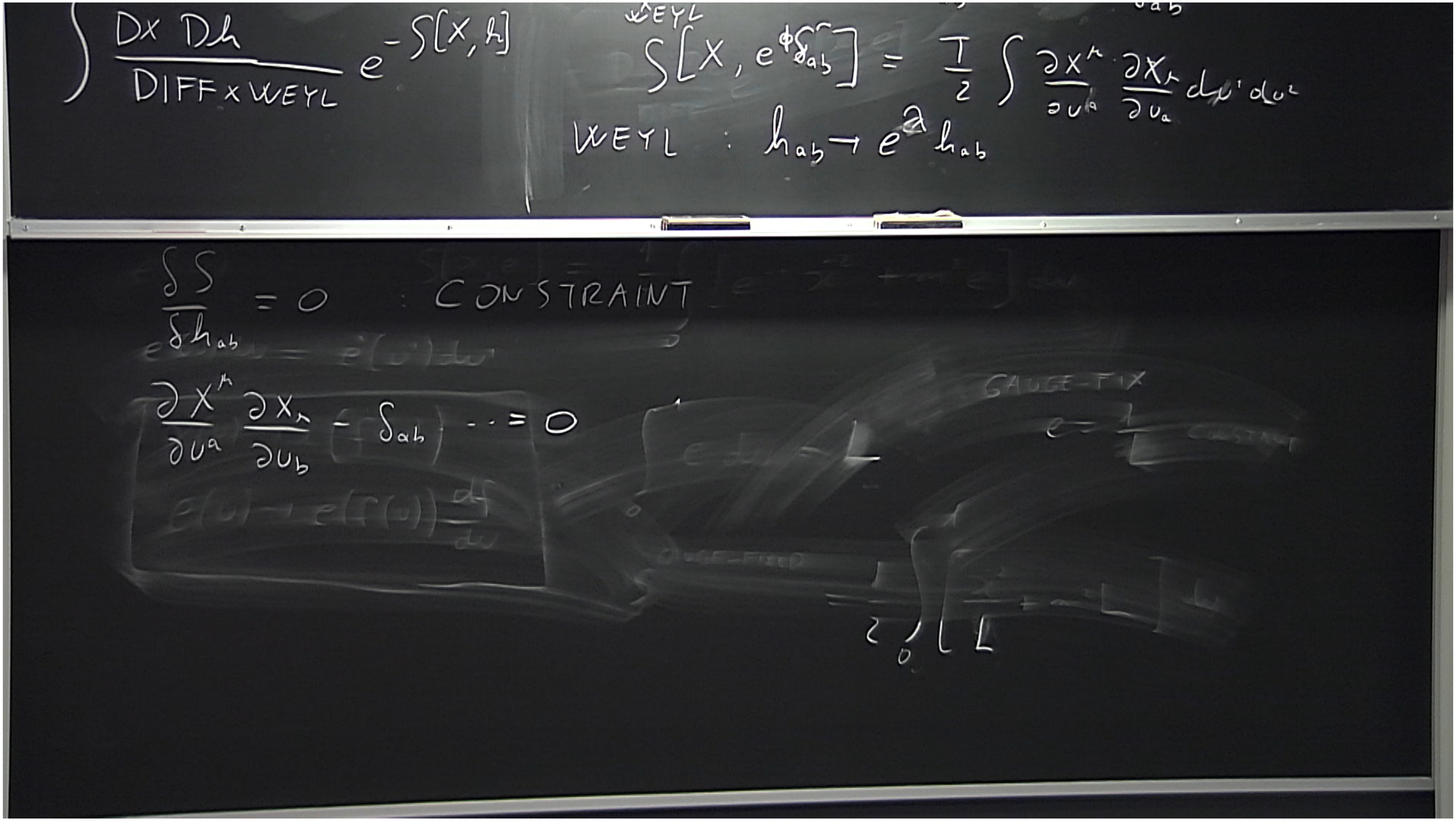
$$\boxed{\begin{aligned} x(u) &\to x(f(u)) \\ e(u) &\to e(f(u))\frac{df}{du} \end{aligned}}$$

$$\int_0^1 e\, du = L$$

GAUGE-FIX

$e = \frac{1}{L}$ CONSTANT

$$S^{\text{GAUGE-FIXED}} = \frac{1}{2}\int_0^1 \left[\frac{\dot{x}^2}{L} + m^2 L\right] du$$


∫ Dx Dh / DIFF x WEYL e^{-S[X,h]}
WEYL
S[X, e^φ δ_ab] = T/2 ∫ ∂X^μ/∂u^a ∂X_μ/∂u_a du^1 du^2
WEYL : h_ab → e^∂ h_ab
δS/δh_ab = 0 : CONSTRAINT
∂X^μ/∂u^a ∂X_μ/∂u_b − δ_ab ( ) ... = 0

