Title: PSI 2018/2019 - Beyond Standard Model - Lecture 1

Speakers: Latham Boyle

Collection: PSI 2018/2019 - Beyond Standard Model (Boyle)

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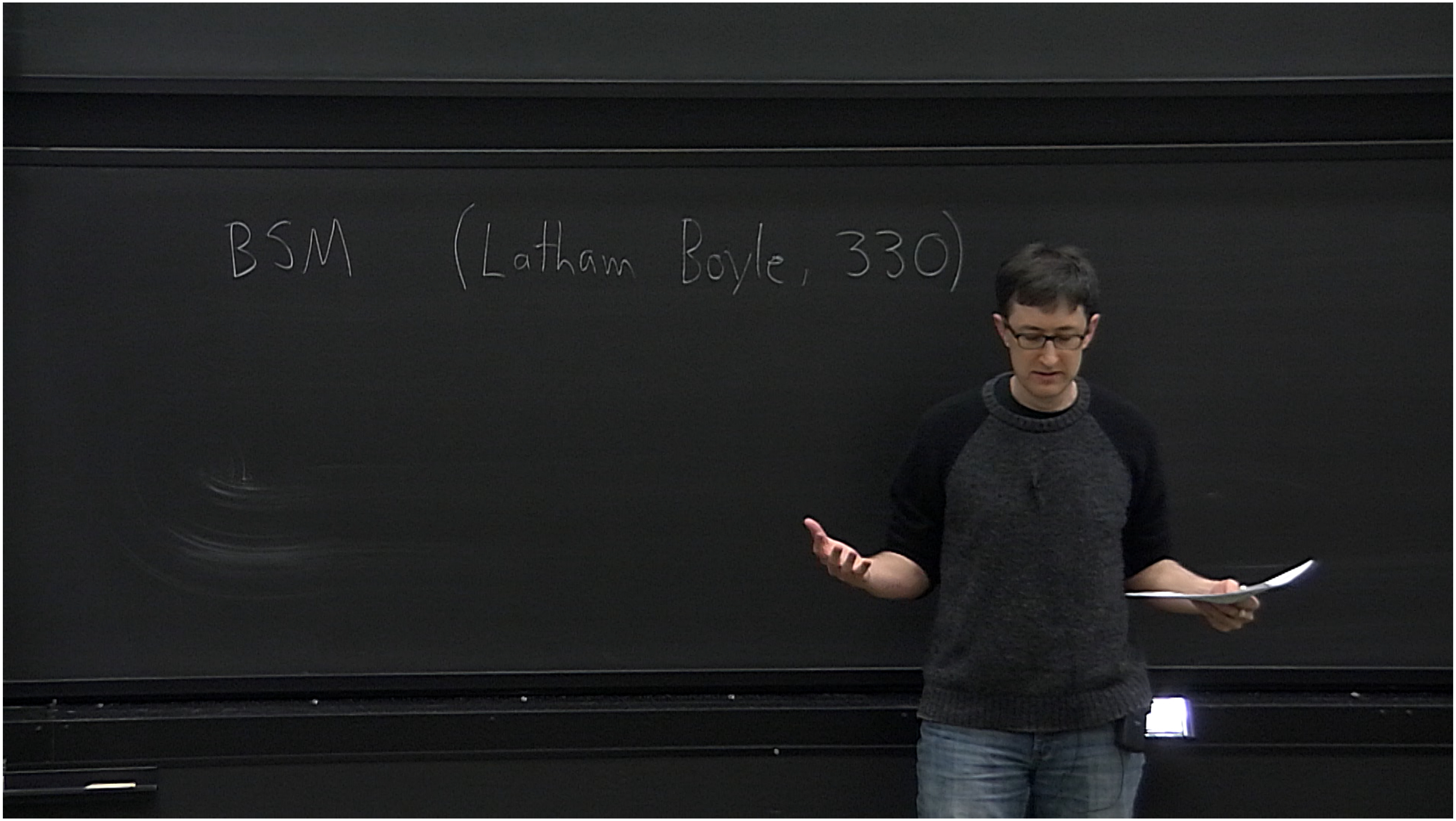
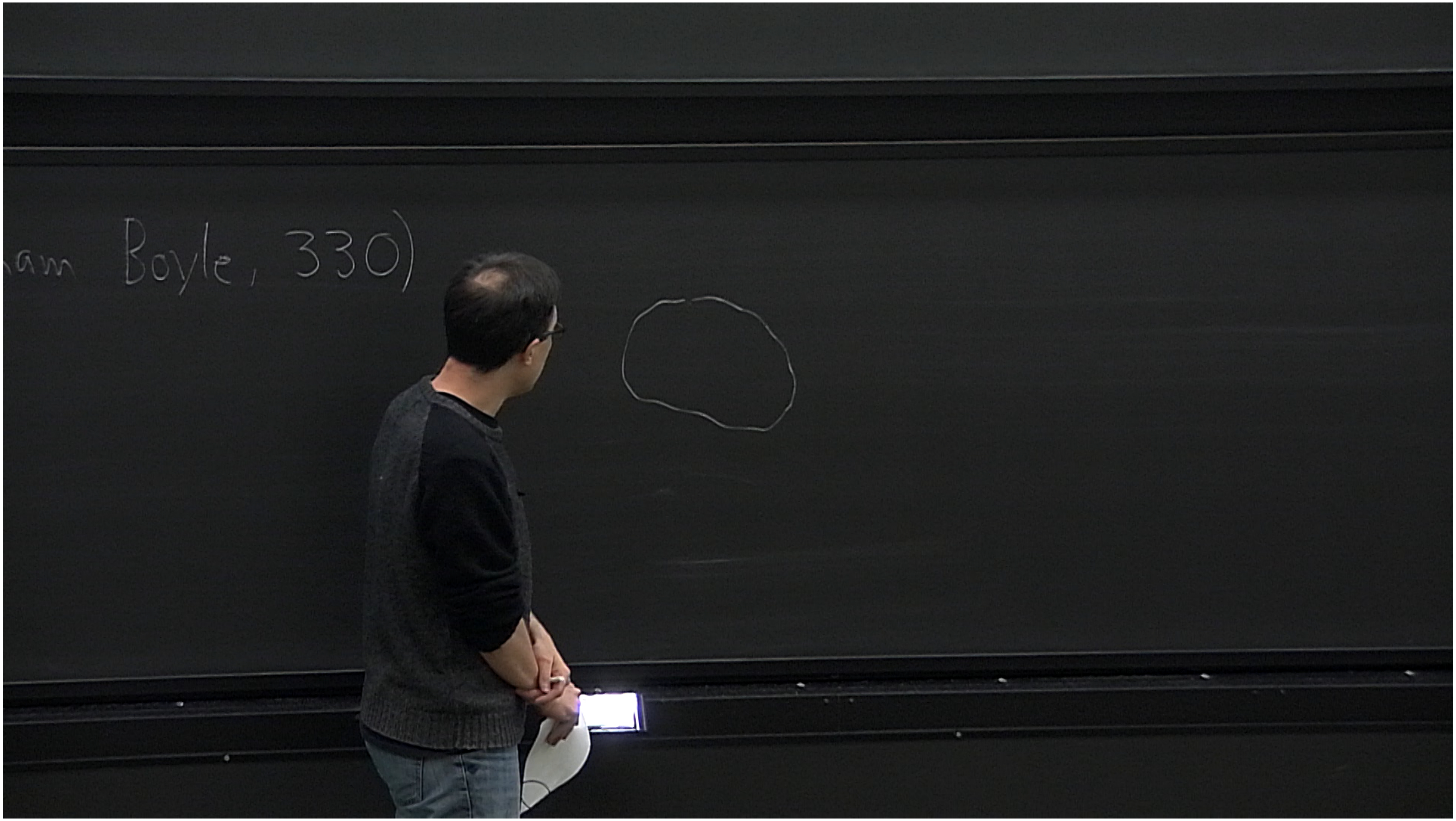
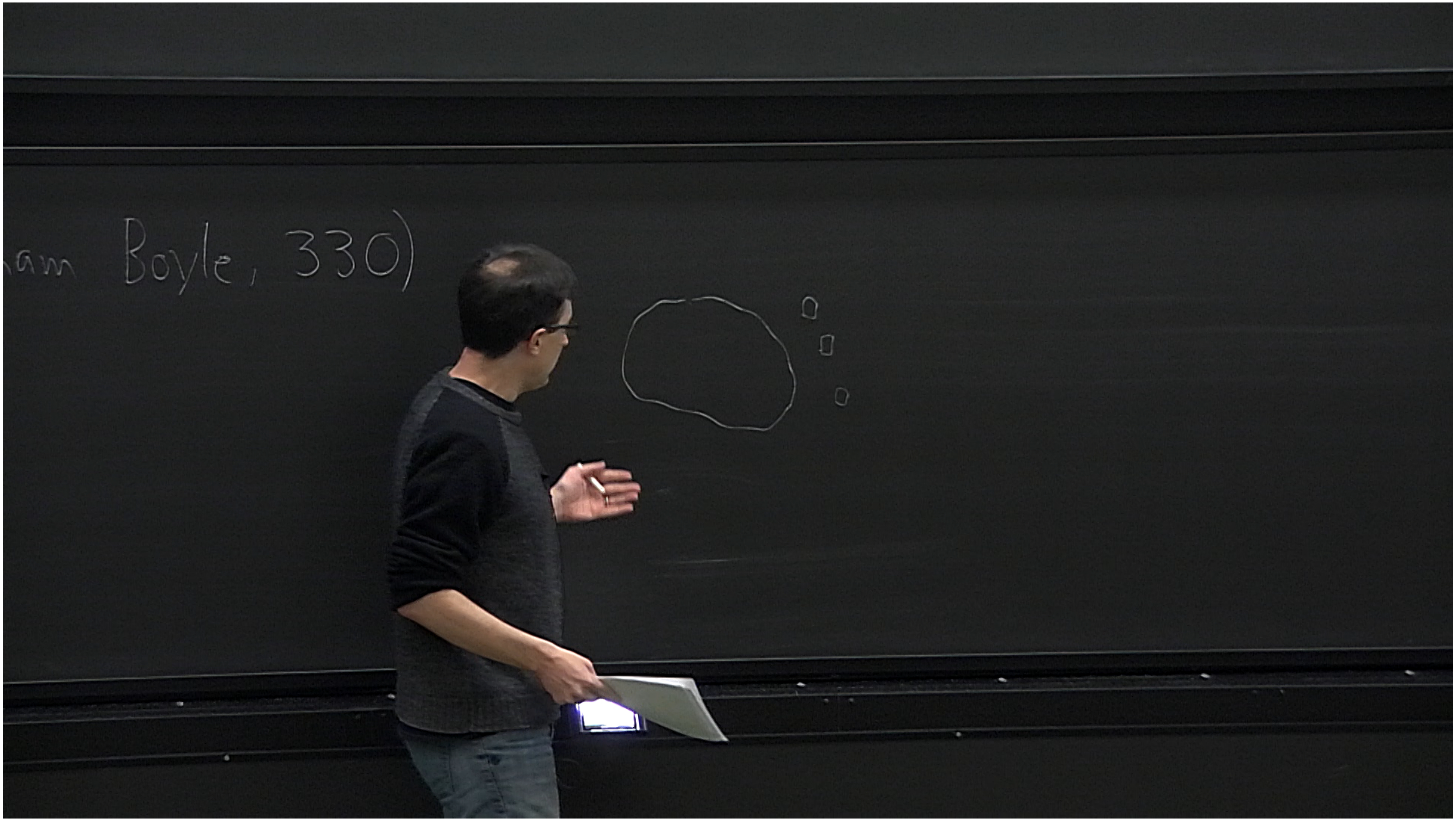
URL: http://pirsa.org/19030063


