

Title: PSI 2018/2019 - Explorations in Quantum Gravity - Lecture 13

Speakers: Maïtê Dupuis

Collection: PSI 2018/2019 - Explorations in Quantum Gravity (Dupuis)

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URL: <http://pirsa.org/19030080>

TODAY: Spinfoam approach.

LQG Canonical quantization

$$4d - \mathcal{H}^{\text{kin}} = \bigcup_{\gamma} \mathcal{H}_{\gamma} / \gamma$$

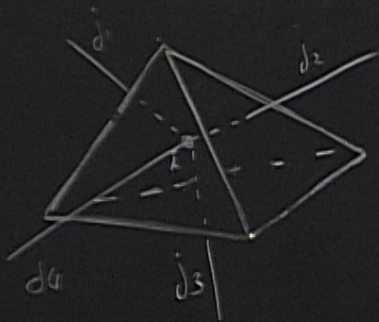
well-defined, well-understood.

+ constraints * Gauss

* Spatial diffeo

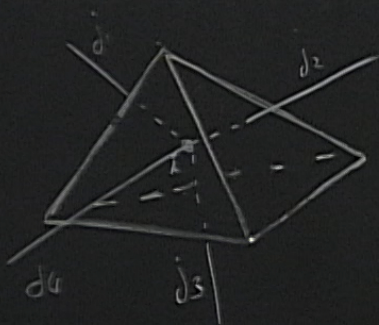
* Scalar/Hamiltonian constraint





$$\hat{A}|\psi\rangle = -8\pi \ell_p^2 \gamma \sqrt{j(j+1)} |\psi\rangle$$





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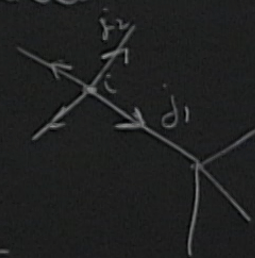


canonical quantization

$$H^{\text{kin}} = \bigcup_{\gamma} H_{\gamma} / \gamma$$

well-defined, well-understood.

+ constraints : + Gauss



Diffco. { * Spatial diffeo $\hat{V}^a \longrightarrow$ s-knot
* Scalar/Hamiltonian constraint : open pb?

Spinfoam

GR: path integral is about transition between geometries

$$P(\Sigma_{\text{final}}, \Sigma_{\text{initial}}) = \int D[e] D[A] \mu[A, e] e^{iS_{\text{GR}}[A, e]} :$$

↳ SF approach: Reisenberger & Rovelli (1997).

in the spinnetwork basis: $\langle s' | s \rangle_{\text{phy}}$ — as a sum over

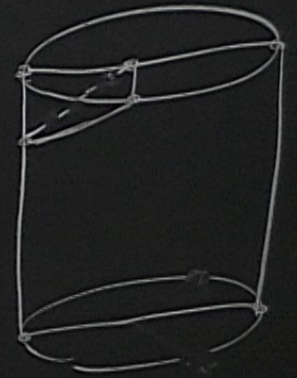
between geometries

$$\langle A \rangle_\mu[A, e] = e^{iS_{GR}[A, e]}.$$

: how to give sense to this?

Ashtekar & Rovelli (1997).

$\langle s | s \rangle_{\text{phy}}$ — as a sum over spinnetwork histories



this?

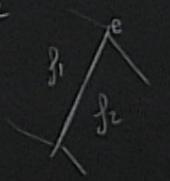


A spinfoam = a colored
2-complex

Standard ansatz for local SF amplitudes.

$$K[\Delta] = \sum_{C/\partial C = \sqrt{\Delta}} \omega(C) \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(j_f, i_e) \prod_v \underbrace{A_v(j_f, i_e)}_{\text{all dynamical info encoded in the vertex amplitude}}$$

graph of spinnetwork



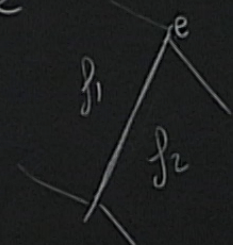
local SF amplitudes.

$$= \sum_{c/\partial c = \sqrt{s}} \omega(c) \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(j_f, i_e) \prod_{\nu} A_{\nu}(j_f, i_e)$$

↑ amplitude

graph of spinnetwork

all dynamical info ends in the



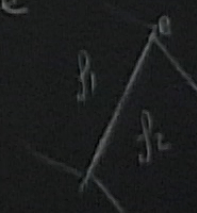
Standard ansatz for local SF amplitudes.

can be derived directly as a regularization of the covariant path integral

$$K[\Delta] = \sum_{C/\partial C = \Gamma_\Delta} \omega(C) \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(j_e, i_e) \prod_v A_v$$

$\omega(C)$ ^{statistical weight}
 Γ_Δ ^{graph of spinnetwork}

amplitude.