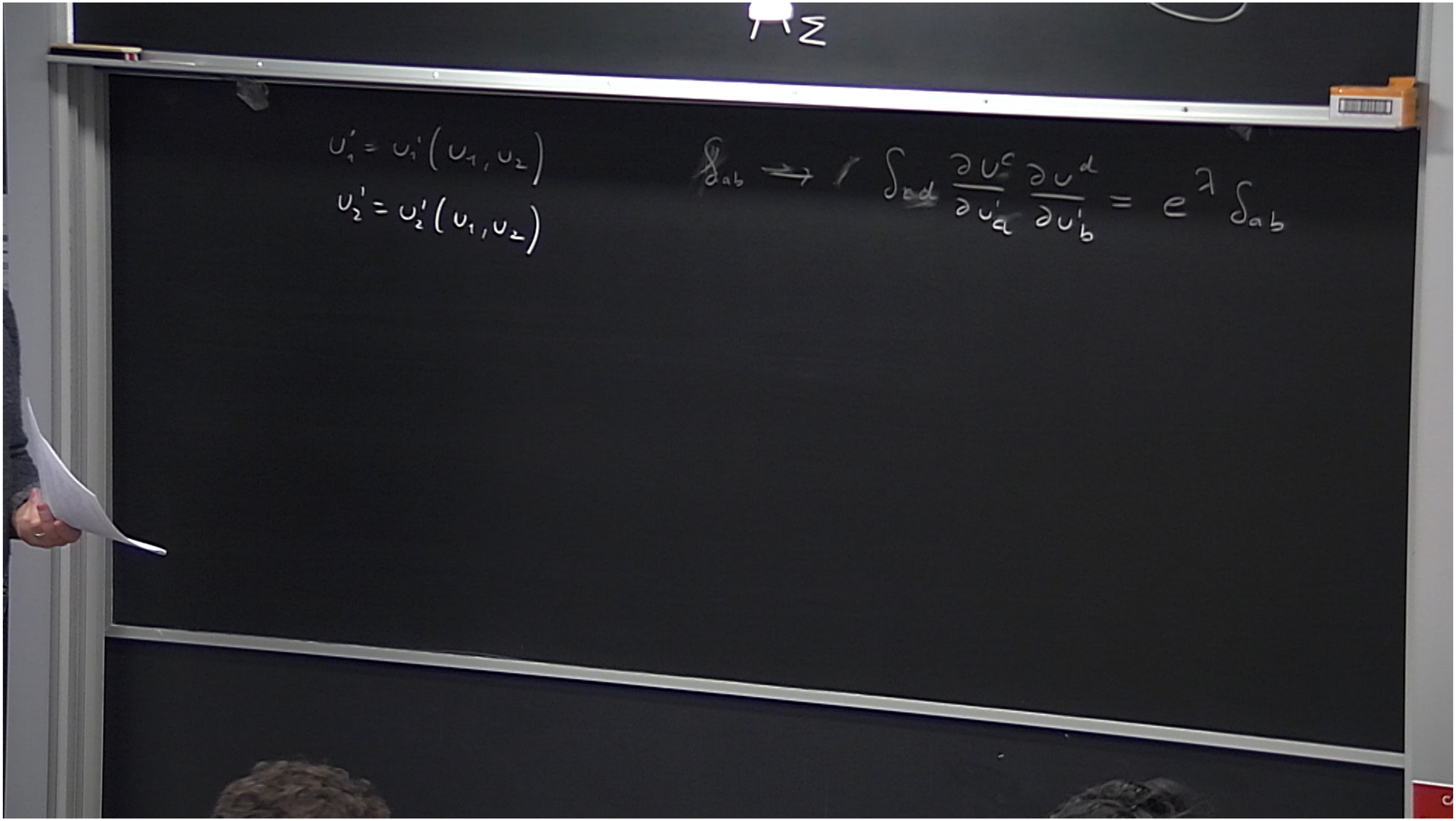
$$\int \frac{DX\, Dh}{\mathrm{DIFF} \times \mathrm{WEYL}} e^{-S[X,h]}$$

