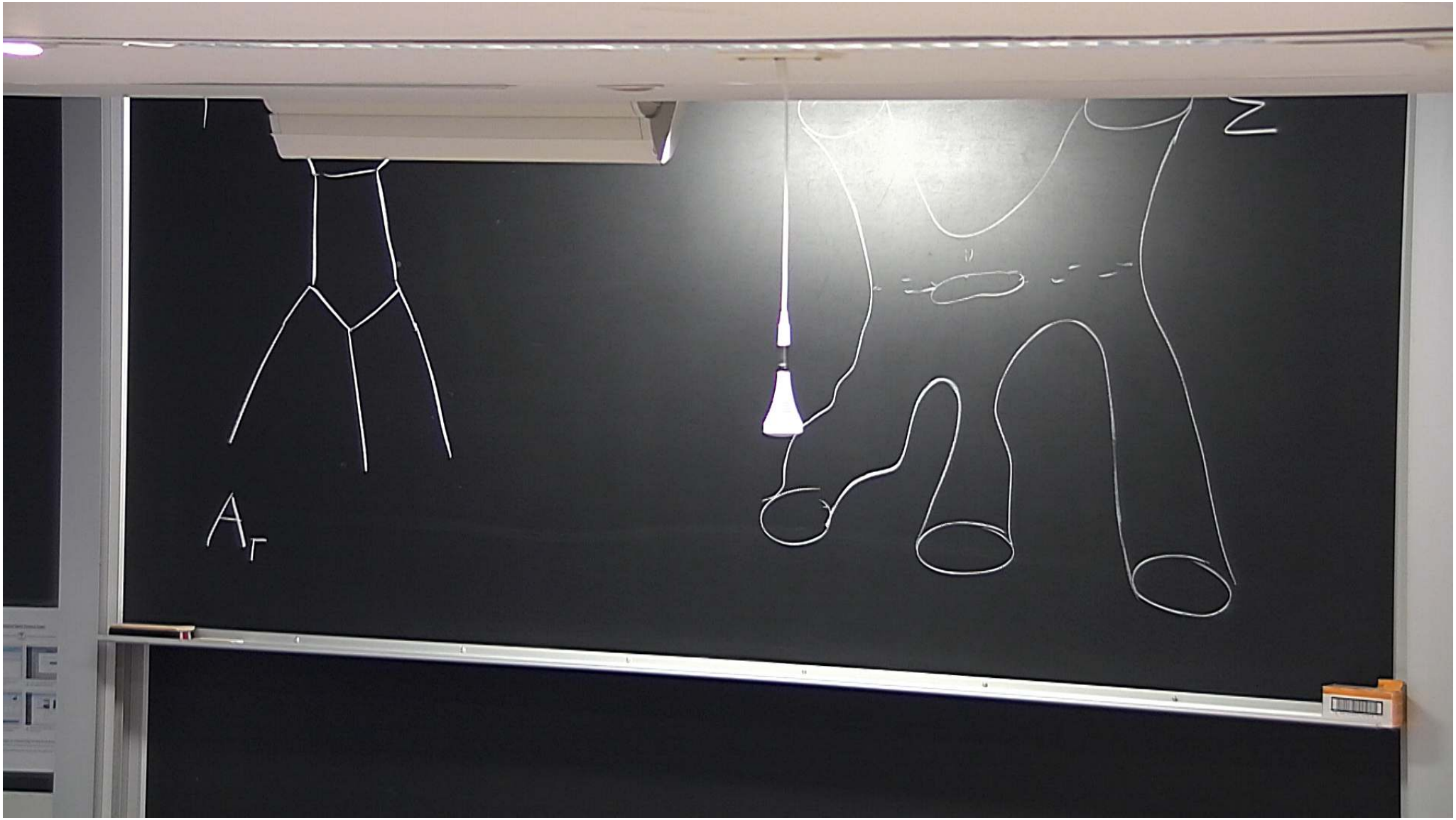
Title: PSI 2018/2019 - String Theory Review - Lecture 2

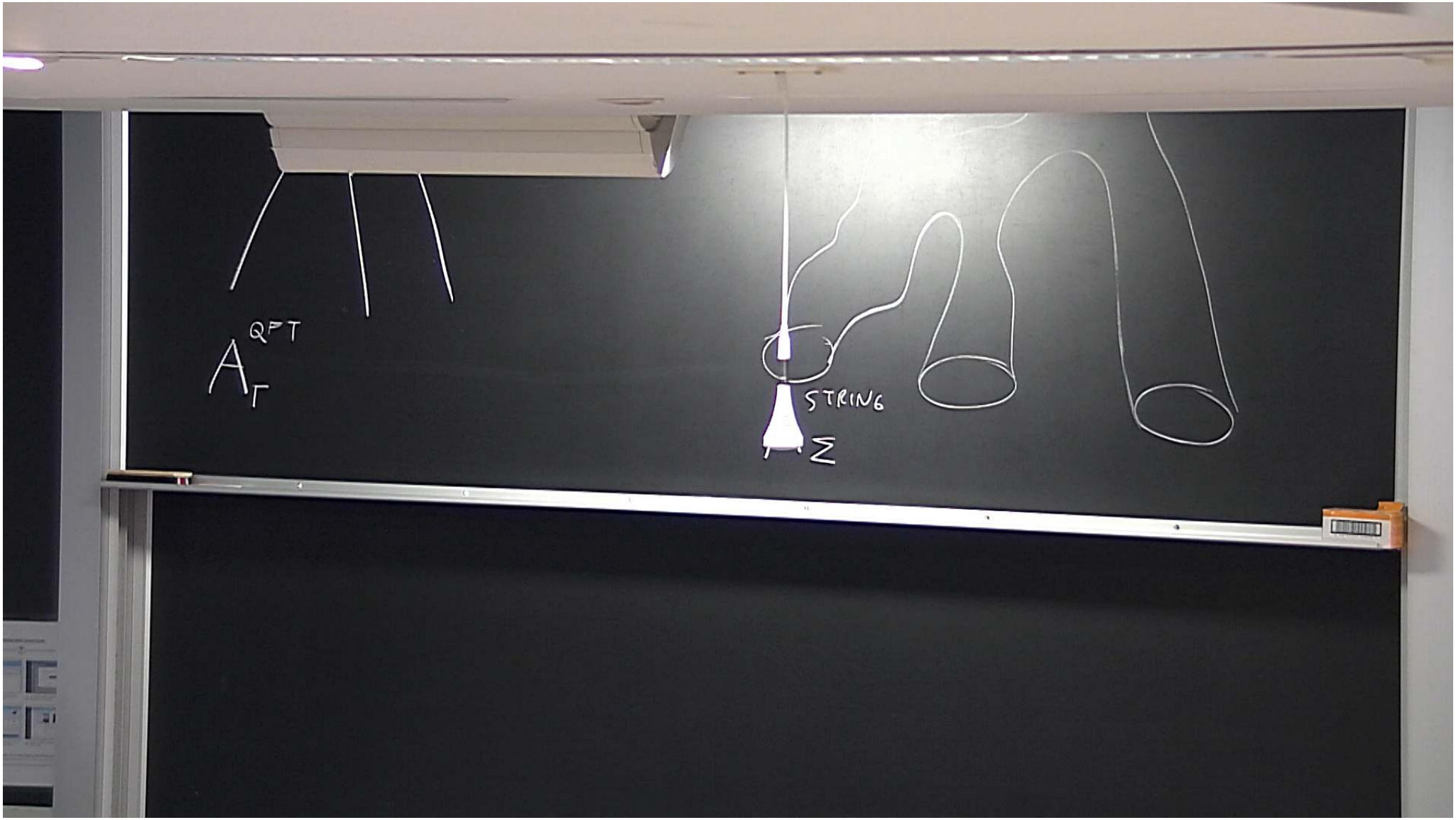
Speakers: Davide Gaiotto

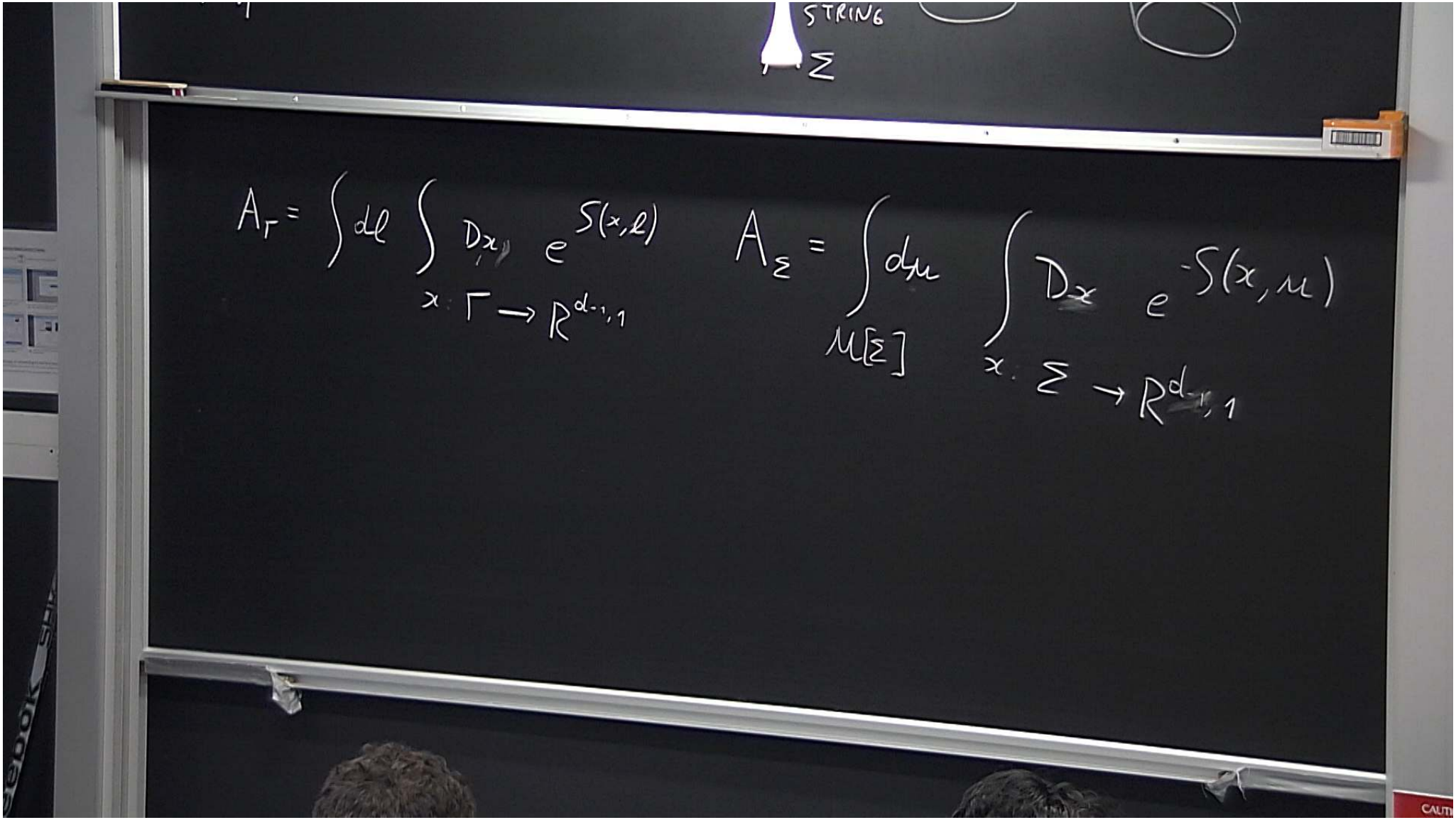
Collection: PSI 2018/2019 - String Theory Review (Gaiotto)

