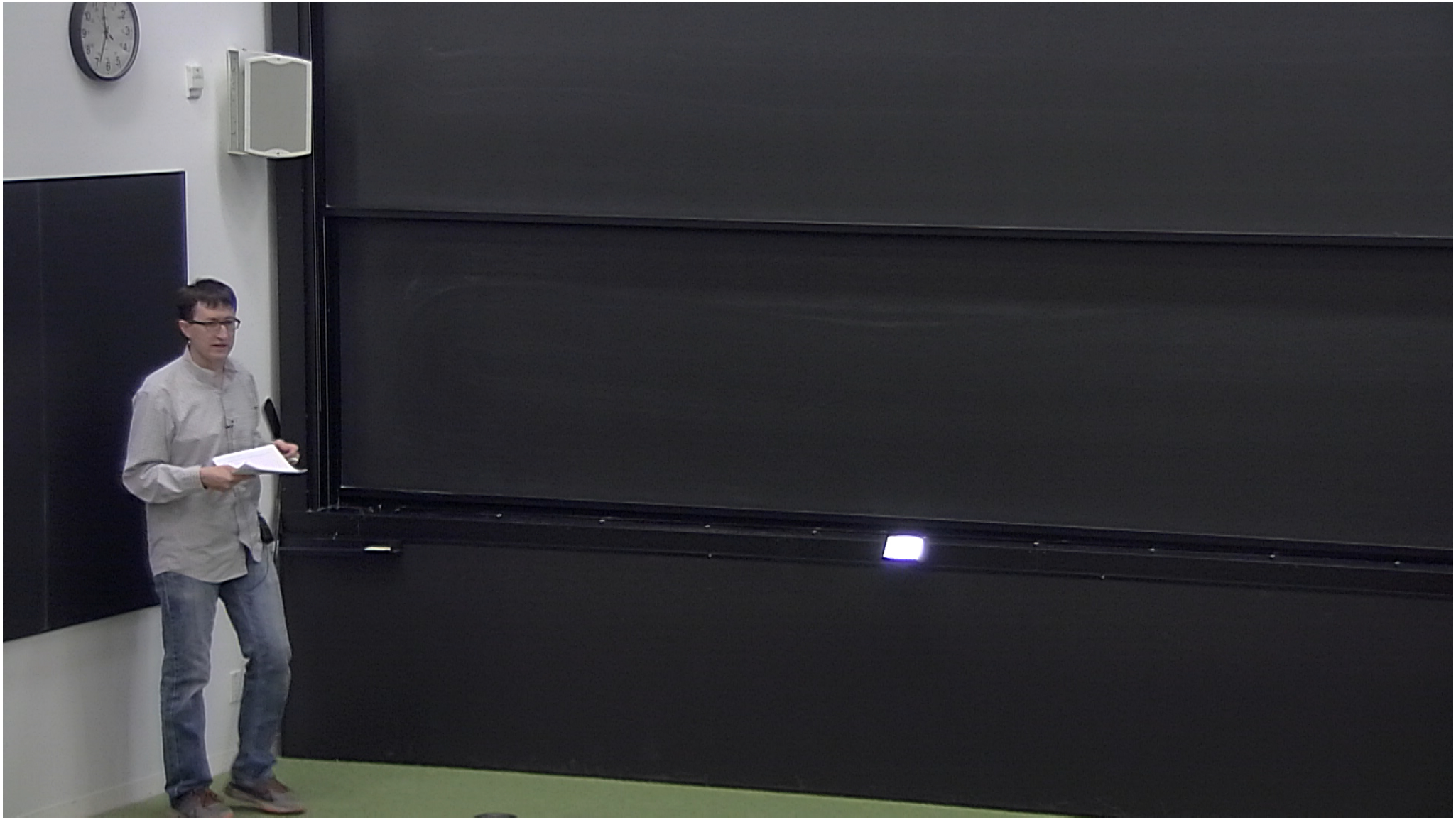
Title: PSI 2018/2019 - Beyond Standard Model - Lecture 2

Speakers: Latham Boyle

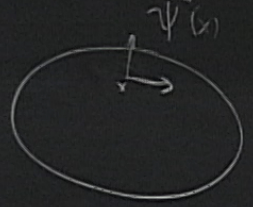
Collection: PSI 2018/2019 - Beyond Standard Model (Boyle)

Date: March 26, 2019 - 11:30 AM

URL: http://pirsa.org/19030064



GR:



