Title: PSI 2018/2019 - Explorations in Quantum Gravity - Lecture 3

Speakers: MaÃ¯tÃ© Dupuis

Collection: PSI 2018/2019 - Explorations in Quantum Gravity (Dupuis)

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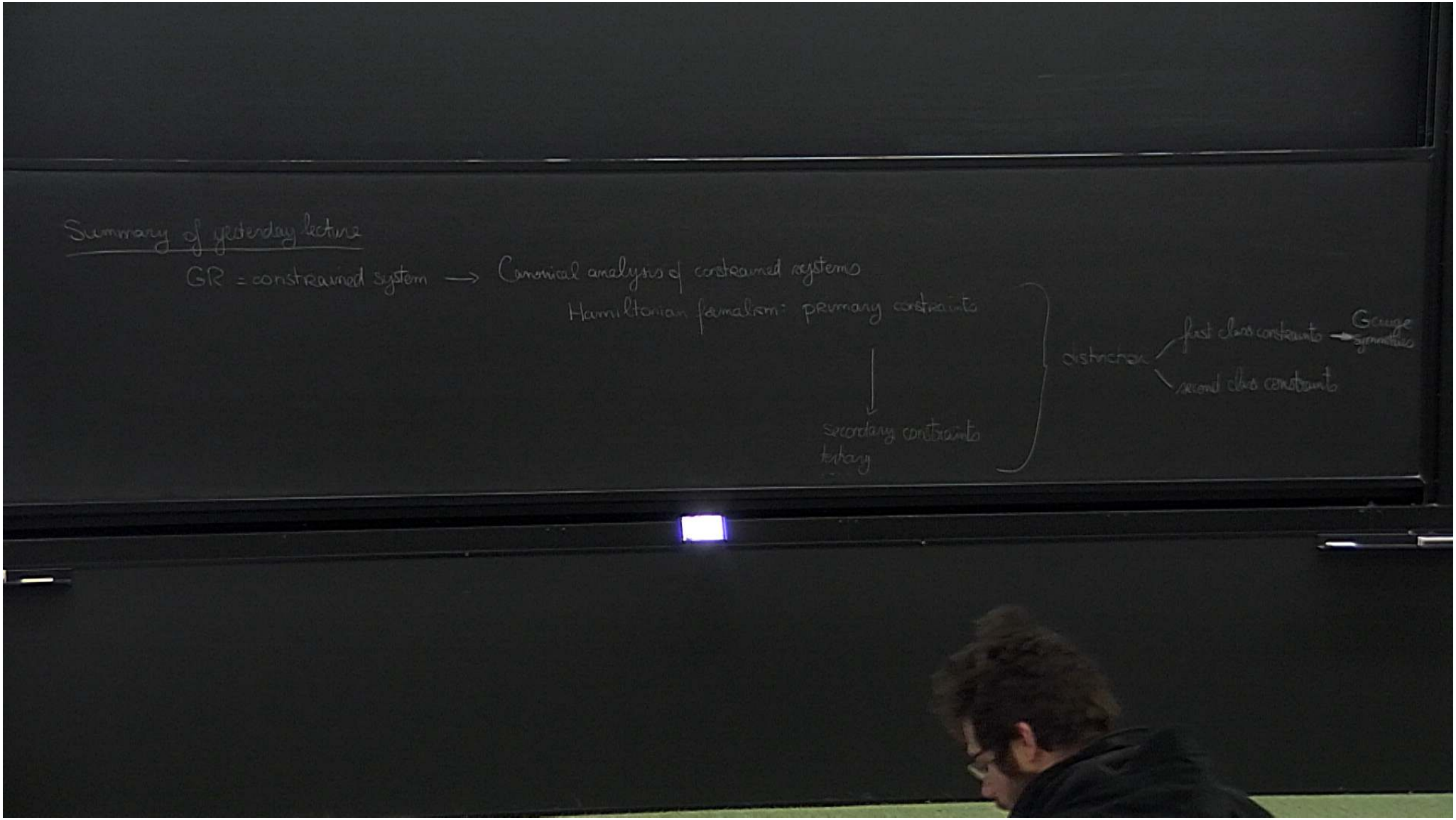
URL: http://pirsa.org/19030070