(GFT)
Group Field Theory

$$K = \int D[\rho] D[A] \mu[A, \rho] \exp(i S_{GR}(\rho, A))$$

3D case: Ponzano-Regge model

$$S[e, \omega] = \int G(e, \omega)$$

Formal path

case: Ponzano-Regge model

$$S[e, \omega] = \int G(e, \omega)$$

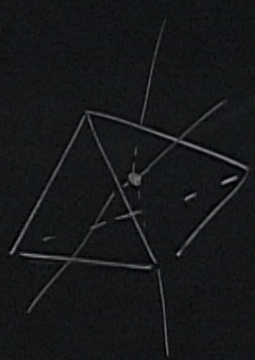
Foamed path integral for 3d gravity: $Z_{\text{FR}} = \int \mathcal{D}e \mathcal{D}\omega e^{iS(e, \omega)}$

① discretization of $M \rightarrow$ simplicial manifold with similar topology.
= triangulation Δ_3 by gluing tetrahedra together.

$\chi = \int d\mathbf{x} d\mathbf{y} d\mathbf{z} d\mathbf{w} d\mathbf{e}$

similar topology
 gluing tetrahedra together

Δ_3	Δ_3^*
tetrahedron	vertex
triangle	edge
segment	face
point	3d region



$$R = \int \text{D} \psi \text{D} \bar{\psi} \text{D} \omega \text{D} \omega^\dagger \text{D} \omega^\dagger \text{D} \omega^\dagger \text{D} \omega^\dagger$$

case: Ponzano-Regge model

$$S[e, \omega] = \int \text{tr}(\omega F \omega)$$

Formal path integral for 3d gravity: $Z_{\text{PI}} = \int \text{d}e \text{d}\omega e^{iS[e, \omega]}$

① discretization of $M \rightarrow$ simplicial manifold with similar topology.
 $=$ triangulation Δ_3 by gluing tetrahedra together.

