Title: PSI 2018/2019 - Explorations in Quantum Gravity - Lecture 2

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Collection: PSI 2018/2019 - Explorations in Quantum Gravity (Dupuis)

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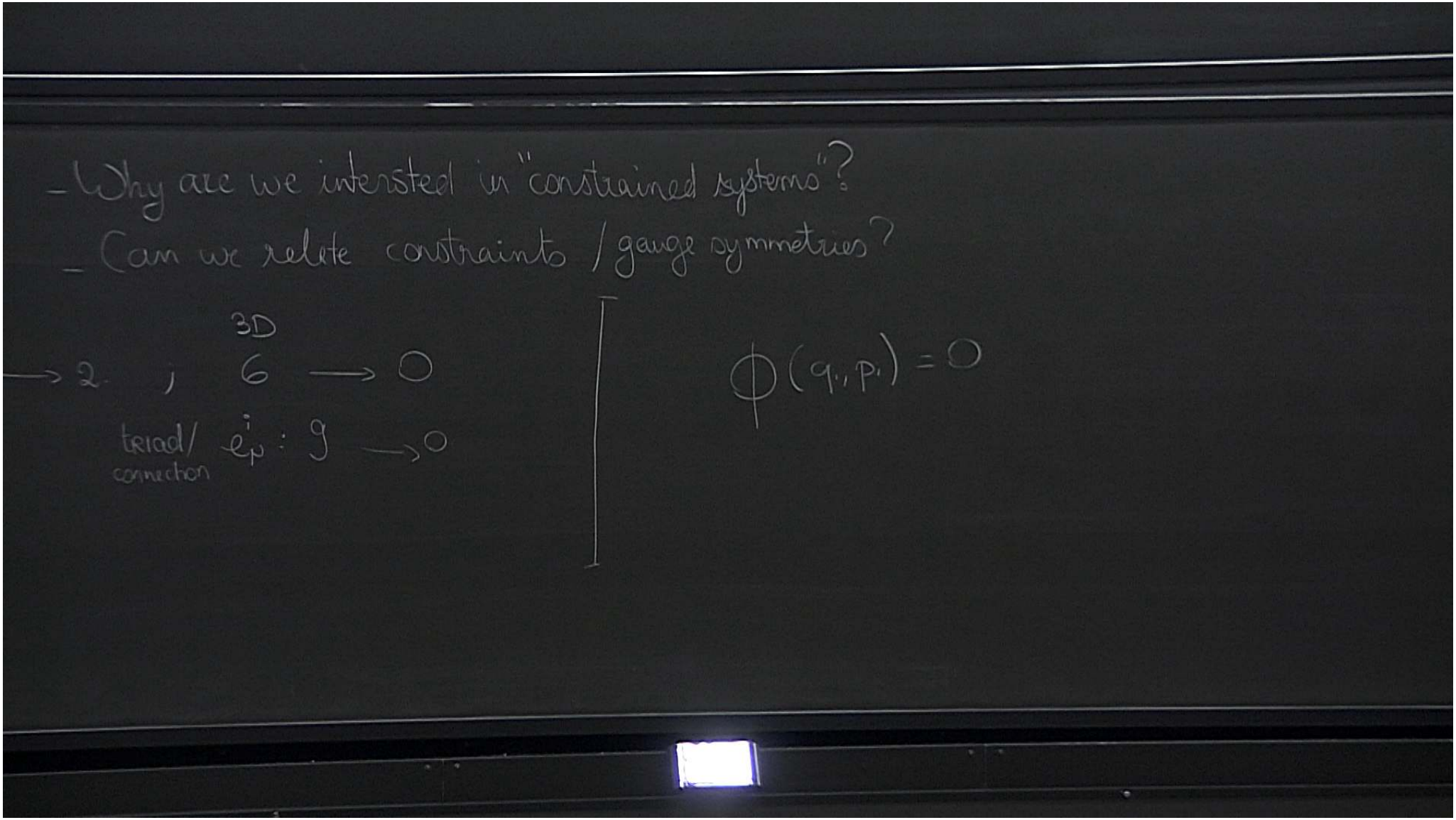
URL: http://pirsa.org/19030069

