Title: PSI 2018/2019 - Explorations in Quantum Gravity - Lecture 12

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Collection: PSI 2018/2019 - Explorations in Quantum Gravity (Dupuis)

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Flatness constrain

$$h_\gamma = \mathbb{1} - \int_{S(\gamma)} F + O(\varepsilon^3) \Rightarrow \mathbb{1}$$

↑ Closed curve

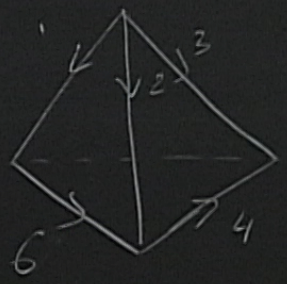


$$H_1 = g_2^{-1} g_6 g_1$$
$$H_2 = g_3^{-1} g_4 g_2$$
$$H_3 = g_1^{-1} g_2 g_3$$
$$H_4 = g_6 g_5 g_4$$

$H_4 =$

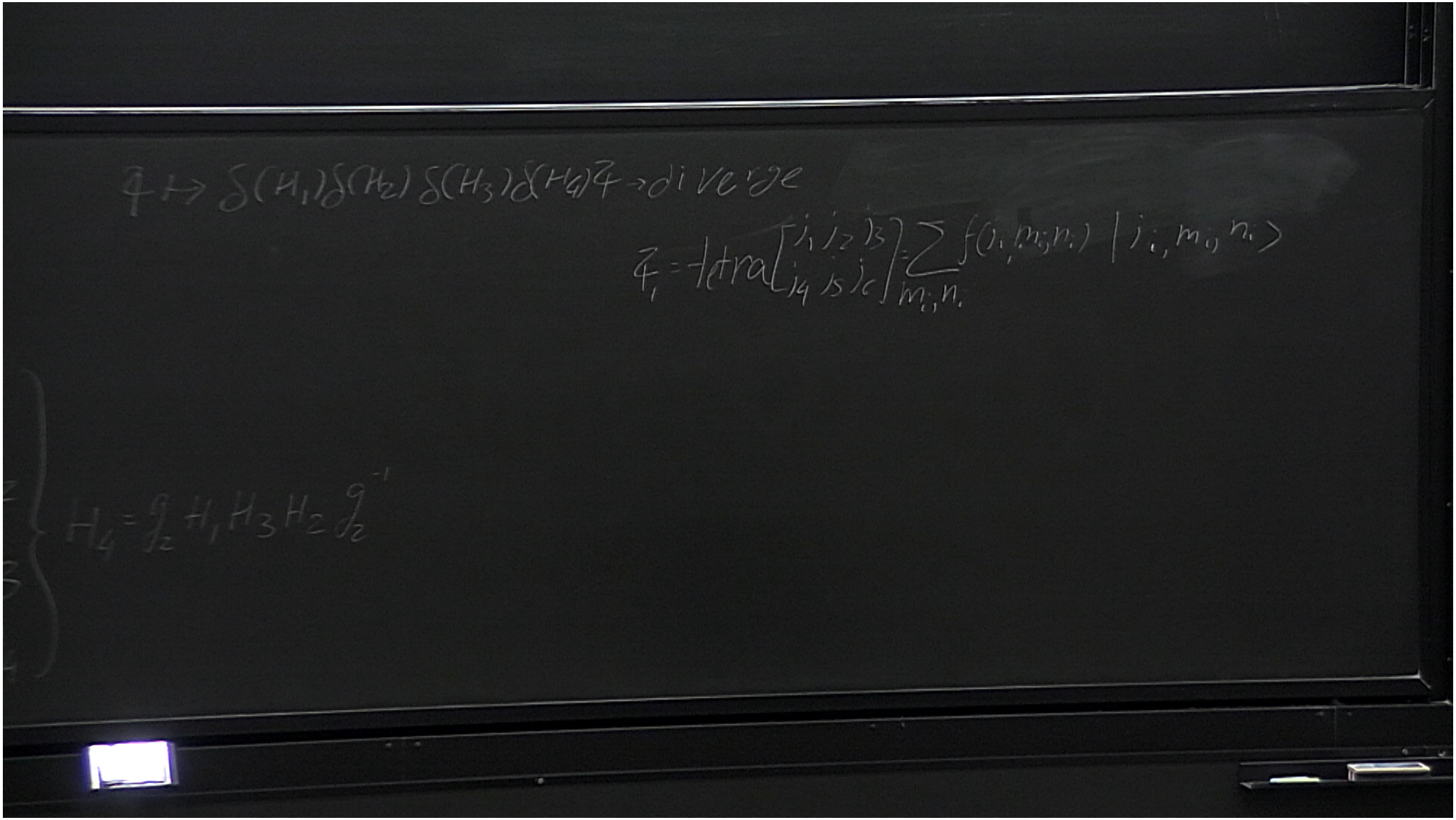
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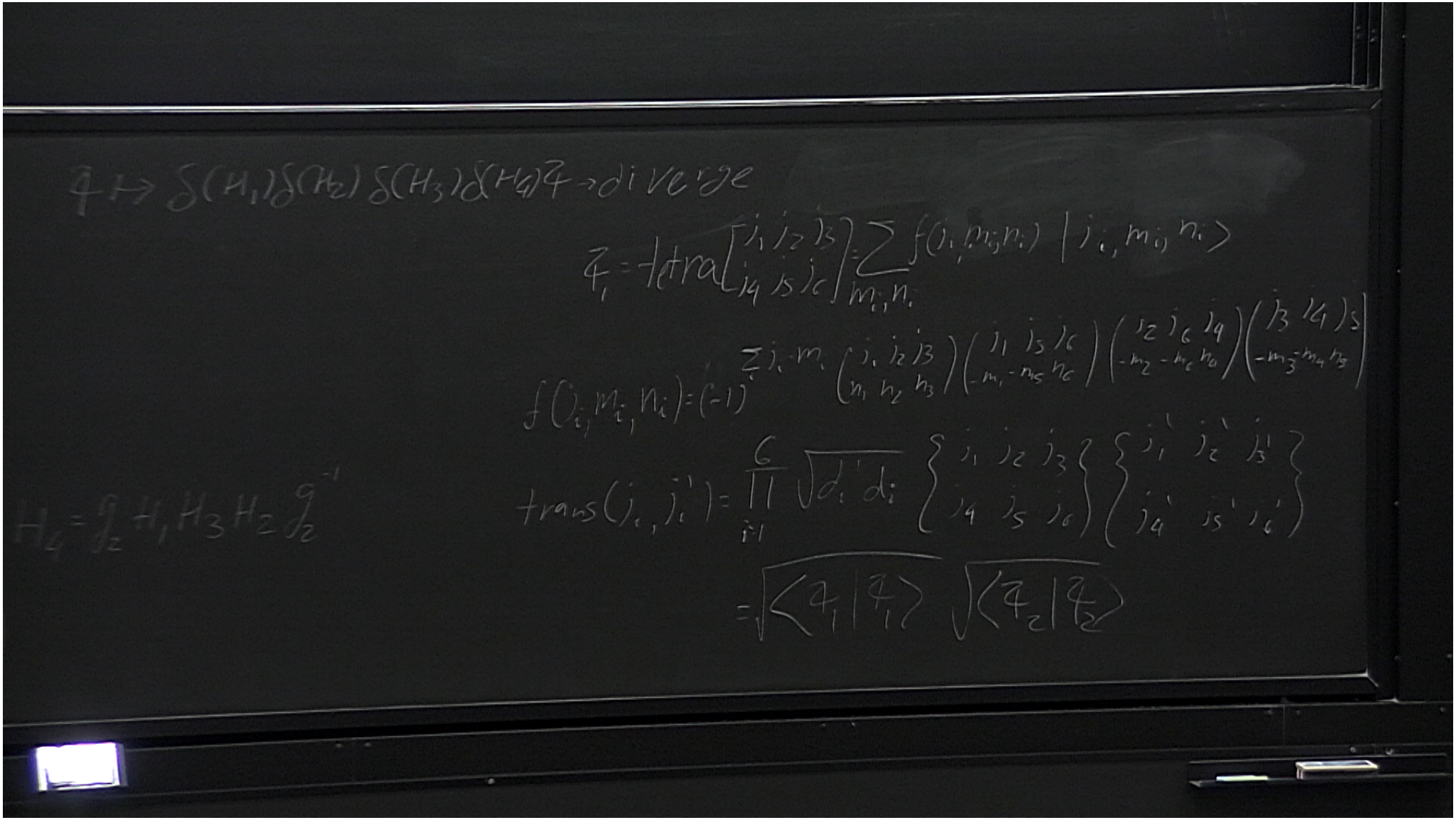
$\xi^3) \Rightarrow 1$