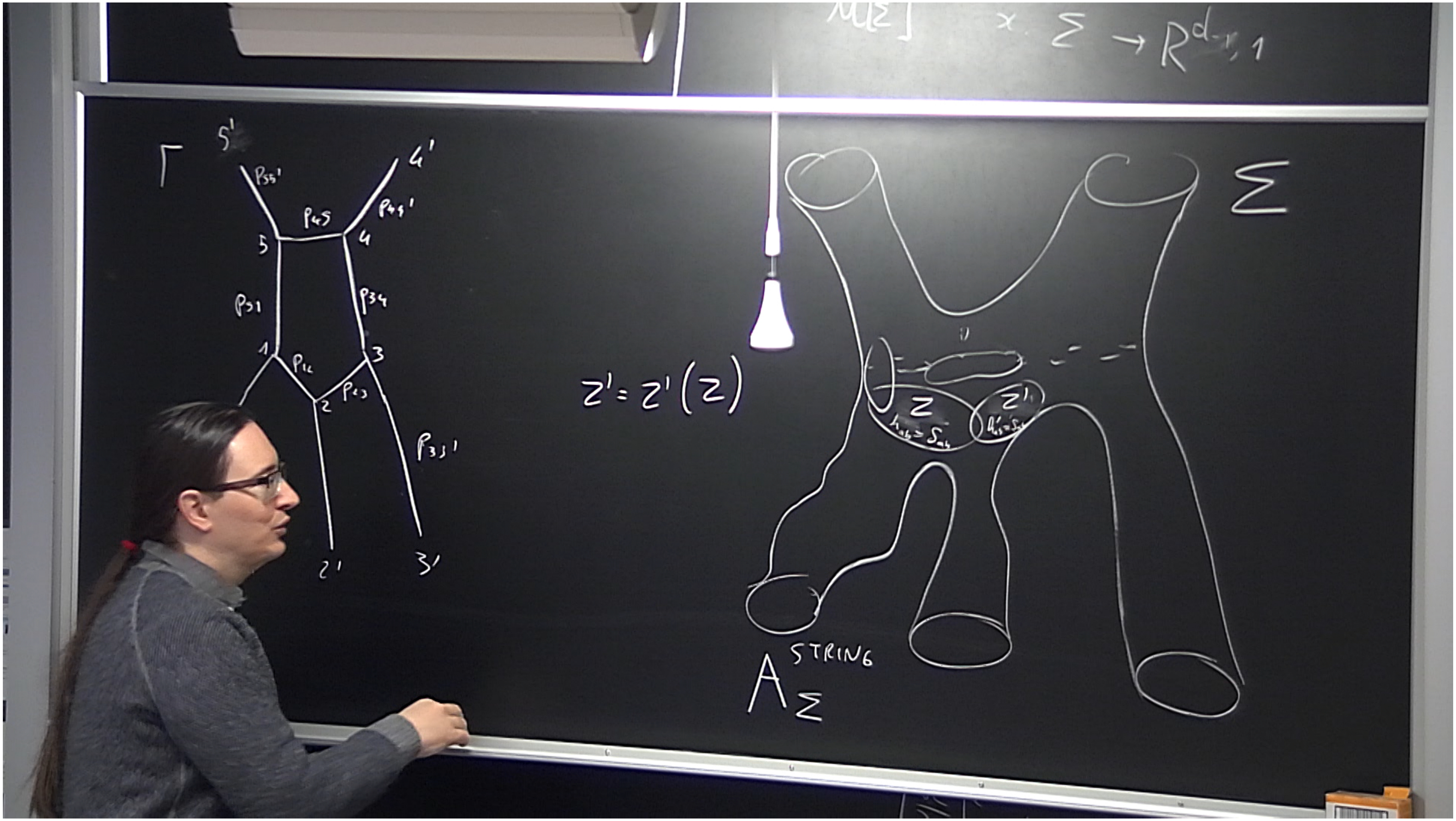
$$S[X, e^{\phi}\delta_{ab}] = \frac{T}{2} \int \frac{\partial X^{\mu}}{\partial u^{a}} \frac{\partial X_{\mu}}{\partial u_{a}} du^1 du^2$$