BSM (Latham Boyle, 330)


am Boyle, 330)



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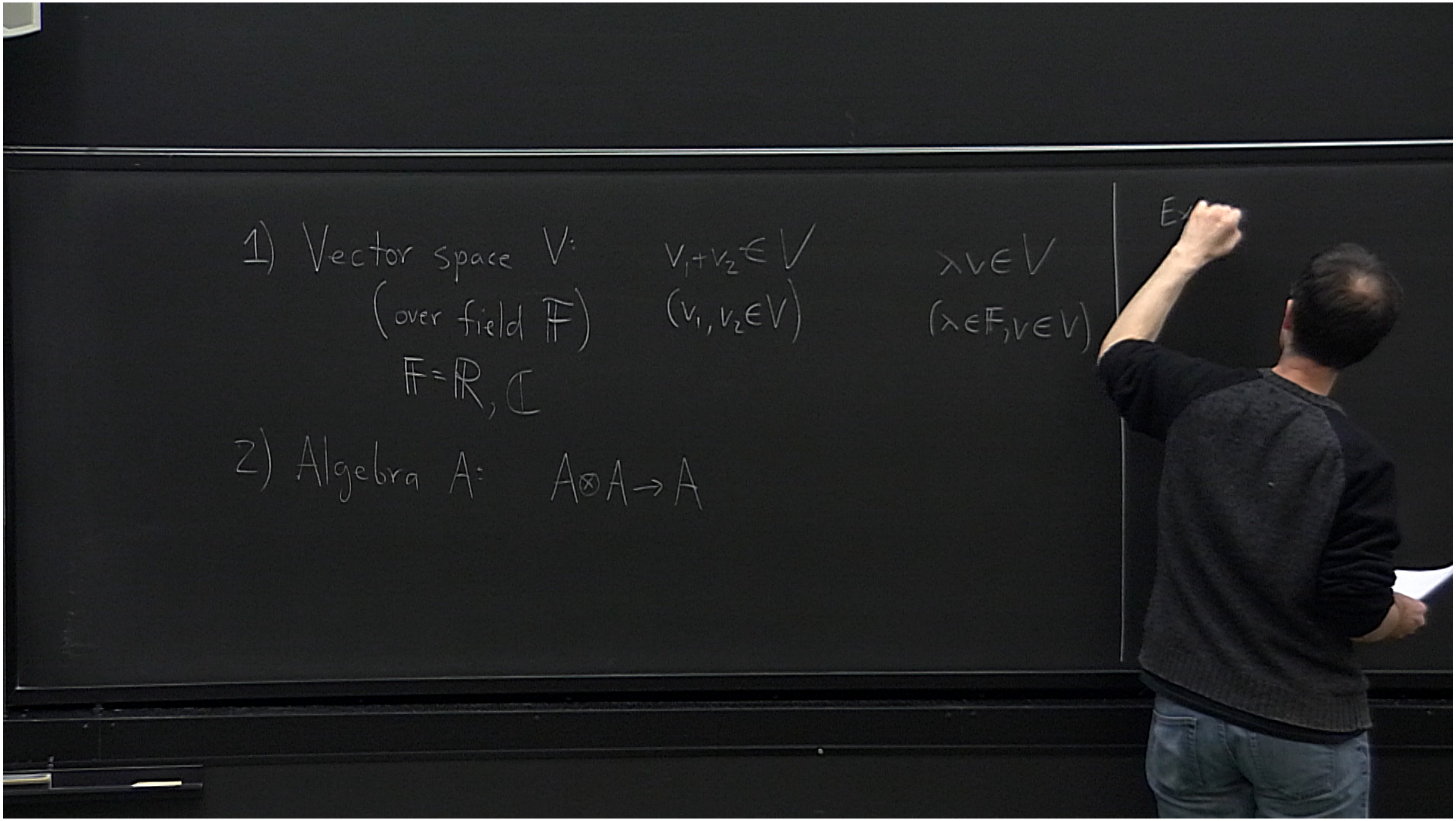
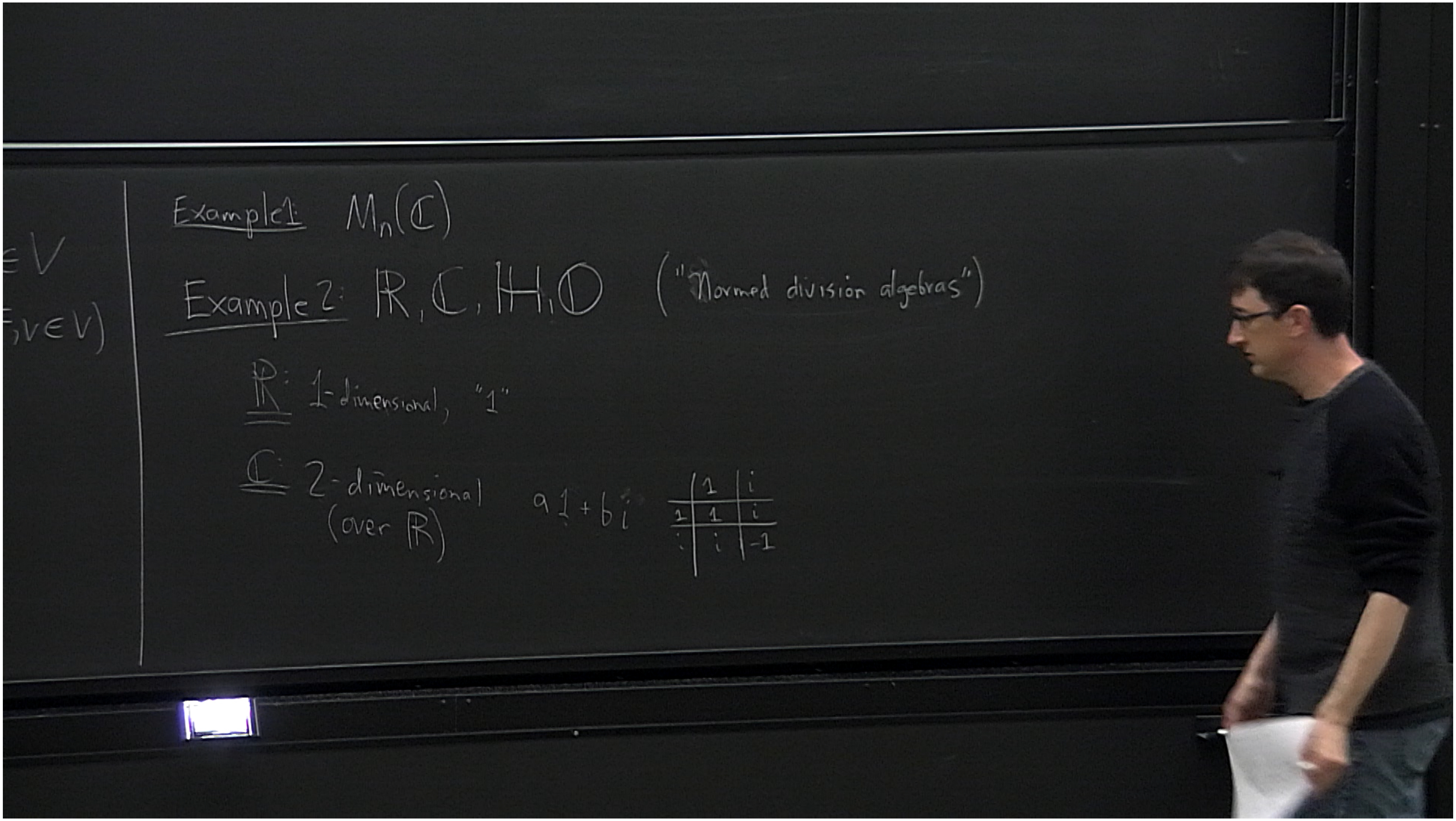
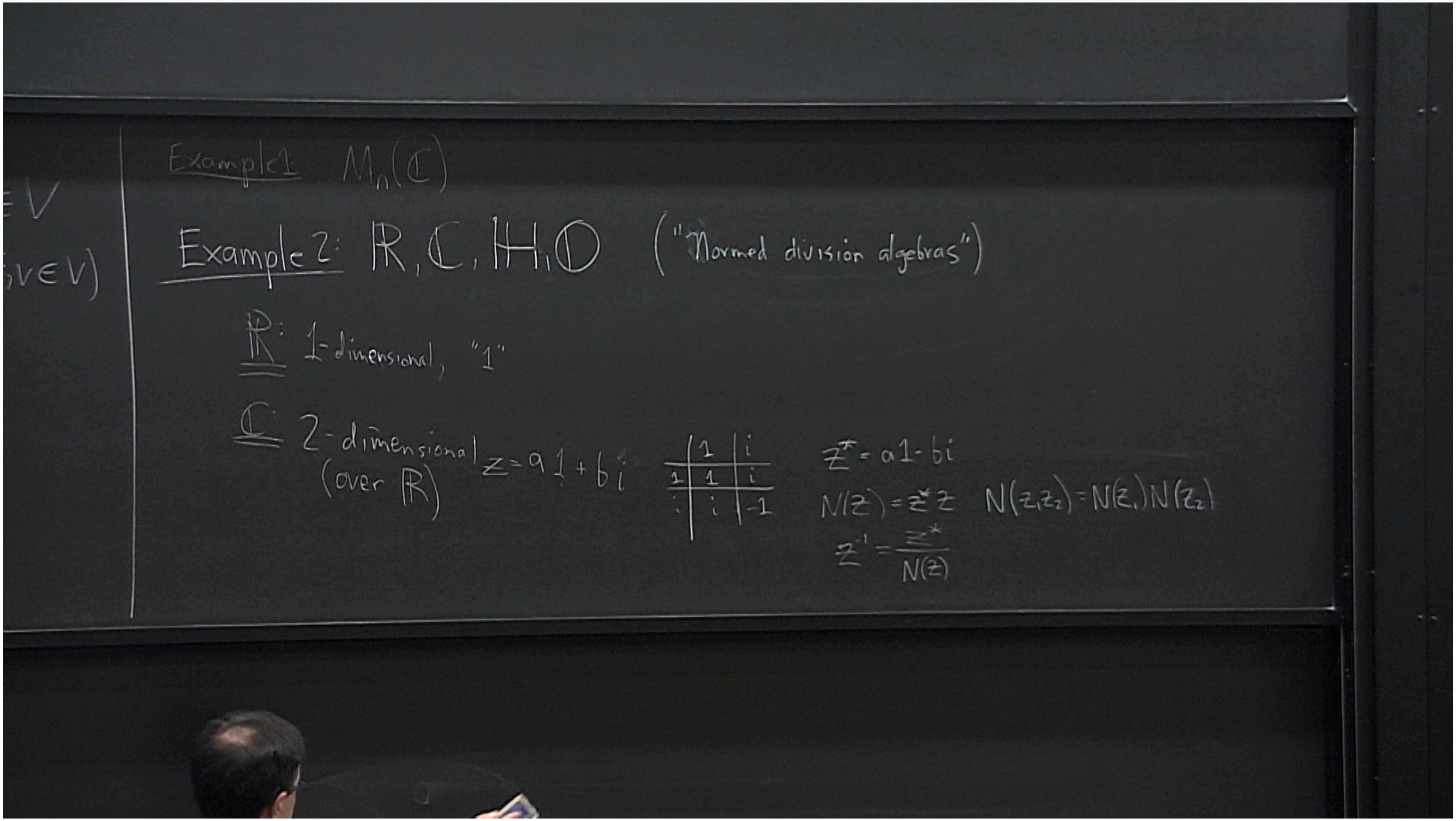
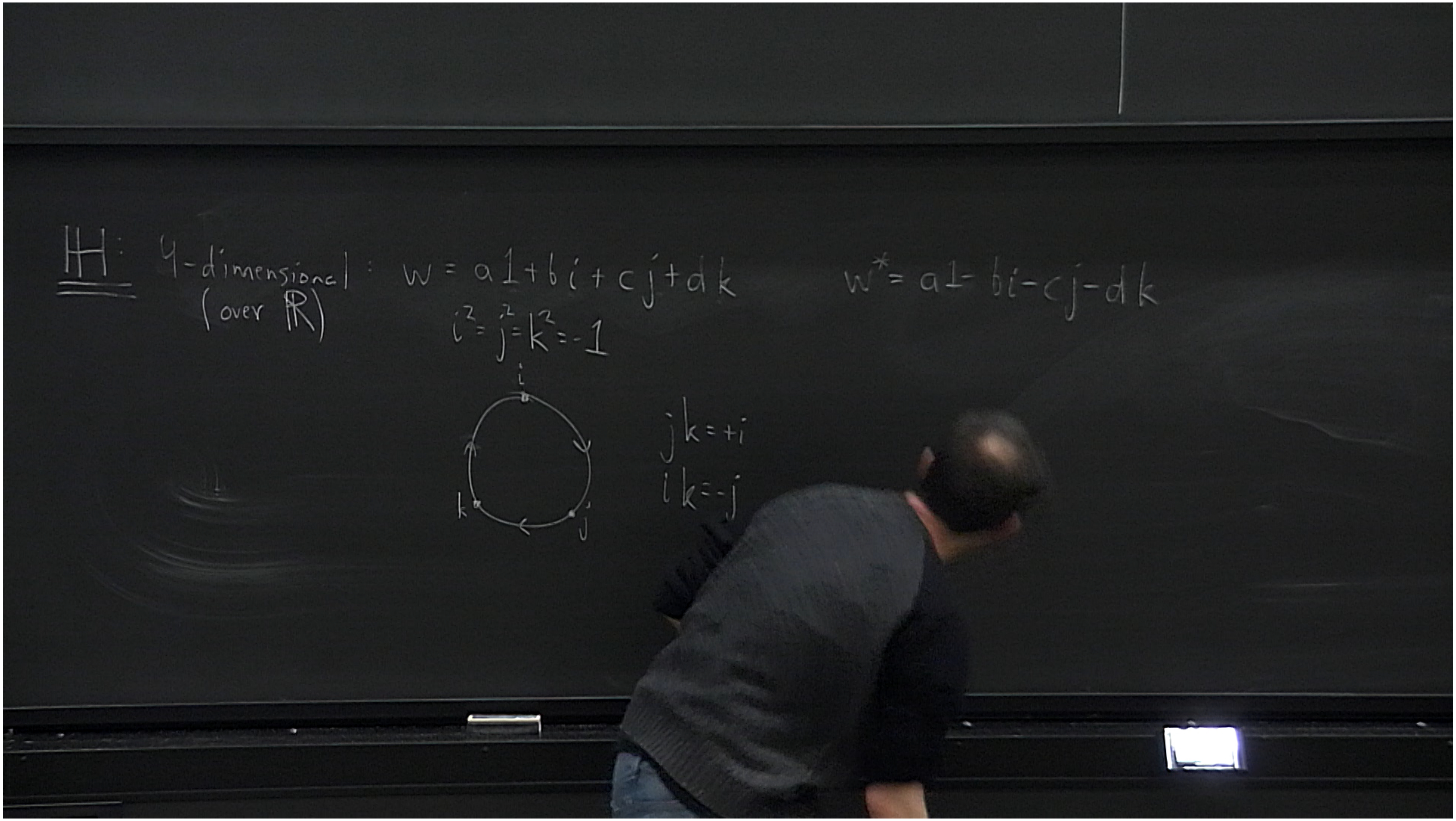

1) Vector space V: (over field $\mathbb{F}$)
$\mathbb{F} = \mathbb{R}, \mathbb{C}$
$v_1 + v_2 \in V$ $(v_1, v_2 \in V)$
$\lambda v \in V$ $(\lambda \in \mathbb{F}, v \in V)$
2) Algebra A: $A \otimes A \to A$