Date: March 26, 2019 - 10:15 AM

URL: http://pirsa.org/19030084


A_r


A_Γ^{QPT}
STRING
Σ

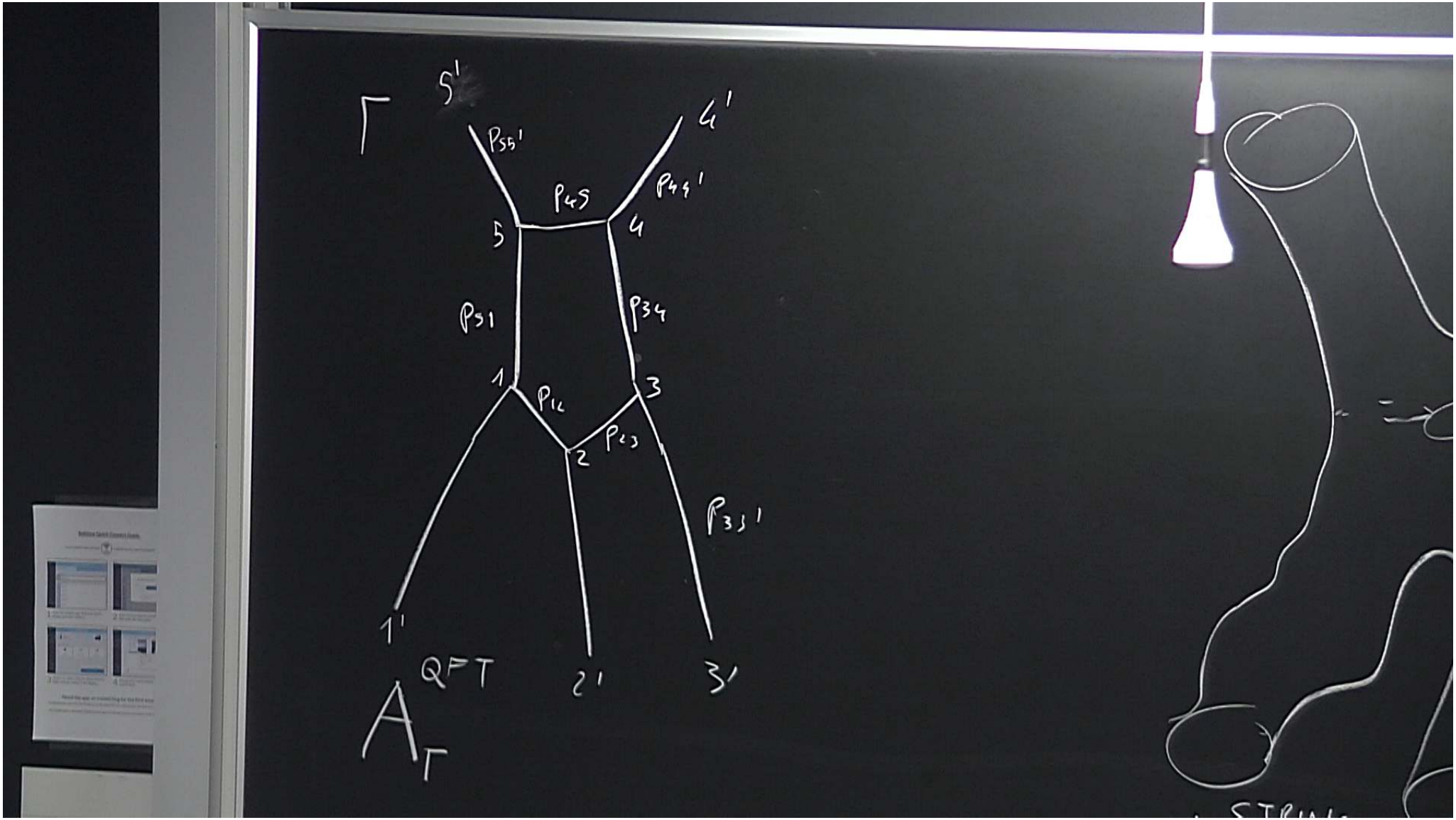

STRING
Σ
$A_\Gamma = \int d\ell \int_{x:\Gamma \to R^{d-1,1}} Dx\, e^{S(x,\ell)}$
$A_\Sigma = \int_{\mathcal{M}[\Sigma]} d\mu \int_{x:\Sigma \to R^{d-1,1}} Dx\, e^{-S(x,\mu)}$

$$\int Dx \; e^{S_{\text{PARTICLE}}(x,\ell)}$$

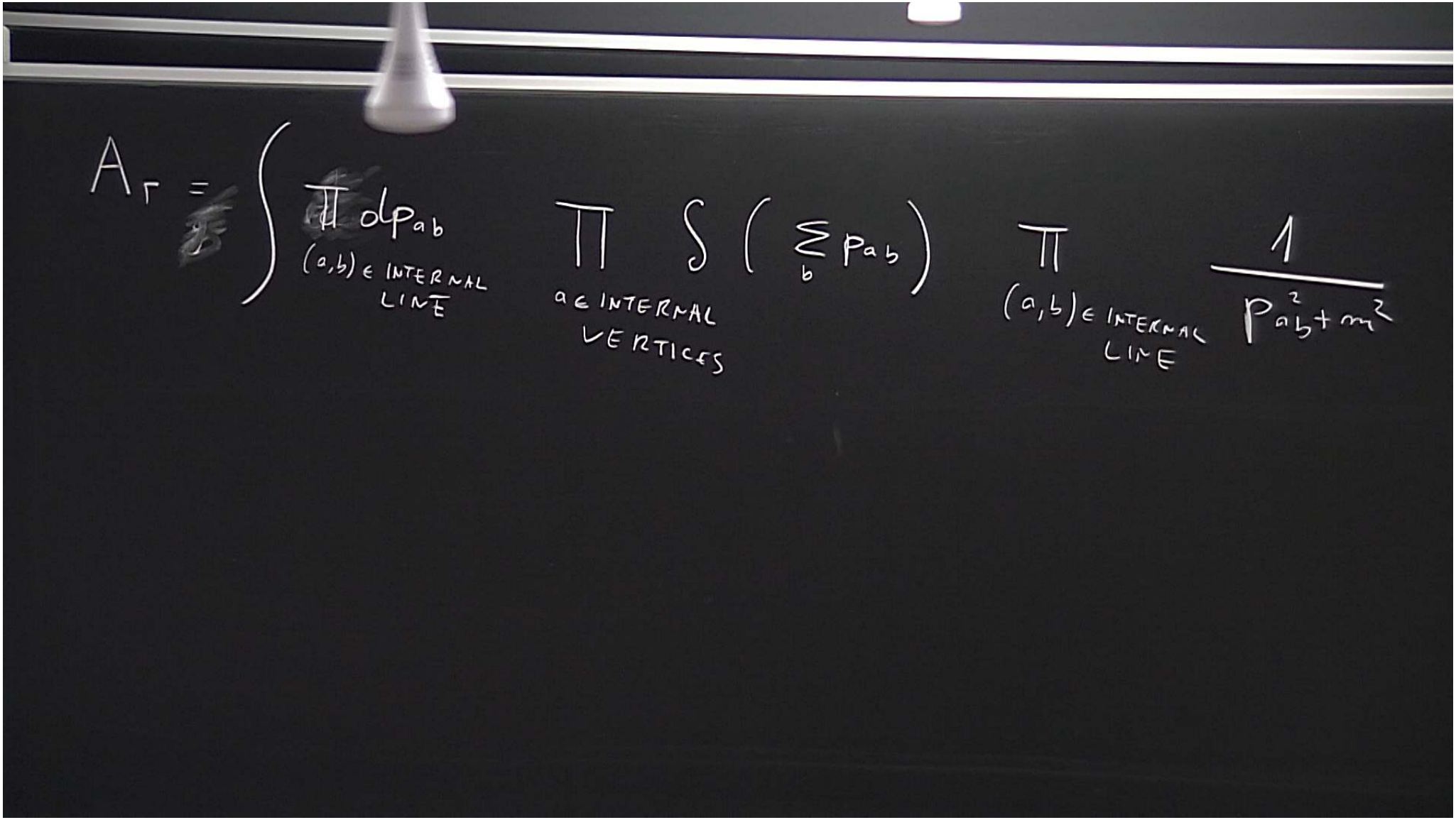
$$x : \Gamma \to R^{d-1,1}$$

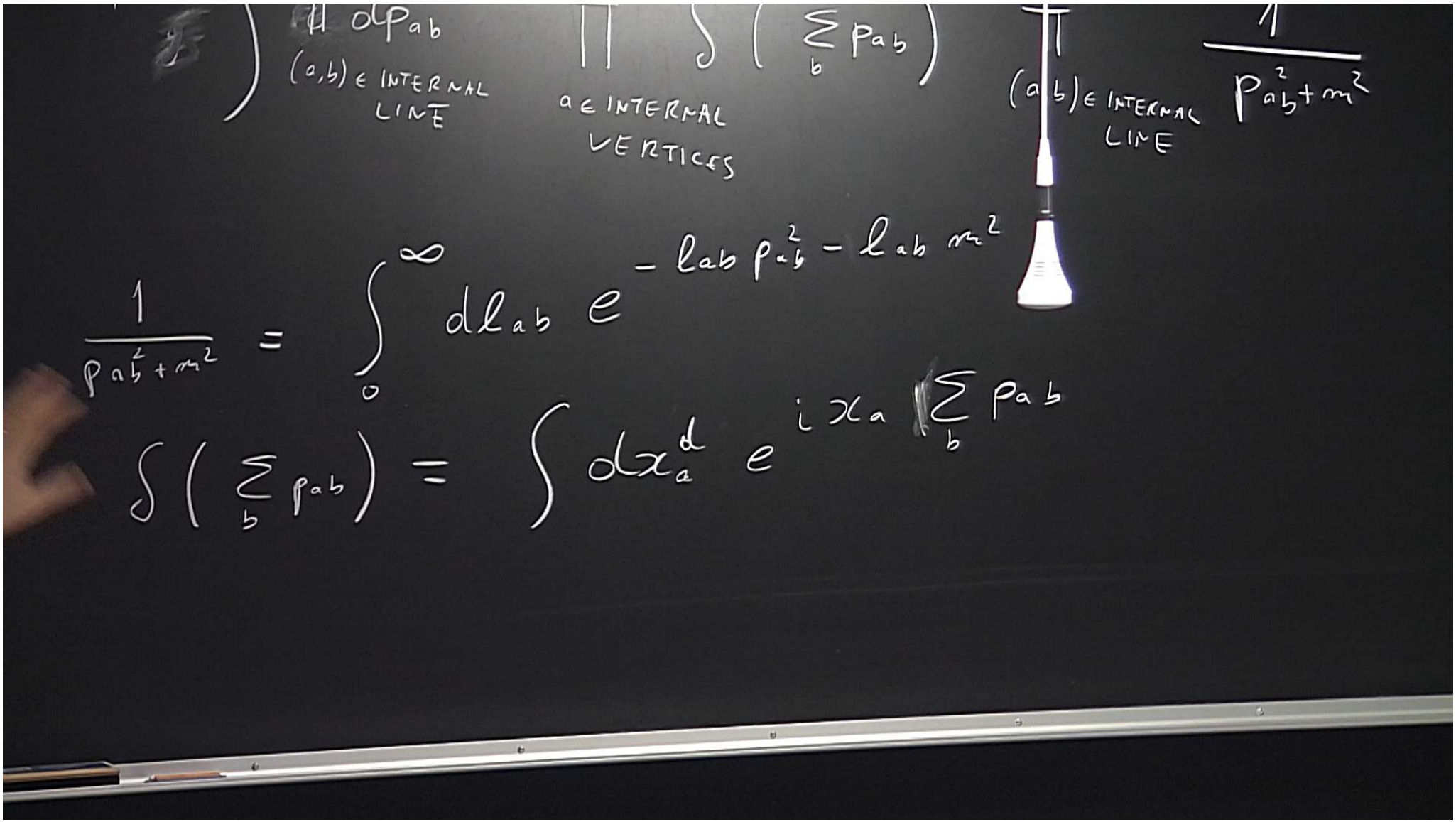
$$A_\Sigma = \int_{\mathcal{M}[\Sigma]} d\mu \int Dx \; e^{-S_{\text{STRING}}(x,\mu)}$$

$$x : \Sigma \to R^{d-1,1}$$


Γ
5'
4'
$p_{55'}$
p_{45}
$p_{44'}$
5
4
p_{51}
p_{34}
1
3
p_{12}
p_{23}
2
$p_{33'}$
1'
2'
3'
QFT
A_Γ

$$A_\Gamma = \int \prod_{(a,b)\in \text{INTERNAL LINE}} dp_{ab} \prod_{a\in \text{INTERNAL VERTICES}} \delta\left(\sum_b p_{ab}\right) \prod_{(a,b)\in \text{INTERNAL LINE}} \frac{1}{p_{ab}^2 + m^2}$$