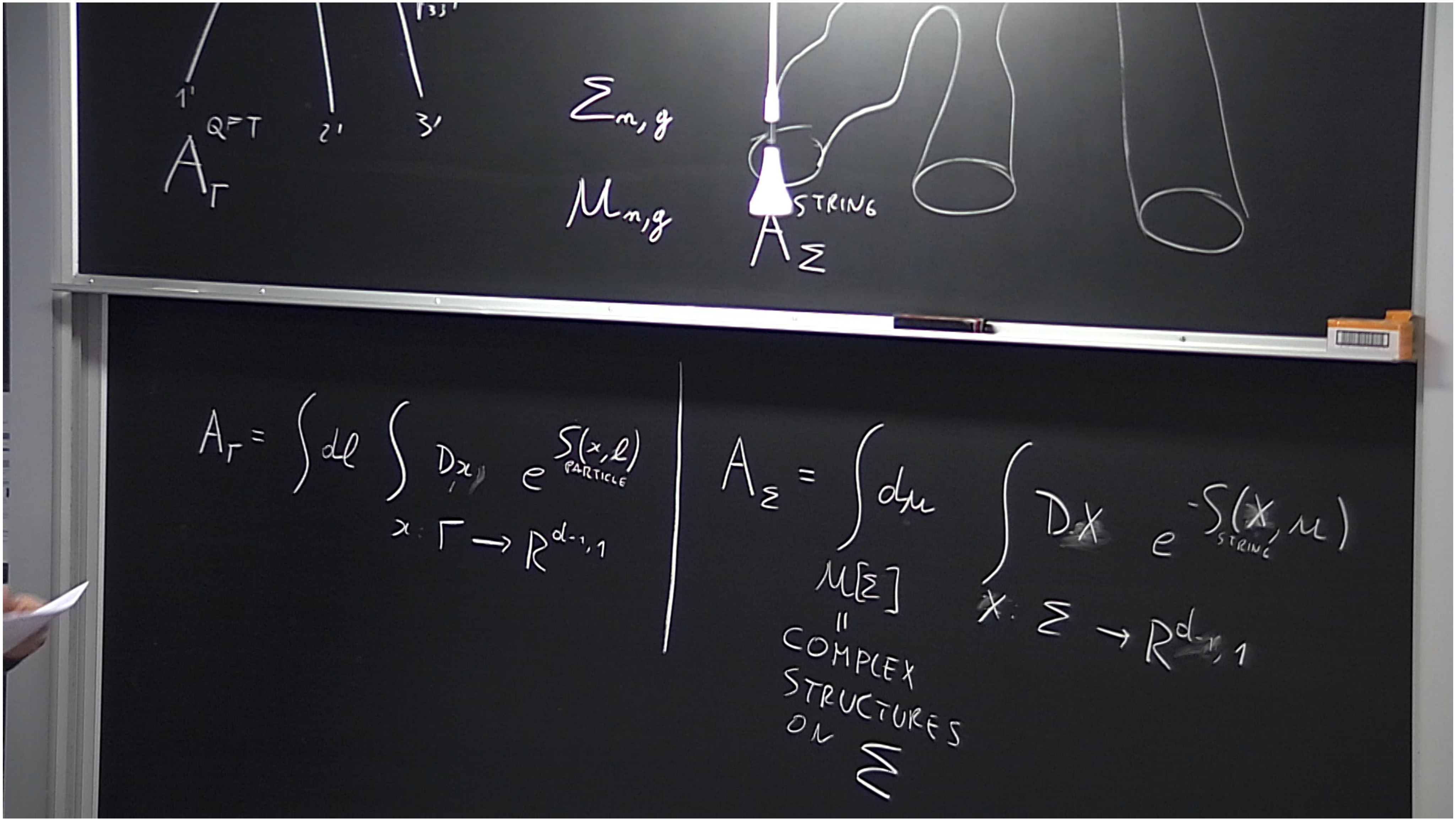
WEYL : $h_{ab} \to e^{\partial} h_{ab}$

$$\frac{\delta S}{\delta h_{ab}} = 0 \quad : \text{CONSTRAINT}$$

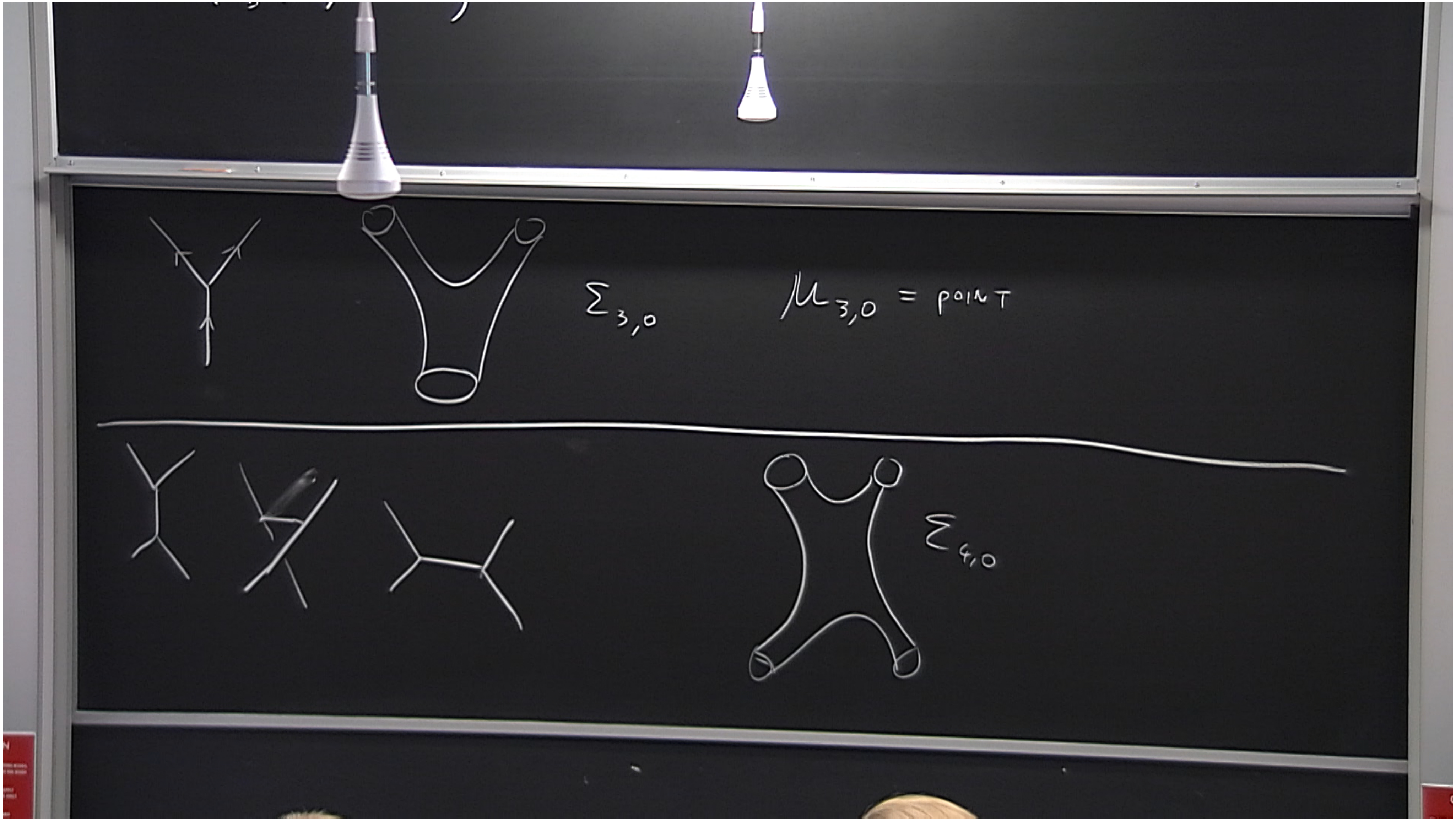
$$\frac{\partial X^{\mu}}{\partial u^{a}} \frac{\partial X_{\mu}}{\partial u_{b}} - \frac{1}{2}\delta_{ab} \frac{\partial X^{\mu}}{\partial u^{c}} \frac{\partial X_{\mu}}{\partial u_{c}} = 0$$