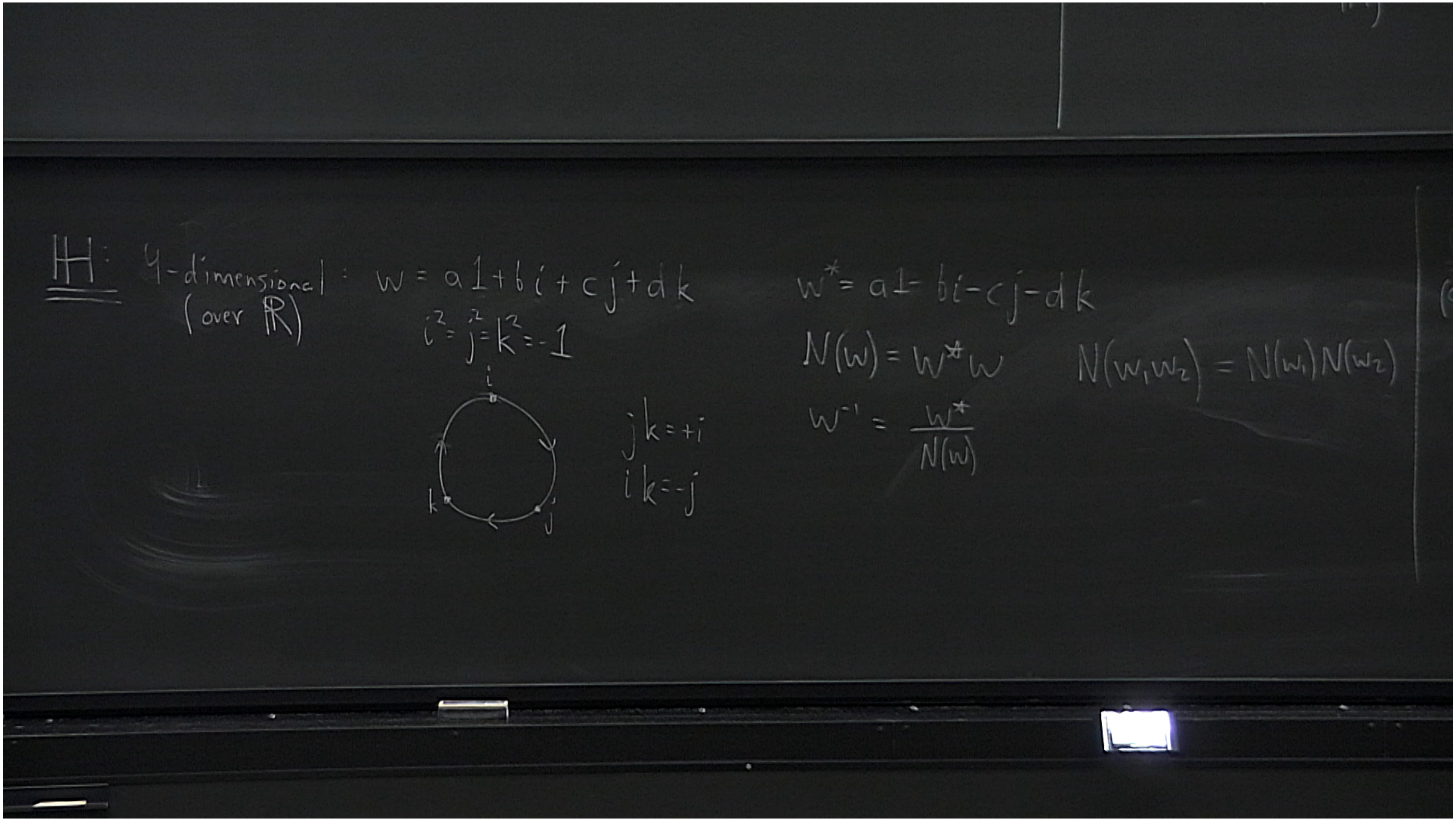

Example 1: Mn(ℂ)
Example 2: ℝ, ℂ, ℍ, 𝕆 ("Normed division algebras")
ℝ: 1-dimensional, "1"
ℂ: 2-dimensional (over ℝ)
a1 + bi
1 i
1 1 i
i i -1