$A_\Gamma = \int \prod_{(a,b)\in \text{INTERNAL LINE}} dp_{ab} \prod_{a\in \text{INTERNAL VERTICES}} \delta\left(\sum_b p_{ab}\right) \prod_{(a,b)\in \text{INTERNAL LINE}} \frac{1}{p_{ab}^2 + m^2}$


$\int \prod_{(a,b)\in \text{INTERNAL LINE}} d\ell_{ab} \prod_{a\in \text{INTERNAL VERTICES}} \delta\left(\sum_b p_{ab}\right) \prod_{(a,b)\in \text{INTERNAL LINE}} \frac{1}{p_{ab}^2+m^2}$
$\frac{1}{p_{ab}^2+m^2} = \int_0^\infty d\ell_{ab}\, e^{-\ell_{ab} p_{ab}^2 - \ell_{ab} m^2}$
$\delta\left(\sum_b p_{ab}\right) = \int dx_a^d\, e^{i x_a \sum_b p_{ab}}$

$$A_\Gamma = \prod_{a \in \text{INTERNAL}} \int dx_a^D \, e^{i x_a \sum_{b \in \text{EXTERNAL}} p_{ab}} \prod_{\substack{(a,b) \\ \in \\ \text{INTERNAL}}} \int_0^\infty dl_{ab} \, e^{-\frac{(x_a - x_b)^2}{4 l_{ab}} - l_{ab} m^2}$$

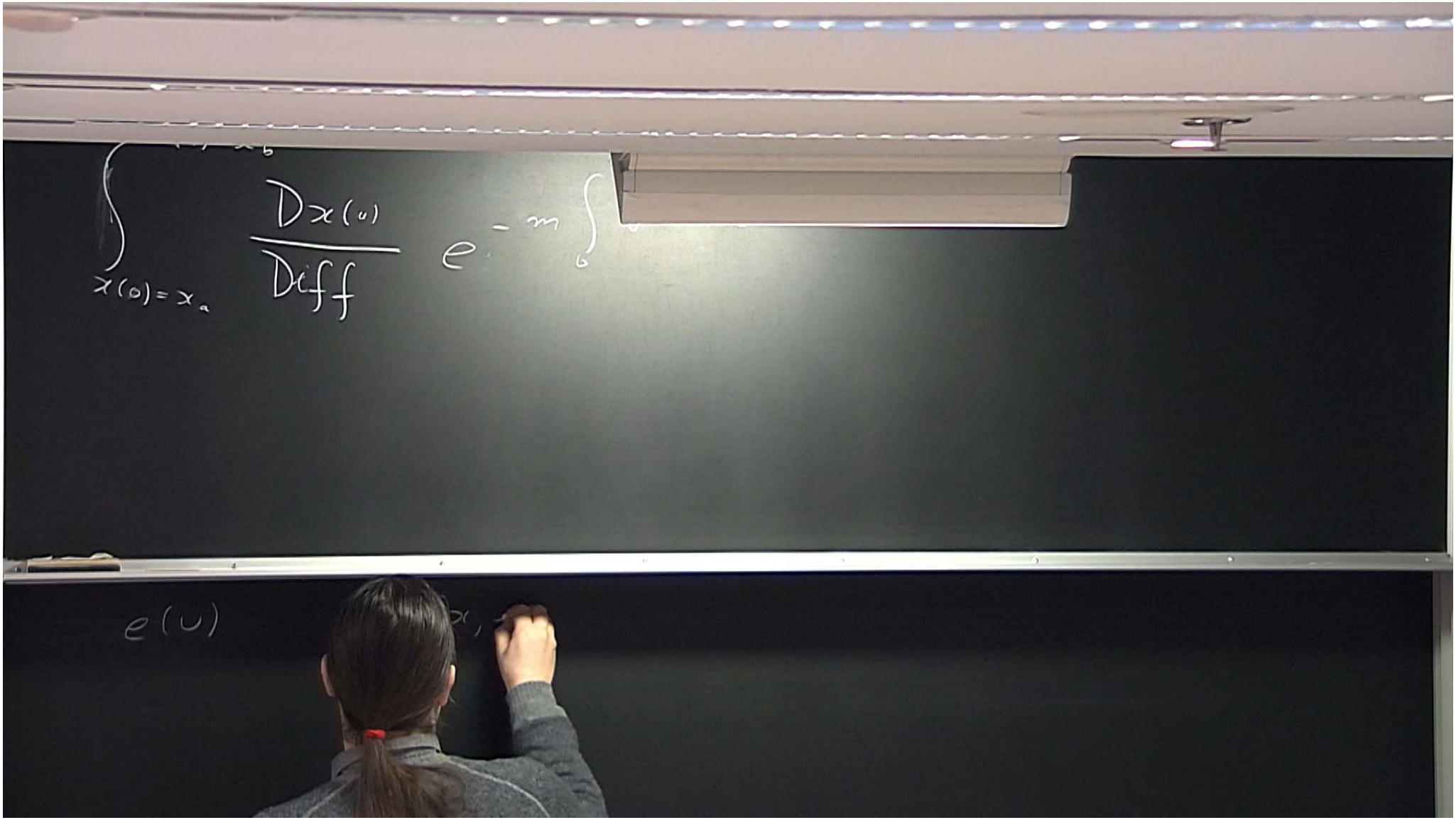
$$A_\Gamma = \prod_{a \in \text{INTERNAL}} \int dx_a^D \, e^{i x_a \sum_{b \in \text{EXTERNAL}} p_{ab}} \prod_{(a,b) \in \text{INTERNAL}} \int_0^\infty dl_{ab} \, e^{-\frac{(x_a - x_b)^2}{4 l_{ab}} - l_{ab} m^2}$$

$$G(x_a, b) = \int_0^\infty dl_{ab} \, e^{-\frac{(x_a - x_b)^2}{4 l_{ab}} - l_{ab} m^2}$$

$$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^\mu}{du} \frac{dx_\mu}{du}}\, du$$

$x(u) : [0,1] \to \mathbb{R}^{d-1,1}$

$u = f(u')$


x(0) = x_a
Dx(u)
Diff
e^{-m ∫
e(u)

$e(u)$ $\qquad S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1}\dot{x}^2 + m^2 e\right] du$

$e(u)\,du = e'(u')\,du'$

$$x(u) \to x(f(u))$$
$$e(u) \to e(f(u))\frac{df}{du}$$

$$\int_0^1 e\,du = \ell$$

GAUGE-FIX
$e = \frac{1}{\ell}$ CONSTANT

$e(u)$ $\qquad S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1}\dot{x}^2 + m^2 e\right] du$