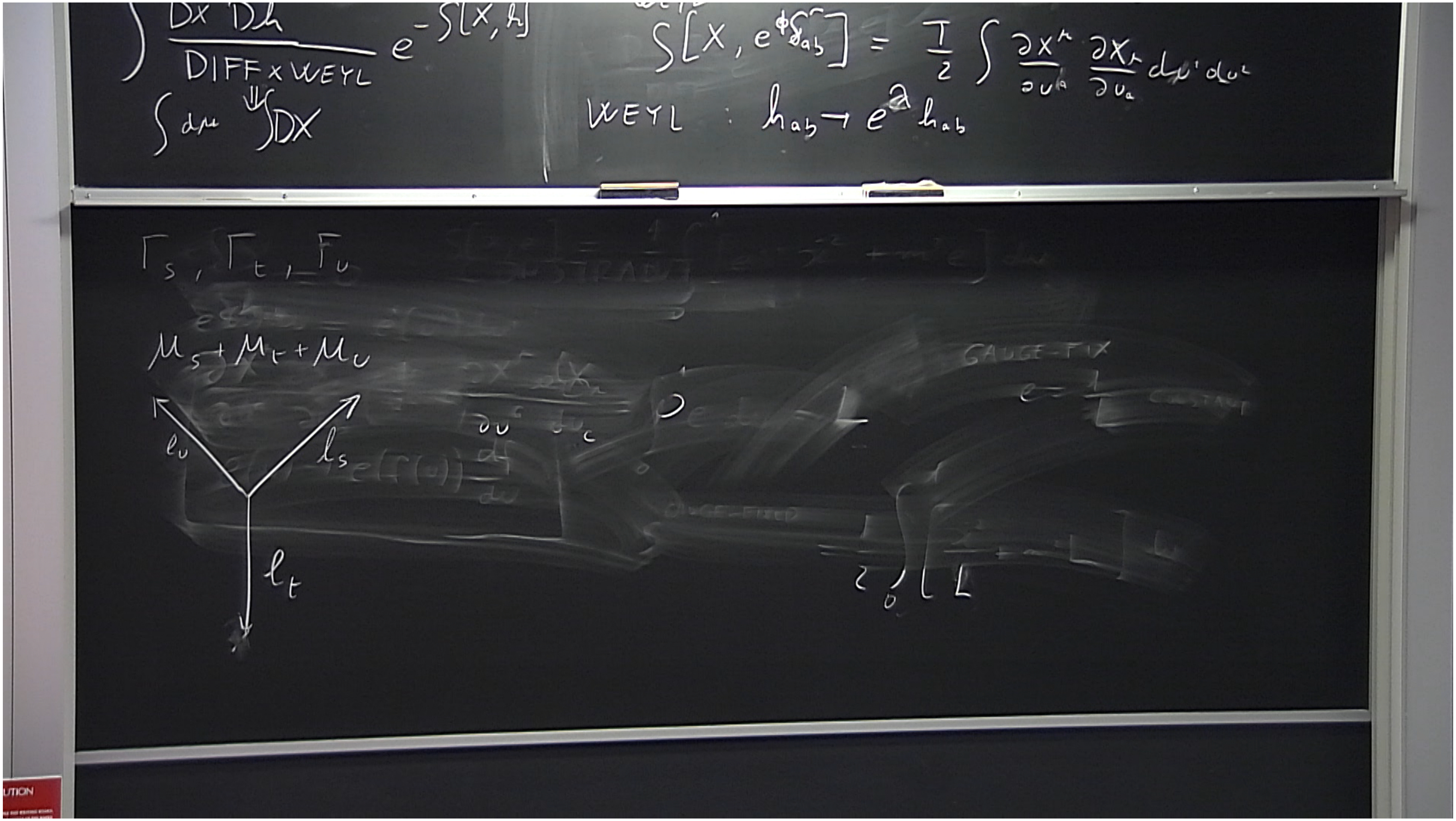

$u_1' = u_1'(u_1, u_2)$
$u_2' = u_2'(u_1, u_2)$
$\delta_{ab} \to \delta_{cd} \frac{\partial u^c}{\partial u'_a} \frac{\partial u^d}{\partial u'_b} = e^{\lambda} \delta_{ab}$