$V^\mu(x)$ $g_{\mu\nu}$ ∇_μ $\Gamma^\gamma_{\alpha\beta}$, R

$\psi^a(x) \to U^a{}_b(x)\psi^b(x)$

A_μ, $F_{\mu\nu}$

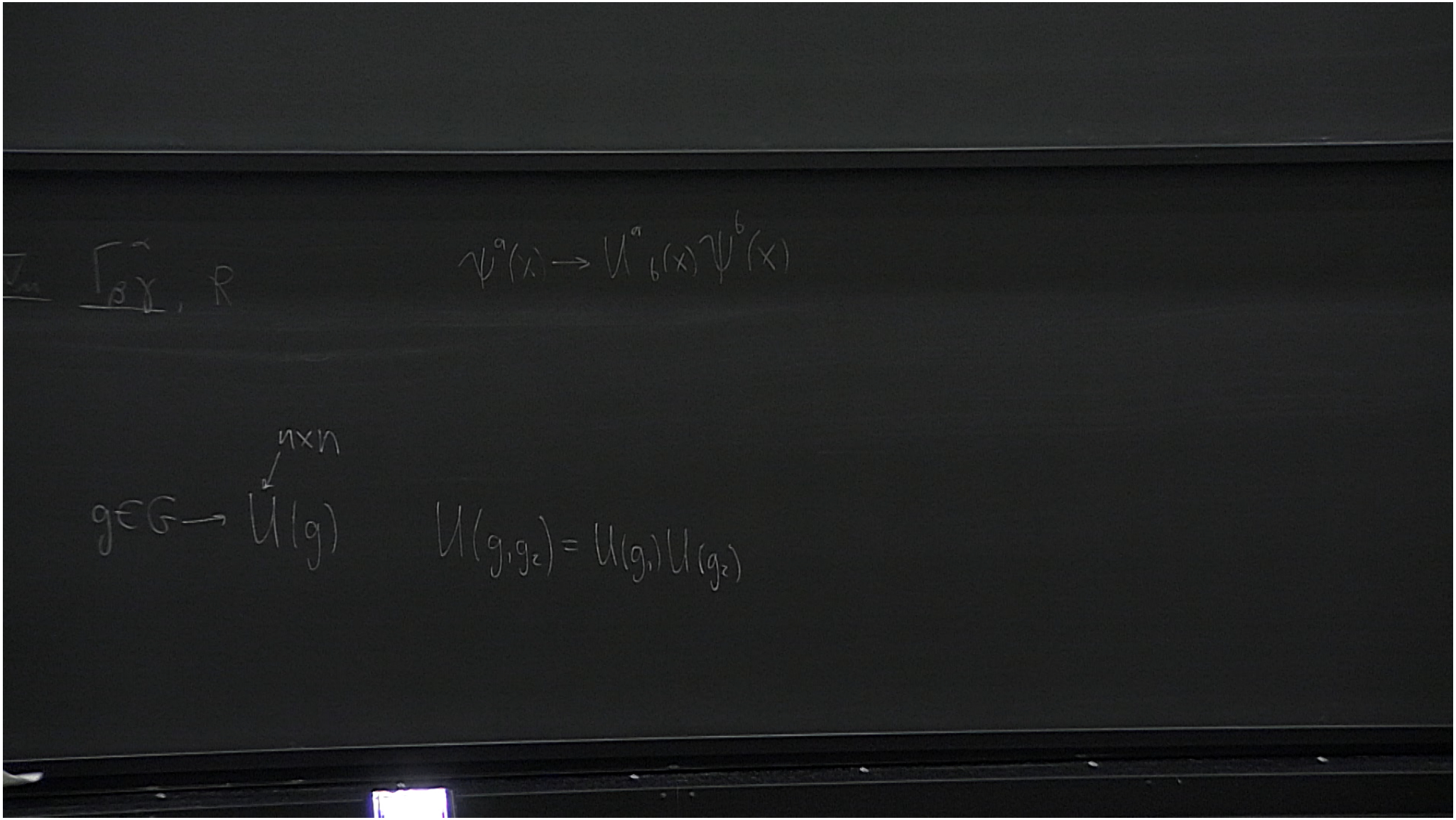
YM: · Pick G e.g. $G = SU(N)$

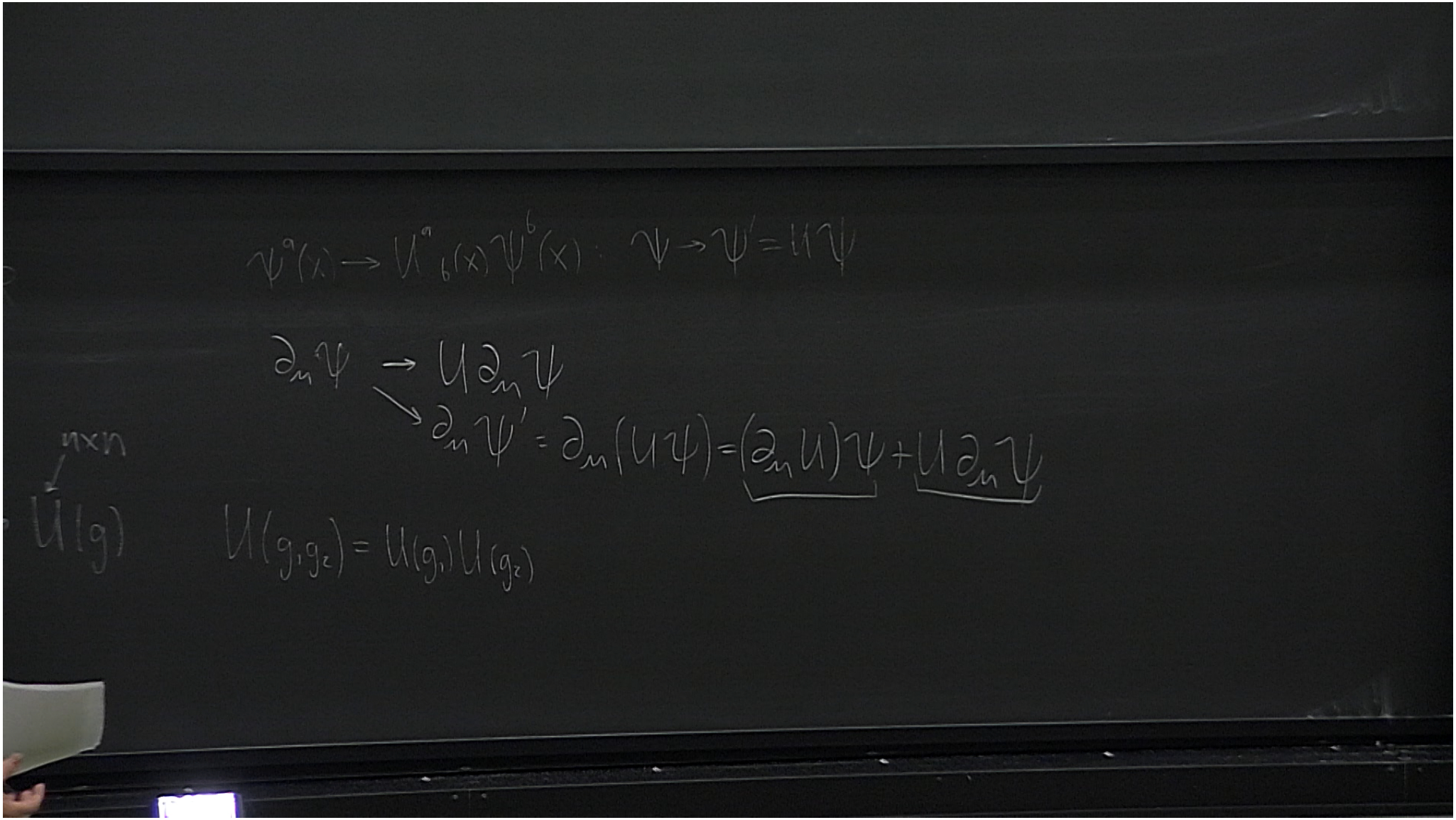
· Pick a representation of G.