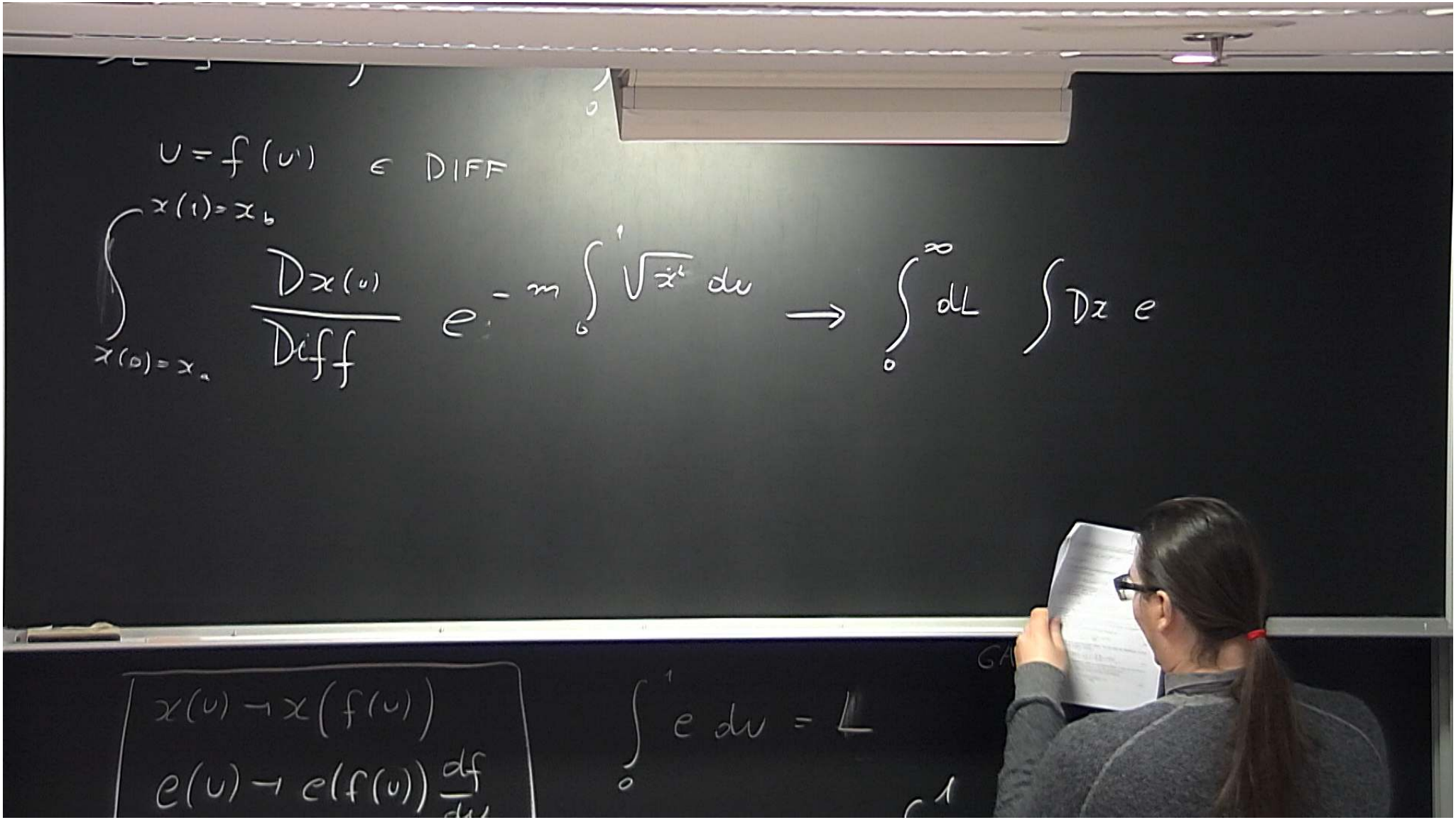
$e(u)\,du = e(u')\,du'$

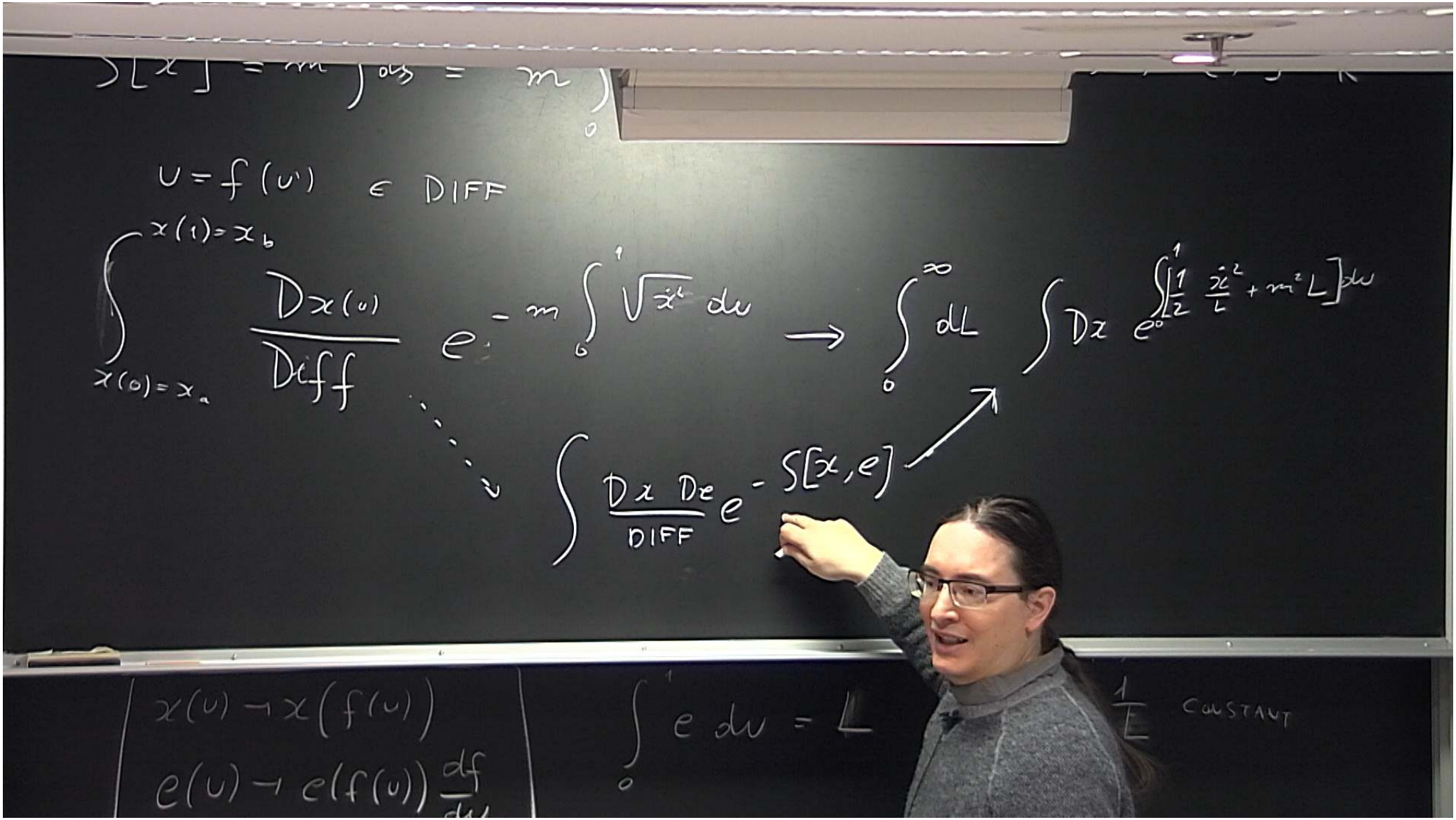
$$x(u) \to x(f(u))$$
$$e(u) \to e(f(u))\frac{df}{du}$$