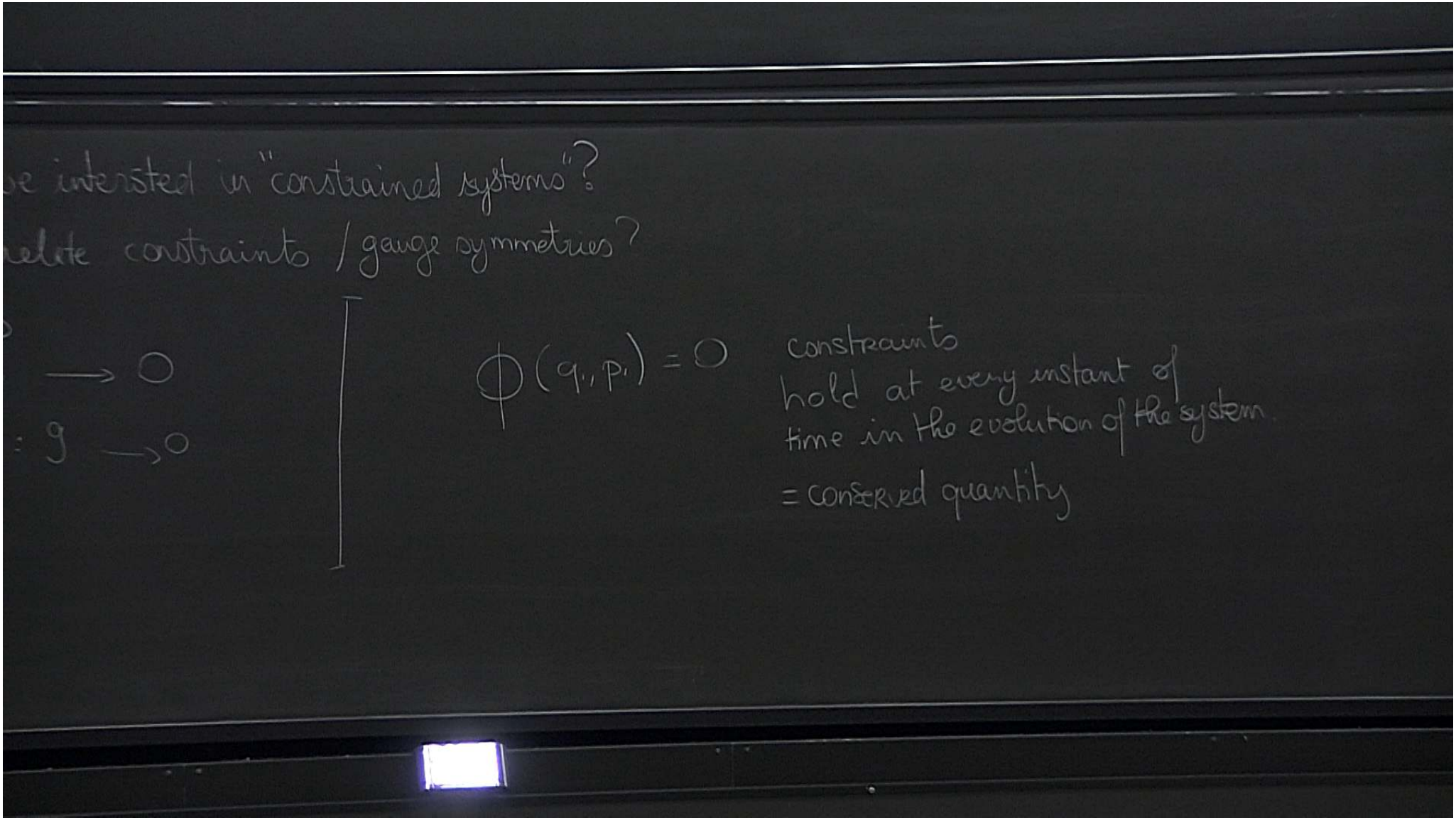
2 questions to start with: - Why are we intersted in "constrained systems"?

- Can we relate constraints / gauge symmetries?