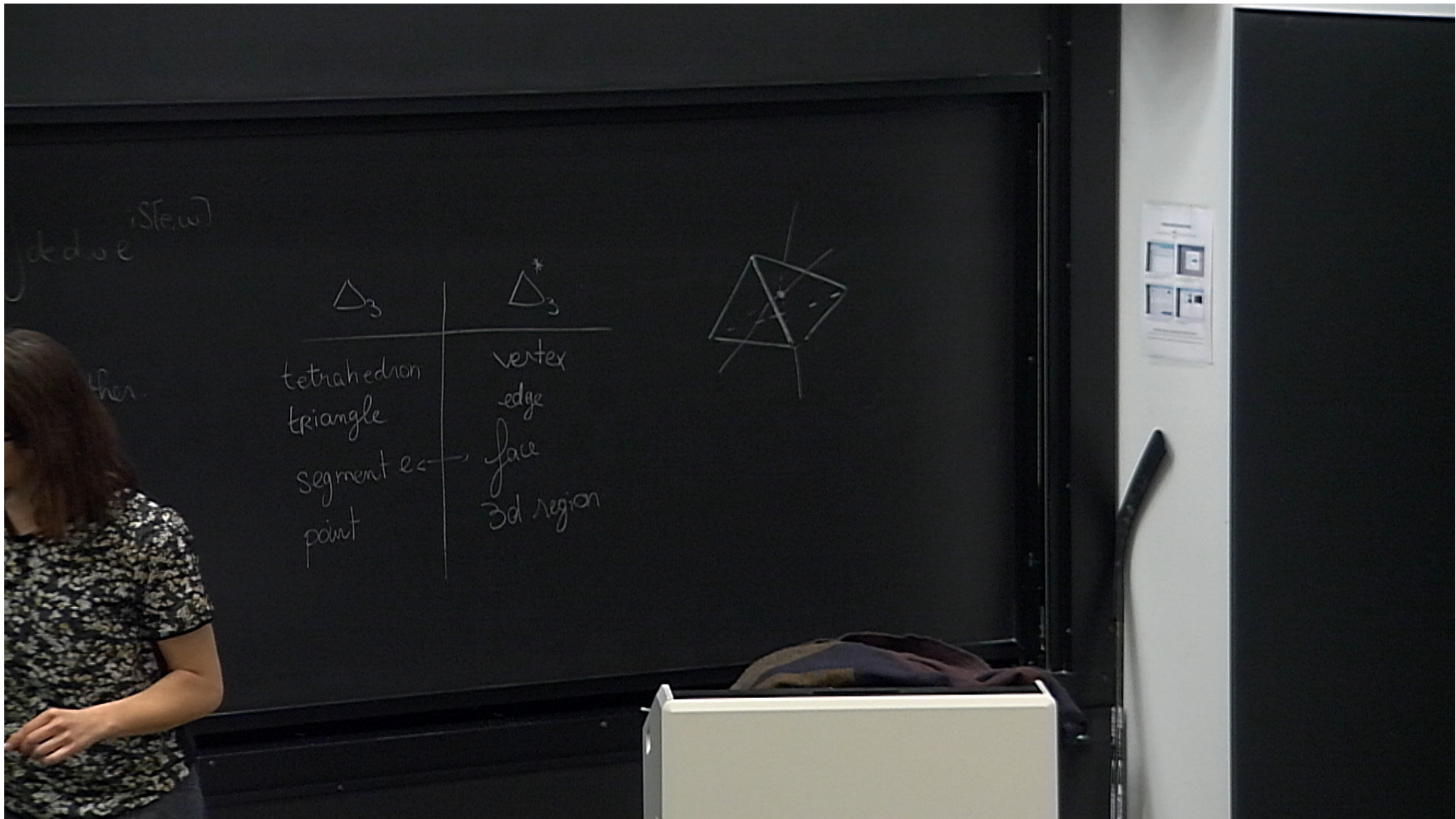
② Discretization of the variables:

$$e \rightarrow X_e = \int_e \omega \in \text{SU}(2)$$

$$\omega \rightarrow g_e = \text{P} \exp \int_e \omega \in \text{SU}(2)$$

$$F(\omega) \rightarrow U_e = \prod_{e^* \subset e} g_{e^*}^{\epsilon(e, e^*)}$$

tetra
trian
seg
poi



$$F(\omega) \rightarrow U_e = \prod_{e^* \subset e} g_i^{\varepsilon(e, e^*)}$$

③ Discrete action, path integral

$$\hookrightarrow S[X_e, g_{e^*}] = \sum_{e \in \Delta_3} \text{tr}(X_e U_e) \quad \uparrow \text{Haar measure}$$

$$\rightarrow Z_{\Delta_3} = \left(\prod_{e \in \Delta_3} \int_{\text{su}(2)} d^3 X_e \right) \left(\prod_{e^* \in \Delta_3^*} \int_{\text{su}(2)} dg_{e^*} \right) e^{iS[X_e, g_{e^*}]}$$

$\uparrow$ Lebesgue measure
 $d^3 X_e$ on $\text{su}(2) \sim \mathbb{R}^3$

$$\prod_{e \in E} g_e^{(x_e)}$$

$$\prod_{e \in E} \int_{\mathbb{R}^3} d\mathbf{x}_e \exp(iS[\mathbf{x}_e, g_e^*])$$

Haar measure

measure
on $\mathbb{R}^3$

→ integration over the x_e variables

$$\int d\mathbf{x}_e \exp(i\text{tr}(\mathbf{x}_e U_e)) = \sum_{\mathbf{d}} \text{tr}(D^{\mathbf{d}}(U_e))$$

over $SU(2)$

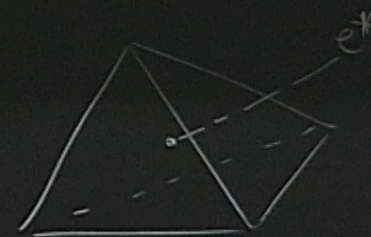
Peter Weyl
th

in the spin network basis: (SIS) phy ...