Summary of yesterday lecture
GR = constrained system → Canonical analysis of constrained systems
Hamiltonian formalism: primary constraints
secondary constraints
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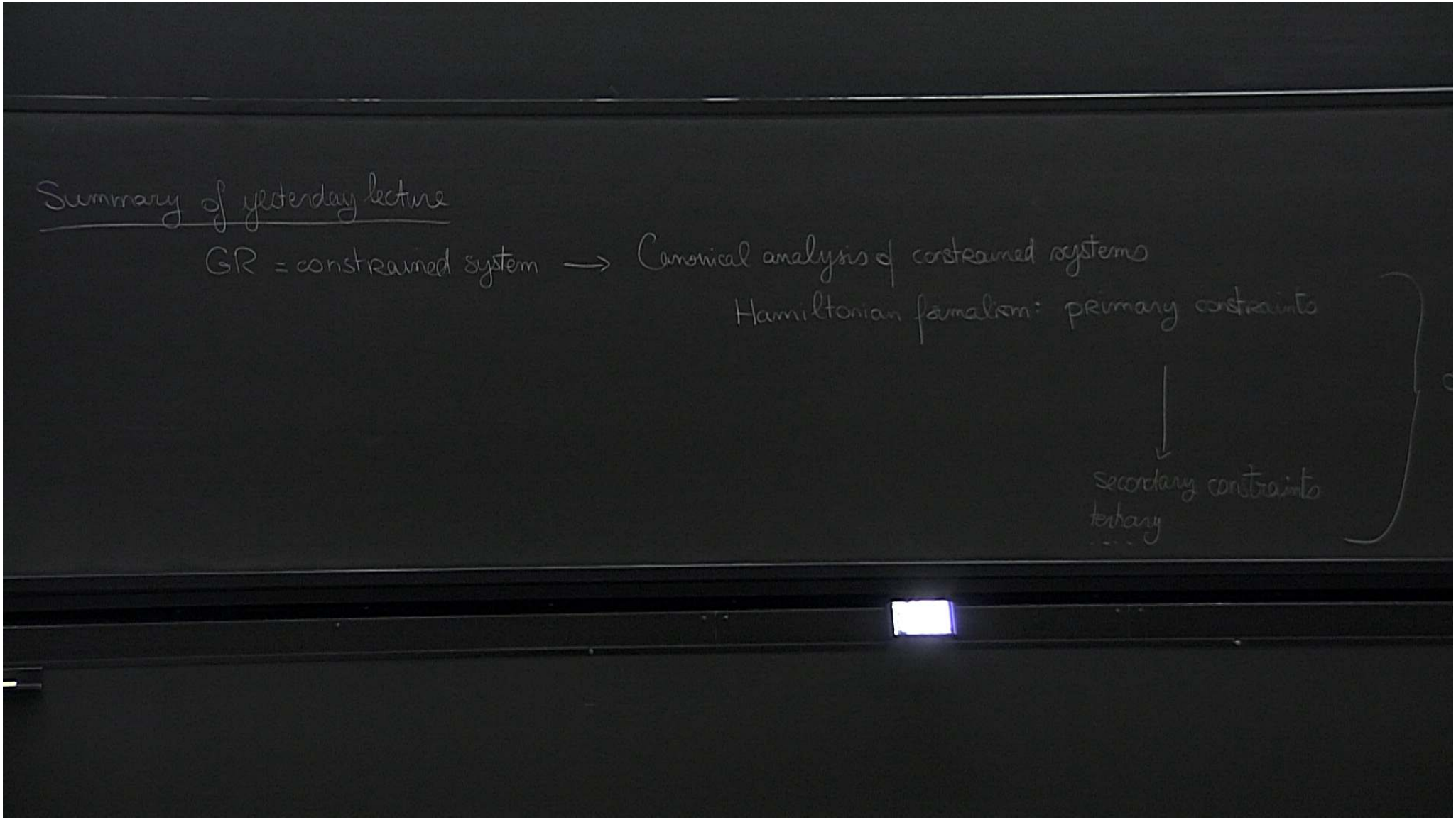