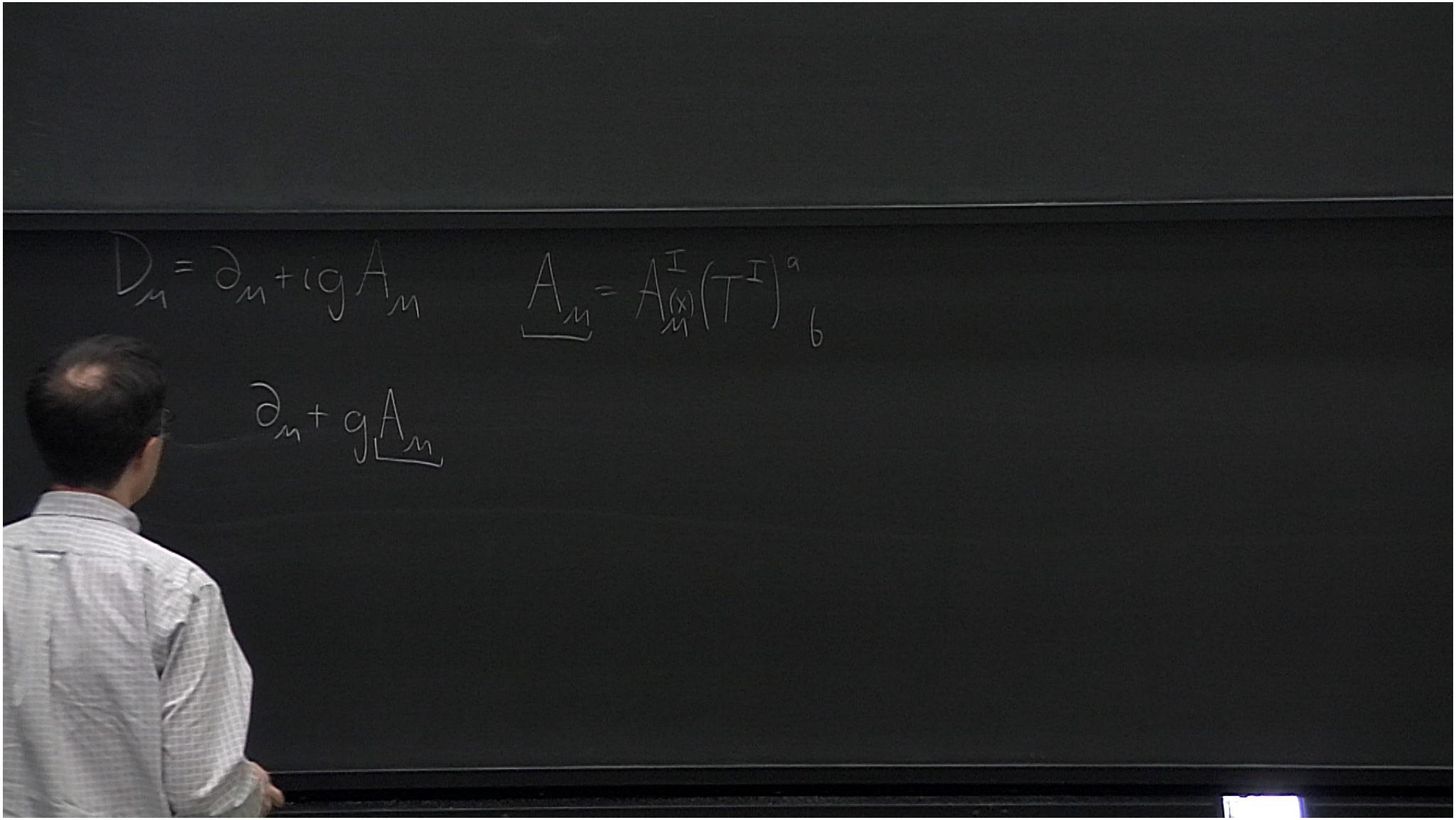
↑ n-dim

$n \times n$

$g \in G \to U(g)$ $\quad U(g_1 g_2) = U(g_1)U(g_2)$


$\psi^a(x) \to U^a{}_b(x)\psi^b(x)$
$n \times n$
$g \in G \to U(g)$
$U(g_1 g_2) = U(g_1)U(g_2)$


$\psi^a(x) \to U^a{}_b(x)\psi^b(x)$: $\psi \to \psi' = U\psi$
$\partial_m \psi \to U\partial_m \psi$
$\partial_m \psi' = \partial_m(U\psi) = (\partial_m U)\psi + U\partial_m \psi$
n×n
$U(g)$
$U(g_1 g_2) = U(g_1)U(g_2)$


$D_\mu = \partial_\mu + igA_\mu$
$\underline{A_\mu} = A^I_\mu(x)(T^I)^a_b$
$\partial_\mu + g\underline{A_\mu}$

$$D_\mu = \partial_\mu + igA_\mu \qquad A_\mu = A^I_\mu (T^I)^a{}_b \qquad A'_\mu = UA_\mu U^{-1} + \frac{i}{g}$$

$$D_\mu \psi \to D'_\mu \psi' = U(D_\mu \psi)$$

$$D_\mu \to D'_\mu = U D_\mu U^{-1} \Rightarrow$$

$$\partial_\mu + igA'_\mu = U(\partial_\mu + igA_\mu)U^{-1} = U(\partial_\mu U^{-1}) + \partial_\mu + igUA_\mu U^{-1}$$

