Title: PSI 2018/2019 - Beyond Standard Model - Lecture 4

Speakers: Latham Boyle

Collection: PSI 2018/2019 - Beyond Standard Model (Boyle)

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$$\mathcal{L}_{SM} = -\frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \frac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$-(D_\mu h)^\dagger (D^\mu h) + m^2(h^\dagger h) - \frac{\lambda}{4}(h^\dagger h)^2$$

$$+ i[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{l}_L \not{D} l_L + \bar{e}_R \not{D} e_R]$$

$$- [\bar{q}_L Y_d h d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{l}_L Y_e h e_R + \text{h.c.}]$$

$$\mathcal{L}_{SM} = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

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$$-\left[\bar{q}_L Y_d h d_R + \bar{q}_L Y_u \tilde{h} u_R + \bar{l}_L Y_e h e_R + \text{h.c.}\right]$$

$$G_\mu = G_\mu^a T^a$$

$$G_{\mu\nu} = G_{\mu\nu}^a T^a \qquad \mathrm{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$$

$$-\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$\tilde{h} = i\sigma^2 h^*$$

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$$-\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\tilde{h} = i\sigma^2 h^*$$

$$Y = V^\dagger \, y \, U \qquad y = \begin{pmatrix} y_1 & & \\ & y_2 & \\ & & y_3 \end{pmatrix}$$

$$Y_d = V_d^\dagger \, y_d \, U_d \rightarrow d_R' = U_d \, d_R$$

$$Y_u = V_u^\dagger \, y_u \, U_u \rightarrow u_R' = U_u \, u_R, \; q_L' = V_u \, q_L$$

$$Y_e = V_e^\dagger \, y_e \, U_e \rightarrow e_R' = U_e \, e_R, \; l_L' = V_e \, l_L$$

$+h.c.]$ $\quad \tilde{h} = i\sigma^2 h^*$

$$-[\bar{q}_L V_{CKM} y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{l}_L y_e h e_R] \qquad V_{CKM} = V_u V_d^\dagger$$

$\bar{q}_L^{i'} = e^{i\alpha_i} q_L^i \qquad u_R^{i'} = e^{i\alpha_i} u_R^i \qquad d_R^{i'} = e^{i\beta_i} d_R^i \qquad 6-1=5 \qquad V_{CKM}: 9-5=4$

3 angles, 1 complex.

$+h.c.]$ $\quad \tilde{h} = i\sigma^2 h^*$

$$-[\bar{q}_L V_{CKM} y_d h d_R + \bar{q}_L y_u \tilde{h} u_R + \bar{l}_L y_e h e_R] \qquad V_{CKM} = V_u V_d^\dagger$$

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3 angles, 1 complex.

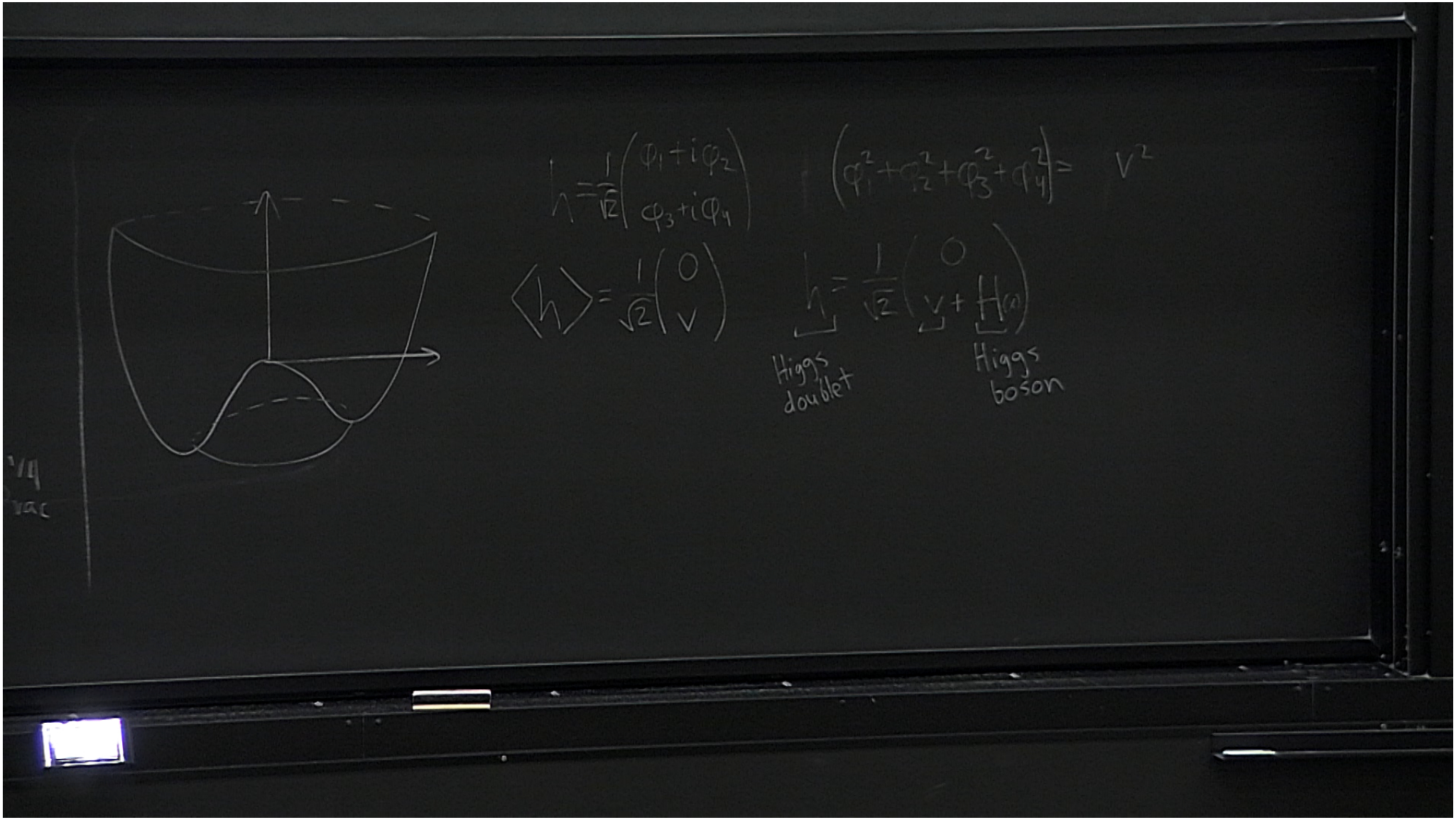
$$\underbrace{g_3, g_2, g_1}_{3}, \underbrace{m, \lambda}_{2}, \underbrace{V_{CKM}}_{4}, \underbrace{y_d, y_u, y_e}_{3\times 3} = 18 \text{ parameters } (19 \text{ including } \theta_{QCD})$$