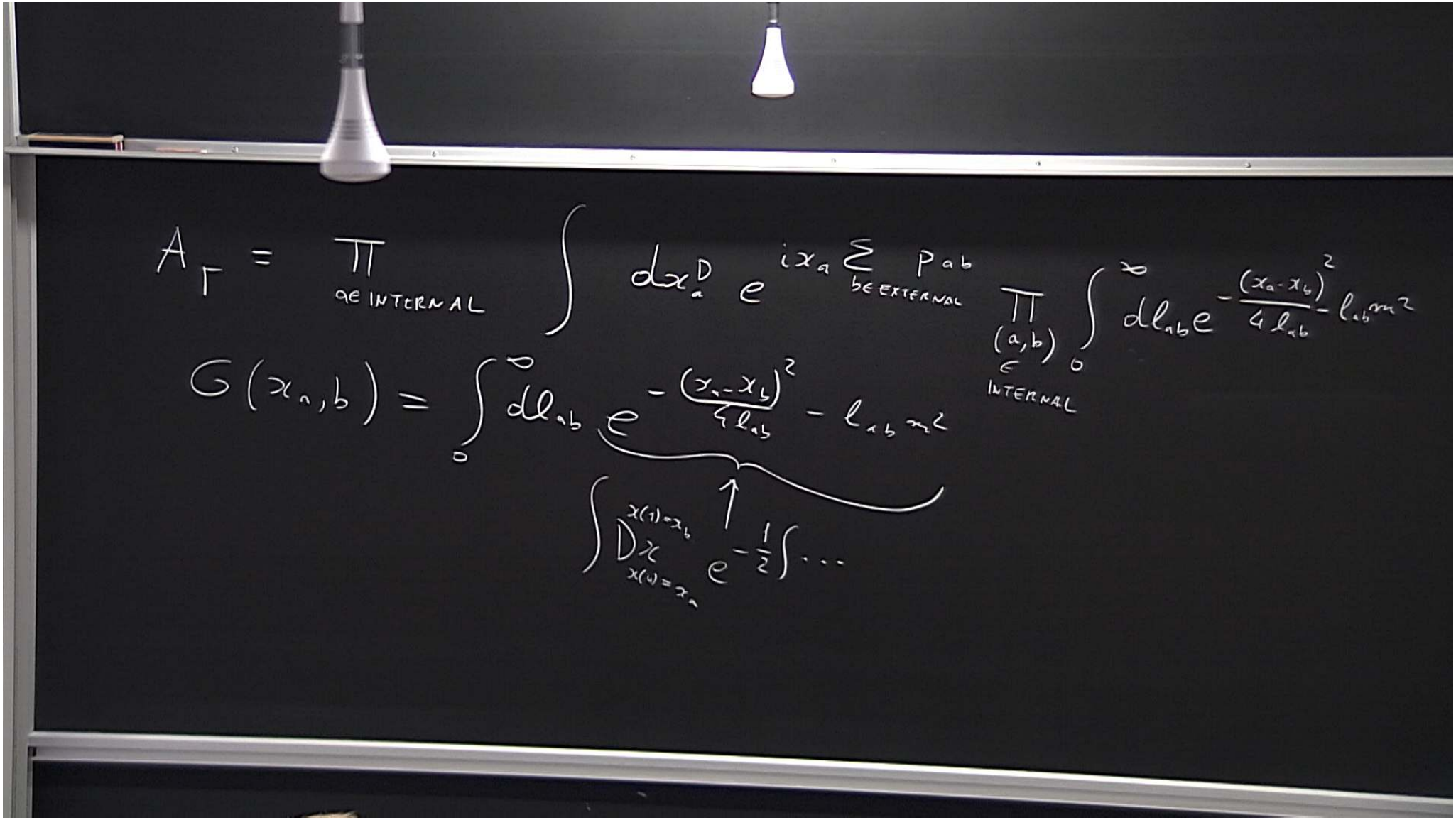
$\int_0^1 e\,du = L$

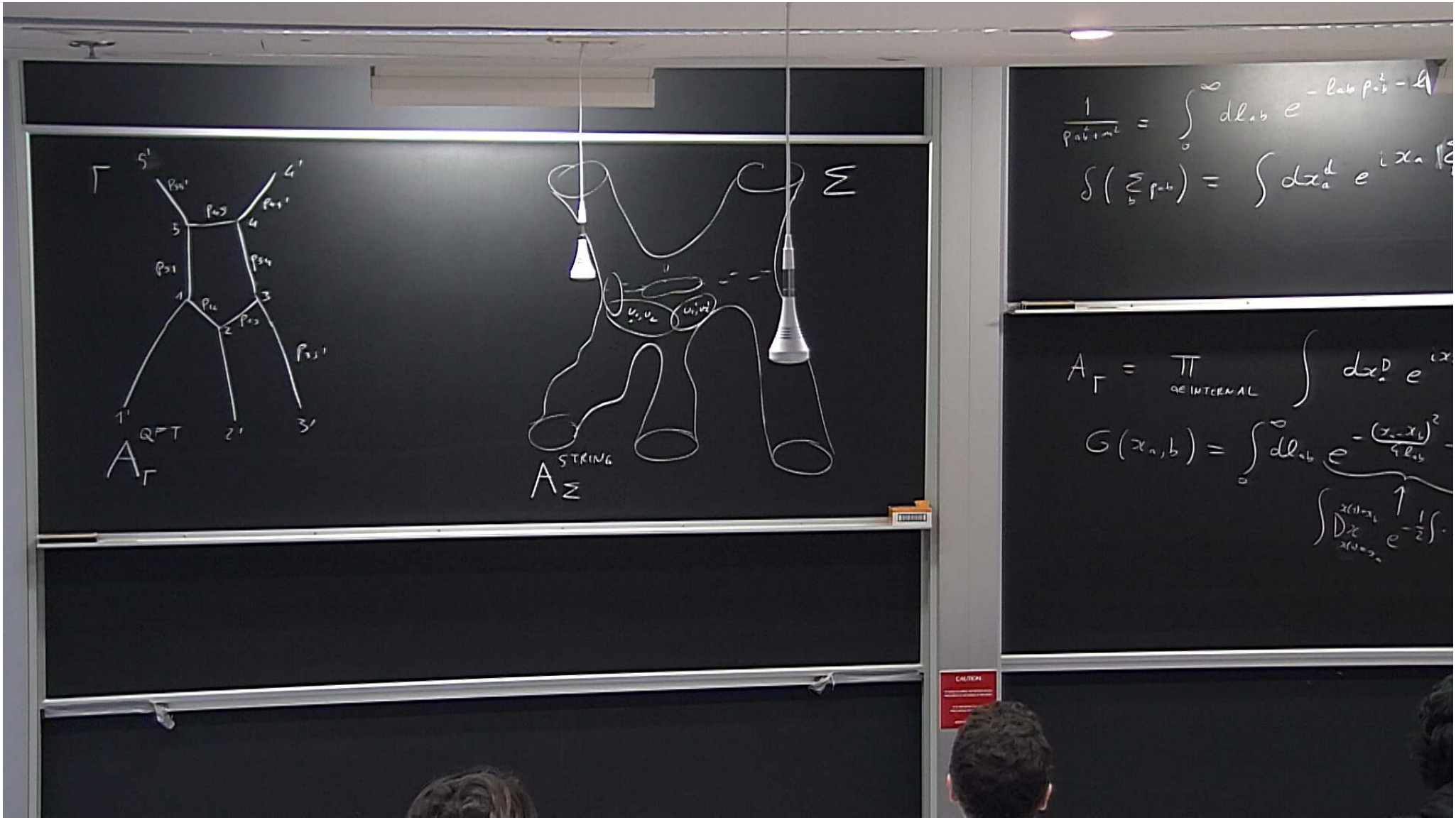
GAUGE-FIX
$e = \frac{1}{L}$ CONSTANT

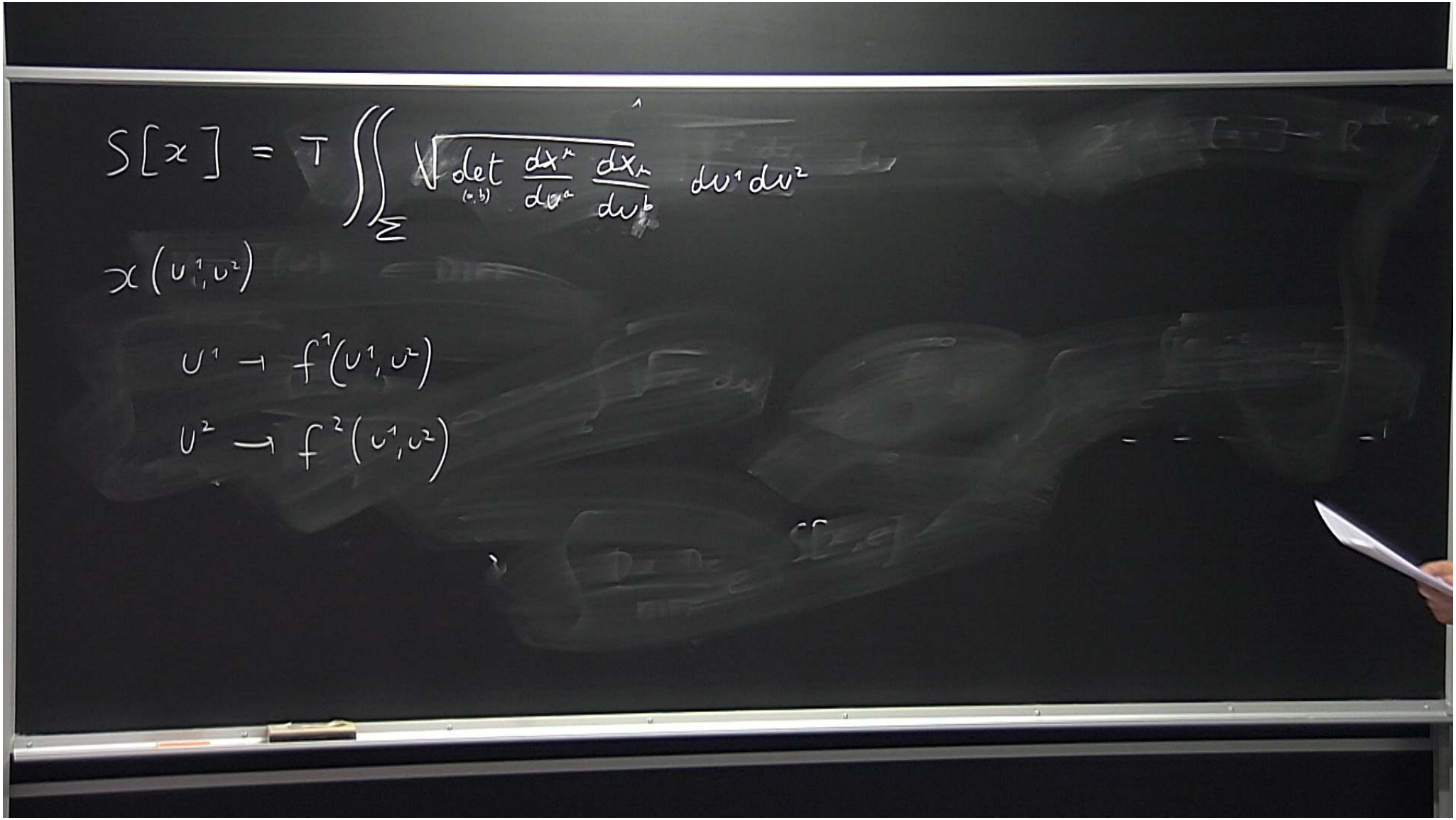
$S^{\text{GAUGE-FIXED}} = \frac{1}{2}\int_0^1 \left[\frac{\dot{x}^2}{L} + m^2 L\right] du$


$U = f(u) \in \mathrm{DIFF}$
$\int_{x(0)=x_a}^{x(1)=x_b} \frac{Dx(u)}{Diff} e^{-m\int_0^1 \sqrt{\dot{x}^2}\,du} \to \int_0^\infty dL \int Dx\, e$
$x(u) \to x(f(u))$
$e(u) \to e(f(u))\frac{df}{du}$
$\int_0^1 e\,du = L$


$U = f(u) \in \mathrm{DIFF}$
$\int_{x(0)=x_a}^{x(1)=x_b} \frac{Dx(u)}{\mathrm{Diff}} e^{-m\int_0^1 \sqrt{\dot{x}^2}\,du} \longrightarrow \int_0^\infty dL \int Dx\, e^{-\int_0^1 [\frac{1}{2}\frac{\dot{x}^2}{L} + m^2 L]\,du}$
$\int \frac{Dx\,De}{\mathrm{DIFF}} e^{-S[x,e]}$
$x(u) \to x(f(u))$
$e(u) \to e(f(u))\frac{df}{du}$
$\int_0^1 e\,du = L$
$\frac{1}{L}$ CONSTANT


$A_\Gamma = \prod_{a \in \text{INTERNAL}} \int dx_a^D \, e^{i x_a \sum_{b \in \text{EXTERNAL}} p_{ab}} \prod_{(a,b) \in \text{INTERNAL}} \int_0^\infty dl_{ab} \, e^{-\frac{(x_a - x_b)^2}{4 l_{ab}} - l_{ab} m^2}$
$G(x_a, x_b) = \int_0^\infty dl_{ab} \, e^{-\frac{(x_a - x_b)^2}{4 l_{ab}} - l_{ab} m^2}$
$\int_{x(0)=x_a}^{x(1)=x_b} \mathcal{D}x \, e^{-\frac{1}{2}\int \dots}$


QFT
STRING


$S[x] = T \iint_\Sigma \sqrt{\det_{(a,b)} \frac{dx^\mu}{du^a} \frac{dx_\mu}{du^b}}\, du^1 du^2$
$x(u^1, u^2)$
$u^1 \to f^1(u^1, u^2)$
$u^2 \to f^2(u^1, u^2)$

$$S[X] = T \iint_{\Sigma} \sqrt{\det_{(a,b)} \frac{dX^\mu}{du^a} \frac{dX_\mu}{du^b}}\, du^1 du^2 \qquad \int \frac{DX}{DIFF}\, e^{-S[X]}$$

$X(u^1, u^2)$

$u^1 \to f^1(u^1, u^2)$

$u^2 \to f^2(u^1, u^2)$

$X(u^a)\,,\; h_{ab}(u^c)$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u^b}\, du^1 du^2$$

$$= \frac{1}{2} \int \left[\frac{\dot{x}^2}{1} + m^2 L \right] du$$

$$S[X] = T \iint_{\Sigma} \sqrt{\det_{(a,b)} \frac{dX^\mu}{du^a} \frac{dX_\mu}{du^b}}\, du^1 du^2 \qquad \int \frac{DX}{DIFF}\, e^{-S[X]}$$

$X(u^1, u^2)$

$u^1 \to f^1(u^1, u^2)$

$u^2 \to f^2(u^1, u^2)$

$X(u^a)\,,\; h_{ab}(u^c)$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u^b}\, du^1 du^2$$

DIFF FIXING : $h_{ab} \Rightarrow e^{\phi}\, \delta_{ab}$

$$S^{CHOICE-FIXED} = \frac{1}{2} \int \left[\frac{\partial^2}{\partial} + m^2 \right] d\omega$$

$$S[X] = T \iint_{\Sigma} \sqrt{\det_{(a,b)} \frac{dX^\mu}{du^a} \frac{dX_\mu}{du^b}}\, du^1 du^2 \qquad \int \frac{DX}{DIFF}\, e^{-S[X]}$$

$X(u^1, u^2)$

$u^1 \to f^1(u^1, u^2)$

$u^2 \to f^2(u^1, u^2)$

$X(u^a)\,,\; h_{ab}(u^c)$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u^b}\, du^1 du^2$$

DIFF FIXING : $h_{ab} \Rightarrow e^{\phi} \delta_{ab}$

$$S[X, e^{\phi} \delta_{ab}] = \frac{T}{2} \int \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u^a}\, du^1 du^2$$

$$S^{GAUGE-FIXED} = \frac{1}{2} \int \left[\frac{\dot{x}^2}{1} + m^2 L \right] du$$

DIFF

$$u^2 \to f^2(u^1, u^2)$$

$$\int \frac{Dx\, Dh}{DIFF \times WEYL} e^{-S[x,h]}$$

$$\frac{\partial x^\mu}{\partial u^a} \frac{\partial x_\mu}{\partial u^b} du^1 du^2$$

DIFF FIXING : $h_{ab} \Rightarrow \delta_{ab}$

WEYL

$$S[x, e^{\phi} \delta_{ab}] = \frac{T}{2} \int \frac{\partial x^\mu}{\partial u^a} \frac{\partial x_\mu}{\partial u^a} du^1 du^2$$

WEYL : $h_{ab} \to e^{2\phi} h_{ab}$

$e(u)$ $\qquad S[x,e] = \frac{1}{2} \int_0^1 \left[e^{-1} \dot{x}^2 + m^2 e \right] du$

$e(u)du = e'(u')du'$

$$x(u) \to x(f(u))$$

$$e(u) \to e(f(u)) \frac{df}{du}$$

$$\int_0^1 e\, du = L$$

GAUGE-FIX

$e = \frac{1}{L}$ CONSTANT

$$S^{\text{GAUGE-FIXED}} = \frac{1}{2} \int_0^1 \left[\frac{\dot{x}^2}{L} + m^2 L \right] du$$

$$\int \frac{Dx\, Dh}{DIFF \times WEYL} e^{-S[x,h]}$$

WEYL

$$S[X, e^{\phi} \delta_{ab}] = \frac{T}{2} \int \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u_a} du^1 du^2$$

$$WEYL : h_{ab} \to e^{\phi} h_{ab}$$

$$\frac{\delta S}{\delta h_{ab}} = 0 \quad : \text{CONSTRAINT}$$

$$\frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u_b} - \delta_{ab} \cdots = 0$$

GAUGE-FIX

GAUGE-FIXED

$$\int \frac{Dx\,Dh}{DIFF \times WEYL} e^{-S[x,h]}$$

$$S[X, e^{\phi} \delta_{ab}] = \frac{T}{2} \int \frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u_a} du^1 du^2$$