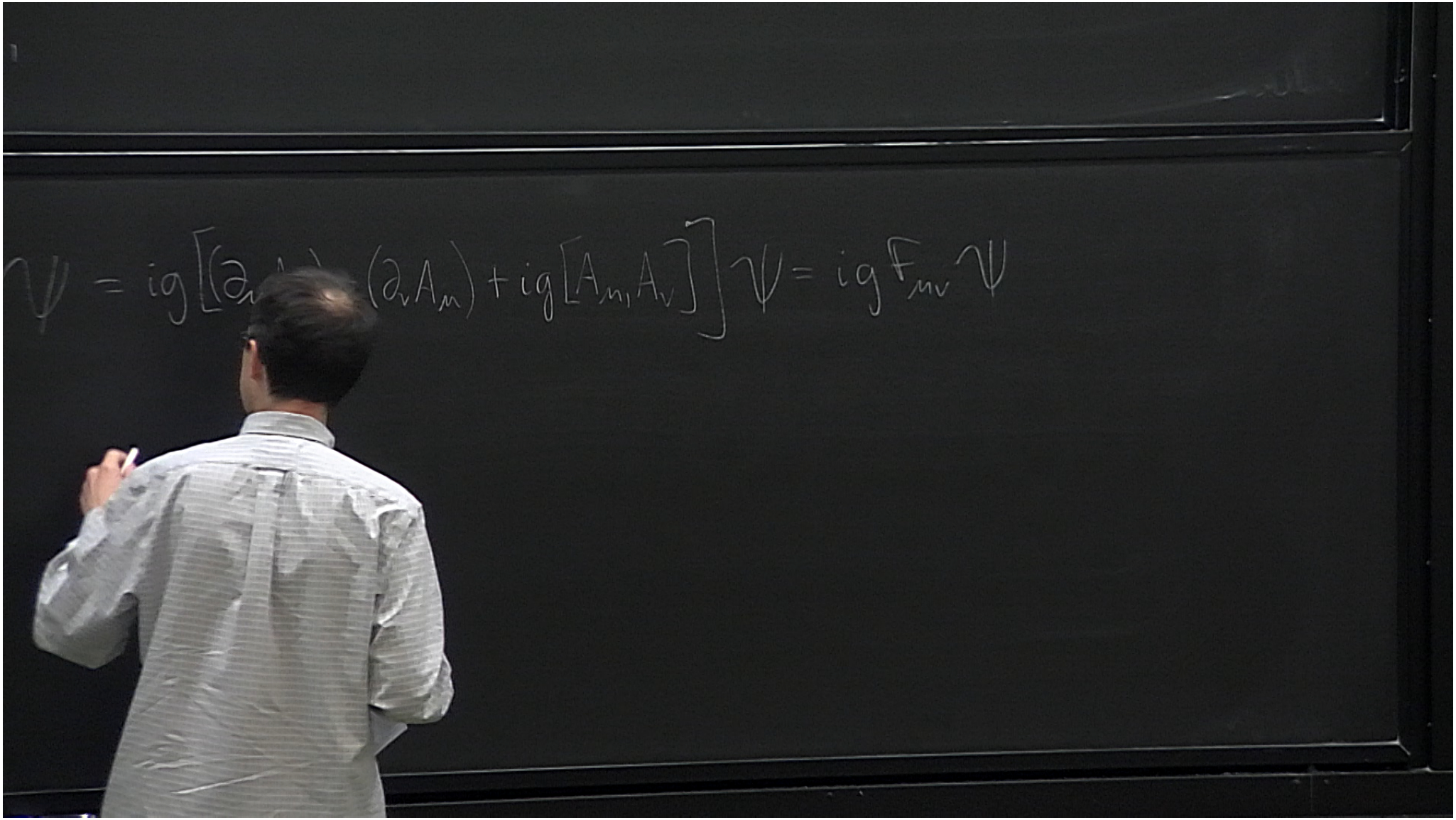
$$A'_\mu = U A_\mu U^{-1} - \frac{i}{g} U(\partial_\mu U^{-1}) = U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U) U^{-1}$$

$\partial_\mu + igUA_\mu U^{-1}$

$$D_\mu \to D'_\mu = U D_\mu U^{-1} \Rightarrow$$

$$\partial_\mu + igA'_\mu = U(\partial_\mu + igA_\mu)U^{-1} = U(\partial_\mu U^{-1}) + \partial_\mu + igUA_\mu U^{-1}$$

$$[D_\mu, D_\nu]\psi = \left[(\partial_\mu + igA_\mu)(\partial_\nu + igA_\nu) - (\partial_\nu + igA_\nu)(\partial_\mu + igA_\mu)\right]\psi = ig\left[(\partial_\mu A_\nu) - (\partial_\nu A_\mu) + ig[A_\mu, A_\nu]\right]\psi$$


$\psi = ig[(\partial_\nu A_\mu) - (\partial_\nu A_\mu) + ig[A_\mu, A_\nu]]\psi = ig F_{\mu\nu}\psi$

$$\psi = ig\left[(\partial_\mu A_\nu) - (\partial_\nu A_\mu) + ig[A_\mu, A_\nu]\right]\psi = ig F_{\mu\nu}\psi$$

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]}$$

$$\psi = ig\left[(\partial_\mu A_\nu) - (\partial_\nu A_\mu) + ig[A_\mu, A_\nu]\right]\psi = ig F_{\mu\nu}\psi$$

