

Title: PSI 2018/2019 - Explorations in Quantum Gravity - Lecture 14

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Collection: PSI 2018/2019 - Explorations in Quantum Gravity (Dupuis)

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TODAY: *Wolfgang on LQC.
*Hal on Black Holes.



Elements of Loop Quantum Cosmology

$$F_{ab}^i = 2\partial_{[a} A_{b]}^i$$

1. LQG in four Lorentzian dimensions

phase space

$$\{\hat{\tilde{E}}_i^a(p), \hat{A}_b^j(q)\} = 8\pi\gamma G \delta_b^a \delta_i^j \delta^{(3)}(p, q)$$

↑
"electric" field

$$G_i[N^i] = \int_{\Sigma} N^i D_a \hat{\tilde{E}}_i^a = 0$$

$$H_a[N^a] = \int_{\Sigma} N^a F_{ab}^i \hat{\tilde{E}}_i^b$$

$$H[N] = \int_{\Sigma} N \epsilon_{abc} F_{ab}^i \hat{\tilde{E}}_i^c$$

$$F_{ab}^i = 2\partial_{[a} A_{b]}^i + i\epsilon^{lmn} A_a^l A_b^m$$

$\gamma \dots \text{Im m m z i}$

$$D_a \hat{E}_i^a = 0$$

$$A_a^i = \Gamma_a^i + i K_a^i$$

$$\hat{E}_i^a = \det(e) e_i^a$$

γ

$$A_a^i = \Gamma_a^i + \gamma K_a^i$$

$$\hat{E}_i^a$$

$$F_{ab}^i \hat{E}_i^b$$

$$F_{ab}^i \hat{E}_i^a \hat{E}_n^b$$

2. Symmetry reduction

homogeneous
isotropic

$$ds^2 = -N^2(t) dt^2 + a^2(t) (dx^2 + dy^2 + dz^2)$$

$$\begin{aligned} \dot{A}^i_a &= c \, dx^i_a \\ \hat{E}_i^a &= p \, d^3x \, \partial_i^a \end{aligned}$$

$$\oplus_z = \frac{1}{8\pi\gamma G} \int_{\Sigma} \tilde{E}_i^a \delta A^i_a = \frac{p \delta c}{8\pi\gamma G} \int_{\Sigma} d^3x \left[\partial_i^a dx^i_a \right] = 3$$

$$V_0 = \ell^3$$

$$\langle \rho \rangle = \frac{8\pi\gamma G}{3}$$

Coupling to matter

$$S[g, \phi] = -\frac{1}{2} \int d^4x \, g^{ab} \partial_a \phi \partial_b \phi$$

$$G_i[N^i] = \int_{\Sigma} N^i D_a E_i^a$$

$$H_a[N^a] = \int_{\Sigma} N^a (F^i_{ab} \tilde{E}_i^b + \nabla_a \phi \tilde{\pi}) ; \quad \{\tilde{\pi}, \phi\} = 1$$

$$\{\rho, c\} = \frac{8\pi\gamma G}{3}$$

$$\phi(\vec{x}) = \varphi$$

$$\tilde{\pi}(\vec{x}) = \pi d^3x$$

$$\{\pi, \varphi\} = 1$$

$$-d\vec{y}^2 + d\vec{z}^2)$$

$$\delta A_a = \frac{p \delta c}{8\pi\gamma G} \int_{\Sigma} d^3x \left[\partial_a^i dx_a^i \right]$$

= 3

$$\{\tilde{E}_i^a(p), \tilde{A}_b(q)\} = 8\pi\gamma C \delta_b^a \delta_i^j \delta^3(p, q)$$

