Title: PSI 2018/2019 - Quantum Information Review - Lecture 15

Speakers: Daniel Gottesman

Collection: PSI 2018/2019 - Quantum Information Review (Gottesman)

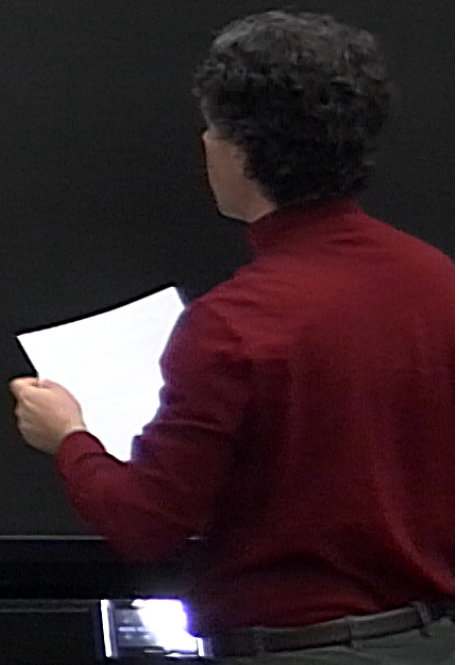
Date: March 22, 2019 - 11:30 AM

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Quantum Channel Capacity:

Max. C s.t. we can send Cn qubits through n uses of the channel w/ high fidelity.

Def: Let $\mathcal{C}^\circ$ be a noisy quantum channel.

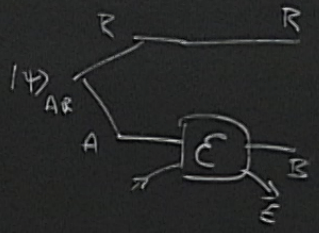


Def: Let $\mathcal{C}$ be a noisy quantum channel. ρ is a quantum state. The coherent information $I(\rho;\mathcal{C})$ for ρ & $\mathcal{E}$ is given as follows:

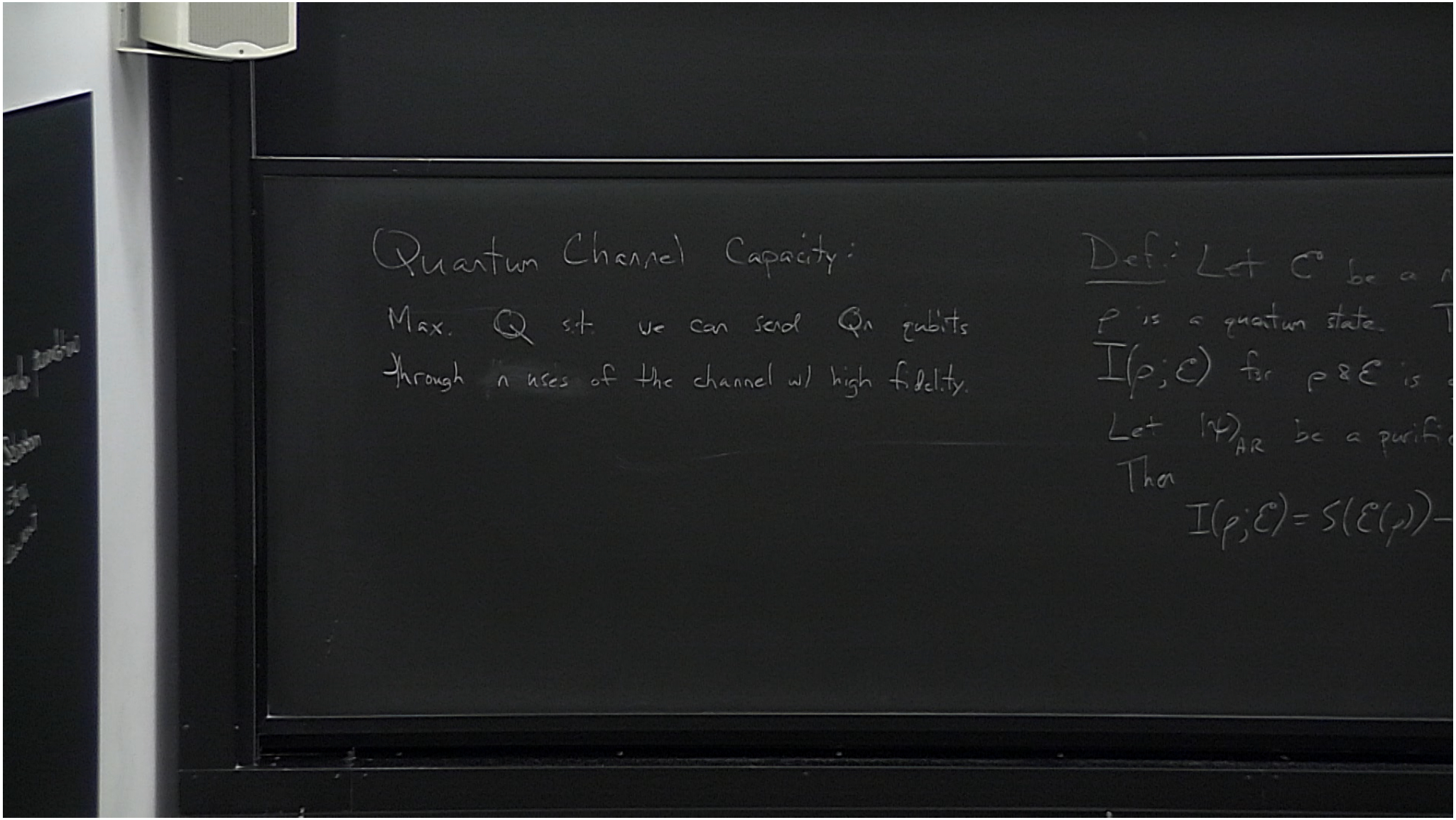
Let $|\psi\rangle_{AR}$ be a purification of ρ using R.