Q1 : 4D $g_{\mu\nu}$: 10 $\longrightarrow$ 2. , 3D 6 $\longrightarrow$ 0

triad/ connection e^i_μ : 9 $\longrightarrow$ 0

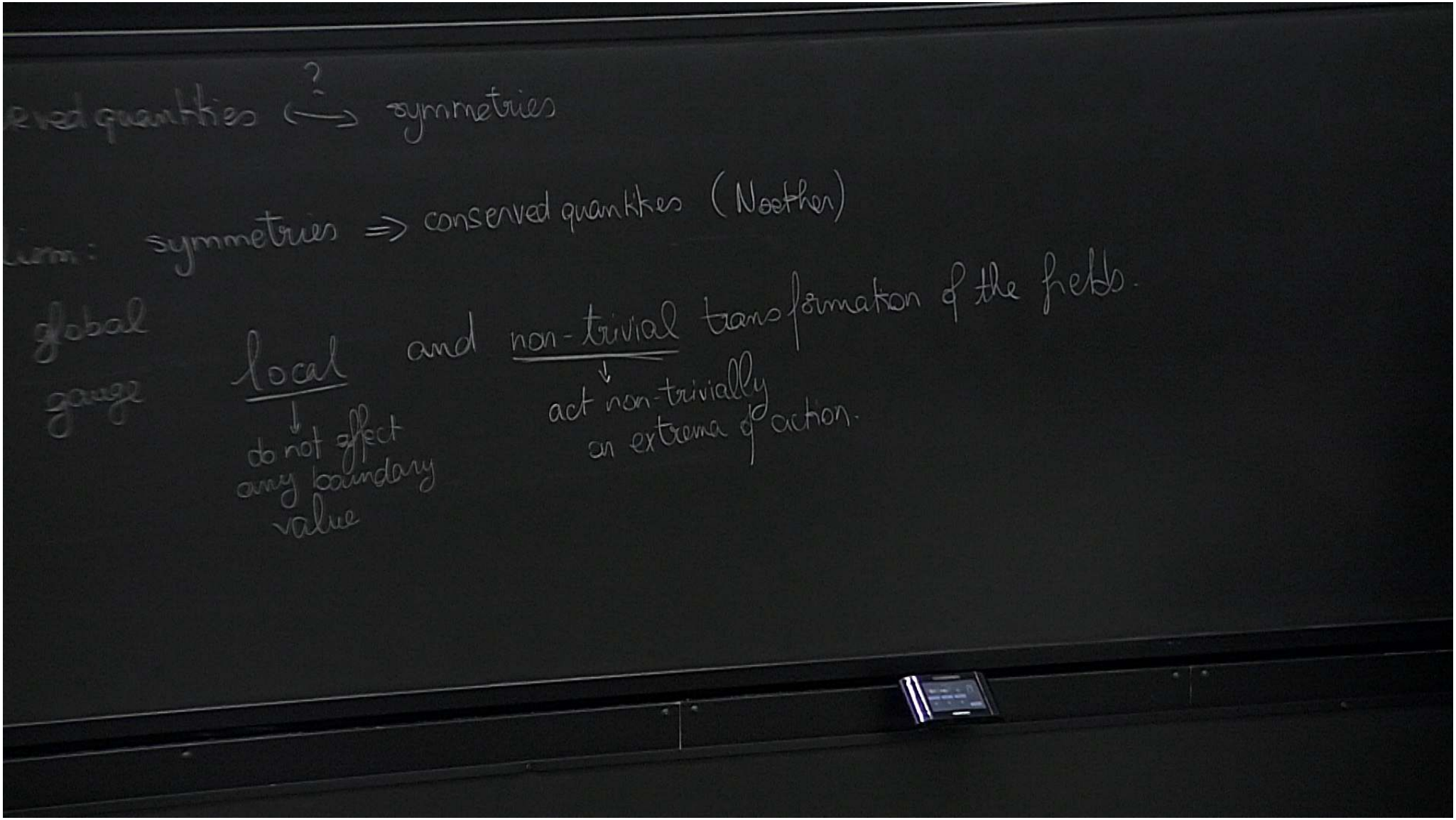

- Why are we interested in "constrained systems"?
- Can we relate constraints / gauge symmetries?
3D
→ 2 , 6 → 0
triad/ connection e^i_μ : 9 → 0
$\phi(q_i, p_i) = 0$


e interested in "constrained systems"?
elate constraints / gauge symmetries?
→ 0
: g → 0
$\phi(q_i, p_i) = 0$
constraints
hold at every instant of
time in the evolution of the system.
= conserved quantity

Q2 conserved quantities $\overset{?}{\longleftrightarrow}$ symmetries

- Lagrangian formalism: symmetries $\Rightarrow$ conserved quantities (Noether)

- Symmetries { global / gauge } local and non-trivial transformation of th



eved quantities $\overset{?}{\longleftrightarrow}$ symmetries

ium: symmetries $\Rightarrow$ conserved quantities (Noether)

global

gauge

local and non-trivial transformation of the fields.