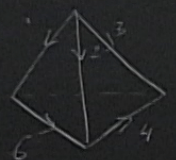


$q \mapsto \delta(H_1)\delta(H_2)\delta(H_3$

$$\left.\begin{aligned} H_1 &= g_2^{-1} s_6 g_1 \\ H_2 &= g_3^{-1} g_4 g_2 \\ H_3 &= g_1^{-1} g_2 g_3 \\ H_4 &= g_6 \; g_5 \; g_4 \end{aligned}\right\} H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$$


$\hat{q} \mapsto \delta(H_1)\delta(H_2)\delta(H_3)\delta(H_4)\hat{q}$ → diverge
$\hat{q}_1 = \text{tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} = \sum_{m_i, n_i} f(j_i, m_i, n_i) \, |j_i, m_i, n_i\rangle$
$H_4 = \hat{q}_2 H_1 H_3 H_2 \hat{q}_2^{-1}$


$\bar{q} \mapsto \delta(H_1)\delta(H_2)\delta(H_3)\delta(H_4)\bar{q}$ → diverge
$q_i = \mathrm{tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} = \sum_{m_i, n_i} f(j_i, m_i, n_i) \, |j_i, m_i, n_i\rangle$
$f(j_i, m_i, n_i) = (-1)^{\sum_i j_i - m_i} \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -n_1 & -n_5 & n_6 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ -n_2 & -n_6 & n_4 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ -n_3 & -n_4 & n_5 \end{pmatrix}$
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
$\mathrm{trans}(j_i, j_i') = \prod_{i=1}^{6} \sqrt{d_i' d_i} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \begin{Bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{Bmatrix}$
$= \sqrt{\langle q_1 | q_1 \rangle} \sqrt{\langle q_2 | q_2 \rangle}$



$$q \mapsto \delta(H_1)\delta(H_2)\delta(H_3)\delta(H_4)q \to \text{diverge}$$

$$q \mapsto \delta(H_1/\delta(H_2/\delta(H_3/q$$

$$\langle \delta(\hat{C})q_1 | \delta(\hat{C})q_2 \rangle_{phys} = \langle q_1 | \delta(\hat{C})q_2 \rangle_{kin}$$