$m \approx 10^2\,\mathrm{GeV} \sim \mathrm{TeV}$

$$S = \int d^4x \sqrt{-g}\left[\mathcal{L}_{SM} + M_{Pl}^2 R - \rho_{vac}\right] \to M_{Pl}^2 \Lambda$$

$M_{Pl} \sim 10^{18}\,\mathrm{GeV}$, $\rho_{vac} \sim 10^{-12}\,\mathrm{GeV}$ $\qquad M_{Pl} \gg m \gg \rho_{vac}^{1/4}$

$$\sqrt{M_{Pl}\,\rho_{vac}^{1/4}} \sim \mathrm{TeV}$$


$h = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4\end{pmatrix}$
$\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2 = v^2$
$\langle h \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}$
$h = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v + H(x)\end{pmatrix}$
Higgs doublet
Higgs boson

$$D_\mu h = \left(\partial_\mu + i g_2 \underbrace{\left(\frac{\sigma^a}{2}\right)}_{T^a} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu\right) h$$

$$\mathrm{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$$

$$D_\mu h = \left(\partial_\mu + i g_2 \underbrace{\left(\frac{\sigma^a}{2}\right)}_{T^a} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu\right) h$$

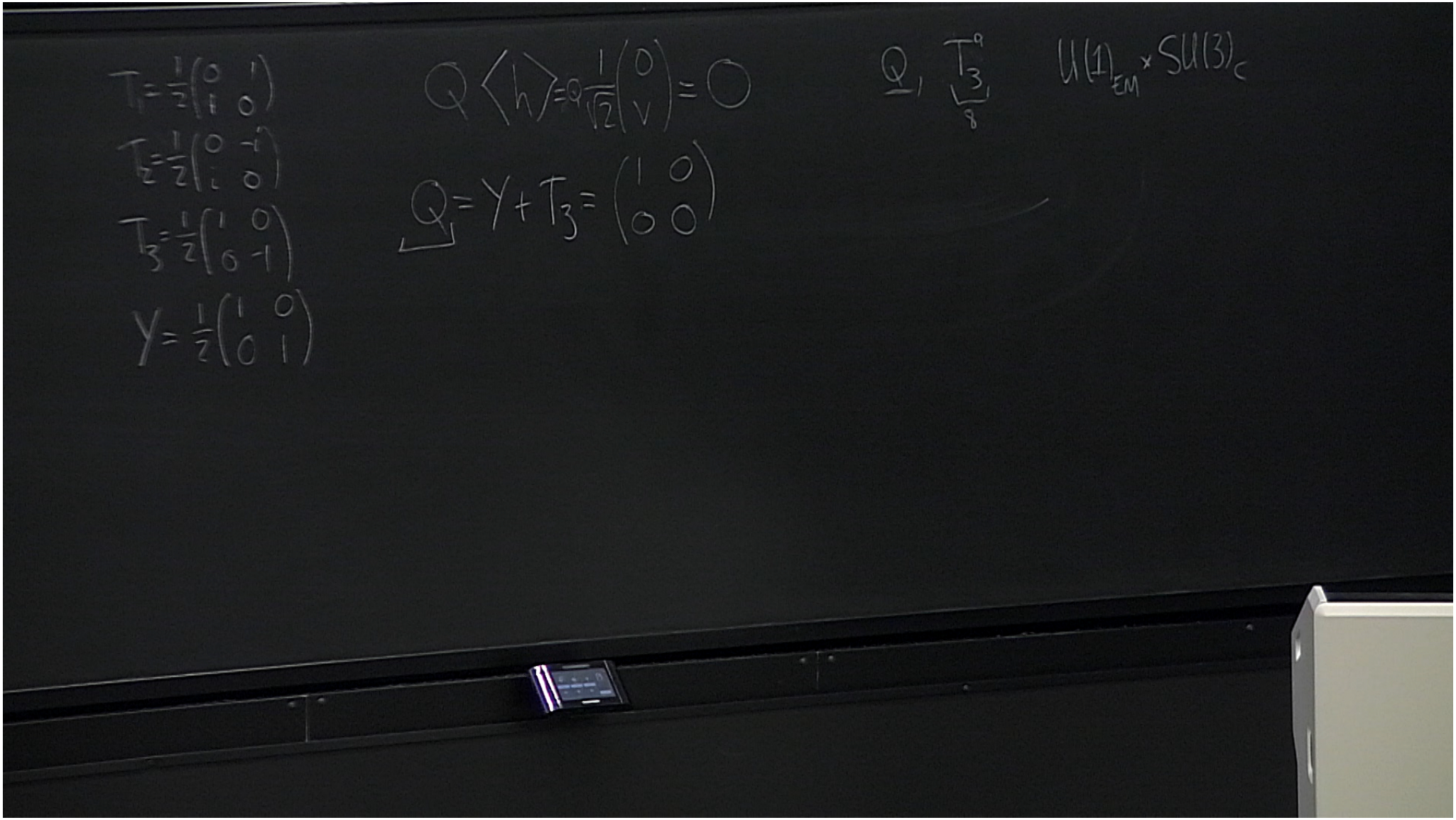
$$\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta^{ab}$$