↑
electric field

$$H_0[N] = \int_{\Sigma} N^2 F_{ab} \tilde{E}_i^a \tilde{E}_i^b$$

$$H[N] = \int_{\Sigma} \epsilon_{ab} F_{cd} \tilde{E}_i^a \tilde{E}_i^b$$

$$\tilde{E}_i^a = \alpha e^a(\mathbf{e}) e_i^a$$

Coupling to matter

$$S[g, \phi] = -\frac{1}{2} \int d^4x g^{ab} \partial_a \phi \partial_b \phi$$

$$\pi^2 + (d^3x)^2 h^{ab} \nabla_a \phi \nabla_b \phi \sim H_{\text{matter}}$$

$$\nabla G_i[N] = \int_{\Sigma} N^2 D_i E_i^a = 0$$

$$\nabla H_i[N] = \int_{\Sigma} N^2 (F_{ab} \tilde{E}_i^a + \nabla_a \phi \pi_a) = 0, \quad \{\pi, \phi\} = 1$$

$$F_{ab}^i = \frac{1}{2} \epsilon^{ijkl} dx_j^l dx_k^l$$

$$\tilde{E}_i^a = \frac{1}{\sqrt{h}} \epsilon^{abc} \partial_b^a, \quad dx_a^b \partial_b^a = \delta_i^a$$

$$H_{\text{grav}}[N] \propto (PC)^2$$

$$H_{\text{matter}}[N]$$

$$H[N] = \int_{\Sigma} N \epsilon_{ab} \tilde{F}^i_{ab} \tilde{E}^a_i \tilde{E}^b_i$$

$$\{p, c\} = 1 \rightarrow v = p^{\frac{3}{2}}$$

$$\{b, v\} = 1$$

$$b = c^{\frac{1}{2}} p^{\frac{1}{2}}$$

$$\boxed{\pi^2 = (bv)^2}$$

Hamiltonian constraint

$$\boxed{\pi^2 = (pc)^2}$$

$$\{p, c\} = 1 \rightarrow v = p^{\frac{2}{3}}$$

$$\{b, v\} = 1 \quad b = p^{\frac{4}{3}} c$$

$$\pi^2 = \dots (bv)^2$$

$$\hat{v}|v\rangle = v|v\rangle$$

$$\hat{b} = \frac{4}{3} \partial_v$$

in LQG geometry is quantized

$$\hat{v}|n\rangle = v_n |n\rangle$$

Hamiltonian constraint

$$\pi^2 = (pc)^2$$

$$\{p, c\} = 1 \rightarrow$$

$$\{b, v\} = 1$$

$$v = p^{\frac{3}{2}}$$

$$b = p^{\frac{1}{2}} c$$

$$a = \sqrt{v} e^{-i\lambda b}$$

$$a^\dagger = \sqrt{v} e^{+i\lambda b}$$

λ ... a fixed parameter

$$\pi^2 = (bv)^2$$

$$\pi^2 = \frac{(a^2 - (a^\dagger)^2)}{\lambda^2}$$

in LQG geometry is quantized

$$\hat{v}|n\rangle = v_0 n|n\rangle$$

$$\frac{a^2 - (a^\dagger)^2}{\lambda} = -v \frac{\sin(\lambda b)}{\lambda} \approx -vb$$