$$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$$

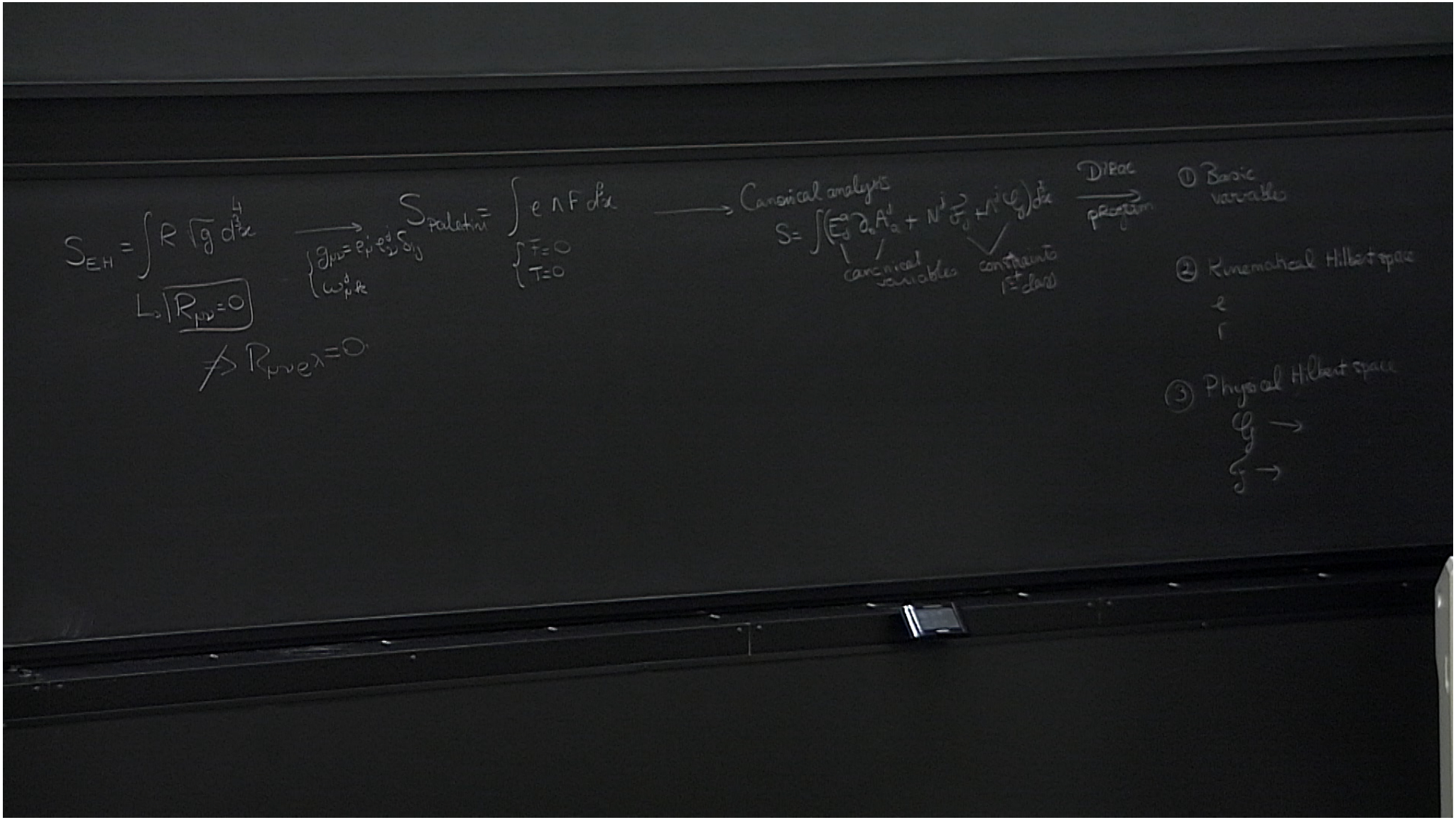
$$\left.\begin{aligned} H_1 &= g_2^{-1} g_6 g_1 \\ H_2 &= g_3^{-1} g_4 g_2 \\ H_3 &= g_1^{-1} g_2 g_3 \\ H_4 &= g_6 g_5 g_4 \end{aligned}\right\}$$

$$q_1 = \text{tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} = \sum_{m_i, n_i} f(j_i, m_i, n_i) \, |j_i, m_i, n_i\rangle$$

$$f(j_i, m_i, n_i) = (-1)^{\sum_i j_i - m_i} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -m_1 & -m_5 & m_6 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ -m_2 & -m_6 & m_4 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ -m_3 & -m_4 & m_5 \end{pmatrix}$$

$$\text{trans}(j_i, j_i') = \prod_{i=1}^{6} \sqrt{d_i d_i'} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \begin{Bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{Bmatrix}$$