distinction
first class constraints → Gauge symmetries
second class constraints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Hamiltonian formalism: primary constraints
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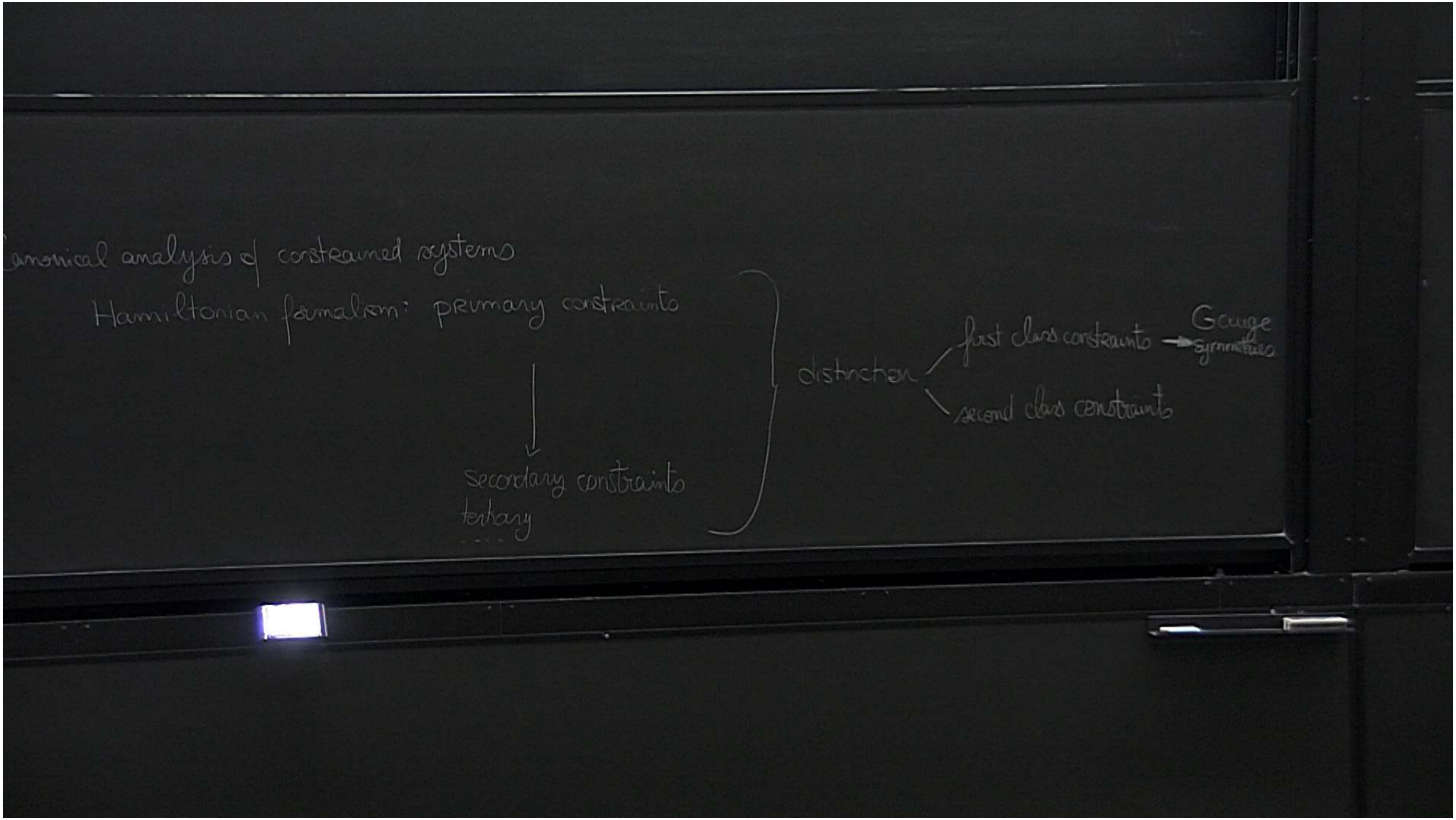
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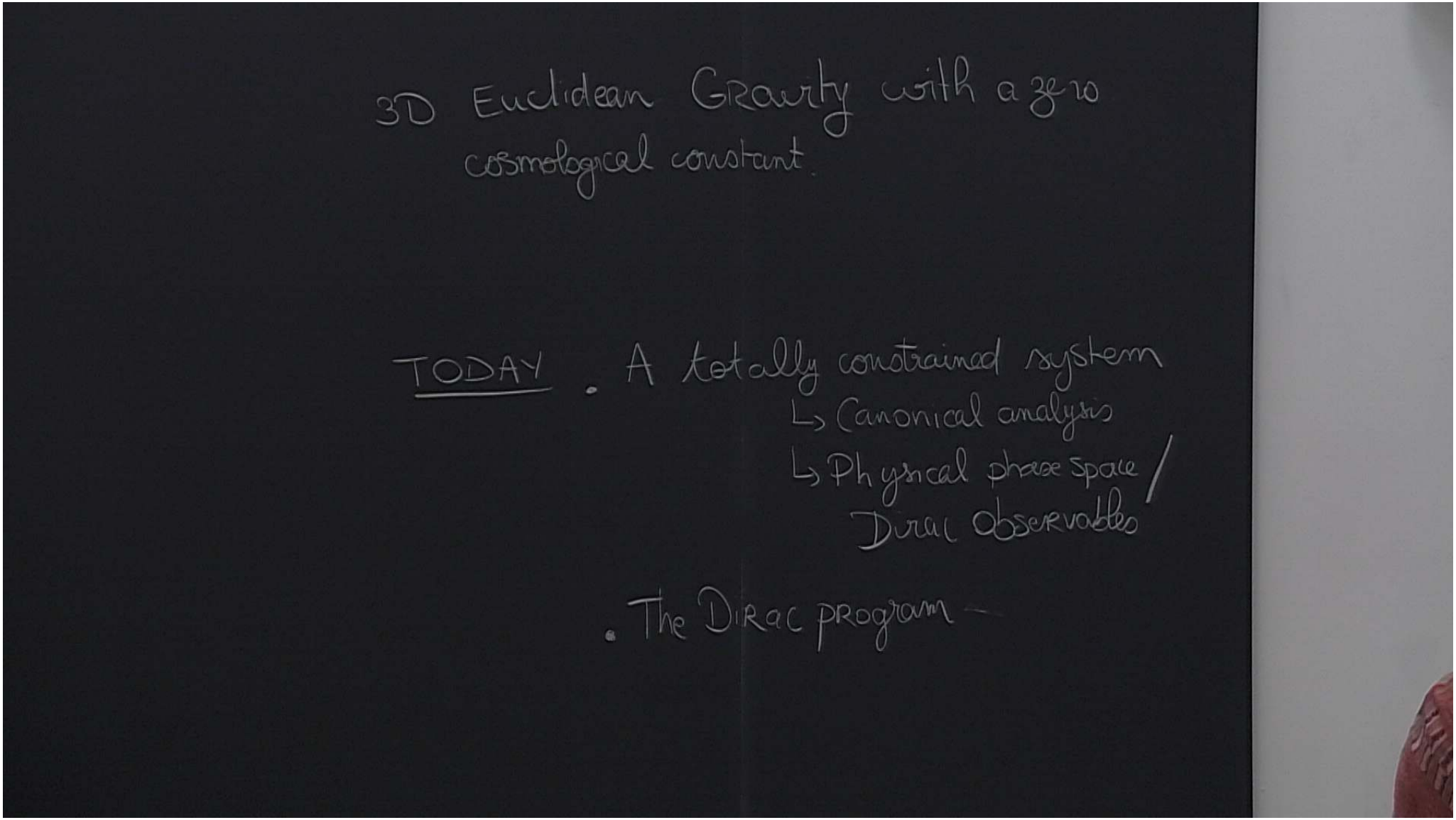
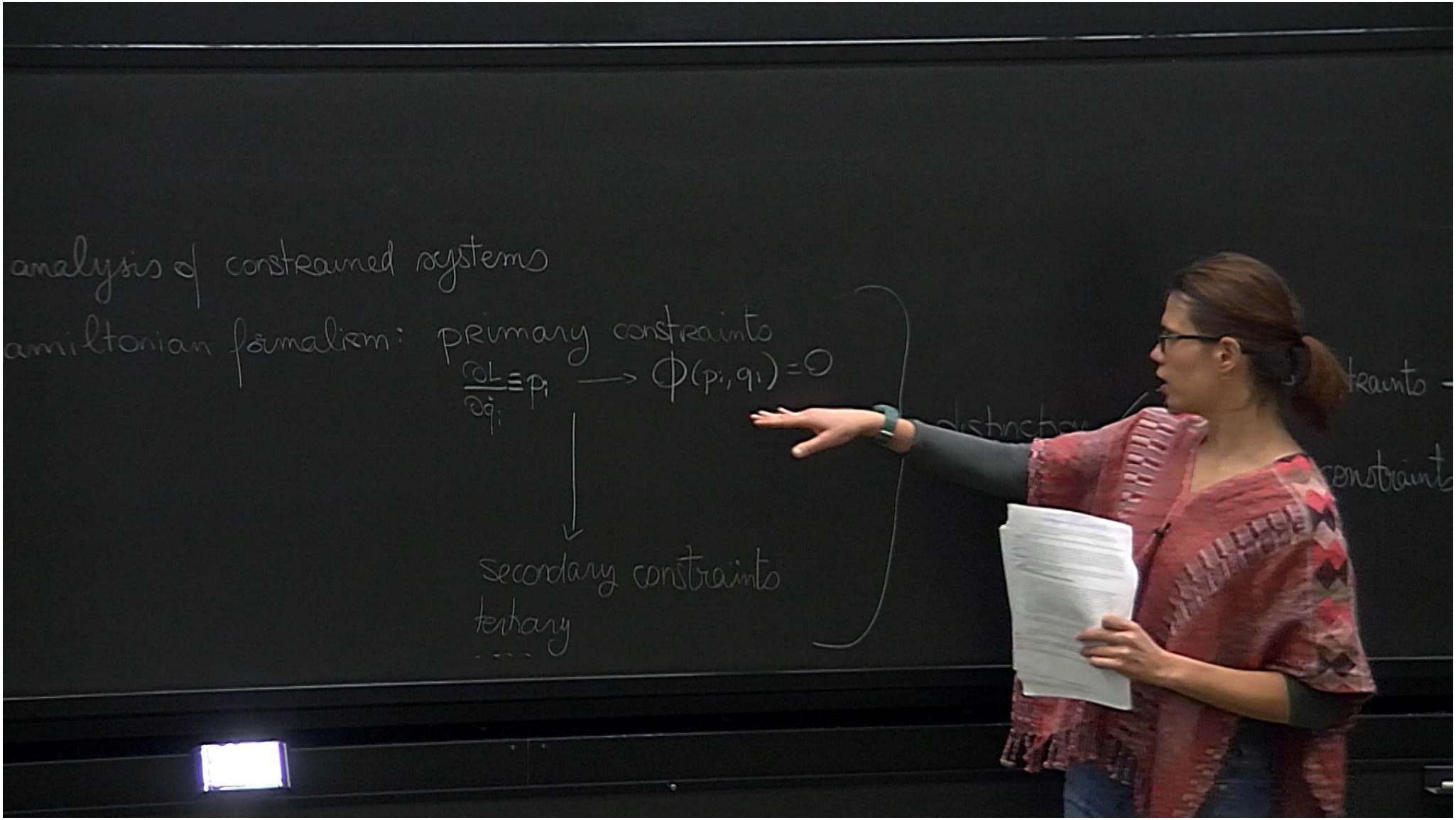
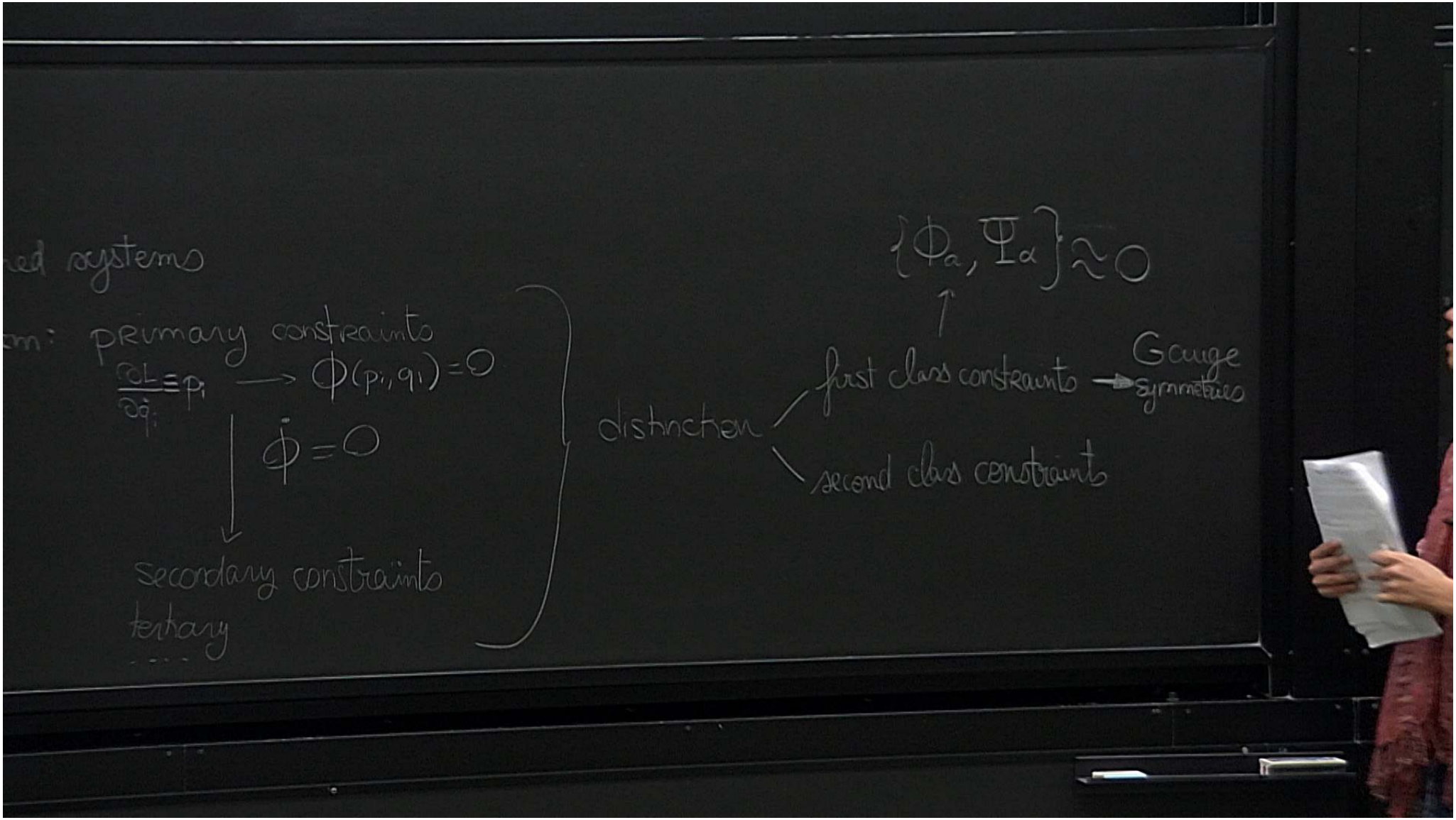