Then

$$I(\rho;\mathcal{C}) = S(\mathcal{C}(\rho)) - S((\mathcal{C}\otimes I)|\psi\rangle_{AR}\langle\psi|).$$



$$I(\rho;\mathcal{C}) = S(B) - S(BR)$$
$$= S(B) - S(E)$$


Quantum Channel Capacity:
Max. Q s.t. we can send Qn qubits
through n uses of the channel w/ high fidelity.
Def: Let C° be a
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I(ρ; E) for ρ & E is
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I(ρ; E) = S(E(ρ))

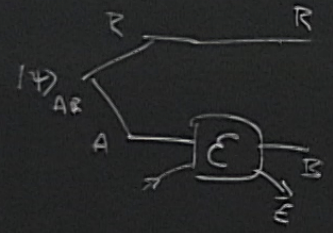
quantum channel

coherent information

as follows

of ρ using R.

$(\mathcal{E}\otimes I)|\psi\rangle_{AR}\langle\psi|$.



$I(\rho;\mathcal{E}) = S(B) - S(BR)$

$= S(B) - S(E)$

Quantum channel capacity of $\mathcal{E}$ is

$$Q = \lim_{n\to\infty} \sup_{\rho} \frac{1}{n} I(\rho; \mathcal{E}^{\otimes n})$$

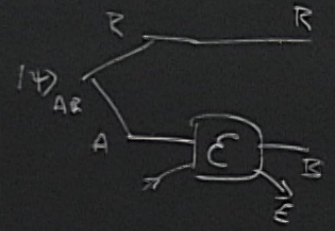
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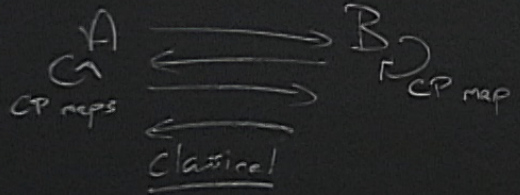
$$Q = \lim_{n\to\infty} \sup_{\rho} \frac{1}{n} I(\rho;\mathcal{E}^{\otimes n})$$