$$T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$


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$Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$Q\langle h \rangle = 0 \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$
$Q = Y + T_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
Q, T_3^a
T_3
8
$U(1)_{EM} \times SU(3)_C$

$$D_\mu h = \left(\partial_\mu + i g_2 \underbrace{\left(\frac{\sigma^a}{2}\right)}_{T^a} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu\right) h$$

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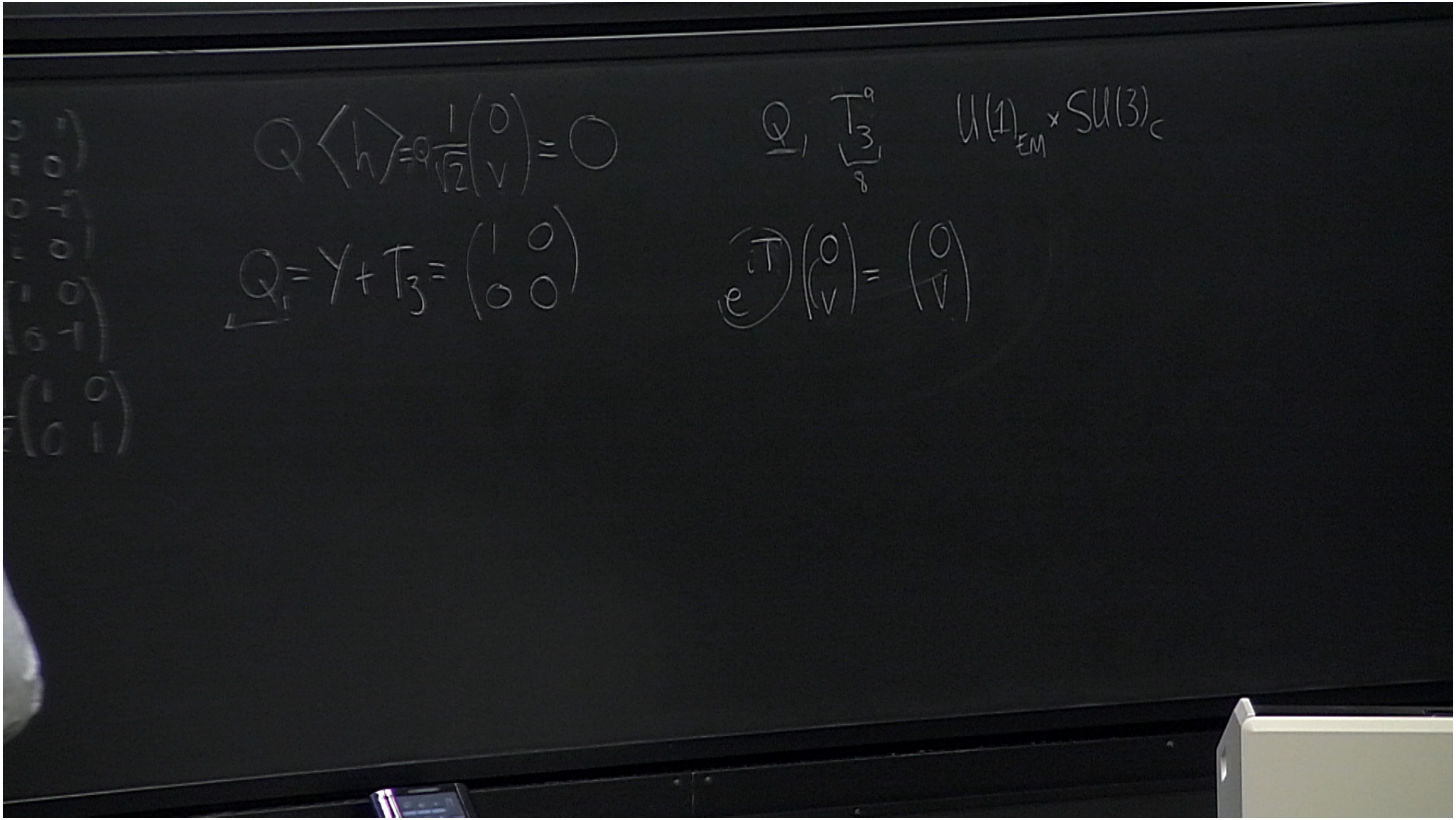
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h: 4 real components.