3D Euclidean Gravity with a zero cosmological constant
TODAY . A totally constrained system
↳ Canonical analysis
↳ Physical phase space / Dirac observables
. The Dirac program


analysis of constrained systems
amiltonian formalism: primary constraints
$\frac{\partial L}{\partial \dot{q}_i} = p_i \longrightarrow \phi(p_i, q_i) = 0$
secondary constraints
tertiary
....


ed systems
m: primary constraints
$\frac{\partial L}{\partial \dot{q}_i} \equiv p_i \longrightarrow \phi(p_i, q_i) = 0$
$\dot{\phi} = 0$
secondary constraints
tertiary
distinction
first class constraints
Gauge symmetries
second class constraints
$\{\Phi_a, \Psi_\alpha\} \approx 0$

A totally constrained system - example of the parametrized particle.

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free non-relativistic particle in 1d $\quad S[q] = \int_{t_1}^{t_2} dt \, \frac{1}{2} m \dot{q}^2 \qquad \dot{q} = \frac{dq}{dt}$

cle.

$$\dot{q} = \frac{dq}{dt} \longrightarrow q(t) = q(\tau) + \frac{p(\tau)}{m}(t-\tau)$$

totally constrained system – example of the parametrized particle.

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. $t \to$ independent variable $\quad S_p[q(s), t(s)] = \int_{s_1}^{s_2} \left(\frac{1}{2} m \frac{q'^2}{t'} \right) ds$

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$$S_s[q(s), t(s)] = \int_{s_1}^{s_2} \left(\frac{1}{2} m \frac{q'^2}{t'}\right) ds$$

' derivative with respect to s

gstem – example of the parametrized particle.

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variable $\quad S_p[q(s), t(s)] = \int_{s_1}^{s_2} \left(\frac{1}{2} m \frac{q'^2}{t'} \right) ds$ ' derivative

invariant under $s \longrightarrow \tilde{s} := f(s)$

Canonical analysis: $p_t = \frac{\partial S}{\partial t'} = -m\frac{q'^2}{2t'^2}$, $p_q = \frac{\partial S}{\partial q'} = m\frac{q'}{t'}$

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$\Rightarrow$ primary constraint $C = p_t + \frac{p_q^2}{2m} = 0$

Hamiltonian: $H = p_t t' + p_q q' - L = t' C$

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Hamiltonian $H = p_t t' + p_q q' - L = t' C$