Hamiltonian constraint

$$\pi^2 = (pc)^2$$

Coupling to matter

$$\pi \rightarrow \frac{\hbar}{i} \frac{d}{d\varphi}$$

$$(a, a^\dagger) :$$

$$a|n\rangle = \sqrt{n} |n-1\rangle$$

$$a^\dagger|n\rangle = \sqrt{n+1} |n+1\rangle$$

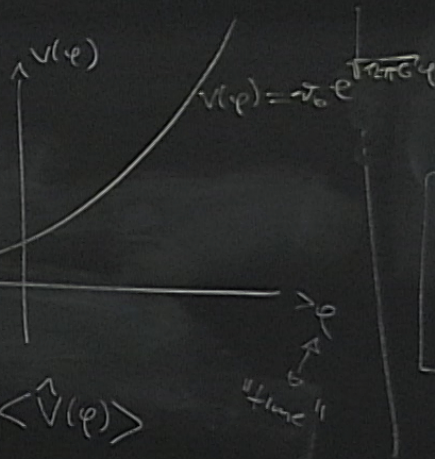
↑
"electric" field

$$U = \phi_b \phi_c \phi_d \phi_e (p, q)$$

$$H[A] = \int d^3x \epsilon_{ab} F_{ab} \tilde{E}^a \tilde{E}^b$$

Coupling to matter

$$\hat{\psi}(\varphi) = e^{\frac{i}{\hbar} A \varphi} \psi_0 a e^{-\frac{i}{\hbar} A \varphi}$$



$$\{p, c\} = 1 \rightarrow$$

$$\{b, v\} = 1$$

$$\pi^2 = (bv)^2$$

$$\pi^2 = \frac{(a^2 - (at)^2)^2}{\lambda^2}$$

$$\pi = \pm \frac{a^2 - (at)^2}{\lambda}$$

$$\frac{\hbar}{i} \frac{d}{d\varphi} |\psi\rangle = \frac{a^2 - (at)^2}{\lambda} |\psi\rangle$$

Top: coupling to LSC
+ field on Black Hole

$$\{c, c\} = 1 \rightarrow v = p^{\frac{3}{2}} \quad \left| \quad \begin{aligned} a &= \sqrt{v} e^{-i\lambda b} \\ a^\dagger &= \sqrt{v} e^{+i\lambda b} \end{aligned} \right| \quad \lambda \dots \text{a fixed parameter}$$