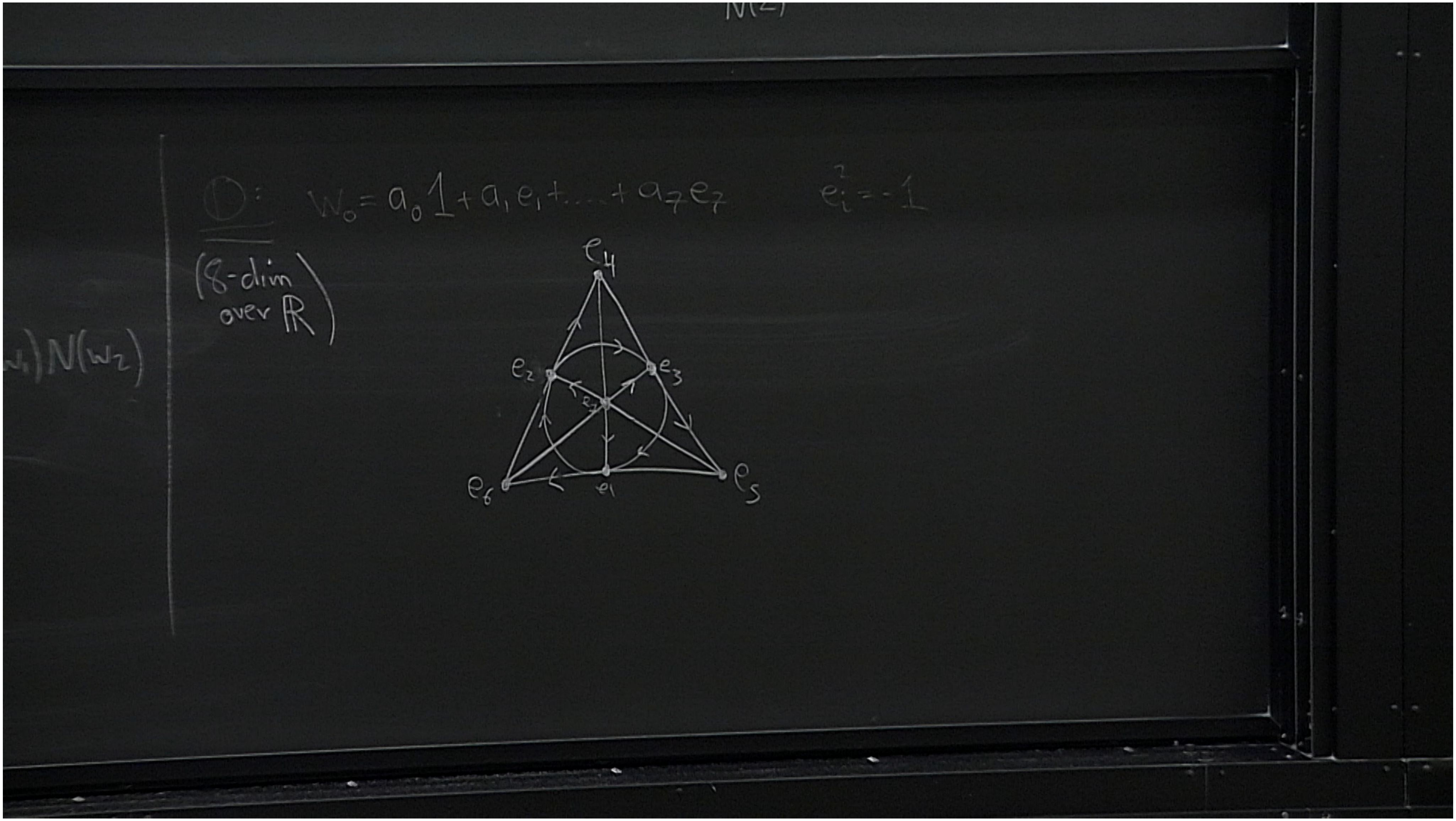


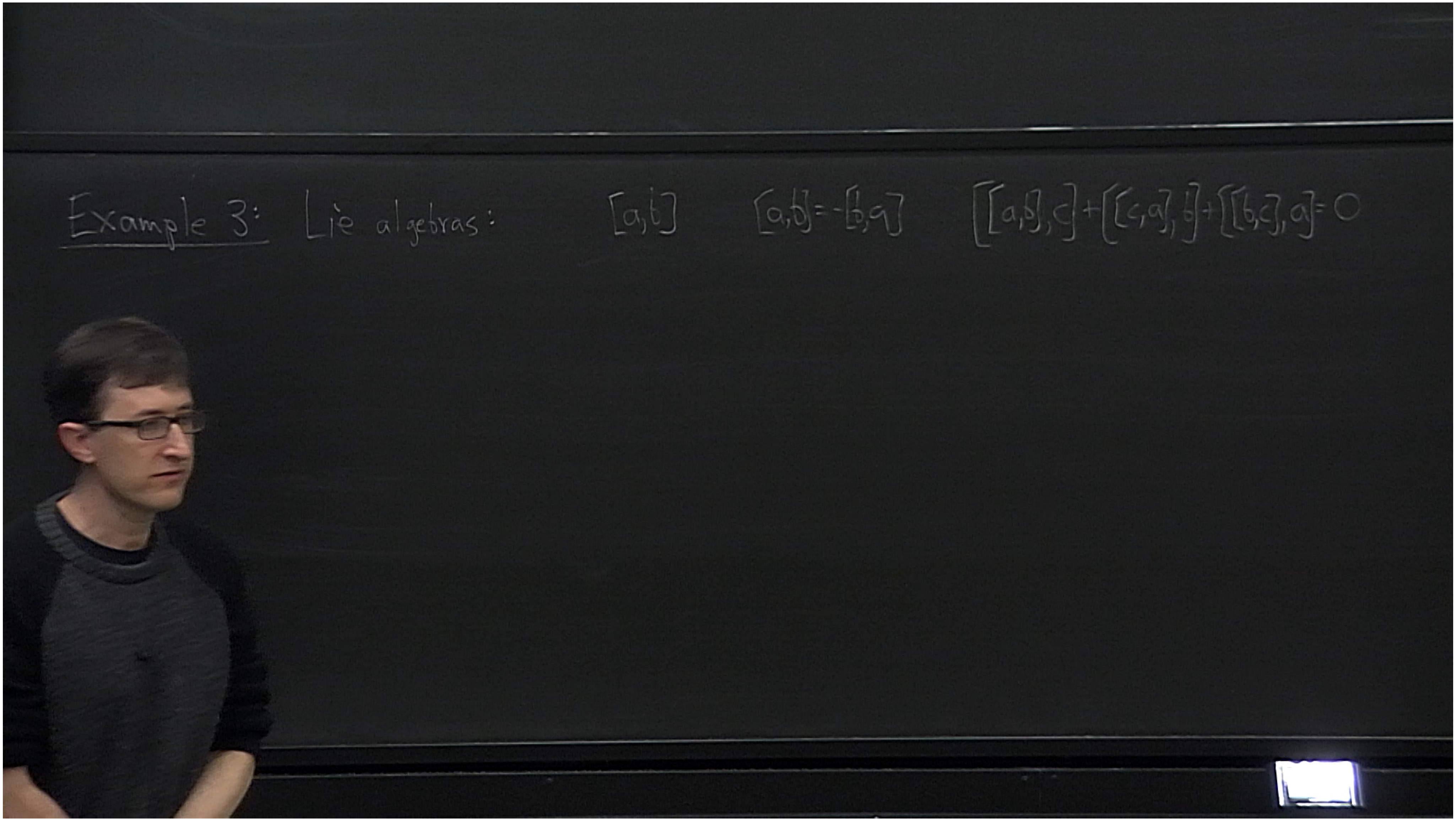
Example 1: Mn(ℂ)
Example 2: ℝ, ℂ, ℍ, 𝕆 ("Normed division algebras")
ℝ: 1-dimensional, "1"
ℂ: 2-dimensional (over ℝ)
z = a1 + bi
| 1 i
1 | 1 i
i | i -1
z* = a1 - bi
N(z) = z*z
N(z1z2) = N(z1)N(z2)
z⁻¹ = z*/N(z)

$\mathbb{H}$: 4-dimensional (over $\mathbb{R}$): $w = a1 + bi + cj + dk$


H: 4-dimensional : w = a1+bi+cj+dk
(over R)
w* = a1-bi-cj-dk
i² = j² = k² = -1
i
k
j
jk=+i
ik=-j


$\mathbb{H}$: 4-dimensional (over $\mathbb{R}$): $w = a\mathbb{1} + bi + cj + dk$
$i^2 = j^2 = k^2 = -1$
i
k
j
$jk = +i$
$ik = -j$
$w^* = a\mathbb{1} - bi - cj - dk$
$N(w) = w^* w$
$N(w_1 w_2) = N(w_1) N(w_2)$
$w^{-1} = \frac{w^*}{N(w)}$


O: $w_0 = a_0 1 + a_1 e_1 + \dots + a_7 e_7$