$Q\langle h\rangle = 0 \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} = 0$
$Q, \; T_3^a \; (a=1,\dots,8) \quad U(1)_{EM} \times SU(3)_C$
$Q = Y + T_3 = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$
$e^{iT}\begin{pmatrix}0\\ v\end{pmatrix} = \begin{pmatrix}0\\ v\end{pmatrix}$

$$D_\mu h = \left(\partial_\mu + i g_2 \underbrace{\left(\frac{\sigma^a}{2}\right)}_{T^a} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu\right) h$$

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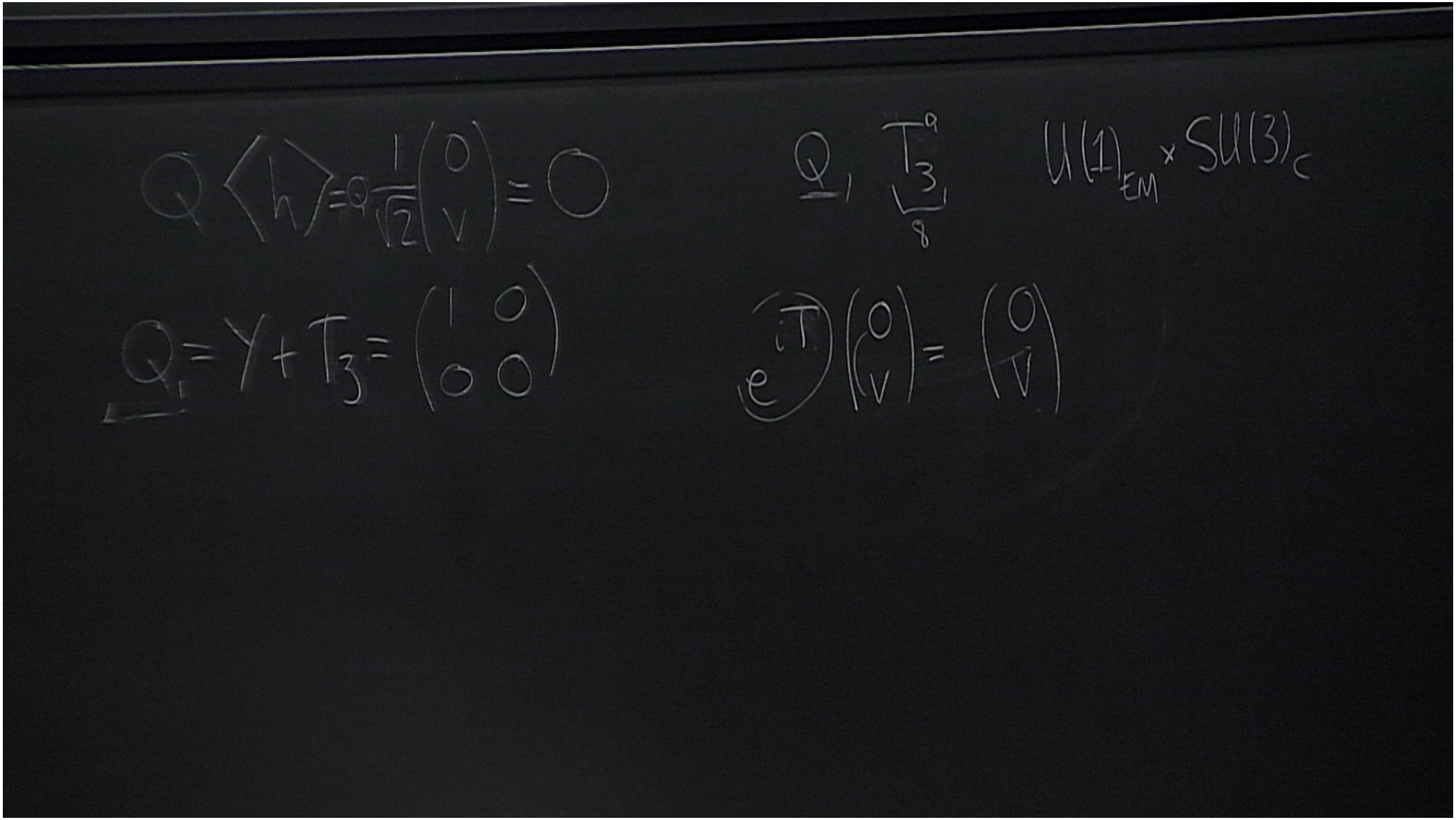
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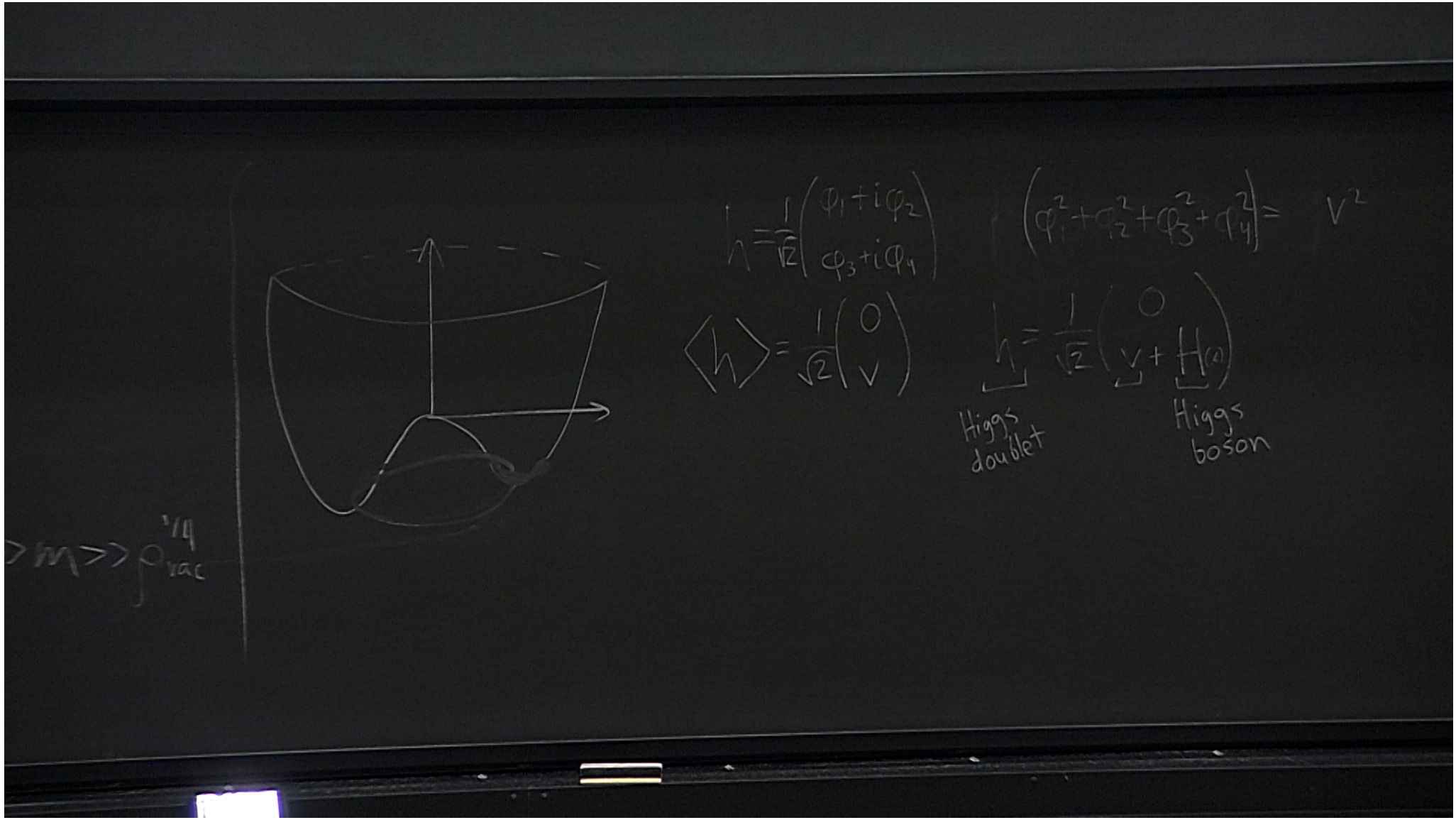
3 goldstones