↓ do not affect any boundary value

↓ act non-trivially on extrema of action.

Review of the Hamiltonian formulation

$$S = S[q_i] = \int L[q_i, \dot{q}_i]\, dt$$

$$p_i \equiv \frac{\partial L}{\partial \dot{q}_i}$$

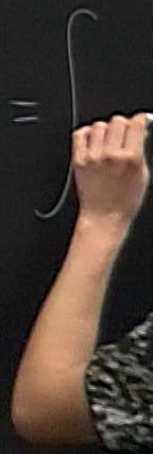
$$H(q_i, p_i) \equiv p_i \dot{q}_i - L$$

$$\dot{f} = \{f, H\}$$

$\{q_i\} \rightarrow$ configuration space

$\{(q_i, p_i)\} \rightarrow$ phase space

$$S = \int$$



Review of the Hamiltonian formulation

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$\{q_i\}$ → configuration space

$\{(q_i, p_i)\}$ → phase space

Time evolution $\dot{f} = \{f, H\}$, $\dot{q}_i, \dot{p}_i$

$S =$

$\{q_i, p_i\} =$

$\delta = \int \left[\sum p_i(q_i, \dot{q}_i)\, \dot{q}_i - H(p_i, q_i) \right] dt$

$\{q_i, p_j\} = \delta_{ij}$

if $\circledast$ not invertible

$\Rightarrow \Psi_\alpha(q_i, p_i) = 0$ primary constraints

↓

primary constraint surface

$H = H_0 + u^\alpha \Psi_\alpha$

$\{f, H\} = \{f, H_0\} + \{f, u^\alpha \Psi_\alpha\}$

$= \{f, H_0\} + \{f, u^\alpha\} \Psi_\alpha + \{f, \Psi_\alpha\} u^\alpha$

$$\delta = \int \left[\sum p_i(q_i, \dot{q}_i)\, \dot{q}_i - H(p_i, q_i) \right] dt$$