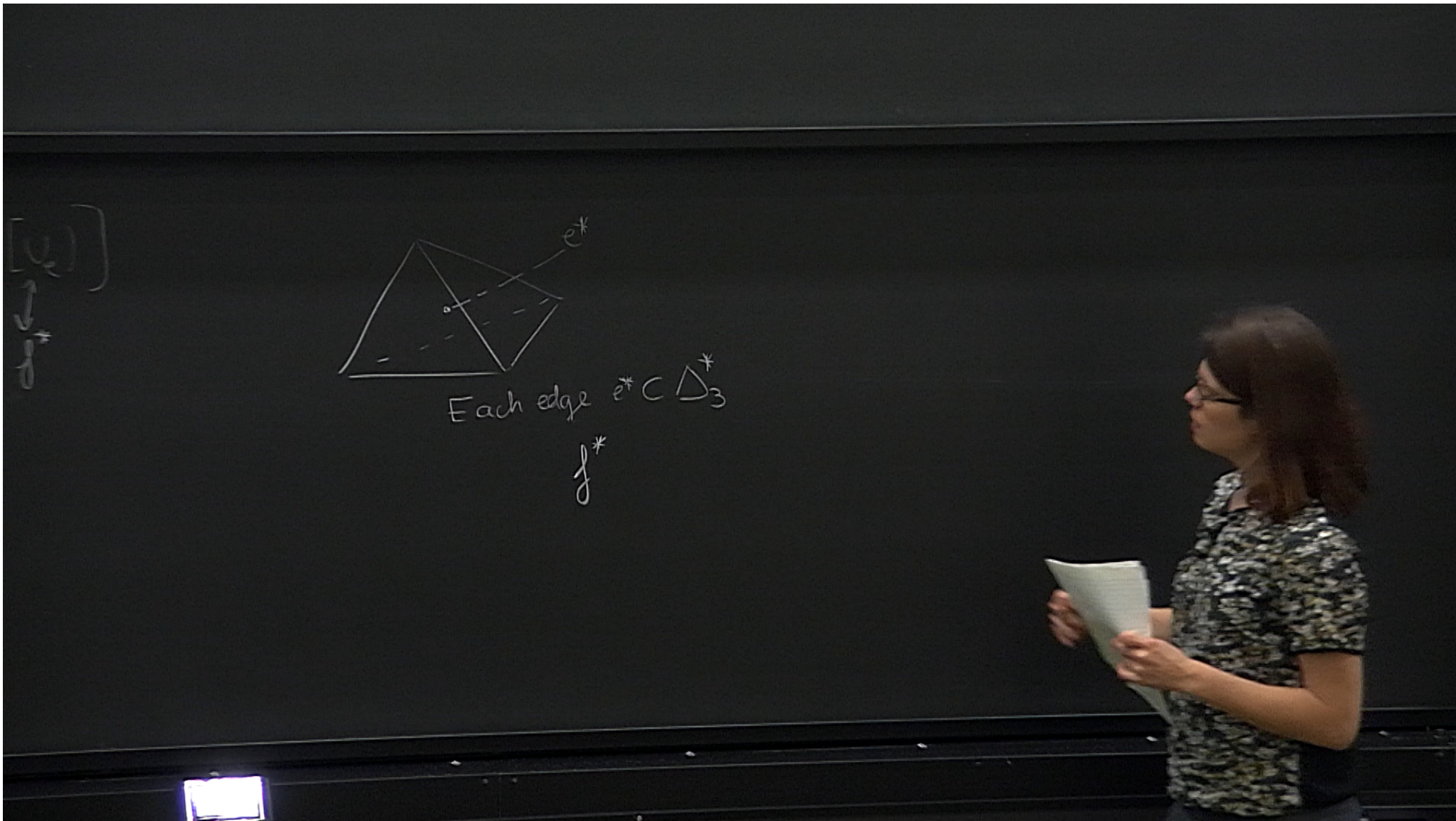
$$Z_{\Delta_3} = \sum_{\{j\}} \left(\prod_{e^* \in \Delta^*} \int_{\text{su}(2)} dg_{e^*} \right) \prod_{e \in \Delta_3} d_{j_e} \text{tr} [D^*[U_e)]$$