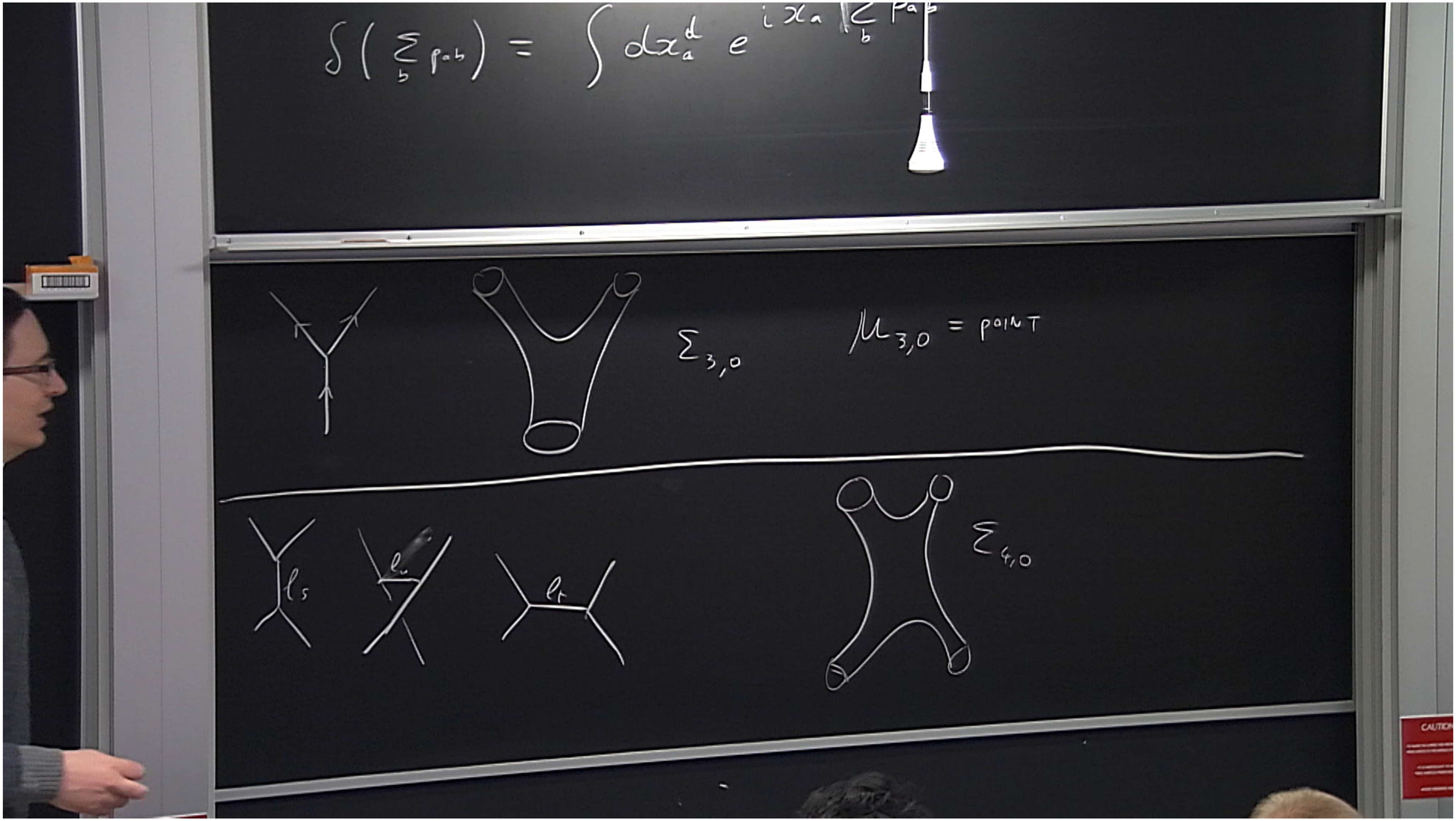

$x: \Sigma \to R^{d-1,1}$
Γ
5'
4'
$p_{55'}$
p_{45}
$p_{44'}$
5
4
p_{51}
p_{34}
1
3
p_{12}
p_{23}
2
$p_{33'}$
2'
3'
$z' = z'(z)$
Σ
z
z'
A_Σ^{STRING}