1 massive

4 generators

3 broken


$Q\langle h\rangle = 0 \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} = 0$
Q, T_3^8
$U(1)_{EM} \times SU(3)_C$
$Q = Y + T_3 = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$
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$h = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1+i\varphi_2\\ \varphi_3+i\varphi_4\end{pmatrix}$
$\left(\varphi_1^2+\varphi_2^2+\varphi_3^2+\varphi_4^2\right) = v^2$
$\langle h\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$
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Higgs doublet
Higgs boson
$m \gg \rho_{vac}^{1/4}$

$$D_\mu h = \left(\partial_\mu + i g_2 \underbrace{\left(\frac{\sigma^a}{2}\right)}_{T^a} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu\right) h$$

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h: 4 real components.

4 generators

3 goldstones ⟵ 3 broken

1 massive

Higgs mechanism:

$$D_\mu h = \left(\partial_\mu + i g_2 \underbrace{\left(\frac{\sigma^a}{2}\right)}_{T^a} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu\right) h$$

$$\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta^{ab}$$

$$T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$Q\langle$

$Q_i = Y$

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h: 4 real components. — 4 generators

3 goldstones — 3 broken

1 massive

Higgs mechanism: 3 massive vector bosons: W^+, W^-, Z

1 massless vector: A_μ

1 massless vector: A_μ

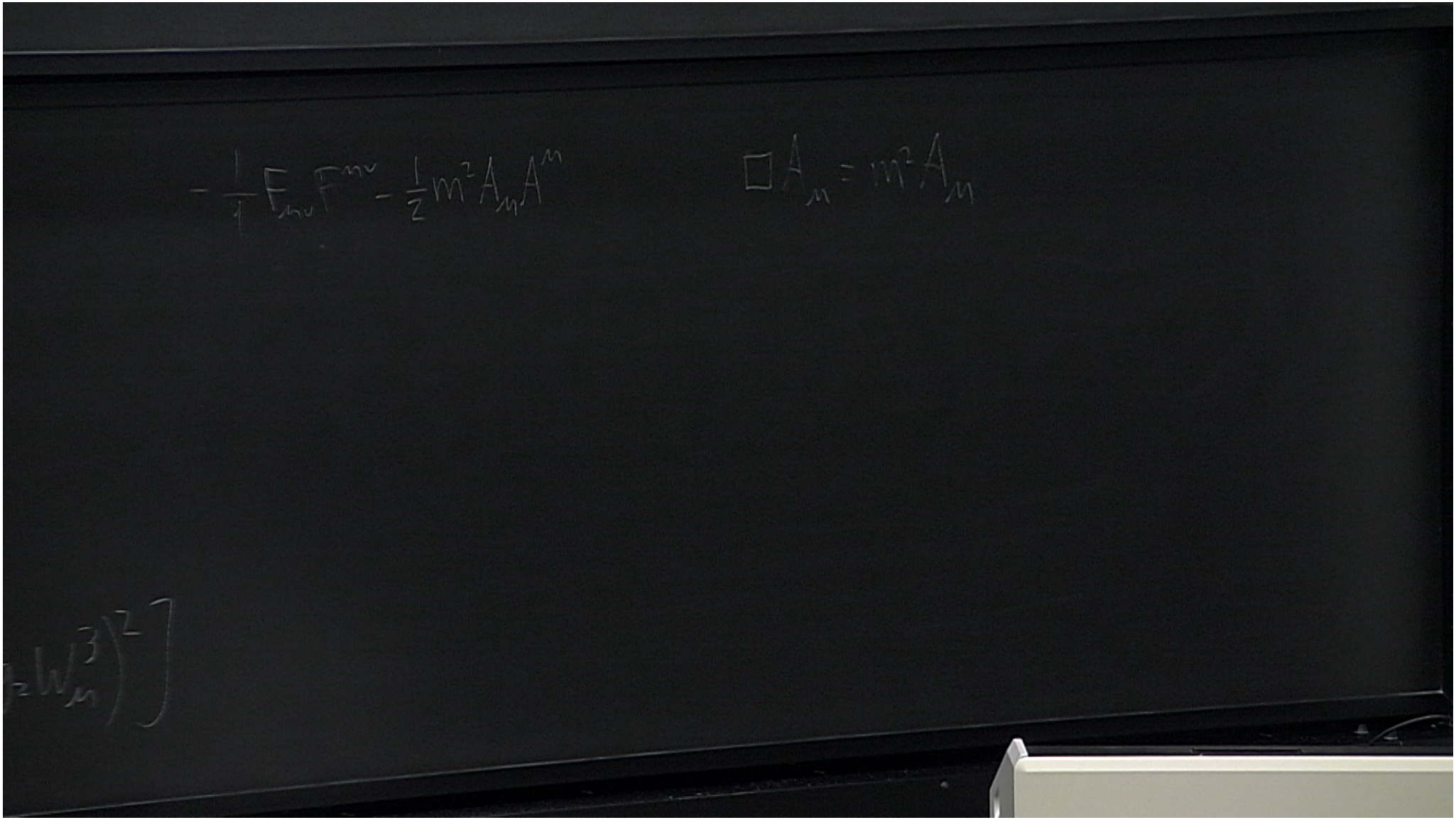
$-(D_\mu H)^\dagger (D^\mu H)$

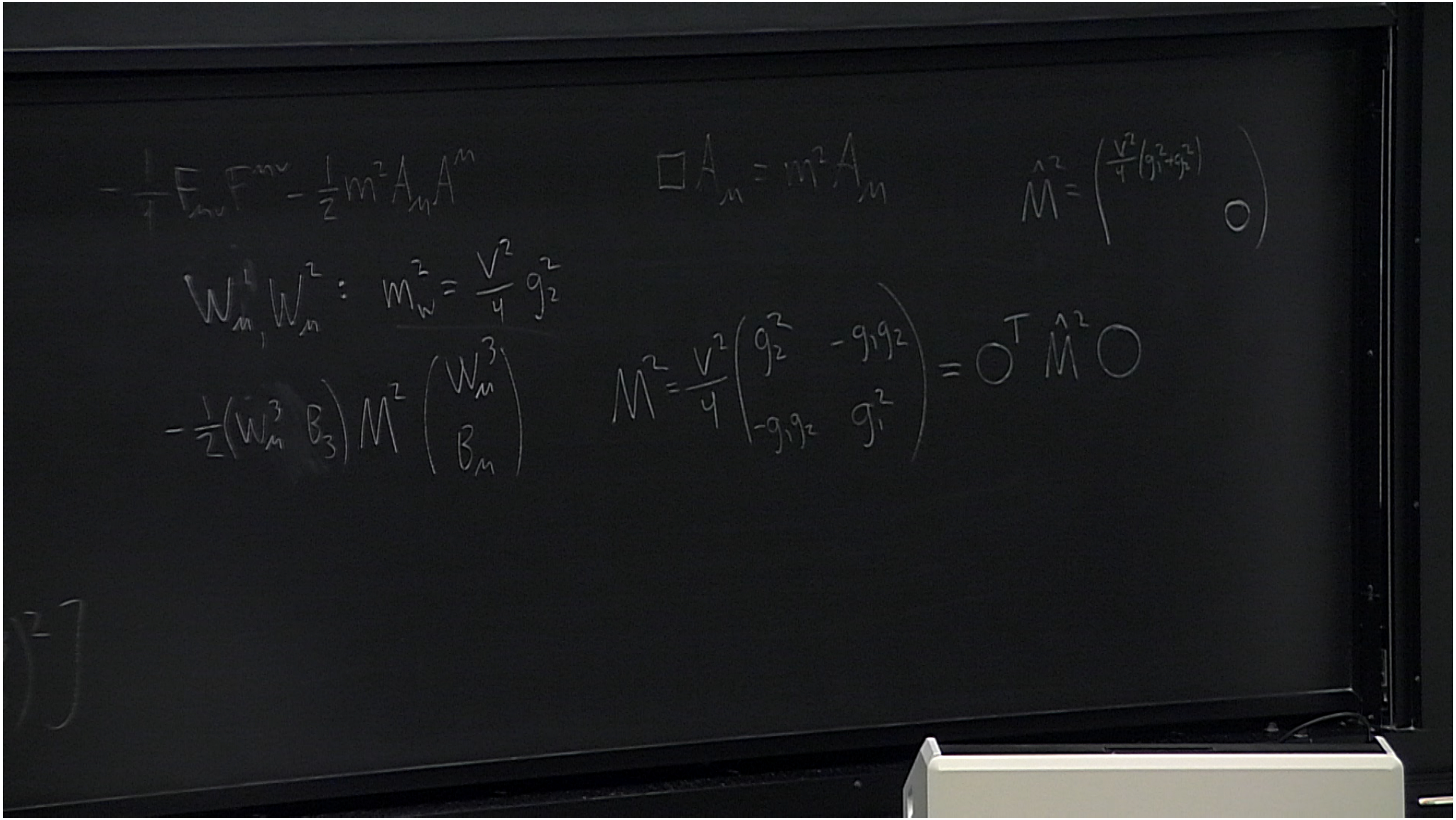
$$D_\mu H = \left(i g_2 \frac{\sigma^a}{2} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

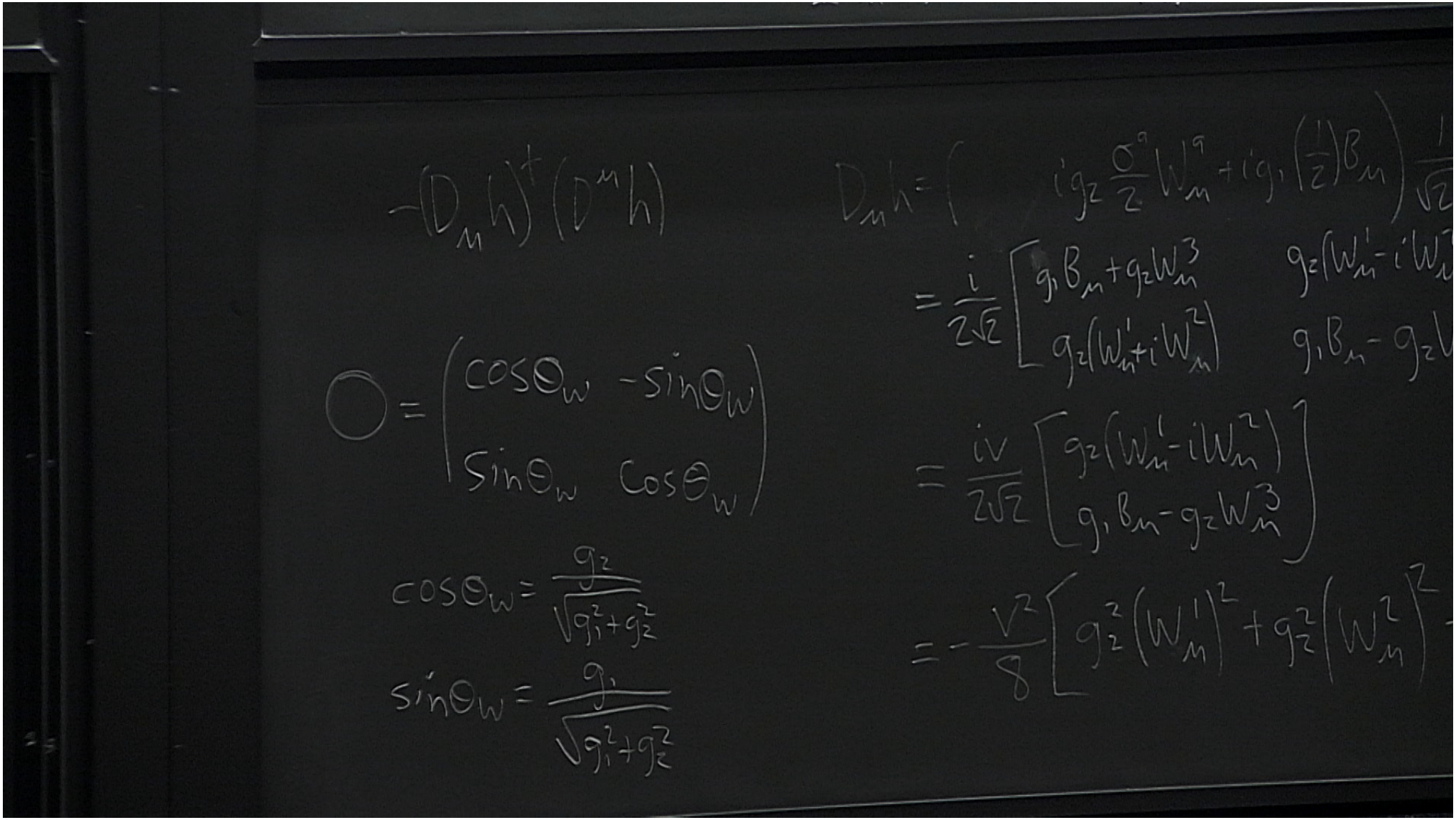
$$= \frac{i}{2\sqrt{2}} \begin{bmatrix} g_1 B_\mu + g_2 W_\mu^3 & g_2 (W_\mu^1 - i W_\mu^2) \\ g_2 (W_\mu^1 + i W_\mu^2) & g_1 B_\mu - g_2 W_\mu^3 \end{bmatrix} \begin{bmatrix} 0 \\ v \end{bmatrix}$$