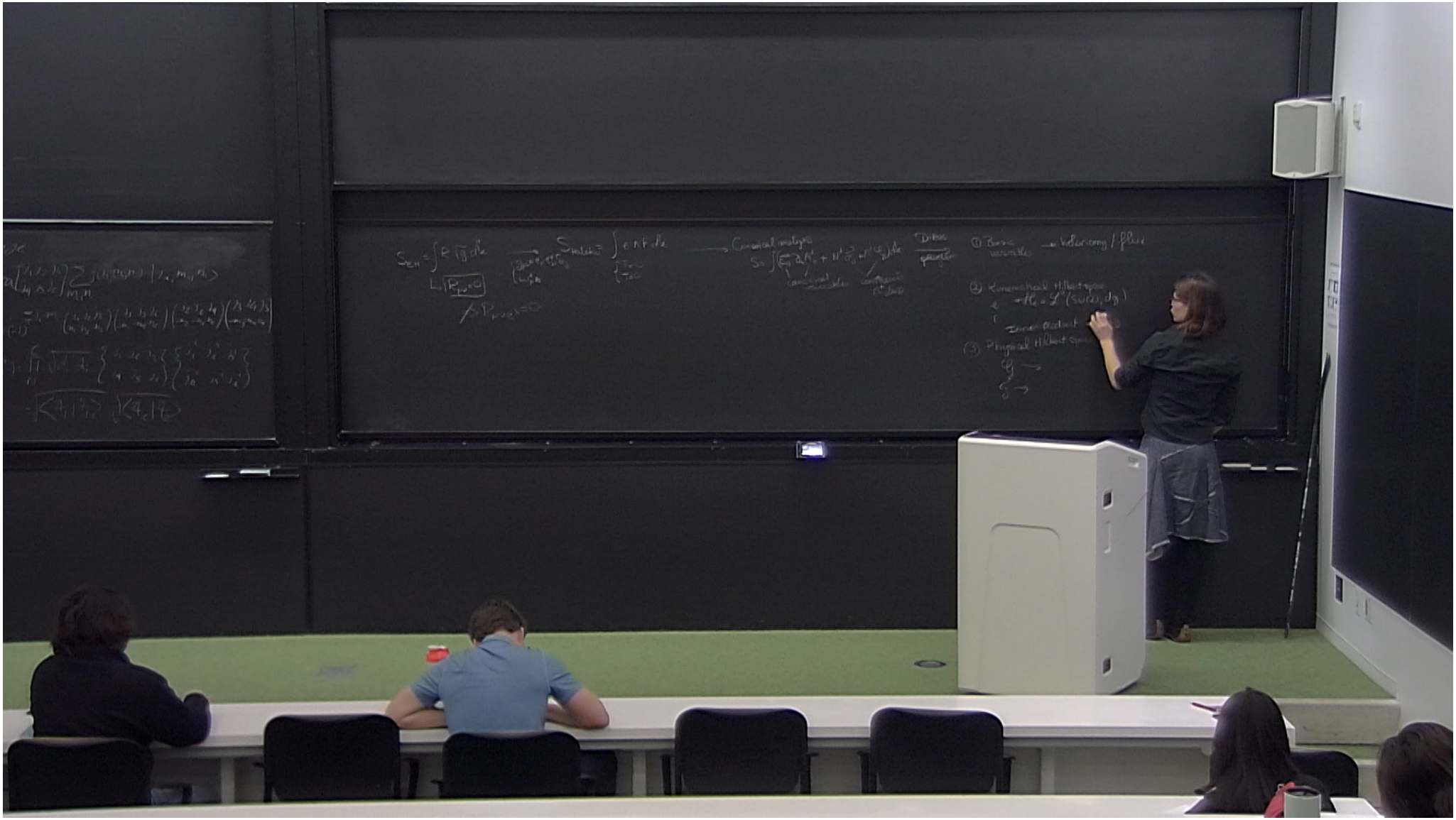
$$= \sqrt{\langle q_1 | q_1 \rangle} \sqrt{\langle q_2 | q_2 \rangle}$$


$S_{E-H} = \int R \sqrt{g}\, d^4x$
$L_0 \boxed{R_{\mu\nu}=0}$
$\not\Rightarrow R_{\mu\nu\rho\lambda}=0$
$\{ g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij}$
$\{ \omega^{ij}_\mu$
$S_{Palatini} = \int e \wedge F\, d^2x$
$\{ T=0$
$\{ T=0$
Canonical analysis
$S = \int (E^a_j \partial_0 A^j_a + N^j G_j + \lambda^i C_i)\, d^3x$
canonical variables
constraints
Dirac program
① Basic variables
② Kinematical Hilbert space
③ Physical Hilbert space

$$S_{EH} = \int R \sqrt{g}\, d^4x \longrightarrow \begin{cases} g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij} \\ \omega^a_{\mu k} \end{cases} S_{Palatini} = \int e \wedge F\, d^2x \longrightarrow \begin{cases} \bar{T} = 0 \\ T = 0 \end{cases}$$

$$\hookrightarrow \boxed{R_{\mu\nu} = 0}$$

$$\not\Rightarrow R_{\mu\nu e\lambda} = 0.$$



Dirac

program

① Basic variables → holonomy / flux

② Kinematical Hilbert space

e $\mathcal{H}_e = L^2(SU(2), dg)$

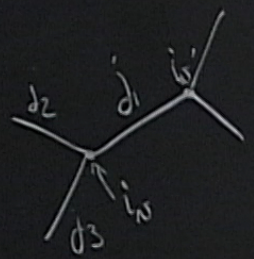
Γ $\mathcal{H}_\Gamma = \bigotimes \mathcal{H}_e$