$$\{v, v\} = 1 \quad b = p^{-\frac{1}{2}} c$$

$$\pi^2 = (bv)^2$$

$$Z = \frac{(a^2 - (a^\dagger)^2)^2}{\lambda^2}$$

$$= \pm \frac{a^2 - (a^\dagger)^2}{\lambda}$$

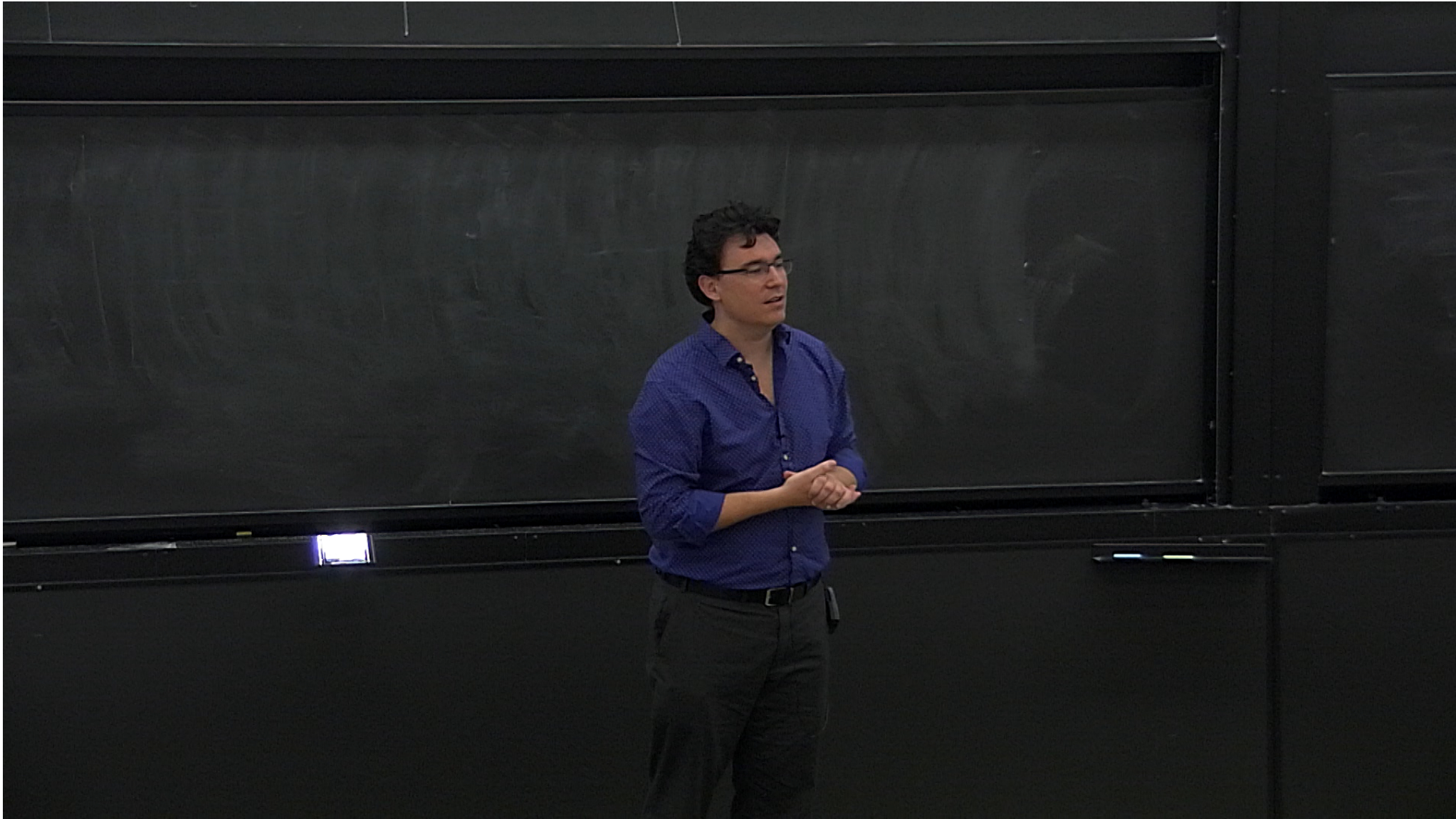
in LQG geometry is quantized

$$\hat{v} |n\rangle = v_0 n |n\rangle$$

$$\frac{a^2 - (a^\dagger)^2}{\lambda} = -v \frac{\sin(\lambda b)}{\lambda} \approx -v b$$

Hamiltonian constraint

$$\pi^2 = (pc)^2$$



For $S_g A^* : 4 \times 10^6 M_\odot$

$$\frac{2GM_\odot}{c^2} = 3 \text{ km}$$

$$K^2 = R_{abcd} R^{abcd} \text{ at } r_s$$

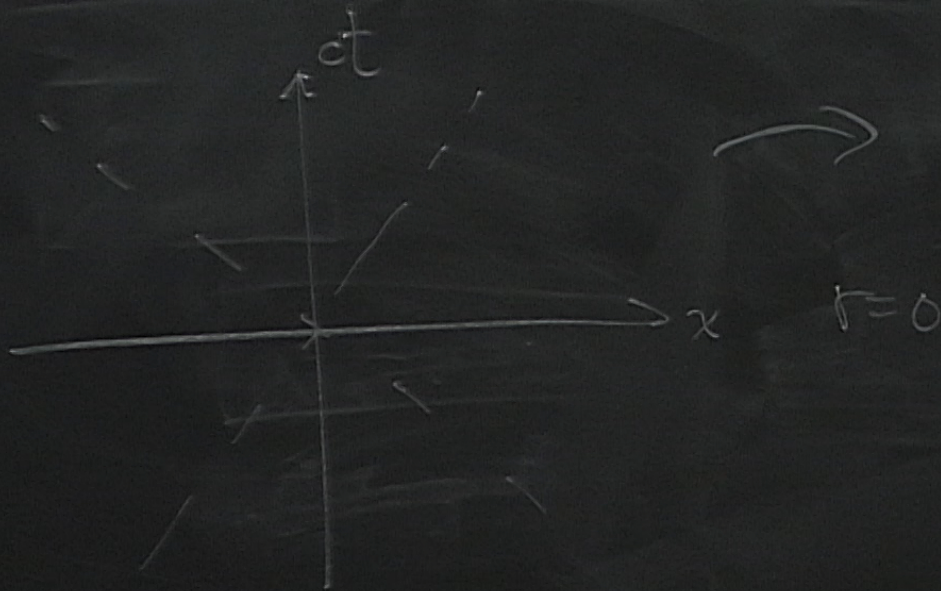
$$\text{Sch. Btl.} = \frac{12 r_s^2}{c^4 r^6} \sim \frac{1}{r_s^4}$$