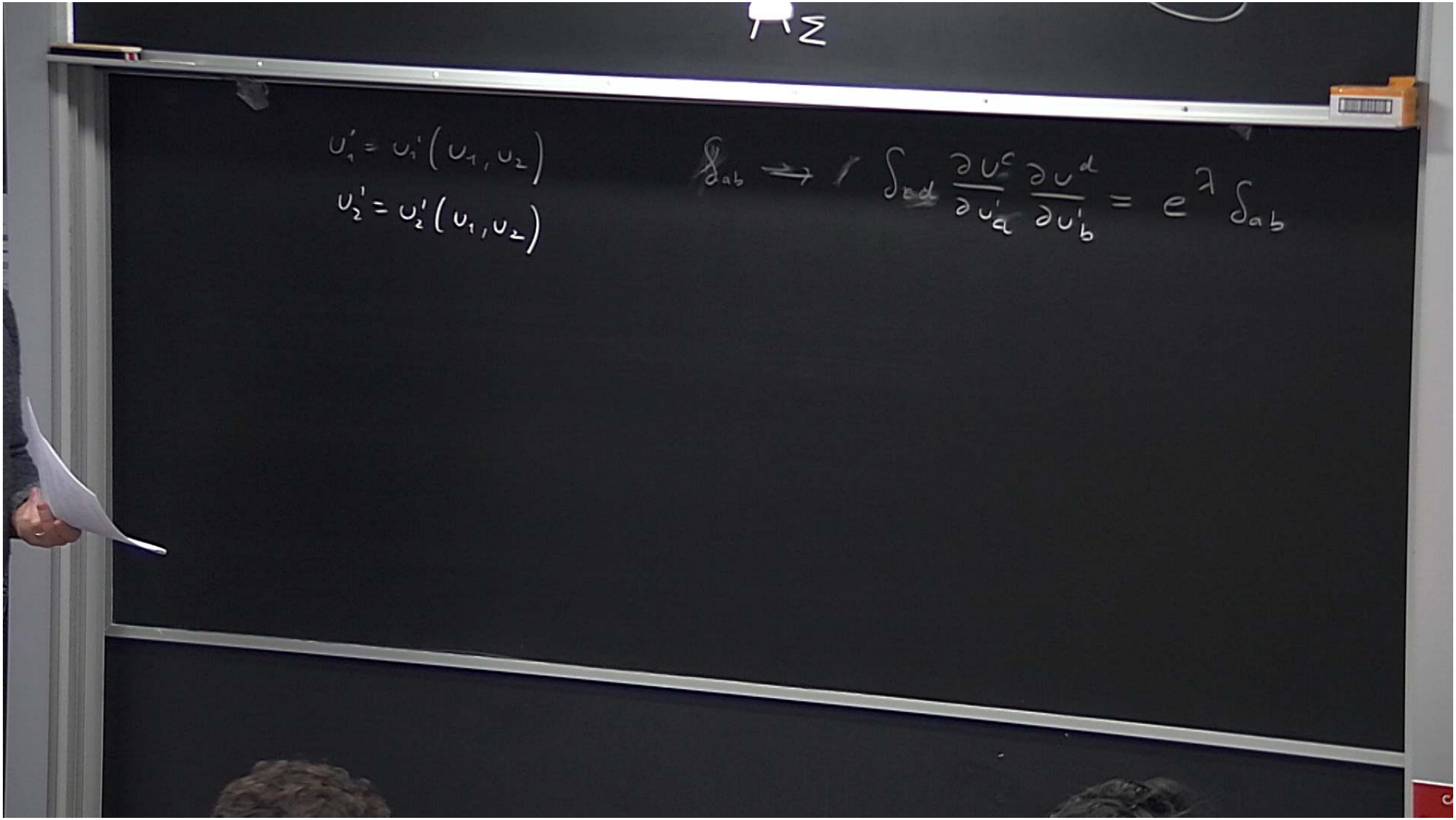
WEYL : $h_{ab} \to e^{\phi} h_{ab}$

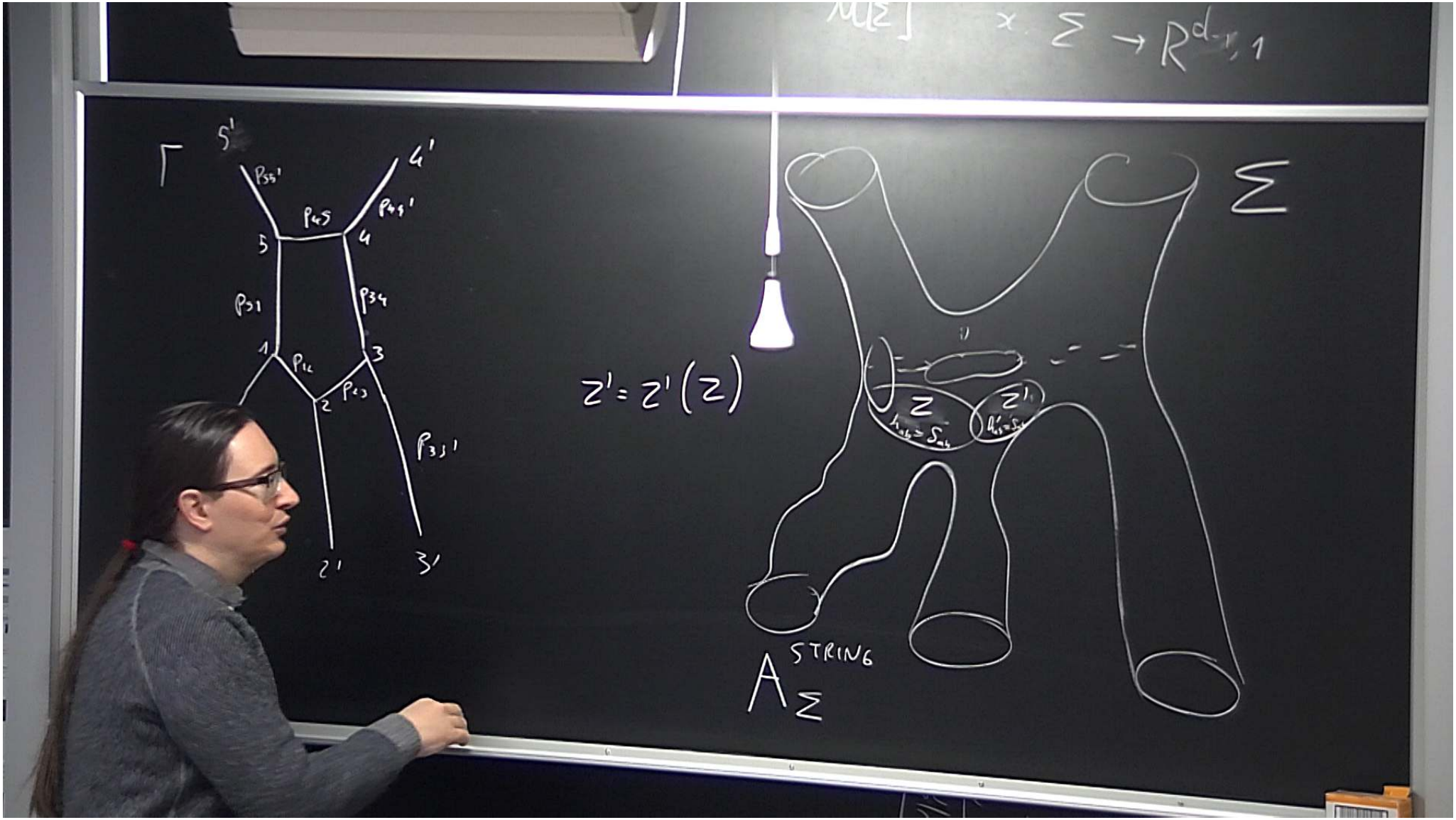
$$\frac{\delta S}{\delta h_{ab}} = 0 \quad : \text{CONSTRAINT}$$

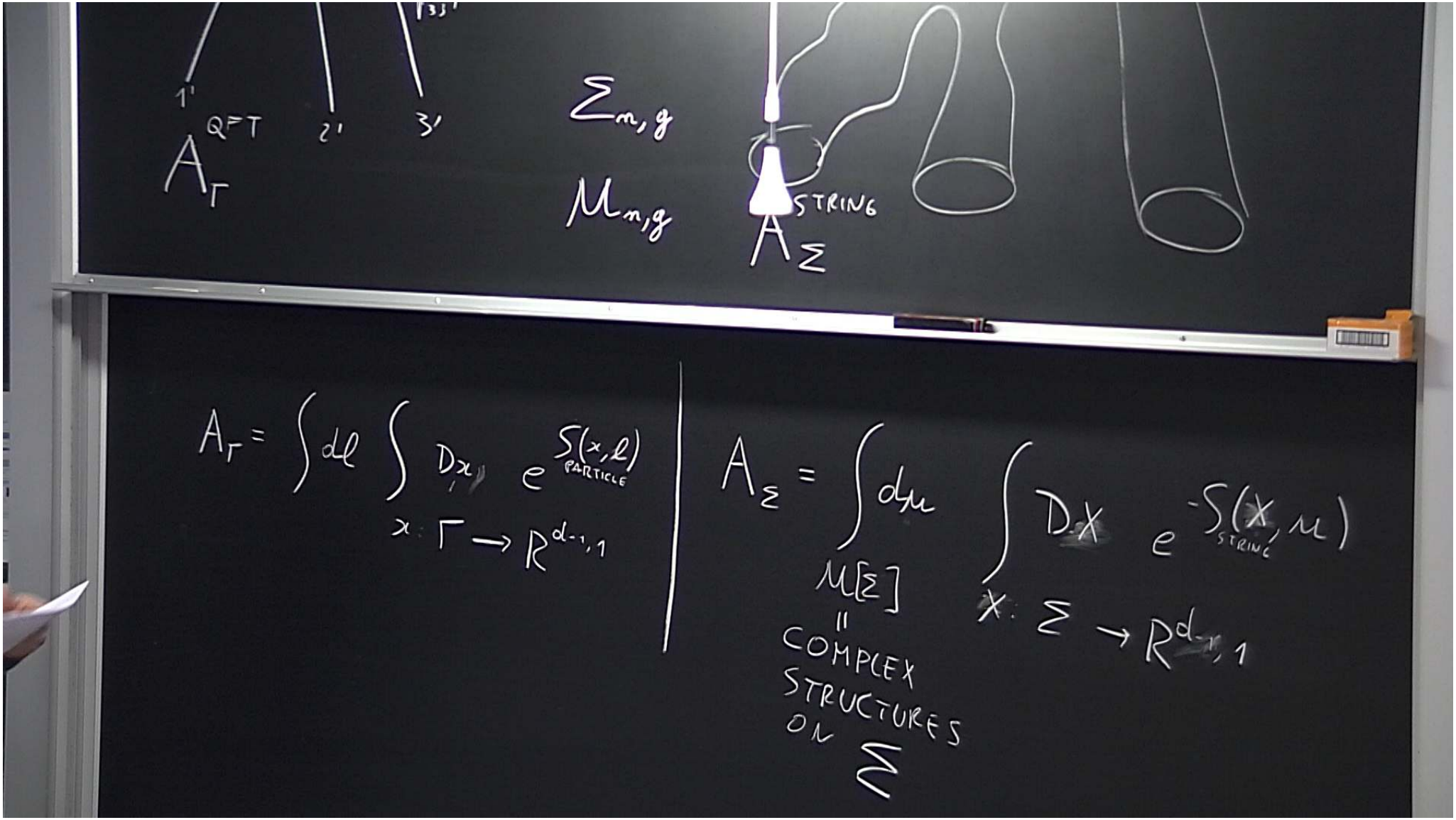
$$\frac{\partial X^\mu}{\partial u^a} \frac{\partial X_\mu}{\partial u_b} - \frac{1}{2} \delta_{ab} \frac{\partial X^\mu}{\partial u^c} \frac{\partial X_\mu}{\partial u_c} = 0$$