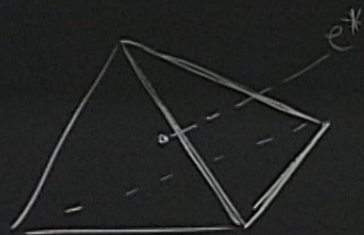
$$\left(\prod_{e^* \in \Delta^*} \int_{\text{su}(2)} dg_{e^*} \right) \prod_{e \in \Delta_3} d_{g_e} \text{tr} \left[D^* [V_e] \right]$$

$\downarrow$
 f^*



Each edge $e^* \in \Delta_3^*$



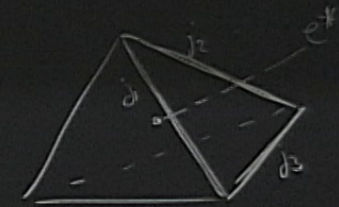


Each edge $e^* \in \Delta_3^*$
 bounds 3 faces $f^* \in \Delta_3^*$

in the spinnetwork basis $\langle S | S \rangle_{\text{phys}} = \text{as a sum over spinnetworks}$

$$Z_{\Delta_3} = \sum_{\{d\}} \left(\prod_{e^* \in \Delta^*} \int_{\text{su}(2)} dg_{e^*} \right) \prod_{e \in \Delta_3} d_{j_e} \text{tr} [D^{j_e}(V_e)]$$

$\downarrow$
 f^*



Each edge $e^* \in \Delta_3^*$

bounds 3 faces $f^* \in \Delta_3^*$

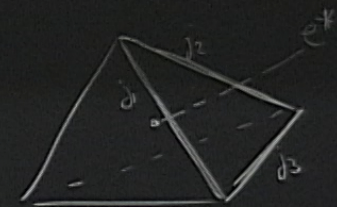
for each g_{e^*} , integral

$\text{tr} D^{j_e}[\dots g_{e^*} \dots]$ containing g_{e^*} in argument
 $\int dg_{e^*} D_{m_1 n_1}^{j_1}(g_{e^*}) D_{m_2 n_2}^{j_2}(g_{e^*}) D_{m_3 n_3}^{j_3}(g_{e^*})$

in the spin network basis: $\langle S | S \rangle_{\text{phys}} = \text{as a sum over spin network states}$

$$Z_{\Delta_3} = \sum_{\{d\}} \left(\prod_{e^* \in \Delta^*} \int_{\text{SU}(2)} dg_{e^*} \right) \prod_{e \in \Delta_3} d_{j_e} \text{tr} [D^{j_e}(U_e)]$$

$\downarrow$
 f^*



Each edge $e^* \in \Delta_3^*$

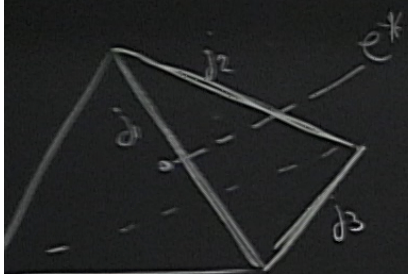
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for each g_{e^*} , integral

$$\text{tr} D^{j_e}[\dots g_{e^*} \dots] \text{ containing } g_{e^*} \text{ in argument}$$

$$\int dg_{e^*} D_{m_1 n_1}^{j_1}(g_{e^*}) D_{m_2 n_2}^{j_2}(g_{e^*}) D_{m_3 n_3}^{j_3}(g_{e^*})$$