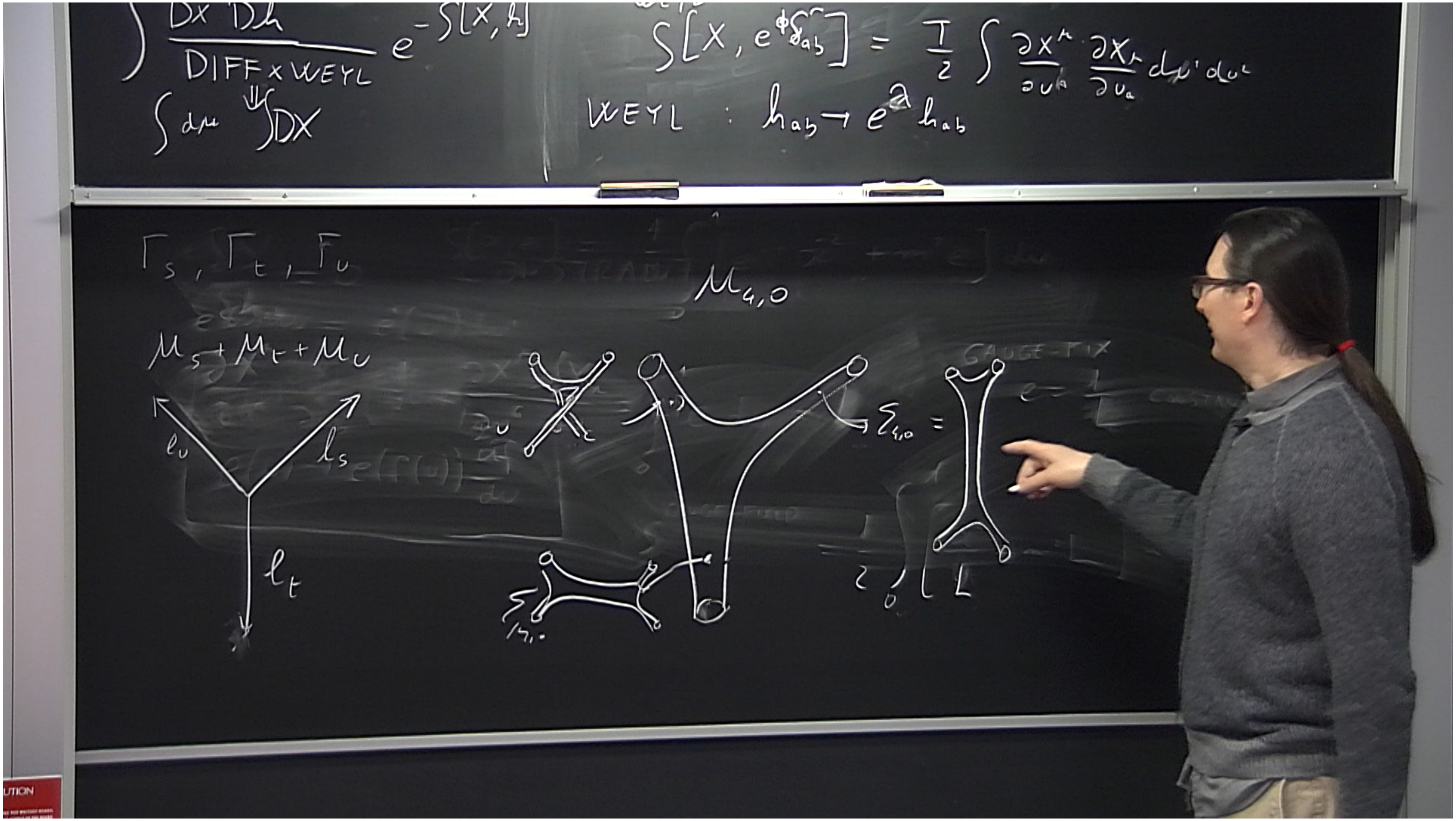


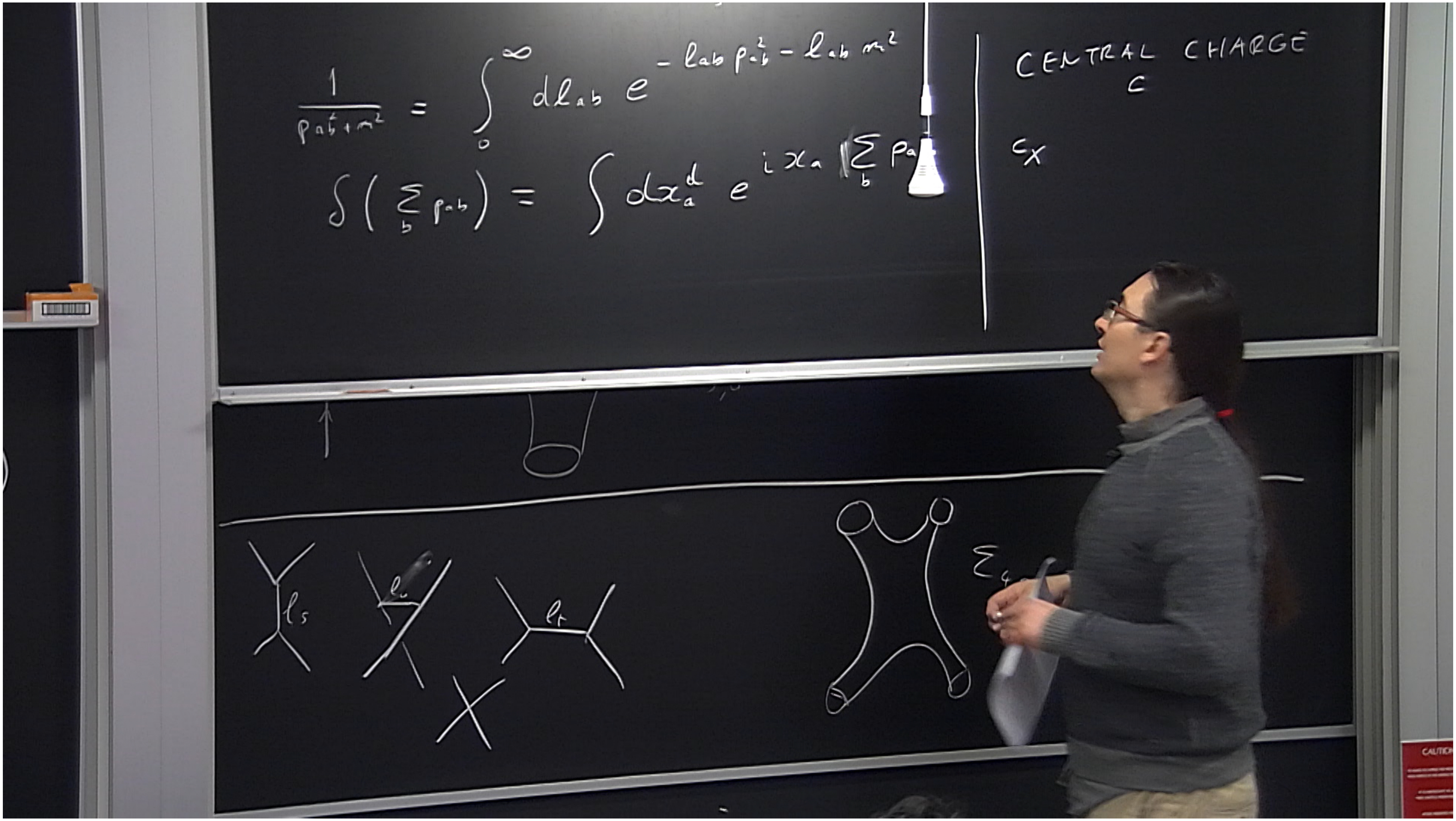
1'
2'
3'
QFT
A_Γ
$\Sigma_{n,g}$
$\mathcal{M}_{n,g}$
STRING
A_Σ
$A_\Gamma = \int d\ell \int Dx \; e^{S_{\text{PARTICLE}}(x,\ell)}$
$x : \Gamma \to R^{d-1,1}$
$A_\Sigma = \int_{\mathcal{M}[\Sigma]} d\mu \int DX \; e^{-S_{\text{STRING}}(X,\mu)}$
$\mathcal{M}[\Sigma]$ = COMPLEX STRUCTURES ON Σ
$X : \Sigma \to R^{d-1,1}$


$\Sigma_{3,0}$
$\mathcal{M}_{3,0}$ = POINT
$\Sigma_{4,0}$


$\int \frac{DX\,Dh}{\text{DIFF} \times \text{WEYL}} e^{-S[X,h]}$
$\int d\mu \Downarrow \int DX$
$S[X, e^{\phi}\delta_{ab}] = \frac{T}{2} \int \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u_a} du^1 du^2$
WEYL : $h_{ab} \to e^{\phi} h_{ab}$
$\Gamma_s, \Gamma_t, \Gamma_u$
$M_s + M_t + M_u$
ℓ_u
ℓ_s
ℓ_t