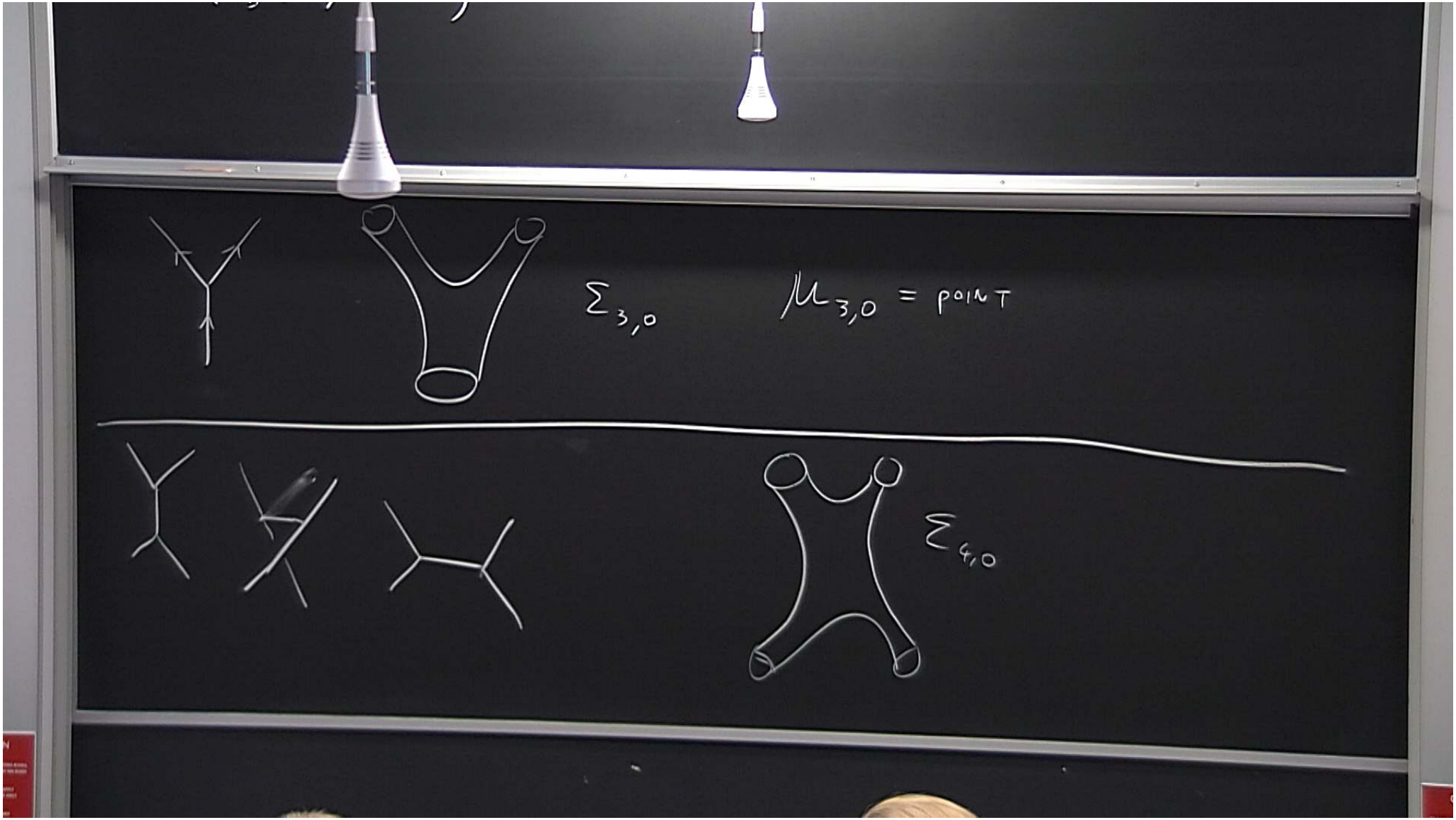
GAUGE-FIX

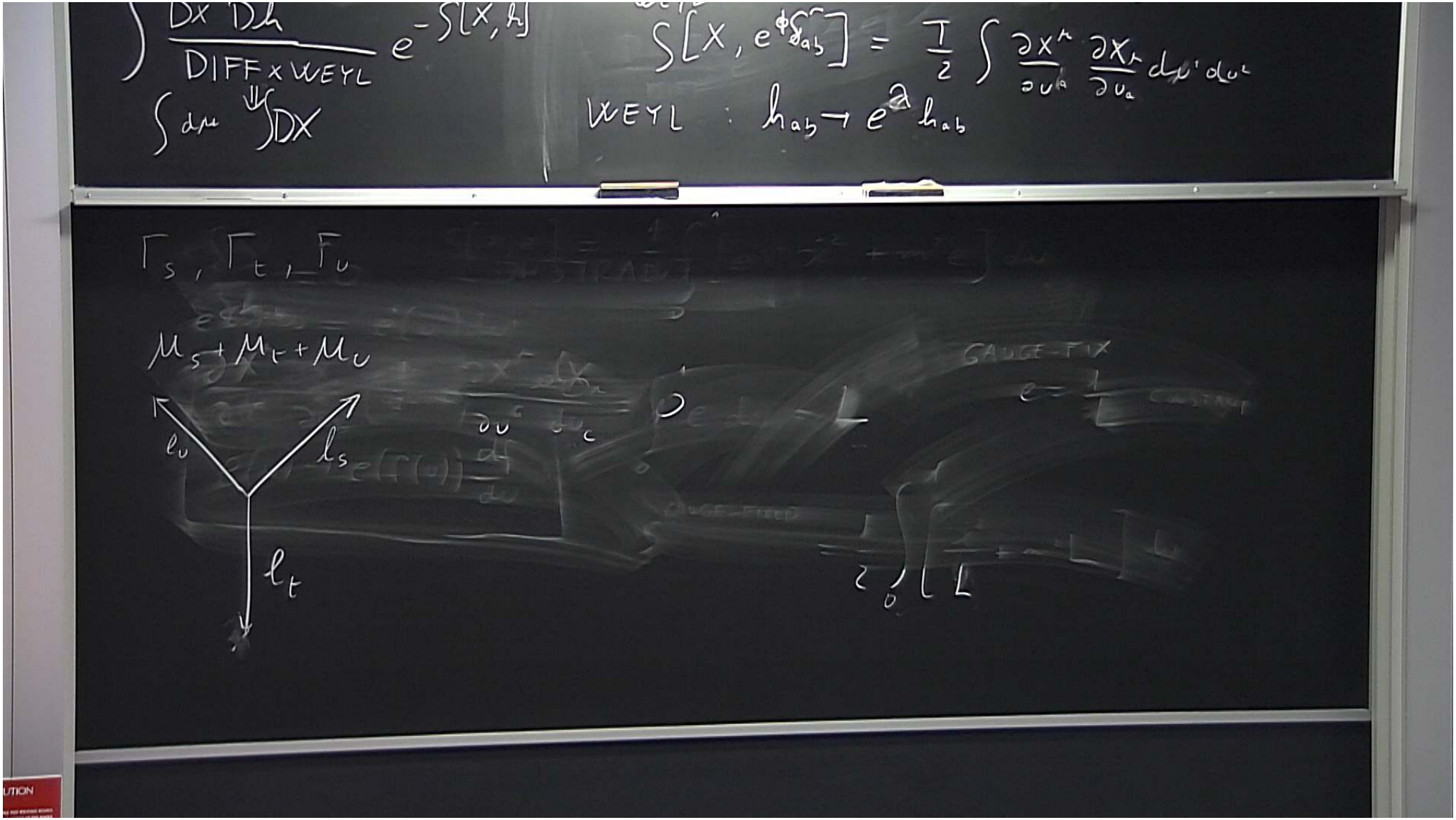
$$\int_0^L$$

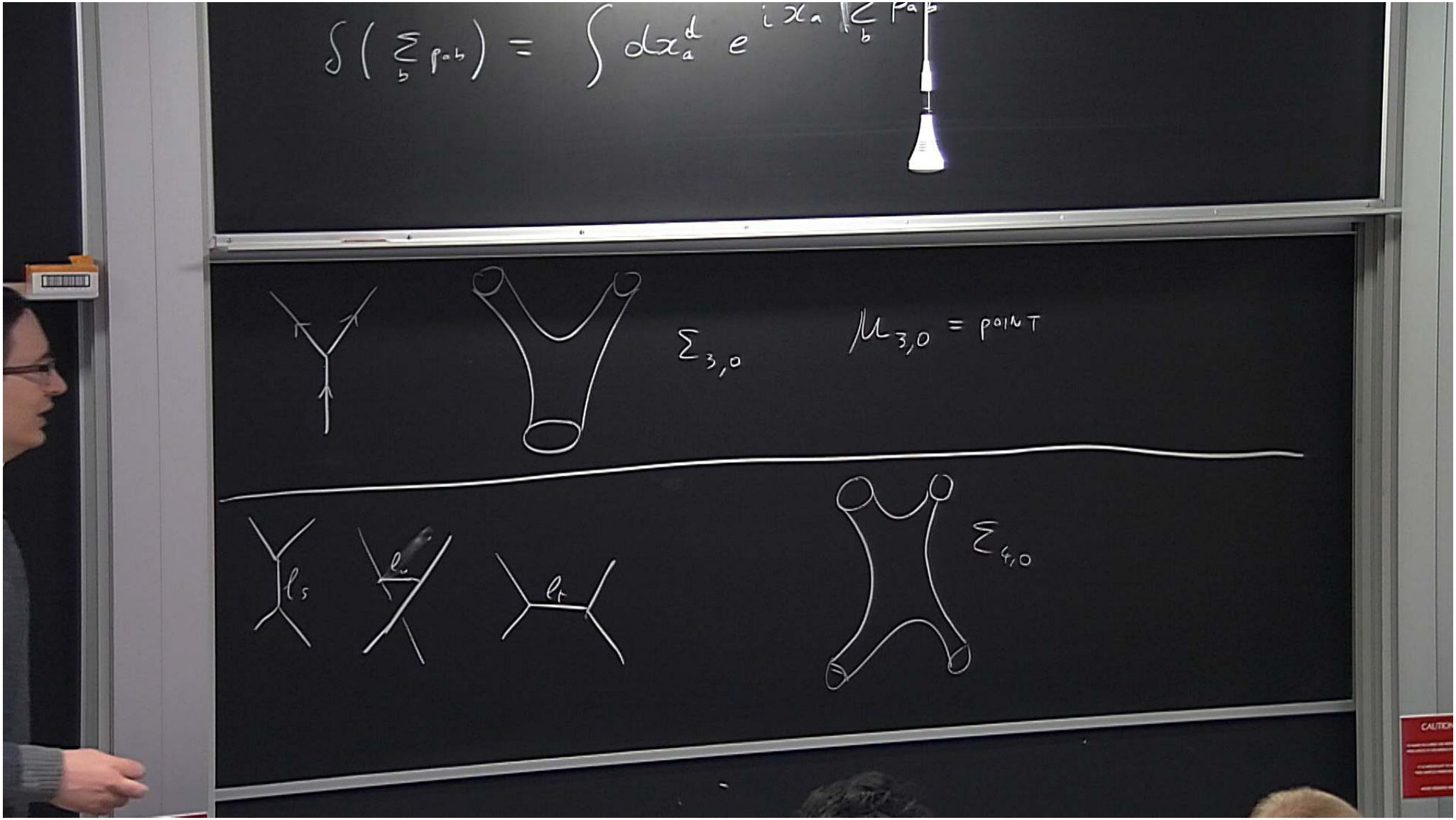

Σ
$u_1' = u_1'(u_1, u_2)$
$u_2' = u_2'(u_1, u_2)$
$g_{ab} \to g_{cd} \frac{\partial u^c}{\partial u'^a} \frac{\partial u^d}{\partial u'^b} = e^{\lambda} g_{ab}$

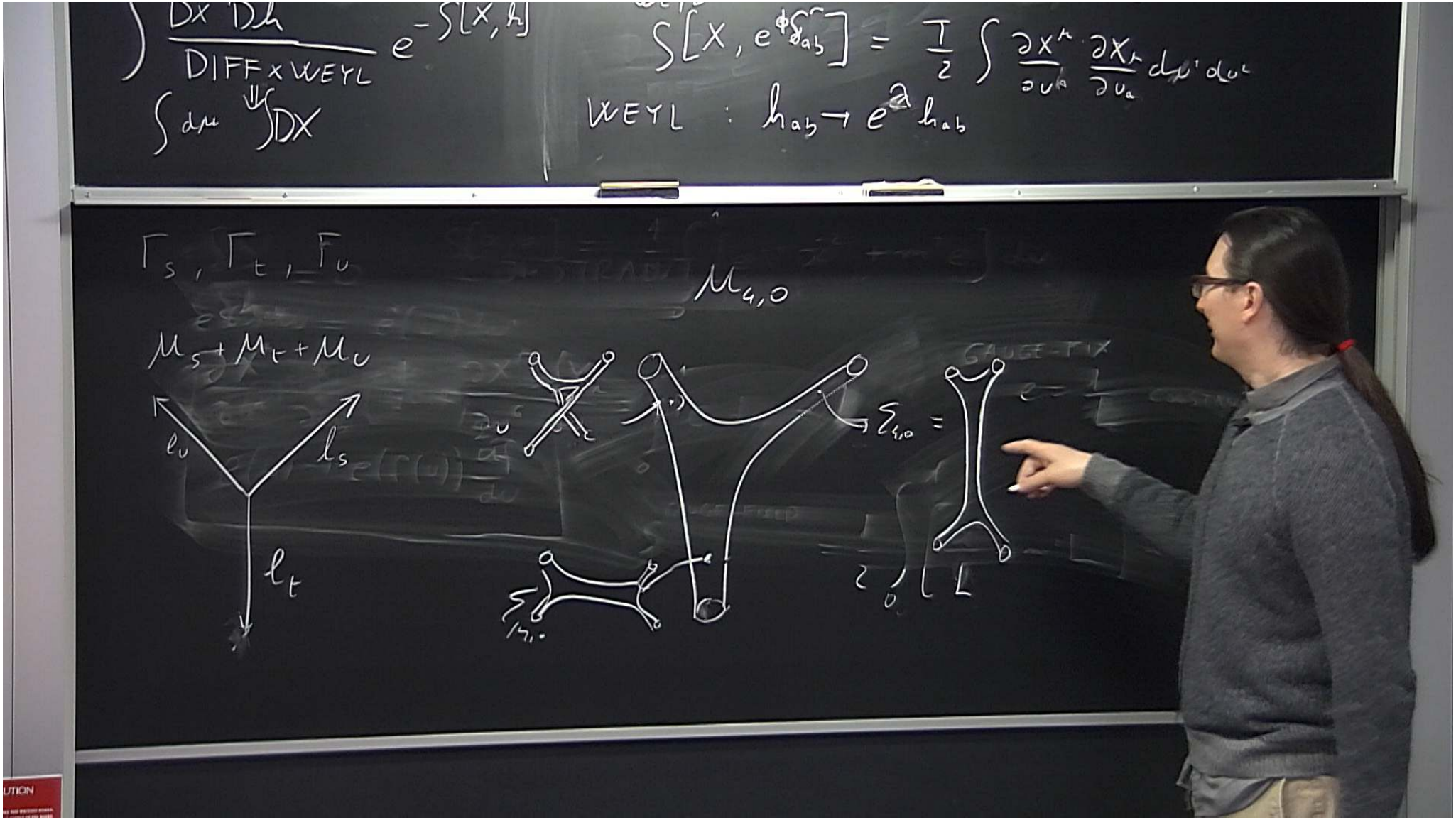

Γ
5'
4'
$P_{55'}$
P_{45}
$P_{44'}$
5
4
P_{51}
P_{34}
1
3
P_{12}
2
P_{23}
$P_{33'}$
2'
3'
$z' = z'(z)$
Σ
z
z'
STRING
A_Σ