$$\{q_i, p_j\} = \delta_{ij}$$

if $\circledast$ not invertible

$\Rightarrow \Psi_\alpha(q_i, p_i) = 0$ primary constraints

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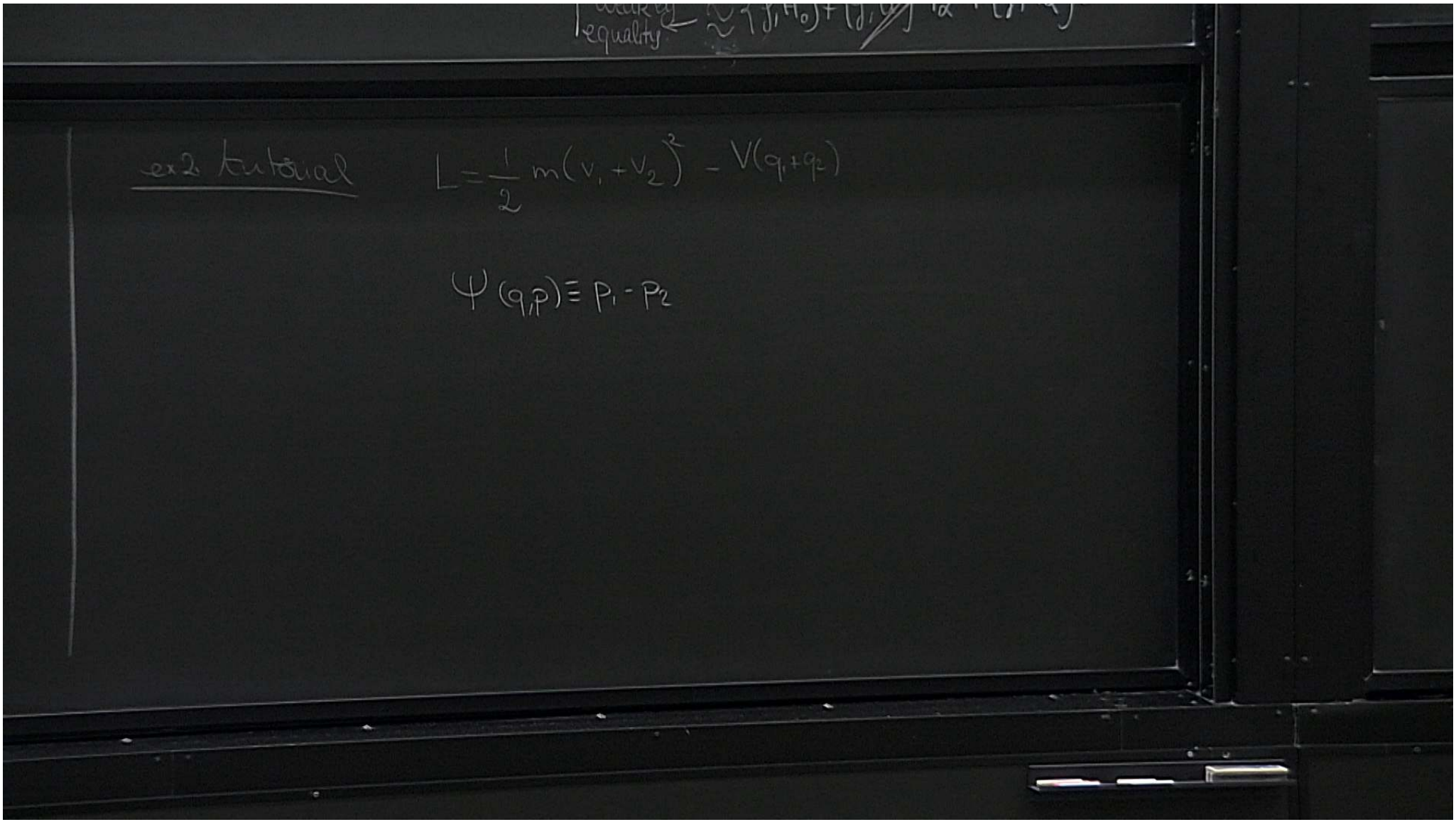
primary constraint surface

$$H = H_0 + u^\alpha \Psi_\alpha$$

$$\{f, H\} = \{f, H_0\} + \{f, u^\alpha \Psi_\alpha\}$$

$$\{f, H\} \underset{\text{weakly equality}}{\approx} \{f, H_0\} + \{f, u^\alpha\}\Psi_\alpha + \{f, \Psi_\alpha\} u^\alpha$$


equality
ex 2. tutorial
$L = \frac{1}{2} m (v_1 + v_2)^2 - V(q_1 + q_2)$
$\Psi(q,p) \equiv p_1 - p_2$

equality

$= 6$

$l : 2$

ex2 tutorial

- $L = \frac{1}{2} m (v_1 + v_2)^2 - V(q_1 + q_2)$ $\quad Q_1 = q_1 + q_2$, $\quad Q_2 = q_1 - q_2$