$H = N(\lambda)\, C$

flow generated by H? $q' = \{q, H\} = N\frac{p_q}{m}$, $p_q' = 0$

$t' = \{t, H\} = N$, $p_q' = 0$

Physical phase space, Dirac observables

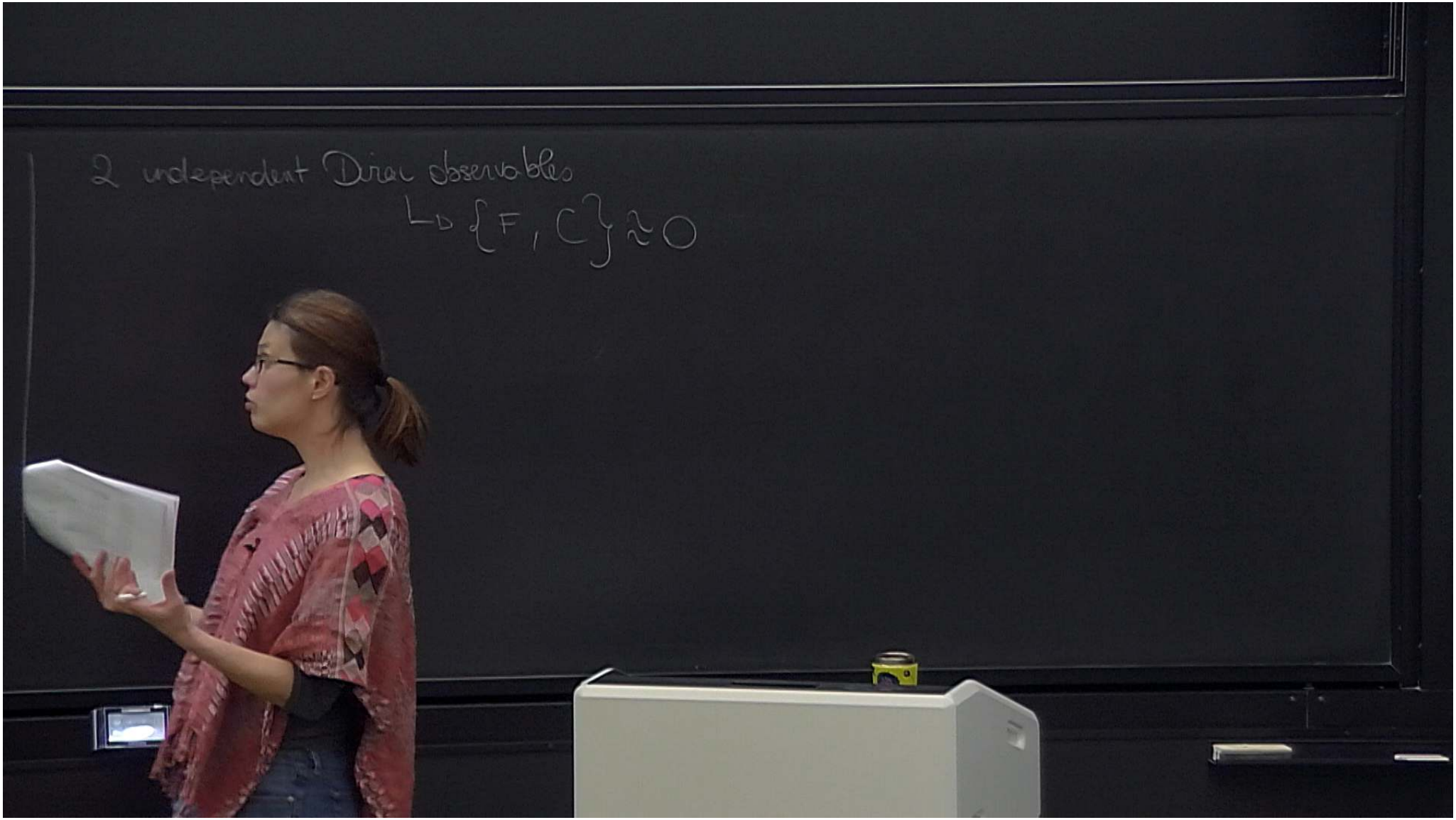
Gauge orbits of the phase space variables generated by C.

$$\frac{dq}{ds} = \{q, C\} = \frac{p_q}{m} \longrightarrow q(s) = q + \frac{p_q}{m} s$$

$$\frac{dt}{ds} = \{t, C\} = 1 \longrightarrow t(s) = s + t$$

$$\frac{dp_q}{ds} = \{p_q, C\} = 0 \Rightarrow p_q = p_{q0}$$

$$\frac{dp_t}{ds} = \{p_t, C\} = 0 \Rightarrow p_t = -\frac{p_q^2}{2m}$$


2 independent Dirac observables
$L_D\{F, C\} \approx 0$

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To construct a physical observable:

- As many gauge fixing as constraints

$$t(s) = \tau$$

- $f($

"Good" gauge fixing

↳ J. Tamborino.

2 independent Dirac observables

$$L_D \{F, C\} \approx 0$$

To construct a physical observable:

- As many gauge fixing as constraints

$$t(s) = \tau$$

"Good" gauge fixing

↳ J. Tambornino.

- $f(s) \longrightarrow F(\tau)$

$q(s) \longrightarrow$

$$F_q^\tau = q_0 + \frac{P_0}{m}(\tau - t_0)$$

$$F_{P_q} = P_{q_0}$$

$$\{F_q^\tau, F_p^\tau\} = 1$$

Dirac program

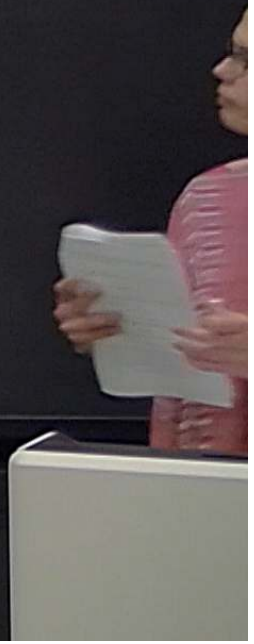
a) Find a representation of the phase space variables as operators acting on $\mathcal{H}_{kin}$ satisfying the standard commutation relations.