$e_i^2 = -1$
(8-dim over $\mathbb{R}$)
$)N(w_2)$
e_4
e_2
e_3
e_7
e_6
e_1
e_5


Example 3: Lie algebras:
[a,b]
[a,b] = -[b,a]
[[a,b],c] + [[c,a],b] + [[b,c],a] = 0

Example 3: Lie algebras: $[a,b]$ $\quad [a,b] = -[b,a]$ $\quad [[a,b],c] + [[c,a],b] + [[b,c],a] = 0$

i) $so(n) = \{ x \in \mathbb{R}[n],\ x^t = -x,\ tr(x) = 0 \}$

ii) $su(n) = \{ x \in \mathbb{C}[n],\ x^\dagger = -x,\ tr(x) = 0 \}$

iii) $sp(n) = \{ x \in \mathbb{H}[n],\ x^\dagger = -x$

Example 3: Lie algebras: $[a,b]$ $\quad [a,b] = -[b,a]$ $\quad [[a,b],c] + [[c,a],b] + [[b,c],a] = 0$

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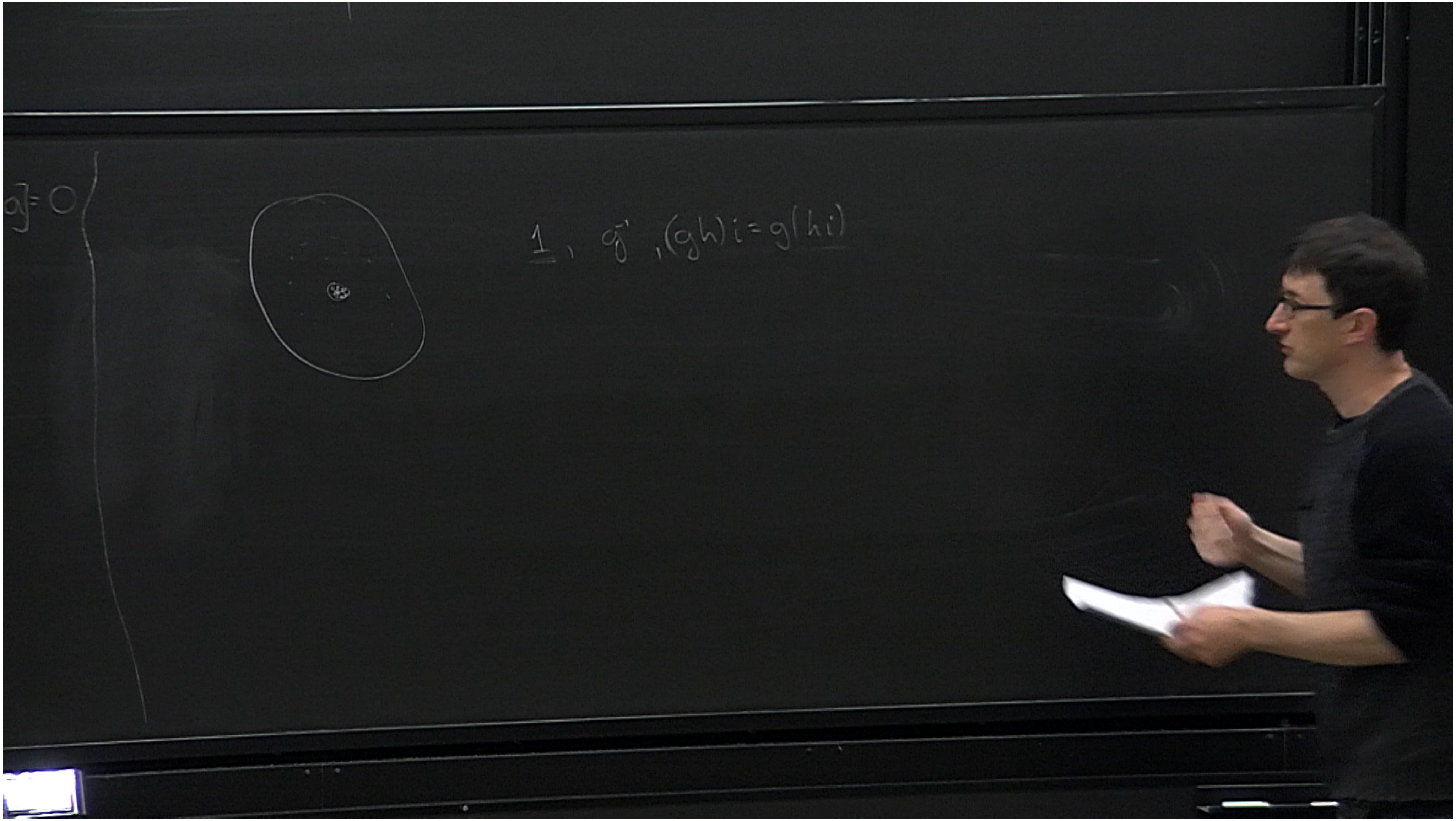
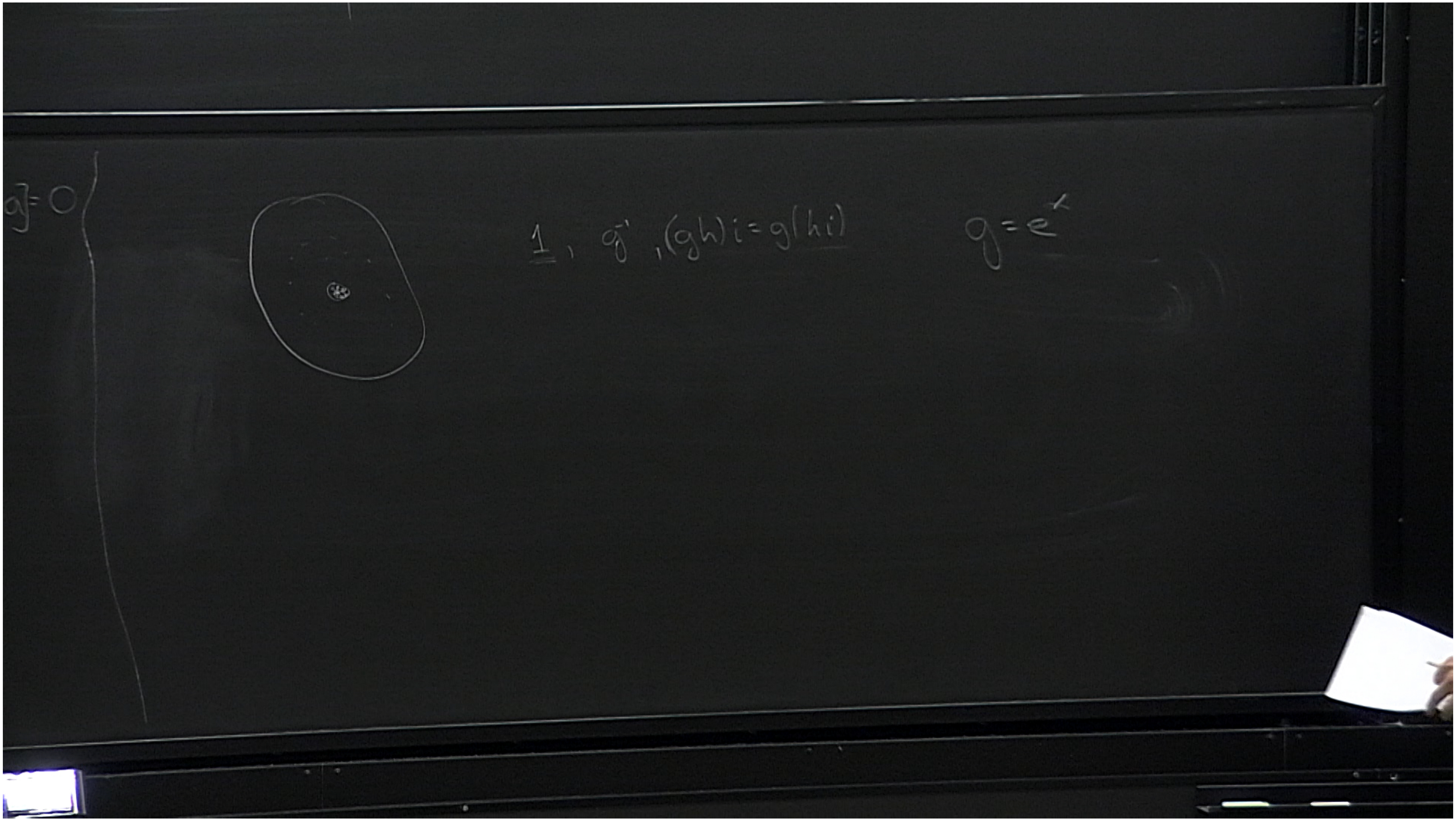
iii) $sp(n) = \{ x \in \mathbb{H}[n],\ x^t = -x \}$