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]}$$

$$F_{\mu\nu} \to F'_{\mu\nu} = U F_{\mu\nu} U^{-1}$$

$$\partial_\mu + igA_\mu = U(\partial_\mu + igA_\mu)U^{-1} = U(\partial_\mu U^{-1}) + \partial_\mu + igUA_\mu U^{-1}$$

$$[D_\mu, D_\nu]\psi = \left[(\partial_\mu + igA_\mu)(\partial_\nu + igA_\nu) - (\partial_\nu + igA_\nu)(\partial_\mu + igA_\mu)\right]\psi =$$

$$i\bar{\psi}\not{\partial}\psi \qquad \not{\partial} = \gamma^\mu\partial_\mu \qquad \{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$\partial_\mu + igA_\mu = U(\partial_\mu + igA_\mu)U^{-1} = U(\partial_\mu U^{-1}) + \partial_\mu + igUA_\mu U^{-1}$$

$$[D_\mu, D_\nu]\psi = \left[(\partial_\mu + igA_\mu)(\partial_\nu + igA_\nu) - (\partial_\nu + igA_\nu)(\partial_\mu + igA_\mu)\right]\psi =$$

$$i\bar{\psi}\not{\partial}\psi \qquad \not{\partial} = \gamma^\mu\partial_\mu \qquad \{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi \qquad \not{D} = \gamma^\mu D_\mu$$

$$\partial_\mu + igA_\mu = U(\partial_\mu + igA_\mu)U^{-1} = U(\partial_\mu U^{-1}) + \partial_\mu + igUA_\mu U^{-1}$$

$$[D_\mu, D_\nu]\psi = \left[(\partial_\mu + igA_\mu)(\partial_\nu + igA_\nu) - (\partial_\nu + igA_\nu)(\partial_\mu + igA_\mu)\right]\psi =$$

$$i\bar{\psi}\not{\partial}\psi \qquad \not{\partial} = \gamma^\mu\partial_\mu \qquad \{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

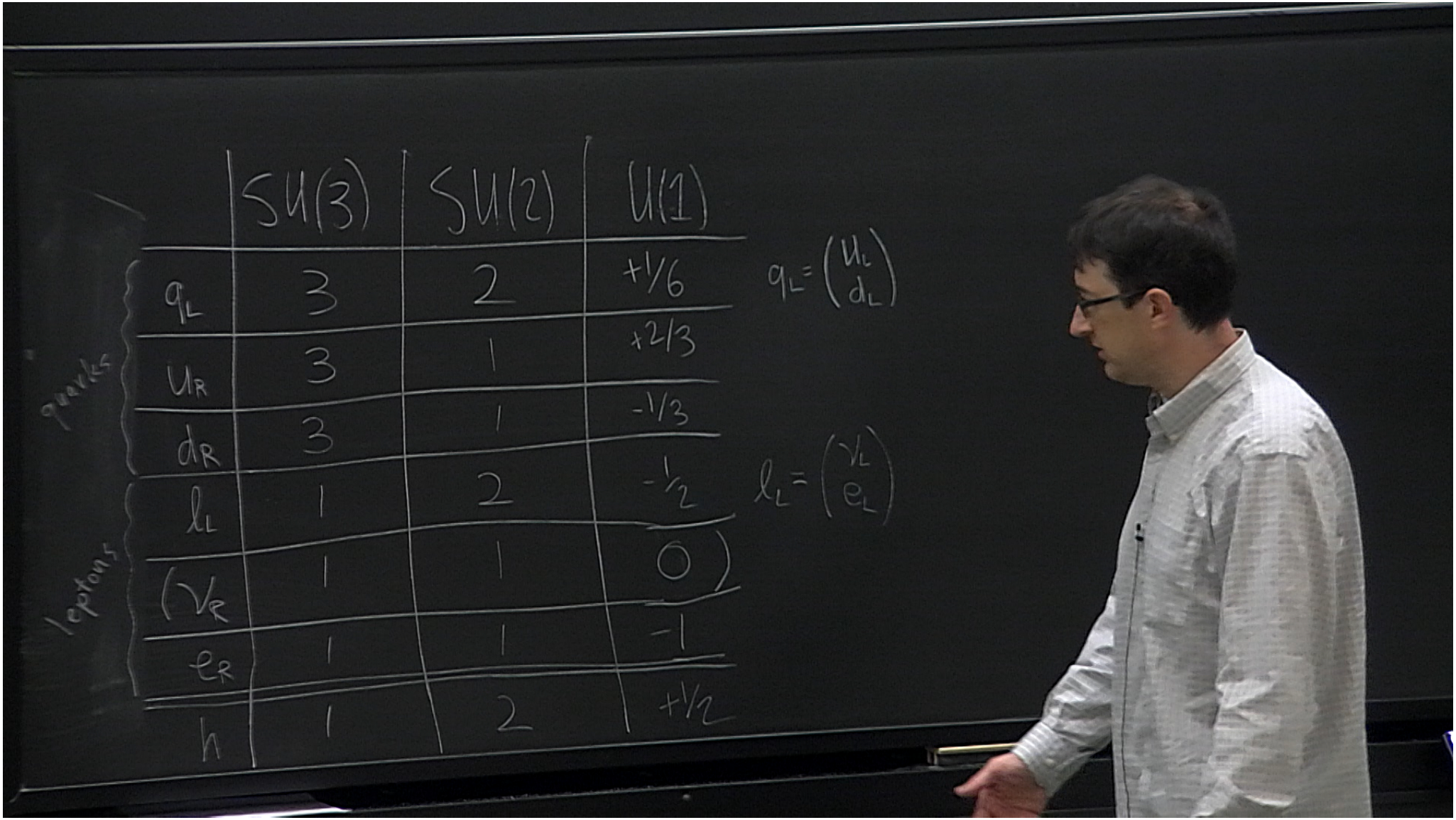
$$-\frac{1}{4}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] + i\bar{\psi}\not{D}\psi \qquad \not{D} = \gamma^\mu D_\mu$$