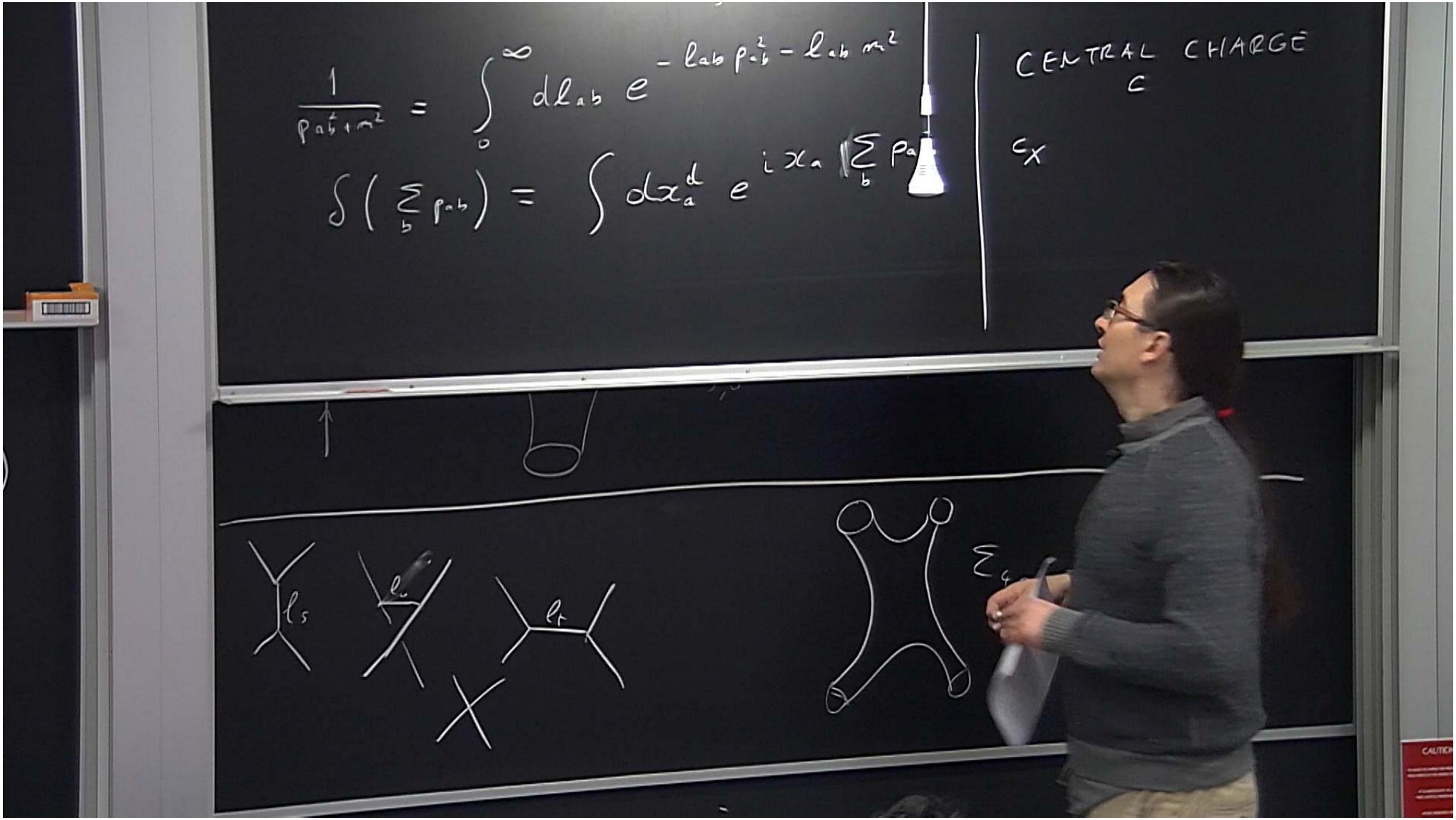

1'
2'
3'
QFT
A_Γ
$\Sigma_{m,g}$
$\mathcal{M}_{m,g}$
STRING
A_Σ
$A_\Gamma = \int d\ell \int_{x:\Gamma\to R^{d-1,1}} Dx\, e^{S_{\text{PARTICLE}}(x,\ell)}$
$A_\Sigma = \int_{\mathcal{M}[\Sigma]} d\mu \int_{X:\Sigma\to R^{d-1,1}} DX\, e^{-S_{\text{STRING}}(X,\mu)}$
$\mathcal{M}[\Sigma]$
=
COMPLEX
STRUCTURES
ON
Σ

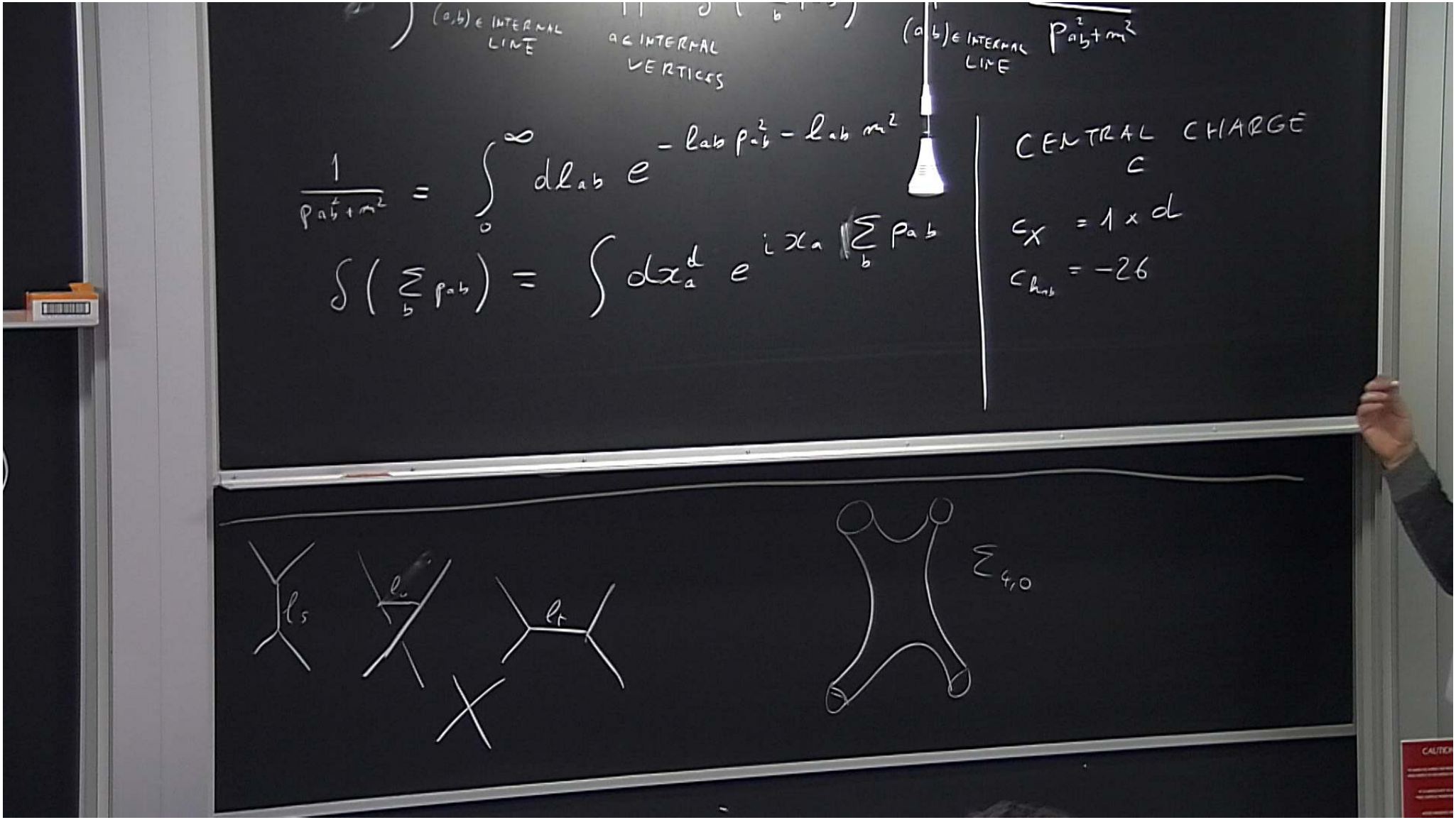

$\Sigma_{3,0}$
$\mathcal{M}_{3,0}$ = POINT
$\Sigma_{4,0}$


$\int \frac{Dx\,Dh}{DIFF \times WEYL} e^{-S[x,h]}$
$\int d\mu \Downarrow \int DX$
$S[X, e^{\phi} \hat{g}_{ab}] = \frac{I}{2} \int \frac{\partial X^\mu}{\partial v^a} \frac{\partial X_\mu}{\partial v_a} dv^1 dv^2$
WEYL : $h_{ab} \to e^{\phi} h_{ab}$
Γ_s, Γ_t, F_v
$M_s + M_t + M_v$
l_v
l_s
l_t
GAUGE-FIX
$\epsilon = \frac{1}{L}$
$\int_0^L$


$\delta\left(\sum_b p_{ab}\right) = \int dx_a^d \, e^{i x_a \sum_b p_{ab}}$
$\Sigma_{3,0}$
$\mathcal{M}_{3,0} = \text{POINT}$
l_s
l_u
l_t
$\Sigma_{4,0}$


$\int \frac{Dx\,Dh}{DIFF \times WEYL} e^{-S[x,h]}$
$\int d\mu \Rightarrow \int DX$
$S[X, e^{\phi} \hat{g}_{ab}] = \frac{I}{2} \int \frac{\partial X^\mu}{\partial v^a} \frac{\partial X_\mu}{\partial v_a} dv^1 dv^2$
WEYL : $h_{ab} \to e^{\phi} h_{ab}$
$\Gamma_s, \Gamma_t, \Gamma_u$
$M_{4,0}$
$M_s + M_t + M_u$
l_u
l_s
l_t
$\Sigma_{4,0} =$
GAUGE FIX


$\frac{1}{p_{ab}^2 + m^2} = \int_0^\infty d\ell_{ab}\, e^{-\ell_{ab} p_{ab}^2 - \ell_{ab} m^2}$
$\delta\left(\sum_b p_{ab}\right) = \int dx_a^d\, e^{i x_a \left(\sum_b p_a\right.}$
CENTRAL CHARGE
C
c_X
ℓ_s
ℓ_u
ℓ_t


(a,b) ∈ INTERNAL LINE
a ∈ INTERNAL VERTICES
(a,b) ∈ INTERNAL LINE
$p_{ab}^2 + m^2$
$\frac{1}{p_{ab}^2 + m^2} = \int_0^\infty d\ell_{ab}\, e^{-\ell_{ab} p_{ab}^2 - \ell_{ab} m^2}$
$\delta\left(\sum_b p_{ab}\right) = \int dx_a^d\, e^{i x_a \sum_b p_{ab}}$
CENTRAL CHARGE
c
$c_X = 1 \times d$
$c_{\ell_{ab}} = -26$
ℓ_s
ℓ_u
ℓ_t
$\Sigma_{4,0}$