- $\Psi(q,p) \equiv p_1 - p_2 \rightarrow$ primary constraint

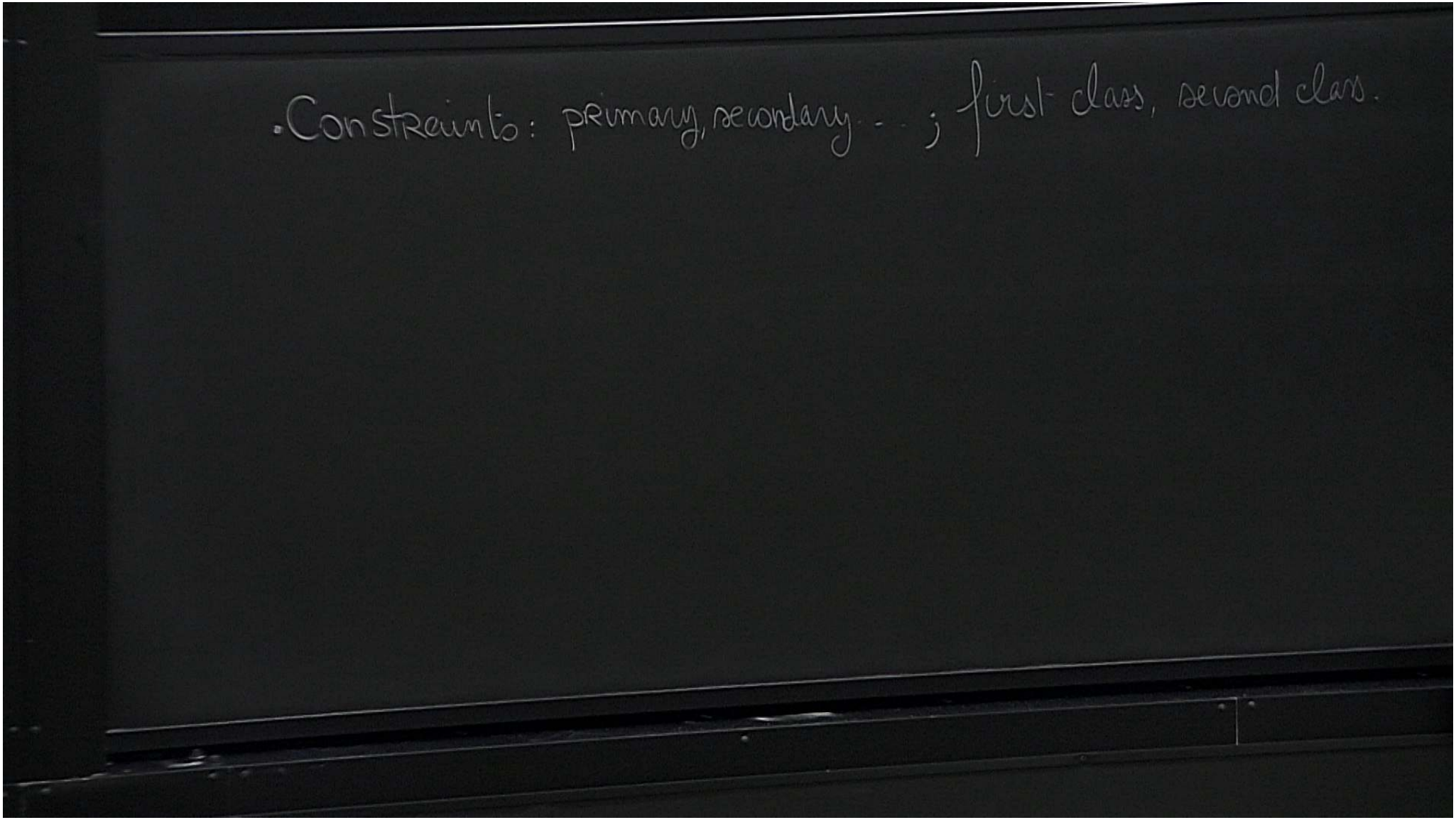