Inner product → ONB

③ Physical Hilbert space

$\mathcal{G}$ → spin network state

$\mathcal{F}$ →



Dirac program

① Basic variables → holonomy / flux

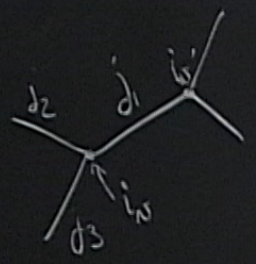
② Kinematical Hilbert space

$\mathcal{H}_e = L^2(SU(2), dg)$

$\mathcal{H}_\Gamma = \bigotimes_e \mathcal{H}_e$

Inner product → ONB

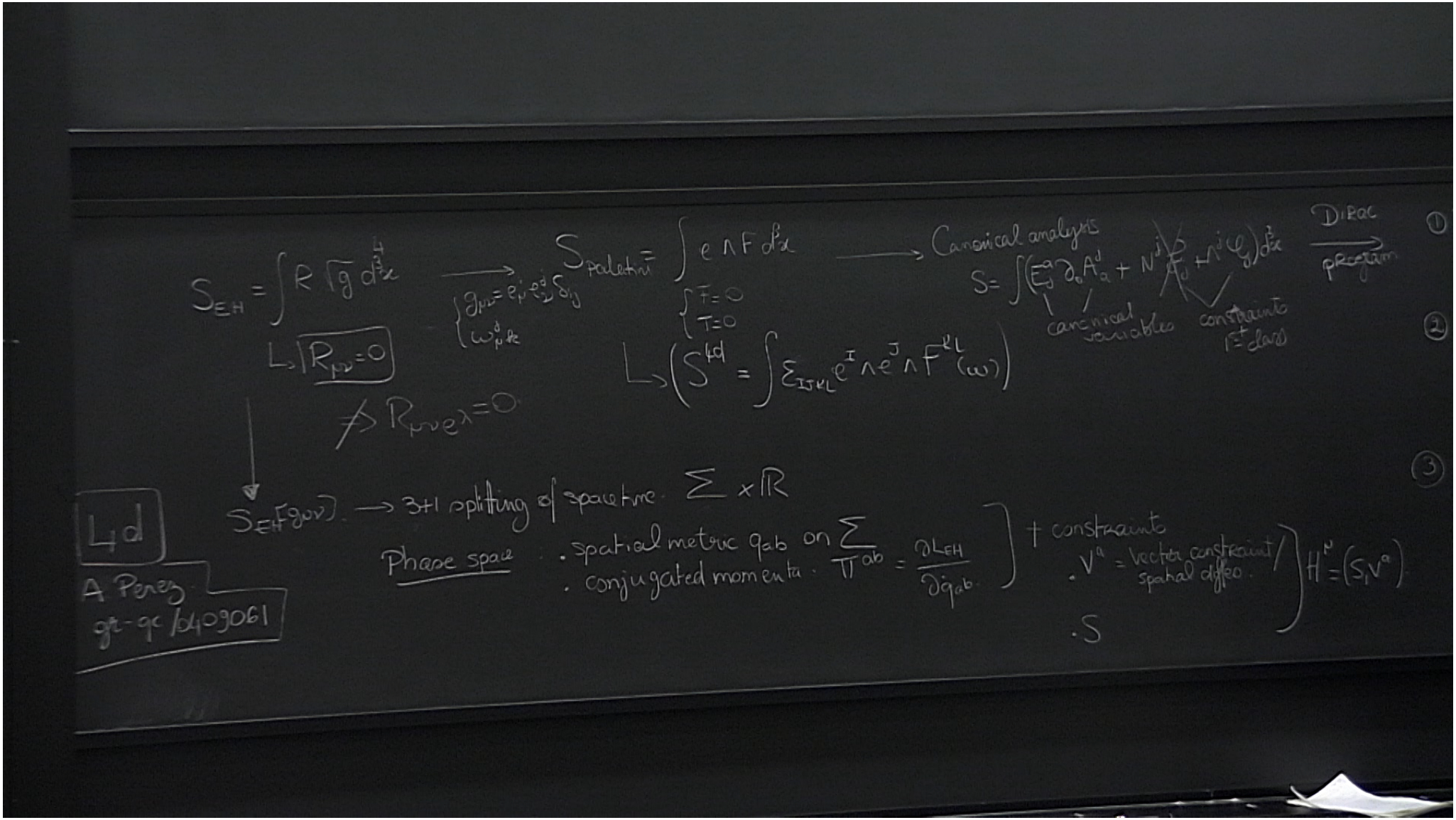
$\left(\mathcal{H} = \overline{\bigcup_\Gamma \mathcal{H}_\Gamma / \sim}\right)$



③ Physical Hilbert space

spin network state

$\left(\begin{matrix}\Gamma \\ j \\ \iota\end{matrix}\right) \rightarrow \{g_j\}$


$S_{EH} = \int R \sqrt{g}\, d^4x$
$\{g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij}$
$\{\omega^{ij}_\mu$
$S_{Palatini} = \int e \wedge F\, d^2x$
$\{T = 0$
$\{T = 0$
Canonical analysis
$S = \int (E^a_i \partial_0 A^i_a + N^i$ ... $+ N^i$ ... $) d^3x$
canonical variables
constraints 1st class
Dirac program
$R_{\mu\nu} = 0$
$\not\Rightarrow R_{\mu\nu\rho\lambda} = 0$
L_d
$S_{EH}[g_{\mu\nu}]$ → 3+1 splitting of spacetime $\Sigma \times \mathbb{R}$
Phase space
. spatial metric q_{ab} on Σ
. conjugated momenta $\pi^{ab} = \frac{\partial L_{EH}}{\partial \dot{q}_{ab}}$
+ constraints
. V^a = vector constraint / spatial diffeo
. S
$H^\perp = (S, V^a)$
A Perez
gr-qc/0409061

A Ashtekar change of variables - triad formulation: $q_{ab} = e^i_a e^j_b \delta_{ij}$ → additional SU(2) gauge symmetry