$$\{\ ,\ \} \longrightarrow -i/\hbar\ [\ ,\ ]$$

b) Promote the constraints to (self-adjoint) operators in $\mathcal{H}_{kin}$

c) Characterize the space of solutions of the constraints → inner product $\mathcal{H}_{phys}$.

d) Find a complete set of gauge inv observables



$\mathcal{H}_{phys}$

d) Find a complete set of gauge inv observables

Case of the parametrized particle

a) $\mathcal{H}_{kin} = \mathcal{L}^2(\mathbb{R}^2)$, $\psi(q,t)$, $\langle \psi, \varphi \rangle = \int dq\,dt\ \overline{\psi(q,t)}\ \varphi(q,t)$

$q \longrightarrow \hat{q}$ $\psi(q,t) =$

$t \longrightarrow \hat{t}$

$p_q \longrightarrow \hat{p}_q$

$p_t \longrightarrow \hat{p}_t$

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$q \longrightarrow \hat{q}\ \psi(q,t) = q\,\psi(q,t)$

$t \longrightarrow \hat{t}\ \psi(q,t) = t\,\psi(q,t)$

$p_q \longrightarrow \hat{p}_q\ \psi(q,t) = -i\hbar \frac{\partial}{\partial q} \psi(q,t)$

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b) $C = p_t + \frac{p_q^2}{2m} \longrightarrow \hat{C} = -i\hbar \frac{\partial}{\partial t} +$

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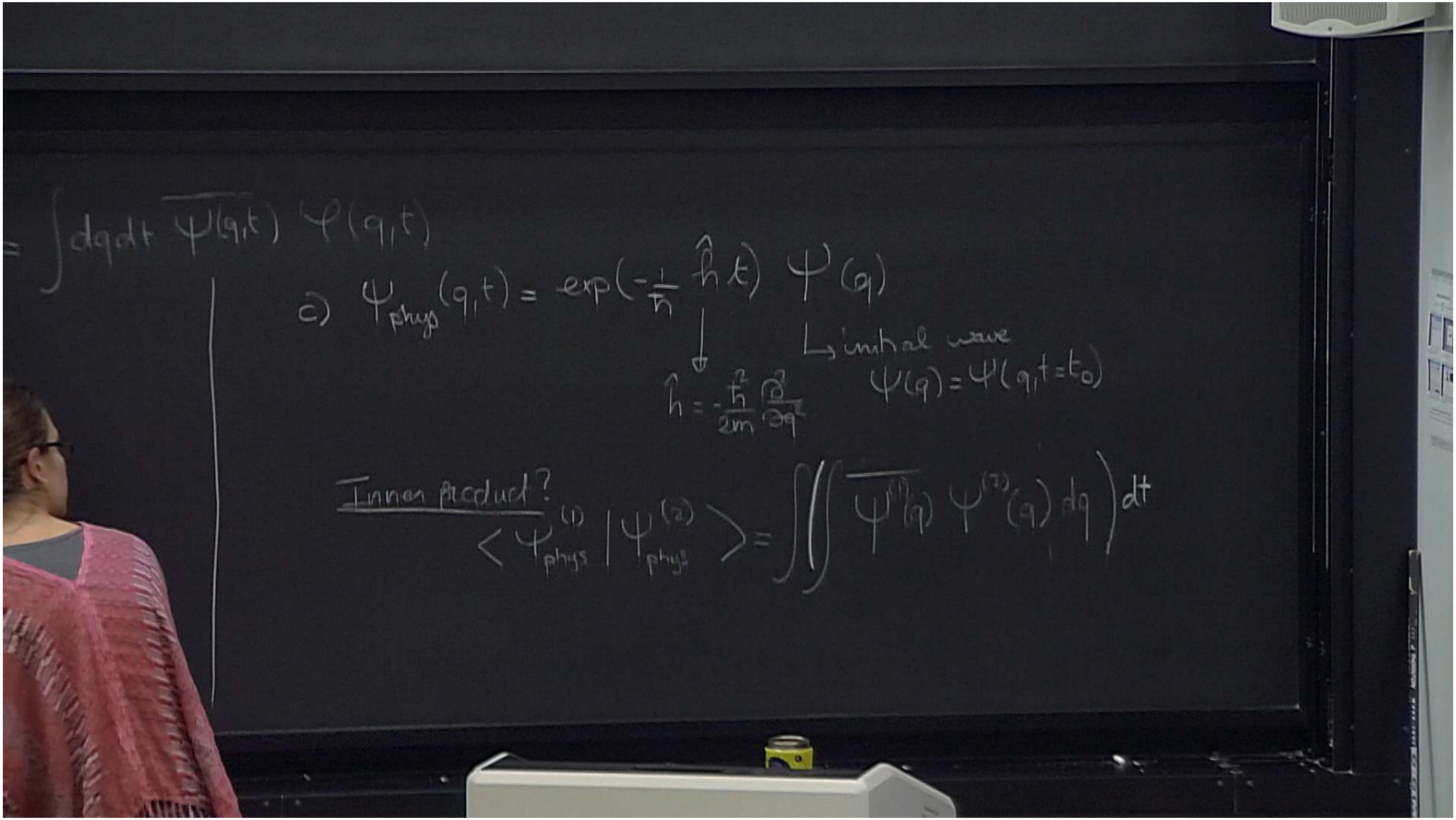
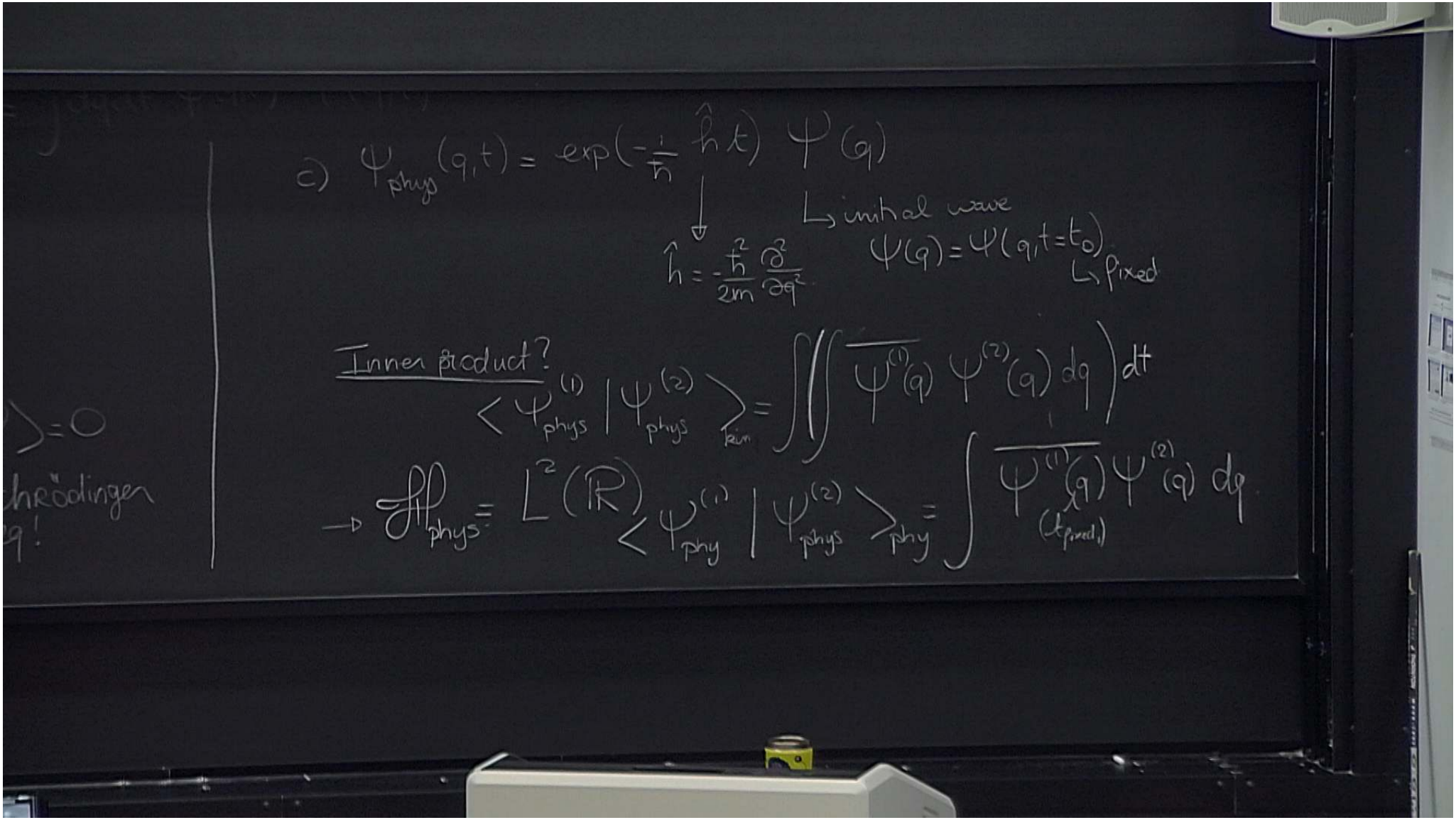
b) $C = p_t + \frac{p_q^2}{2m} \longrightarrow \hat{C} = -i\hbar \frac{\partial}{\partial t} - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial q^2}$