$$= (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$$



Each edge $e^* \in \Delta_3^*$

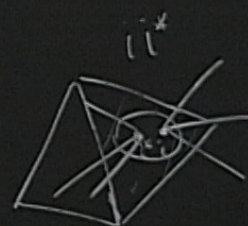
bounds 3 faces $f^* \in \Delta_3^*$

$$\left(\begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} \right)$$

$$\int_{SU(2)} dg$$

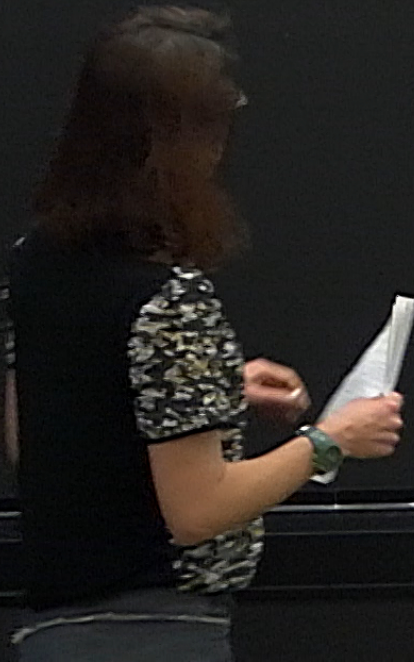


=



$$g^*, \text{ integral } \int dg^* D_{m_1 n_1}^{j_1}(g^*) D_{m_2 n_2}^{j_2}(g^*) D_{m_3 n_3}^{j_3}(g^*) = (-1)^{j_1 + j_2 + j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} \int dg_{SU(2)}$$

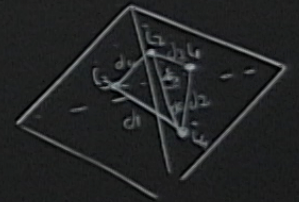
intertwiners associated to the 4 triangle of the tetrahedron
glued together into one spinnetwork which defines the quantum state of the tetrahedron



$$g^*, \text{ integral } \int dg^* D_{m_1 n_1}^{j_1}(g^*) D_{m_2 n_2}^{j_2}(g^*) D_{m_3 n_3}^{j_3}(g^*) = (-1)^{j_1 + j_2 + j_3} \underbrace{\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix}}_{3 \text{ jsymbol / C-G}} dg_{SU(2)}$$

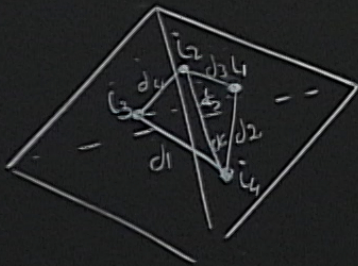
intertwiners associated to the 4 triangle of the tetrahedron
glued together into one spinnetwork which defines the quantum state of the tetrahedron

→ 6j-symbol $\left\{ \begin{matrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{matrix} \right\} = \begin{pmatrix} & & \\ & & \end{pmatrix} \begin{pmatrix} & & \\ & & \end{pmatrix} \begin{pmatrix} & & \\ & & \end{pmatrix} \begin{pmatrix} & & \\ & & \end{pmatrix}$



→ Ponzano-Regge $Z_{\Delta_3} = \sum_{\{j_e\}} \prod_e (-1)^{2j_e} d_{j_e} \prod_e \left\{ \begin{matrix} j_1^{(1)} & j_2^{(1)} & j_3^{(1)} \\ j_4^{(1)} & j_5^{(1)} & j_6^{(1)} \end{matrix} \right\}_N$
 $\xrightarrow{j \rightarrow \infty} \frac{1}{\sqrt{12\pi V}} \cos\left(S_R + \frac{\pi}{4}\right)$

state of the tetrahedron



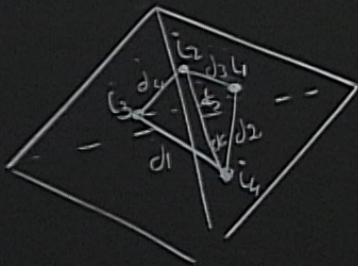
$$\cos\left(S_R + \frac{\pi}{4}\right)$$

4d gravity reformulated
as a BF theory

→ Plebanski's action

$$S[B, \omega, \lambda] = \int B^{IJ} \wedge F_{IJ}(\omega) - \frac{1}{2} \lambda_{IJKL} B^{KL} \wedge B^{IJ}$$

state of the tetrahedron



$$\cos\left(S_R + \frac{\pi}{4}\right)$$

4d gravity reformulated
as a BF theory

→ Plebanski's action

$$S[B, \omega, \lambda] = \int B^{IJ} \wedge F_{IJ}(\omega)$$

$$- \frac{1}{2} \lambda_{IJKL} B^{KL} \wedge B^{IJ}$$

- EPRL-FK model
- holomorphic SF.