→ Densitized triad $= \frac{1}{2} \varepsilon_{ijk} \varepsilon^{abc} e^j_b e^k_c = E^a_i$ → Immirzi parameter $\in \mathbb{R}$

→ Ashtekar Barbero connection: $A^i_a = \Gamma^i_a + \gamma K^i_a$ (extrinsic curvature)

(Γ^i_a: Levi Civita connection)

case $\gamma = i$ self-dual formulation

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→ Ashtekar Barbero connection: $A^i_a = \Gamma^i_a + \gamma K^i_a$ (Γ: Levi Civita connection; K: extrinsic curvature) $= \frac{1}{2} \varepsilon^i_{jk} \omega_a^{jk} + \gamma e^i_a$ (partial gauge fixing time-gauge)

Case $\gamma = i$ self-dual formulation

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(Γ: Levi Civita connection; K: extrinsic curvature)

$$\{E^a_j(x), A^i_b(y)\} = K \gamma \delta^a_b \delta^i_j \delta(x,y)$$

case $\gamma = i$ self-dual formulation

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Γ^i_a: Levi Civita connection; K^i_a: extrinsic curvature

case $\gamma = i$ self-dual formulation

$$\{E^a_j(x), A^i_b(y)\} = K \gamma \delta^a_b \delta^i_j \delta(x,y)$$

+ Constraints

A Ashtekar change of variables – triad formulation: $q_{ab} = e^i_a e^d_b \delta_{ij}$ →

→ Densitized triad $= \frac{1}{2} \varepsilon_{ijk} \varepsilon^{abc} e^d_b e^k_c = E^a_i$ → I

→ Ashtekar Barbero connection: $A^i_a = \Gamma^i_a + \gamma K^i_a$ =
(↑ Levi Civita connection; ↓ extr

case $\gamma = i$ self-dual formulation

$\{E^a_j$(

+ Con

$\delta^{ij}_{ab}\,\delta_{ij}$ ⟶ additional SU(2) gauge symmetry.

$= \tilde{E}^a_i$

$+\gamma K^i_a$ ⟶ Immirzi parameter $\in \mathbb{R}$

$= \frac{1}{2}\varepsilon^i{}_{jk}\,\omega_a^{jk} + \gamma\,\omega_a^{0i}$ (partial gauge fixing: time-gauge)

K^i_a ⟶ extrinsic curvature

Levi Civita connection

$$\{E^a_j(x), A^i_b(y)\} = k\gamma\,\delta^a_b\,\delta^i_j\,\delta(x,y)$$

+ Constraints $\begin{cases} G \\ V^a \\ S \end{cases}$ ⟶ smearing of constraints ⟶

Constraint algebra

$$\{G[\alpha], G[\beta]\} = G([\alpha,\beta])$$

$$\{G[\alpha], V[N^a]\} = -G(\mathcal{L}_N(\alpha))$$

$$\{G[\alpha], S[N]\} = 0$$

$$\{V(N^a), V(M^a)\} = -S(\mathcal{L}_N M)$$

$$\{S[N], S(M)\} = V(S^a) + \text{terms proportional to Gauss}$$

⟵ field dependent structure function.

δ_{ij} → additional SU(2) gauge symmetry.

$= E^a_i$

→ Immirzi parameter $\in \mathbb{R}$

$+\gamma K^i_a = \frac{1}{2}\varepsilon^i{}_{jk}\,\omega_a^{jk} + \gamma\,\omega_a^{0i}$ (partial gauge fixing / time-gauge)

↳ extrinsic curvature

Levi-Civita connection

$$\{E^a_j(x), A^i_b(y)\} = k\gamma\,\delta^a_b\,\delta^i_j\,\delta(x,y)$$

+ Constraints $\begin{cases} G \\ V^a \\ S \end{cases}$ → smearing of constraints →

Constraint algebra

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$\{V(N^a), V(M^a)\} = -S(\mathcal{L}_N M)$

$\{S[N], S(M)\} = V(S^a) +$ terms proportional to Gauss.

Does not close in the BRS sense ← field dependent structure function.

Does not close in the BRS sense ← field dependent structure function

$[S[N], S[M]$

Dirac's program

↳ Kinematical Hilbert space

- $A \to$ holonomies
- Cylindrical functions of generalized connections (holonomies)
- Measure Ashtekar-Lewandowski measure

$$Cyl = \bigcup_{\gamma} Cyl_\gamma$$

↳ union over all possible graphs

$$\mathcal{H}^{LOG}_{kin} = \bigcup_{\gamma} \mathcal{H}_\gamma / \sim$$

$$\Psi_{\gamma_1} \in \mathcal{H}_{\gamma_1}$$

→ Can identify a equivalent representative on any bigger $\gamma_2 \geq \gamma_1$

$$\mu_{AL}(\Psi_{\gamma,f}) = \int_{e \in \gamma} \prod \widetilde{dh_e}\, f(h_{e_1} \dots h_{e_n})$$

Haar measure

$$\langle \Psi^{(1)}_{(\gamma_1, f_1)} \,|\, \Psi^{(2)}_{(\gamma_2, f_2)} \rangle = \int_{e \in \Gamma_{\gamma_1\gamma_2}} \prod dh_e\, f_1(h_{e_1} \dots h_{e_N})\, f_2(h_{e_1} \dots h_{e_N})$$

$\Gamma_{\gamma_1\gamma_2}$ any graph / $\gamma_1 \subset \Gamma_{\gamma_1\gamma_2}$, $\gamma_2 \subset \Gamma_{\gamma_1\gamma_2}$