$$K \sim \frac{1}{r_s^2}$$

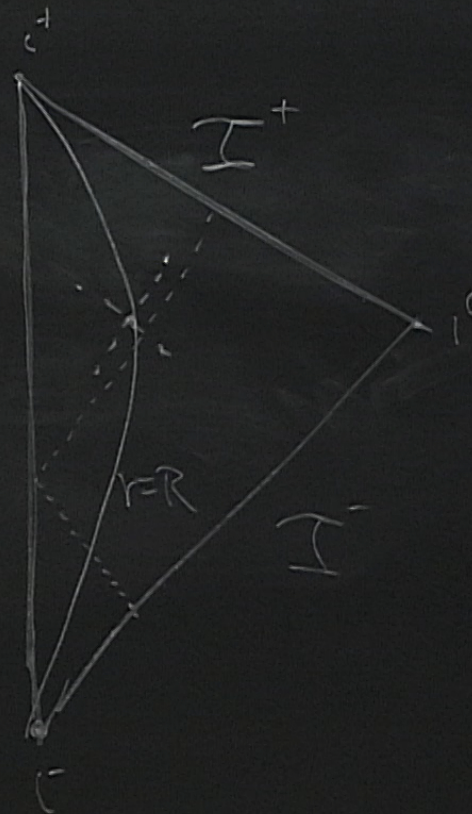
If we follow G.R.
into the B.H. —

our expectation is
that we can't neglect
quantum effects on the interior.