iv) $g_2,\ f_4,\ e_6,\ e_7,\ e_8$

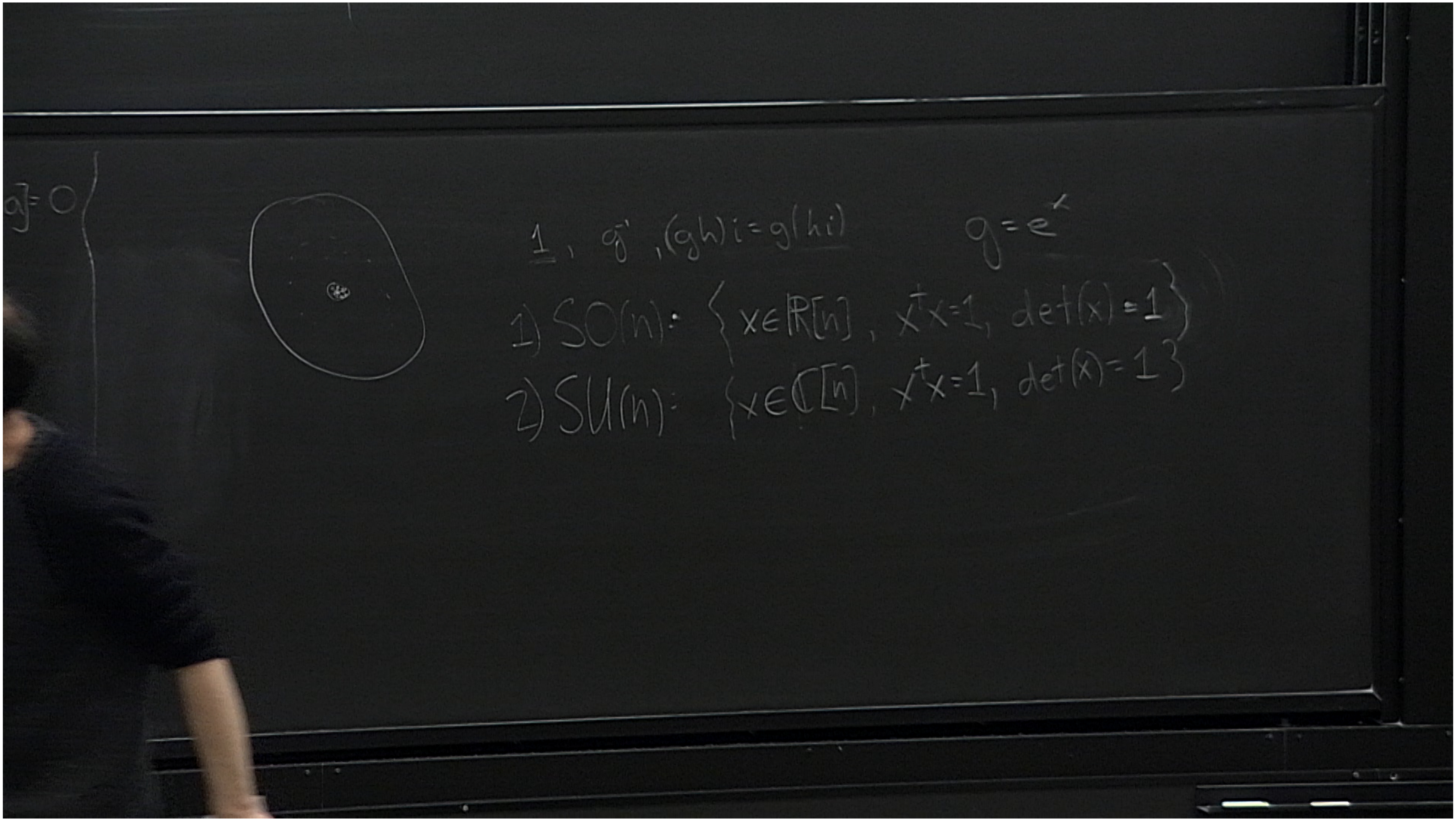
$\mathbb{O} \leftarrow Aut(\mathbb{O}) = g_2$

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$				f_4
$\mathbb{C}$				e_6
$\mathbb{H}$				e_7
$\mathbb{O}$	f_4	e_6	e_7	e_8