Dirac's program

↳ Kinematical Hilbert space

- $A \to$ holonomies
- Cylindrical functions of generalized connections
- Measure: Ashtekar-Lewandowski measure

$$\mu_{AL}(\psi_{\gamma,f}) = \int$$

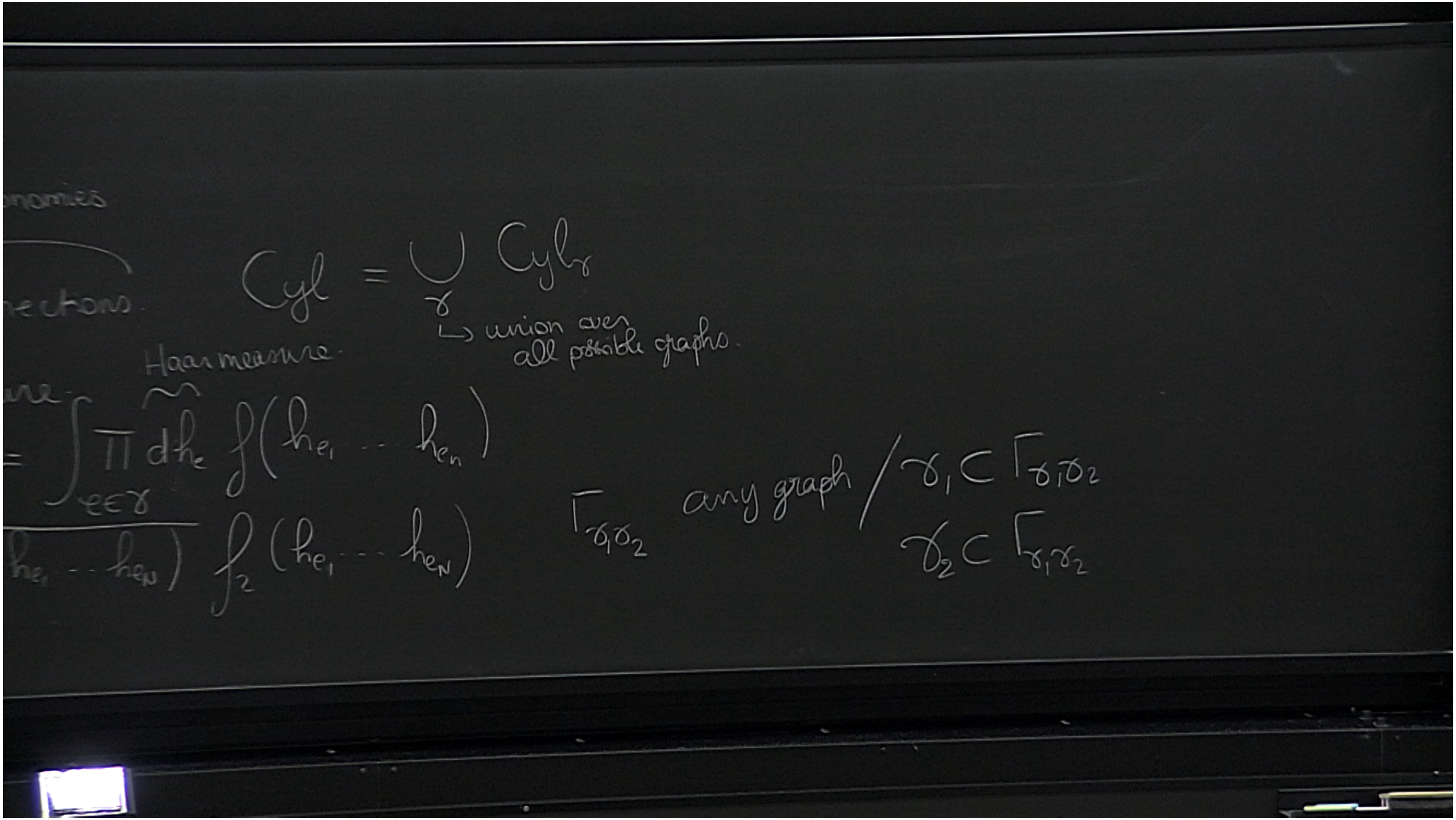
$$\mathcal{H}_{kin} \overset{LQG}{=} \bigcup_{\gamma} \mathcal{H}_{\gamma}/\sim$$

$$\psi_{\gamma_1} \in \mathcal{H}_{\gamma_1}$$

→ can identify a equivalent representative on any bigger $\gamma_2 \gg \gamma_1$

$$\langle \psi^{(1)}_{(\gamma_1, f_1)} \,|\, \psi^{(2)}_{(\gamma_2, f_2)} \rangle = \int_{e \in \Gamma_{\gamma_1,\gamma_2}} \prod_e dh_e \; \bar{f}_1(h_{e_1} \cdots$$

holonomie

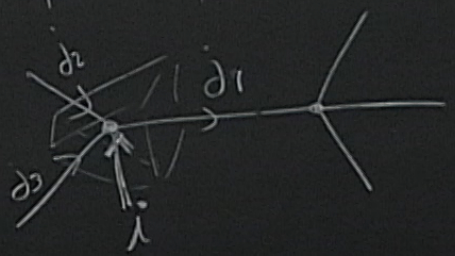

Cyl = ⋃_γ Cyl_γ
union over all possible graphs.
Haar measure
Γ_{γ1,γ2} any graph / γ1 ⊂ Γ_{γ1,γ2}
γ2 ⊂ Γ_{γ1,γ2}