$\hat{C}|\Psi\rangle = 0$ ↳ Schrödinger eq!


$= \int dq\,dt\ \overline{\psi(q,t)}\ \psi(q,t)$
c) $\psi_{phys}(q,t) = \exp(-\frac{i}{\hbar}\hat{h}t)\ \psi(q)$
↳ initial wave
$\psi(q) = \psi(q, t=t_0)$
$\hat{h} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2}$
Inner product?
$\langle \psi^{(1)}_{phys} | \psi^{(2)}_{phys} \rangle = \int \left(\int \overline{\psi^{(1)}(q)}\ \psi^{(2)}(q)\, dq \right) dt$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↳ fixed
$\hat{h} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2}$
Inner product?
$\langle \Psi^{(1)}_{phys} | \Psi^{(2)}_{phys} \rangle_{kin} = \int \left(\int \overline{\Psi^{(1)}(q)}\, \Psi^{(2)}(q)\, dq \right) dt$
$\rightarrow \mathcal{H}_{phys} = L^2(\mathbb{R})$
$\langle \Psi^{(1)}_{phy} | \Psi^{(2)}_{phys} \rangle_{phy} = \int \overline{\Psi^{(1)}(q)}\, \Psi^{(2)}(q)\, dq$
(t_{fixed})

d) Two independent observables

$$\hat{O}_1 = \hat{q} - \frac{\hat{p}_q}{m}(\hat{t} - t_0)$$

$$\hat{O}_2 = \hat{p}_q$$

$$[\hat{O}_1, C] = 0 \text{ : to check}$$

$p_q' = 0$

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