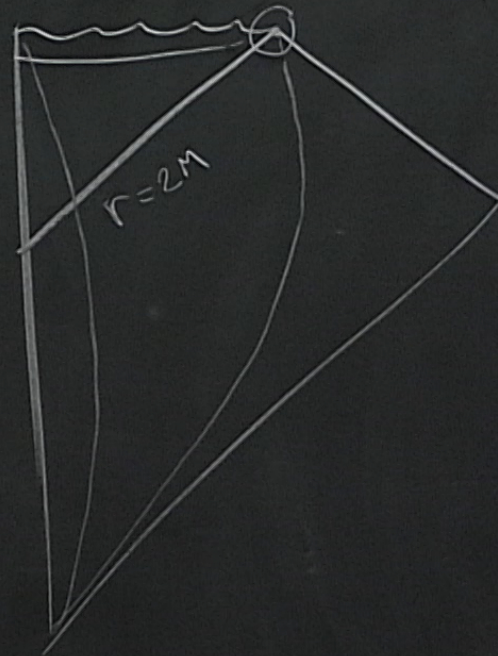
Minkowski Space Penrose



$$r=0$$



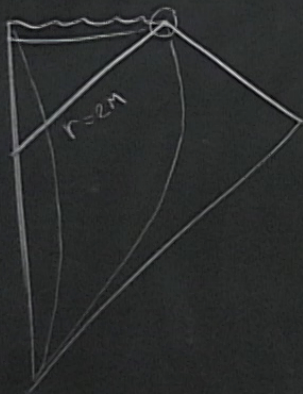
BH diagram



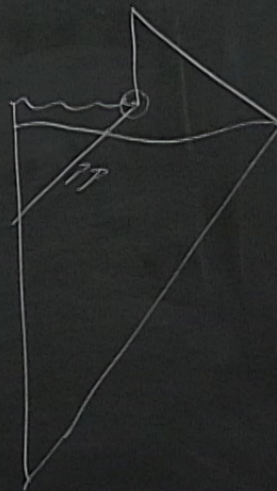
Electric field

$$\frac{1}{\epsilon_0} \epsilon_{ab} E_a E_b$$

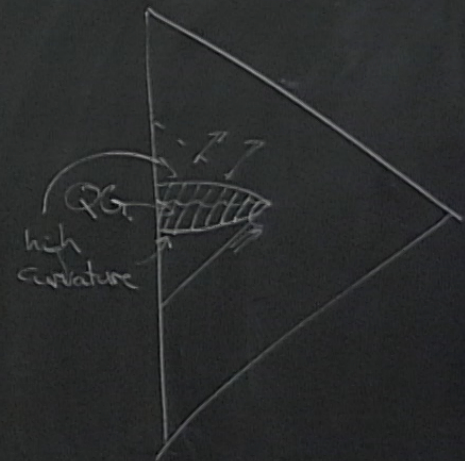
BH diagram



Hawking evaporating BH



Paradigm



$$\{\tilde{E}_i^a(p), A_b^j(q)\} = 8\pi G \int d^3x \delta^3(p-q)$$

↑
"field"

$$G_i[N^a] = \int_{\Sigma} N^a D_a \tilde{E}_i^a = 0$$

$$H_a[N^a] = \int_{\Sigma} N^a F_{ab}^i \tilde{E}_i^b$$

$$H[N] = \int_{\Sigma} \epsilon_{ab} F_{cd}^i \tilde{E}_i^a \tilde{E}_i^b$$

$$\tilde{E}_i^a = \det(e) e_i^a \rightarrow \hat{\tilde{E}}_i^a$$

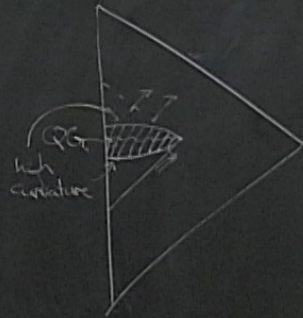
BH diagram



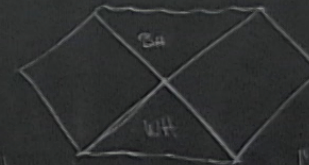
Hawking evaporating BH



Paradigm



Kruskal diagram



What is the amplitude for a BH to tunnel into a WH?

$$\{\tilde{E}_i^a(p), A_b^j(q)\} = 8\pi\gamma G \int d^3x \delta^3(p-q)$$

↑
"field"

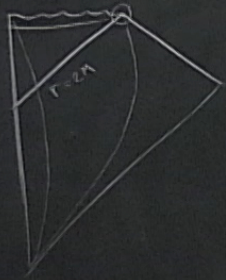
$$G_{ij}[N] = \int_{\Sigma} N^2 D_a E_i^a = 0$$

$$H_a[N] = \int_{\Sigma} N^a F_{ab}^i \tilde{E}_i^b$$

$$H[N] = \int_{\Sigma} \epsilon_a F_{ab}^i \tilde{E}_i^a \tilde{E}_b^b$$

$$\tilde{E}_i^a = \det(e) e_i^a \rightarrow \tilde{E}_i^a$$

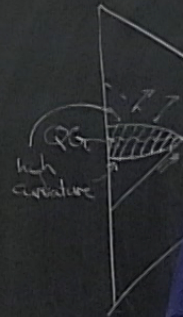
BH diagram



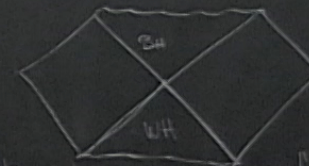
Hawking evaporating BH



Paradigm



Kruskal diagram



$$T_H \sim M^3$$

$$T_H \sim \frac{M_0}{10^{75} \text{ secs}}$$

$$T_0 \sim 10^{17} \text{ secs}$$

What is the amplitude for a BH tunnel, into a WH?