∂q_{ab}) spatial diffeo. $\Gamma = (S$

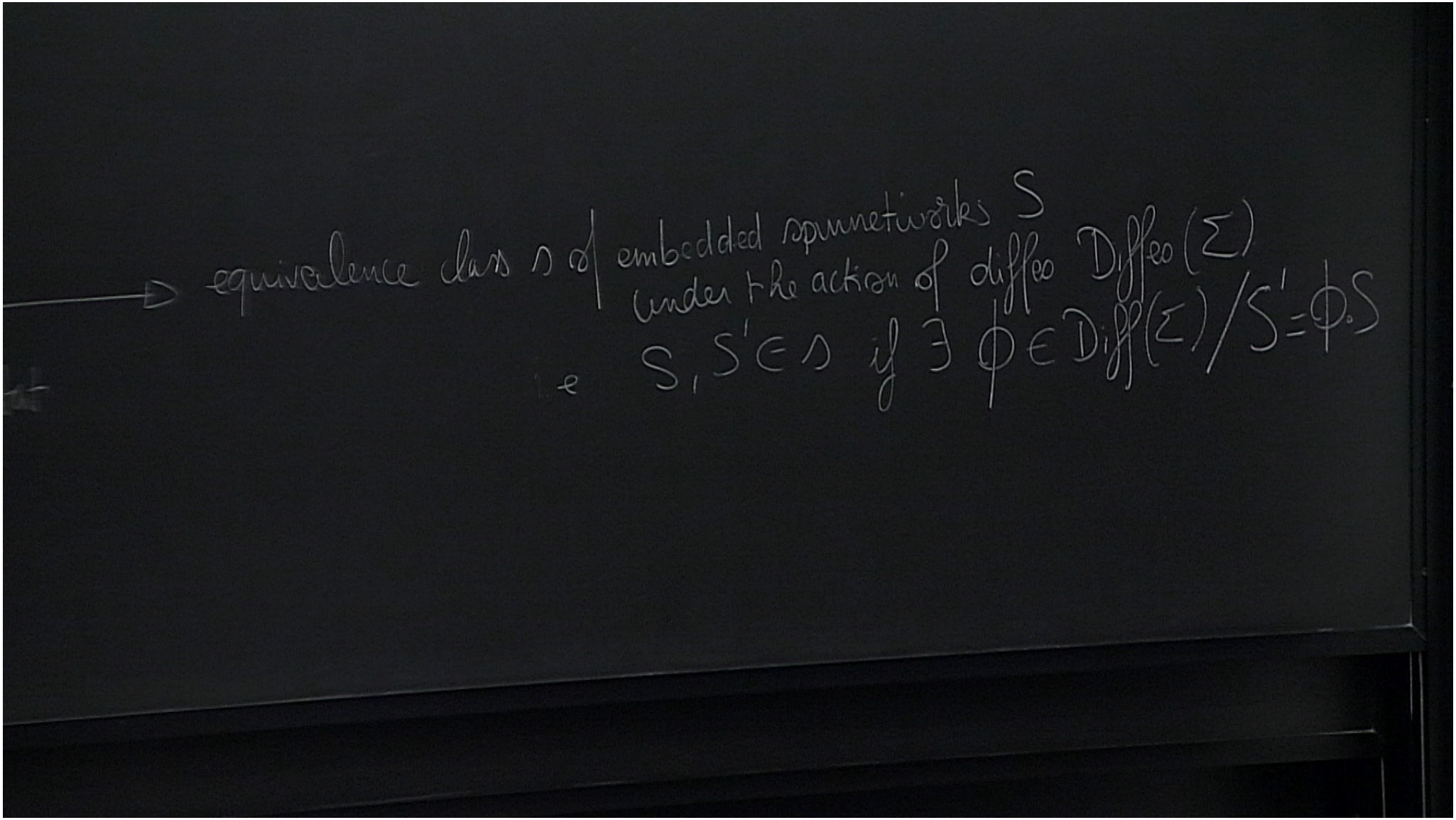
ONB $\rightarrow |j, m, n\rangle$ (Peter-Weyl theorem)

Physical phase space

- Gauss constraint

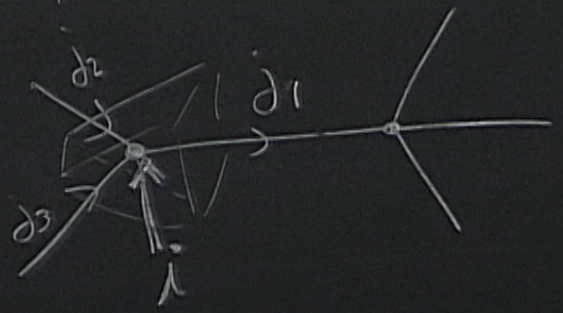


- Spatial diffeo $\hat{V}^a \Psi_s[A] = 0 \rightarrow$ s-knots / abstract spin network stat $\longrightarrow$


equivalence class s of embedded spinnetworks S
under the action of diffeo Diffeo(Σ)
i.e S, S' ∈ s if ∃ φ ∈ Diff(Σ) / S' = φ·S

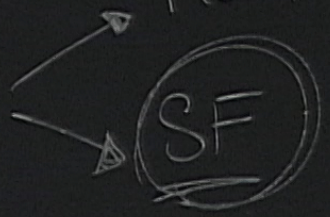
Physical phase space

- Gauss constraint



- Spatial diffeo $\hat{V}^a \Psi_S[A] = 0 \longrightarrow$ | s-knots / abstract spin network stat

- S ?



Spinfoam.