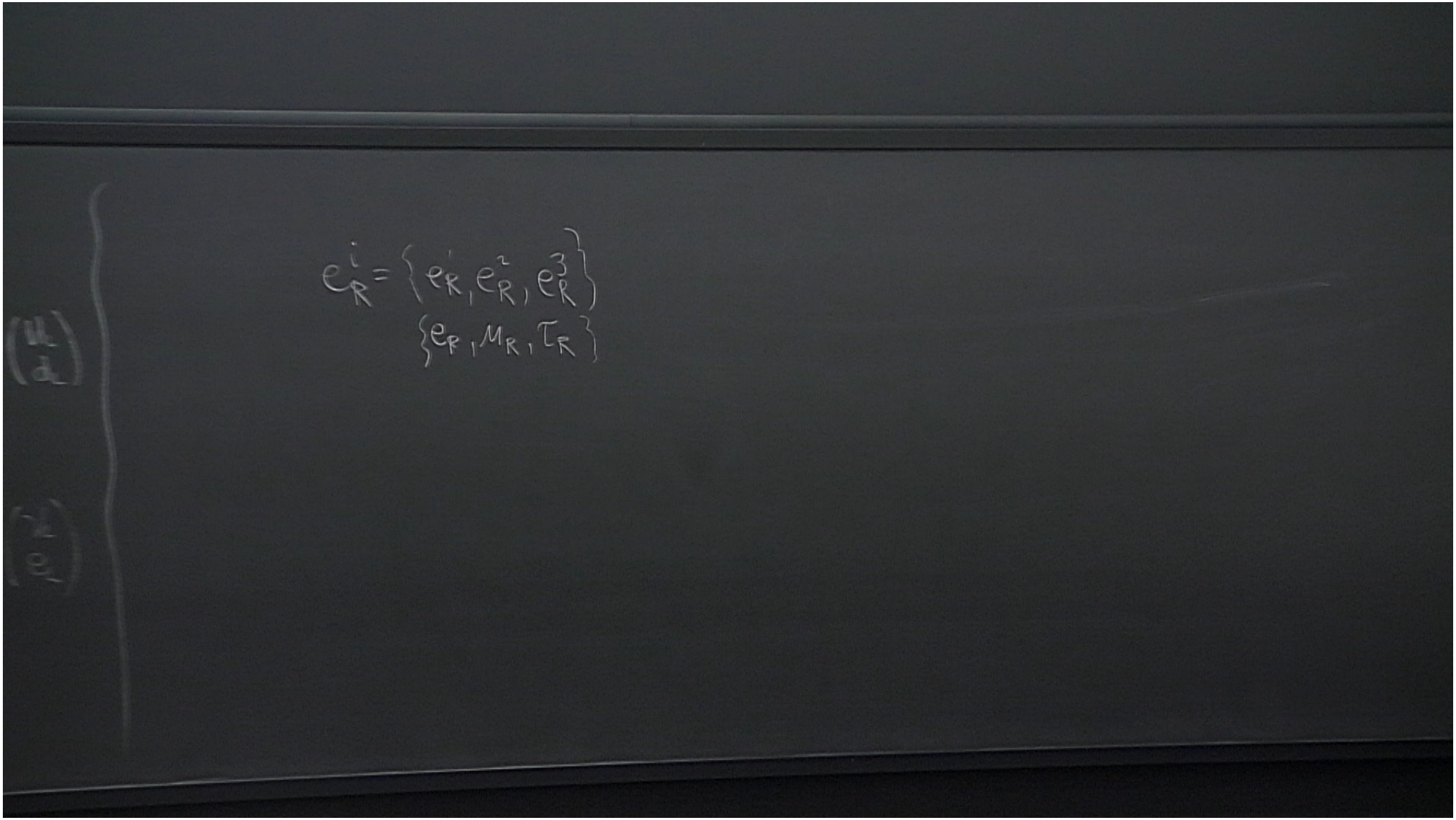
$$i\bar{\psi}U^\dagger U\not{D}\psi$$

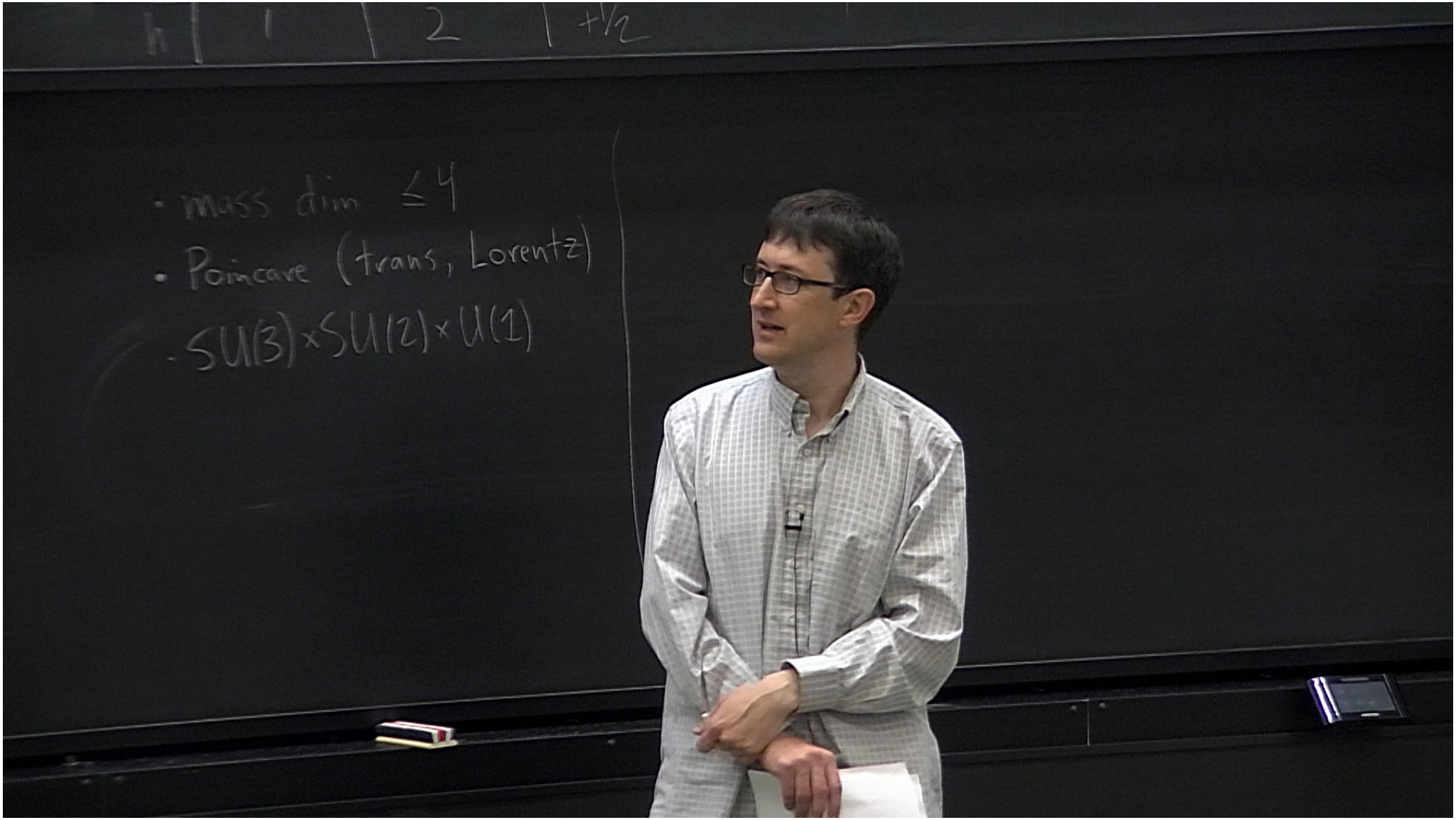
$$\psi \to U\psi$$

$$\psi^\dagger \to \psi^\dagger U^\dagger$$

$$\bar{\psi} = \psi^\dagger\gamma^0 \to \bar{\psi}U^\dagger$$


SU(3) SU(2) U(1)
q_L 3 2 +1/6
u_R 3 1 +2/3
d_R 3 1 -1/3
l_L 1 2 -1/2
ν_R 1 1 0
e_R 1 1 -1
h 1 2 +1/2
quarks
leptons
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$
$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$


$e_R^i = \{e_R^1, e_R^2, e_R^3\}$
$\{e_R, \mu_R, \tau_R\}$


1
2
+1/2
• mass dim ≤ 4
• Poincare (trans, Lorentz)
• SU(3) × SU(2) × U(1)