GAUGE-FIX


$\delta(\sum_b p_{ab}) = \int dx_a^d \, e^{i x_a \sum_b p_{ab}}$
$\Sigma_{3,0}$
$\mathcal{M}_{3,0}$ = POINT
ℓ_s
ℓ_u
ℓ_t
$\Sigma_{4,0}$
CAUTION


DIFF x WEYL
WEYL : h_ab → e^λ h_ab
S[X, e^φ δ_ab] = T/2 ∫ ∂X^μ/∂u^a ∂X_μ/∂u_a du^1 du^2
Γ_s , Γ_t , Γ_u
M_s + M_t + M_u
ℓ_u
ℓ_s
ℓ_t
M_4,0
Σ_4,0 =


CENTRAL CHARGE
c
c_X
1/(p_ab^2 + m^2) = ∫_0^∞ dl_ab e^(−l_ab p_ab^2 − l_ab m^2)
δ(Σ_b p_ab) = ∫ dx_a^d e^(i x_a Σ_b p_a

$(a,b) \in$ INTERNAL LINE

$a \in$ INTERNAL VERTICES

$(a,b) \in$ INTERNAL LINE $\quad p_{ab}^2+m^2$

$$\frac{1}{p_{ab}^2+m^2} = \int_0^\infty d\ell_{ab}\, e^{-\ell_{ab}p_{ab}^2-\ell_{ab}m^2}$$

$$\delta\left(\sum_b p_{ab}\right) = \int dx_a^d\, e^{i x_a \sum_b p_{ab}}$$

CENTRAL CHARGE

c

$c_X = 1 \times d$

$c_{h_{ab}} = -26$

$\Sigma_{4,0}$