- $\dot{\Psi} = \{\Psi, H\} = 0$

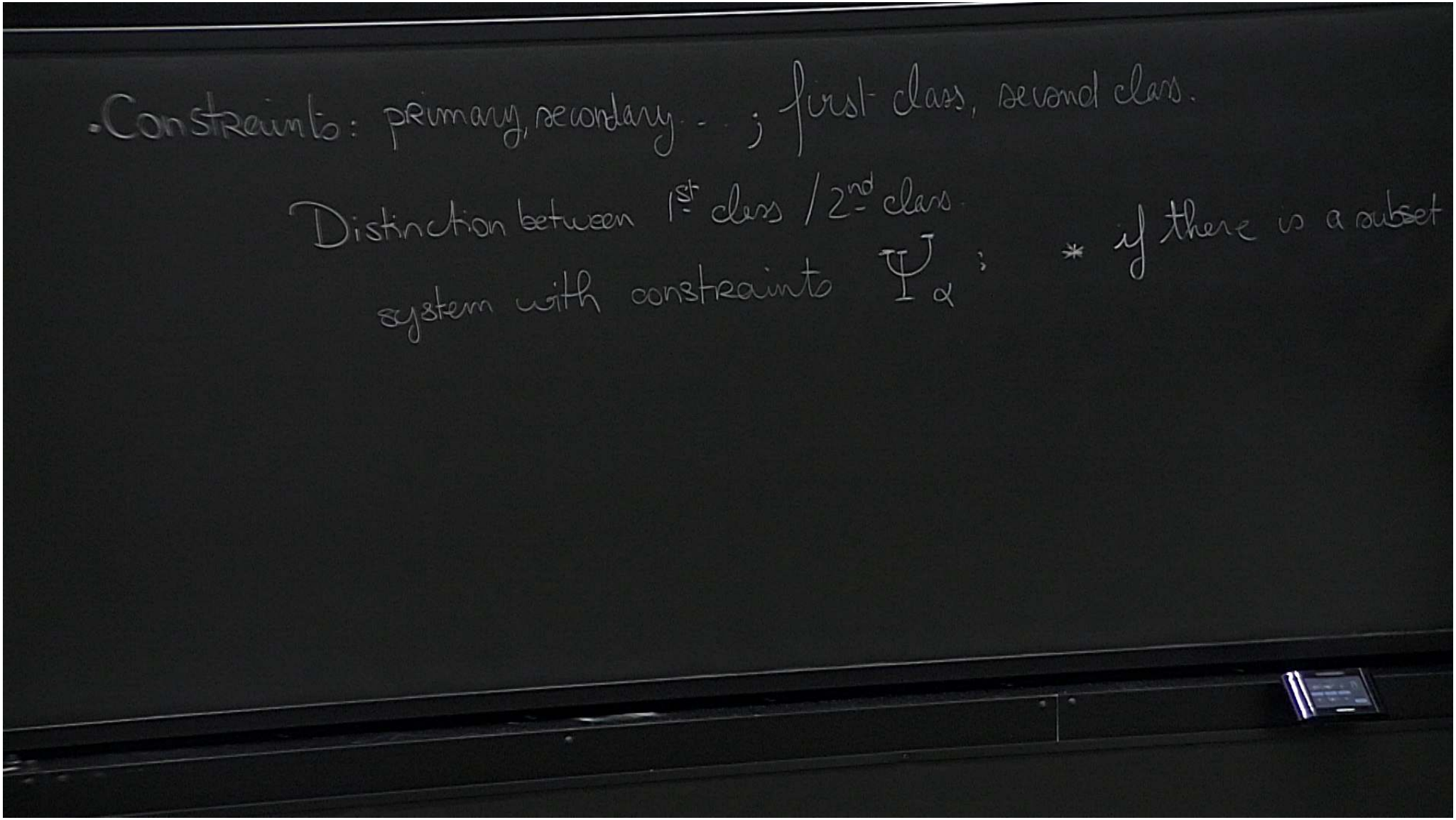
- Symmetry? $\quad \delta q_1 = u\{q_1, \Psi\} = u$

