

Title: PSI 2018/2019 - Explorations in Quantum Gravity - Lecture 2

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Collection: PSI 2018/2019 - Explorations in Quantum Gravity (Dupuis)

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URL: <http://pirsa.org/19030069>

- Why are we interested in "constrained systems"?

- Can we relate constraints / gauge symmetries?

$$\begin{array}{lcl} \rightarrow 2. & , & \overset{3D}{6} \rightarrow 0 \\ \text{triad/} & & \\ \text{connection} & e_p^i : \mathfrak{g} & \rightarrow 0 \end{array}$$

$$\phi(q, p) = 0$$

be interested in "constrained systems"?
relate constraints / gauge symmetries?

$\rightarrow 0$
 $g \rightarrow 0$

$$\phi(q_i, p_i) = 0$$

constraints
hold at every instant of
time in the evolution of the system.
= conserved quantity

Q2 conserved quantities $\overset{?}{\longleftrightarrow}$ symmetries

- Lagrangian formalism: symmetries $\Rightarrow$ conserved quantities (Noether)

- Symmetries $\left\{ \begin{array}{l} \text{global} \\ \text{gauge} \end{array} \right.$ local and non-trivial transformation of the

conserved quantities $\overset{?}{\longleftrightarrow}$ symmetries

theorem: symmetries $\Rightarrow$ conserved quantities (Noether)

global
gauge

local

$\downarrow$
do not affect
any boundary
value

and non-trivial transformation of the fields.

$\downarrow$
act non-trivially
on extrema of action.

Review of the Hamiltonian formulation

$$S = S[q] = \int L[q, \dot{q}] dt$$

$$p_i \equiv \frac{\partial L}{\partial \dot{q}_i}$$

$$H(q, p) \equiv p \cdot \dot{q} - L$$

$$\dot{f} = \{f, H\}$$

$\{q\} \rightarrow$ configuration space

$\{(q, p)\} \rightarrow$ phase space

$$S = \int$$

Review of the Hamiltonian formulation

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Time
evolution

$$\dot{f} = \{f, H\}, \quad \dot{q}_i, \dot{p}_i$$

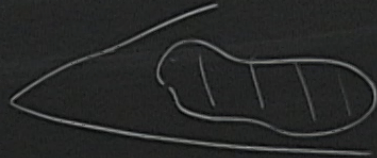
$$S = \int \{q, p\} =$$

$$S = \int \left[\sum p_i(q, \dot{q}) \dot{q}_i - H(p, q) \right] dt$$

$$\{q_i, p_j\} = \delta_{ij}$$

if $\circledast$ not invertible

$$\Rightarrow \Psi_\alpha(q, p) = 0 \quad \text{primary constraints}$$



↓
primary constraint surface

$$H = H_0 + u^\alpha \Psi_\alpha$$

$$\{f, H\} = \{f, H_0\} + \{f, u^\alpha \Psi_\alpha\}$$

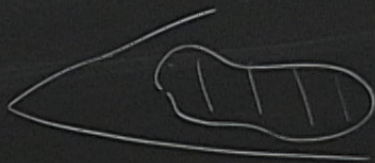
$$= \{f, H_0\} + \{f, u^\alpha\} \Psi_\alpha + \{f, \Psi_\alpha\} u^\alpha$$

$$S = \int \left[\sum p_i(q, \dot{q}) \dot{q}_i - H(p, q) \right] dt$$

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primary constraint surface

$$H = H_0 + u^\alpha \Psi_\alpha$$

$$\{f, H\} = \{f, H_0\} + \{f, u^\alpha \Psi_\alpha\}$$

weakly equality $\leftarrow$

$$\approx \{f, H_0\} + \cancel{\{f, u^\alpha \Psi_\alpha\}} + \{f, \Psi_\alpha\} u^\alpha$$

equality $\leftarrow \sum (p_i, H_0) + (p_i, H_1) \approx \dots$

ex2 tutorial

$$L = \frac{1}{2} m (\dot{q}_1 + \dot{q}_2)^2 - V(q_1, q_2)$$

$$\Psi(q, p) \equiv p_1 - p_2$$

ex 2 tutorial

$$L = \frac{1}{2} m (\dot{q}_1 + \dot{q}_2)^2 - V(q_1 + q_2)$$

$$Q_1 = q_1 + q_2$$

$$Q_2 = q_1 - q_2$$

$$\Psi(q, p) \equiv p_1 - p_2 \rightarrow \text{primary constraint}$$

$$\dot{\Psi} = \{\Psi, H\} = 0$$

$$\text{Symmetry?} \quad \delta q_1 = u \{q_1, \Psi\} = u$$

$$\delta q_2 = u \{q_2, \Psi\} = -u$$

$Q_1 = \text{physical}$

$Q_2 = \text{"internal" degree of freedom}$

• Constraints: primary, secondary - - ; first-class, second class.

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Distinction between 1st class / 2nd class
system with constraints Ψ_{α} : * if there is a subset

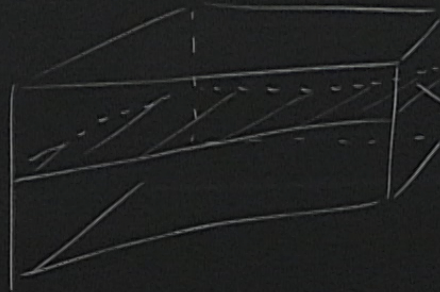
* remaining constraints by X_m , then the sub-matrix
 $\Delta_{mn} = \{X_m, X_n\}$ is invertible.

X_n : 2nd class constraints
→ corresponding Lagrange multipliers are (weakly) fixed.

* remain

$$\{\mathcal{L}, H\} = 0$$

Constraints and physical degrees of freedom



constrained
surface

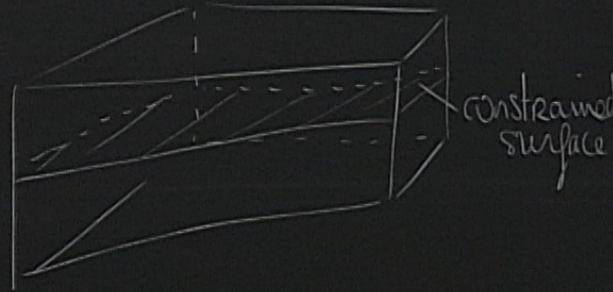
kinematical phase space

↓
physical or reduced
phase space

* remain

$$\{\mathcal{L}, H\} = 0$$

Constraints and physical degrees of freedom



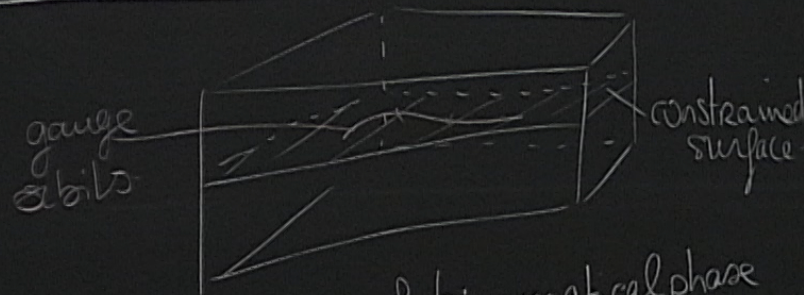
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* remain

$$\{\mathcal{L}, H\} = 0$$

Constraints and physical degrees of freedom



dim of kinematical phase space

$$- \#(1^{st} \text{ class}) \times 2 = \text{dim physical}$$