$$= \frac{iv}{2\sqrt{2}} \begin{bmatrix} g_2 (W_\mu^1 - i W_\mu^2) \\ g_1 B_\mu - g_2 W_\mu^3 \end{bmatrix}$$

$$= -\frac{v^2}{8} \left[g_2^2 (W_\mu^1)^2 + g_2^2 (W_\mu^2)^2 + (g_1 B_\mu - g_2 W_\mu^3)^2 \right]$$


$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$
$\Box A_\mu = m^2 A_\mu$

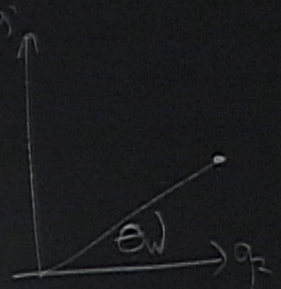

$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$
$\Box A_\mu = m^2 A_\mu$
$\hat{M}^2 = \begin{pmatrix} \frac{v^2}{4}(g_1^2+g_2^2) & \\ & 0 \end{pmatrix}$
$W_\mu^1, W_\mu^2 : m_W^2 = \frac{v^2}{4} g_2^2$
$-\frac{1}{2}(W_\mu^3 \; B_3) M^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$
$M^2 = \frac{v^2}{4}\begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} = O^T \hat{M}^2 O$


$-(D_\mu h)^\dagger (D^\mu h)$
$D_\mu h = \begin{pmatrix} & i g_2 \frac{\sigma^a}{2} W_\mu^a + i g_1 \left(\frac{1}{2}\right) B_\mu \end{pmatrix} \frac{1}{\sqrt{2}}$
$= \frac{i}{2\sqrt{2}} \begin{bmatrix} g_1 B_\mu + g_2 W_\mu^3 & g_2 (W_\mu^1 - i W_\mu \\ g_2 (W_\mu^1 + i W_\mu^2) & g_1 B_\mu - g_2 \end{bmatrix}$
$= \frac{iv}{2\sqrt{2}} \begin{bmatrix} g_2 (W_\mu^1 - i W_\mu^2) \\ g_1 B_\mu - g_2 W_\mu^3 \end{bmatrix}$
$= -\frac{v^2}{8} \left[g_2^2 (W_\mu^1)^2 + g_2^2 (W_\mu^2)^2 \right.$
$\bigcirc = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix}$
$\cos\theta_w = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$
$\sin\theta_w = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu \qquad \Box A_\mu = m^2 A_\mu \qquad \hat{M}^2 = \begin{pmatrix} \frac{v^2}{4}(g_1^2+g_2^2) & \\ & 0 \end{pmatrix}$$

$$W_\mu^1, W_\mu^2 : \; m_W^2 = \frac{v^2}{4} g_2^2$$

$$-\frac{1}{2}\begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} M^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \qquad M^2 = \frac{v^2}{4}\begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} = O^T \hat{M}^2 O$$



$$= \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Q\langle h \rangle = 0 \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

$$Q_1 = Y + T_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$e^{iT}\begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = 0 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$Q,\ T_3^a$ (8) $U(1)_{EM} \times SU(3)_C$

$$-\frac{1}{4} Z_{\mu\nu}^2 - \frac{1}{4} F_{\mu\nu}^2 - m_Z^2 Z_\mu^2$$

$$m_Z^2 = \frac{v^2}{4}\left(g_1^2 + g_2^2\right)$$