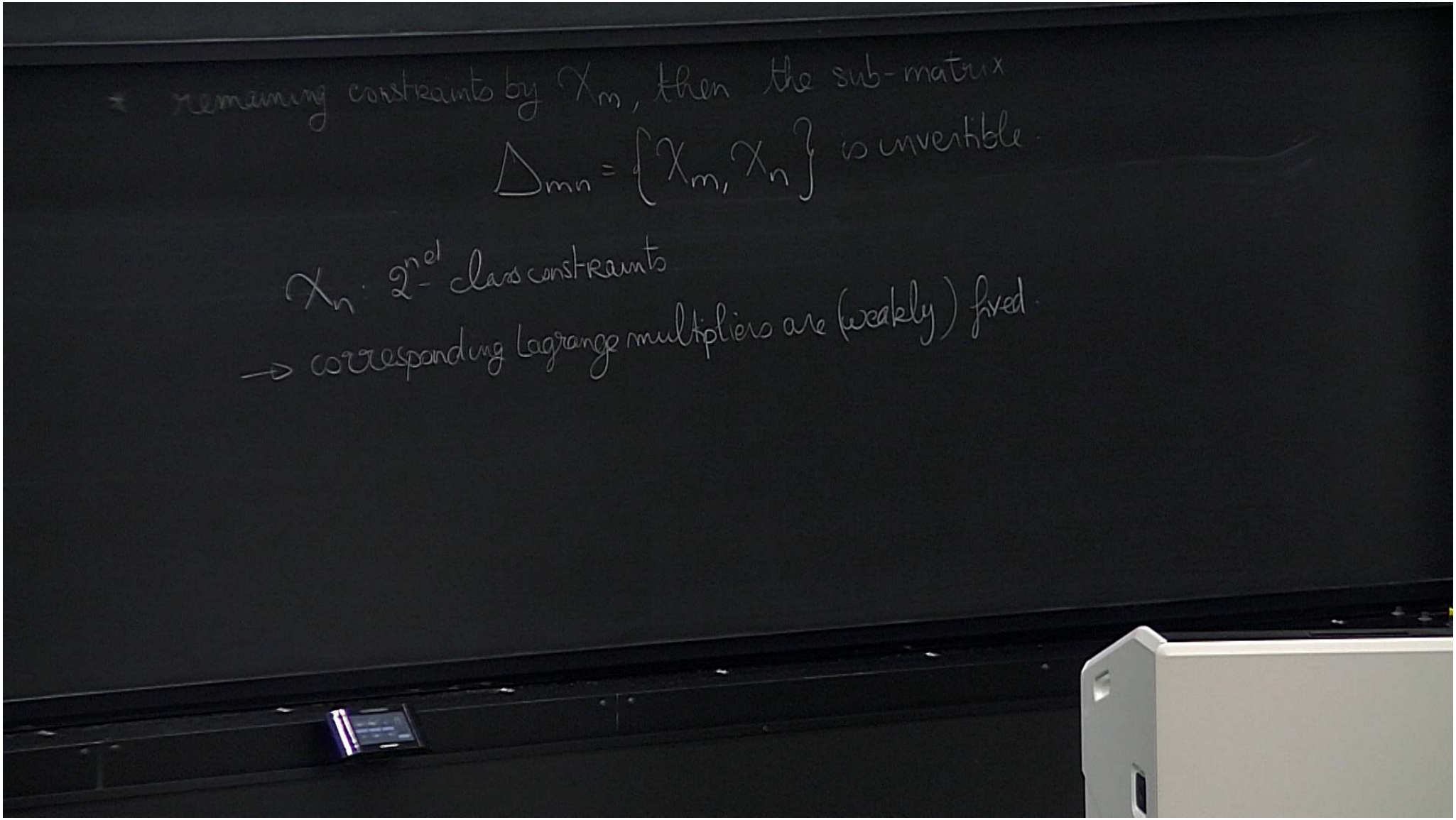
$\delta q_2 = u\{q_2, \Psi\} = -u$

$Q_1 =$ physical

$Q_2 =$ "internal" degree of freedom

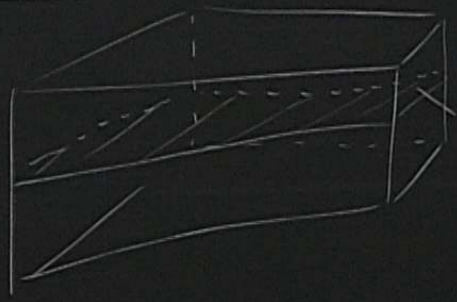

• Constraints: primary, secondary - - ; first-class, second class.


Constraints: primary, secondary - ; first-class, second class.
Distinction between 1st class / 2nd class
system with constraints Ψ_α ; * if there is a subset


remaining constraints by χ_m, then the sub-matrix
$\Delta_{mn} = \{\chi_m, \chi_n\}$ is invertible.
χ_n: 2nd class constraints
→ corresponding Lagrange multipliers are (weakly) fixed.

$\{\mathcal{C}, H\} = 0$

Constraints and physical degrees of freedom



constrained surface

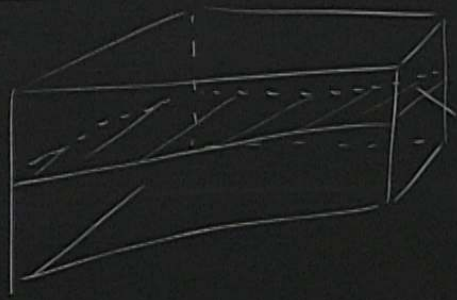
kinematical phase space

↓

physical or reduced phase space.

$\{\mathcal{C}, H\} = 0$

Constraints and physical degrees of freedom



constrained surface

kinematical phase space

↓

physical or reduced phase space.

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Constraints and physical degrees of freedom

kinematical phase space

↓

physical or reduced phase space

dim of kinematical phase space $-\#(1^{st}\ class) \times 2 = \dim$ physical