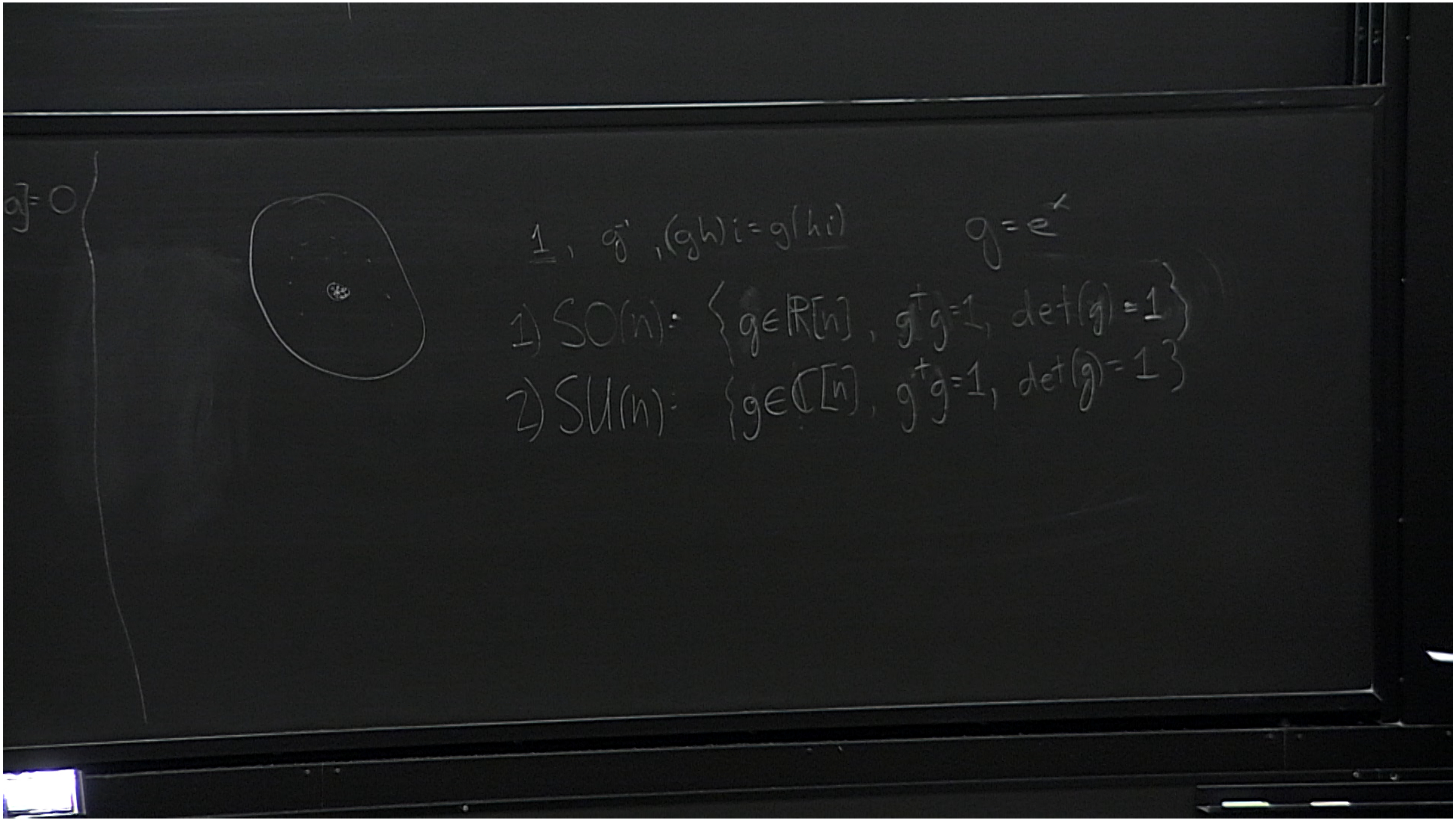

1 , g' , (gh)i = g(hi)



1, g⁻¹, (gh)i = g(hi)
g = e^X



a]=0
1, g⁻¹, (gh)i = g(hi)
g = e^X
1) SO(n): {x ∈ ℝ[n], xᵀx = 1, det(x) = 1}
2) SU(n): {x ∈ ℂ[n], x†x = 1, det(x) = 1}


g]=0
1, g⁻¹, (gh)i = g(hi)
g = e^X
1) SO(n): {g ∈ ℝ[n], gᵀg = 1, det(g) = 1}
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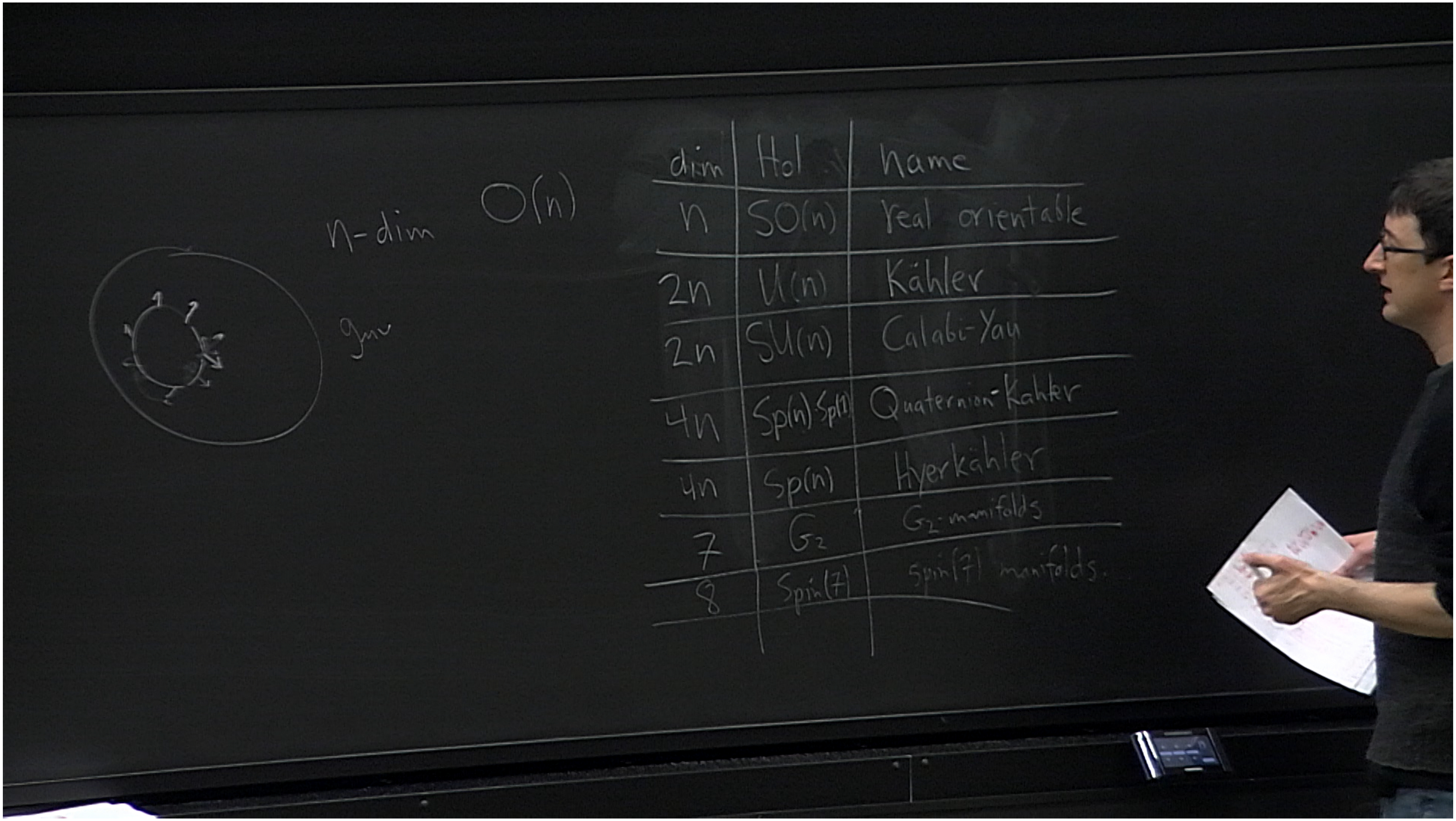
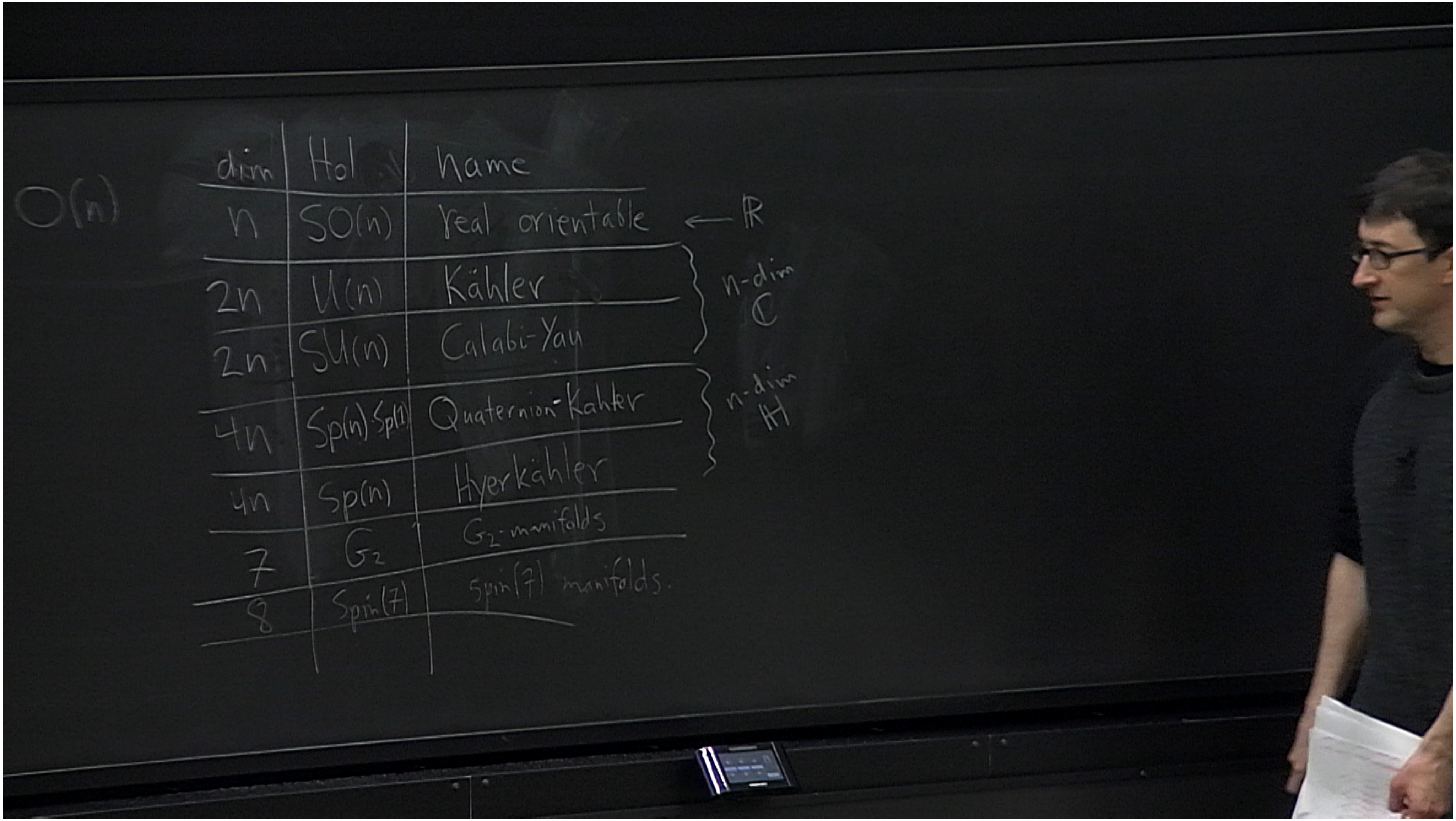
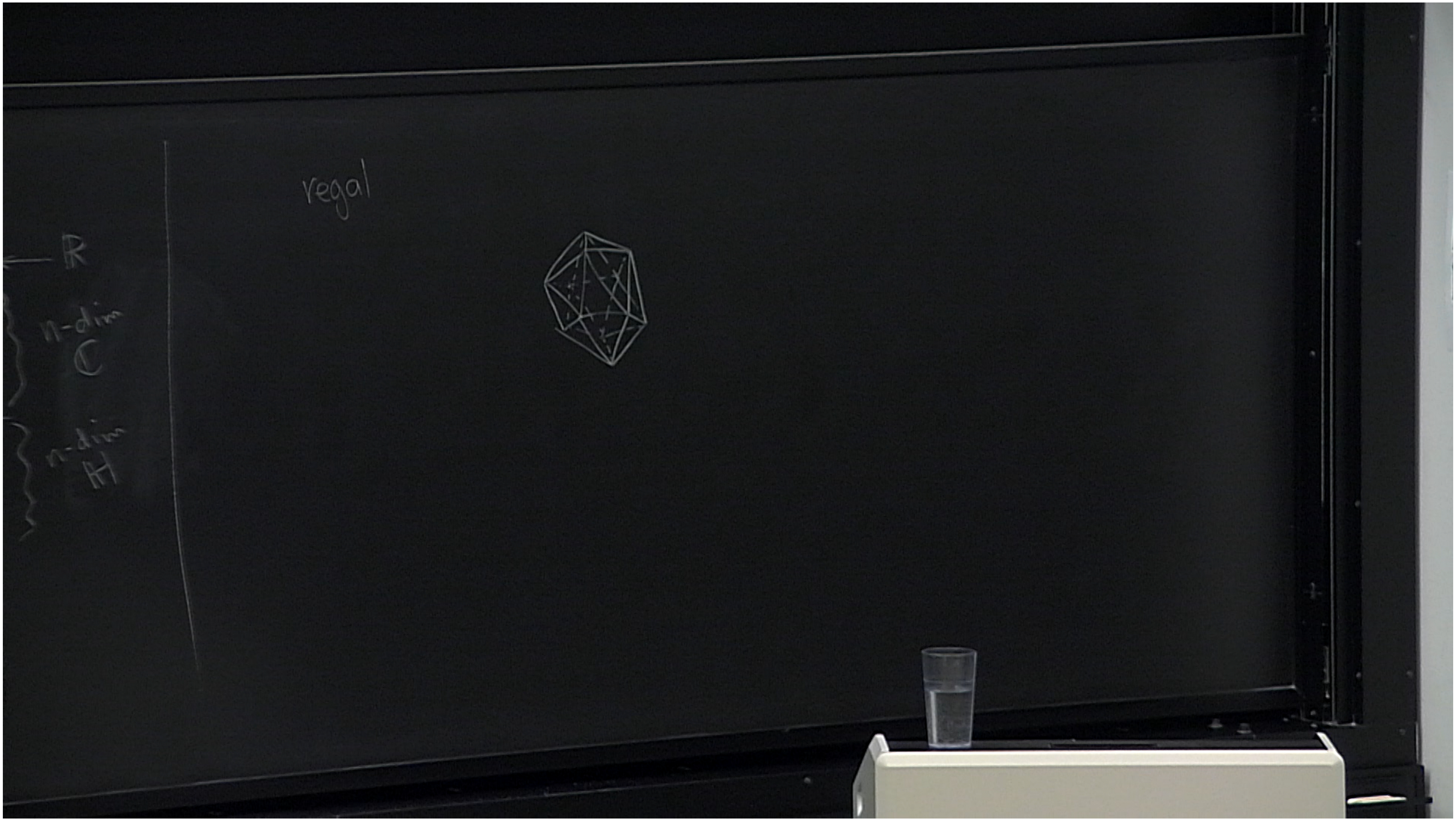
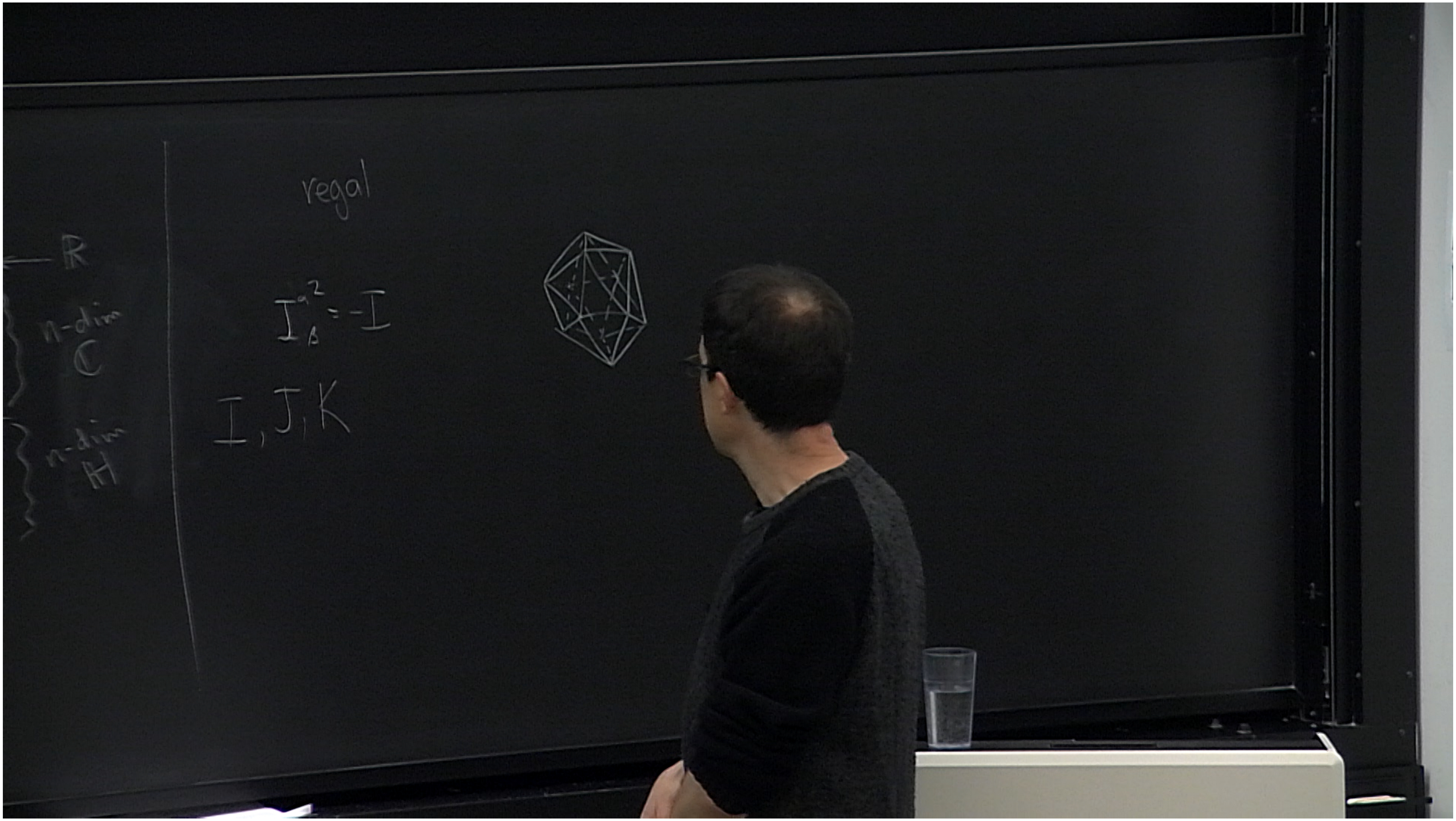

n-dim
O(n)
gμν
dim | Hol | name
n | SO(n) | real orientable
2n | U(n) | Kähler
2n | SU(n) | Calabi-Yau
4n | Sp(n)·Sp(1) | Quaternion-Kahler
4n | Sp(n) | Hyperkähler
7 | G2 | G2-manifolds
8 | Spin(7) | Spin(7) manifolds.


O(n)
dim
Hol
name
n
SO(n)
real orientable
← ℝ
2n
U(n)
Kähler
2n
SU(n)
Calabi-Yau
n-dim
ℂ
4n
Sp(n)·Sp(1)
Quaternion-Kahler
4n
Sp(n)
Hyperkähler
n-dim
ℍ
7
G2
G2-manifolds
8
Spin(7)
spin(7) manifolds.



regal
R
n-dim
C
n-dim
H


regal
n-dim
n-dim