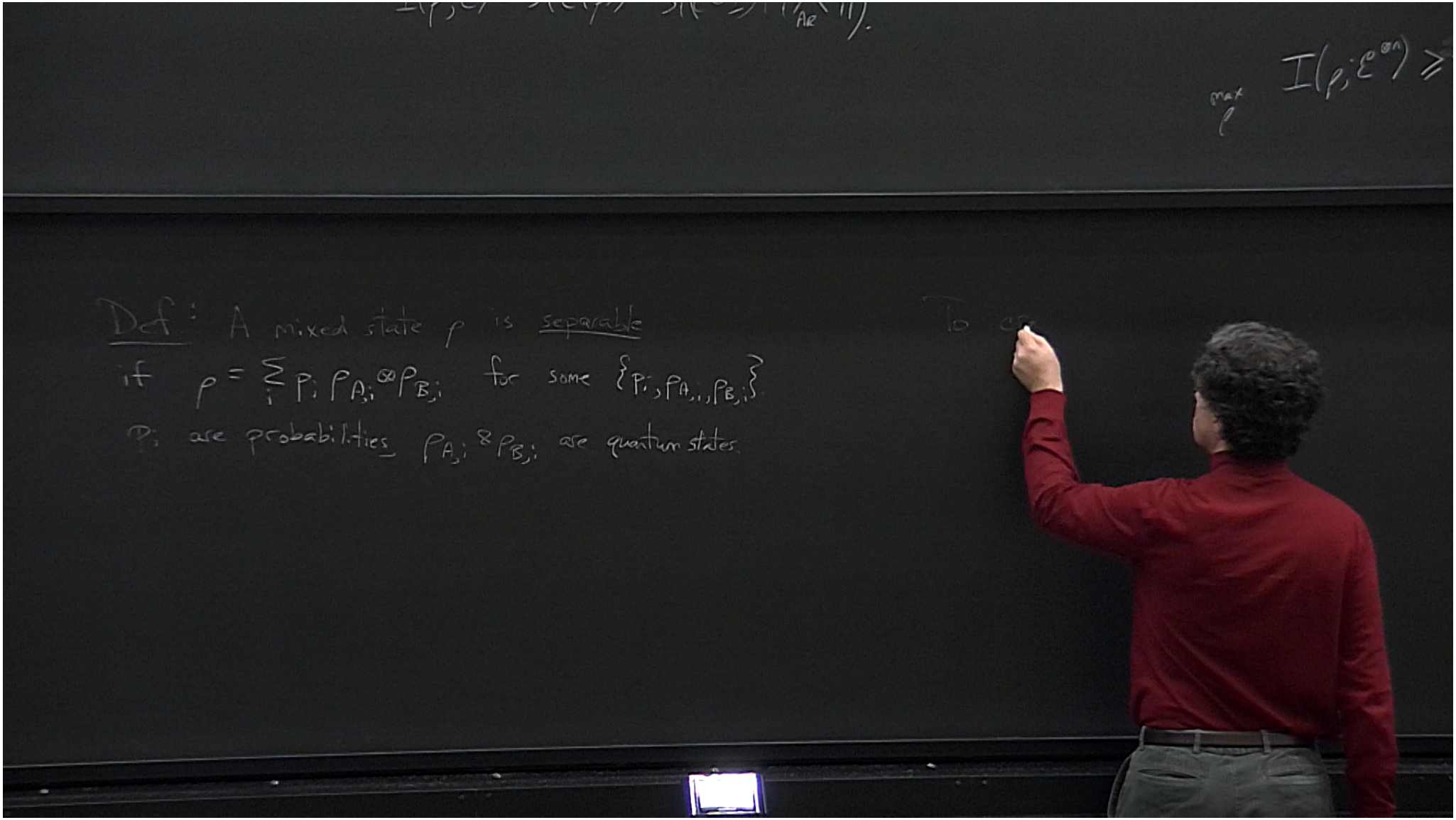
(Classically $C = \max_x I(X:Y)$. Additive.

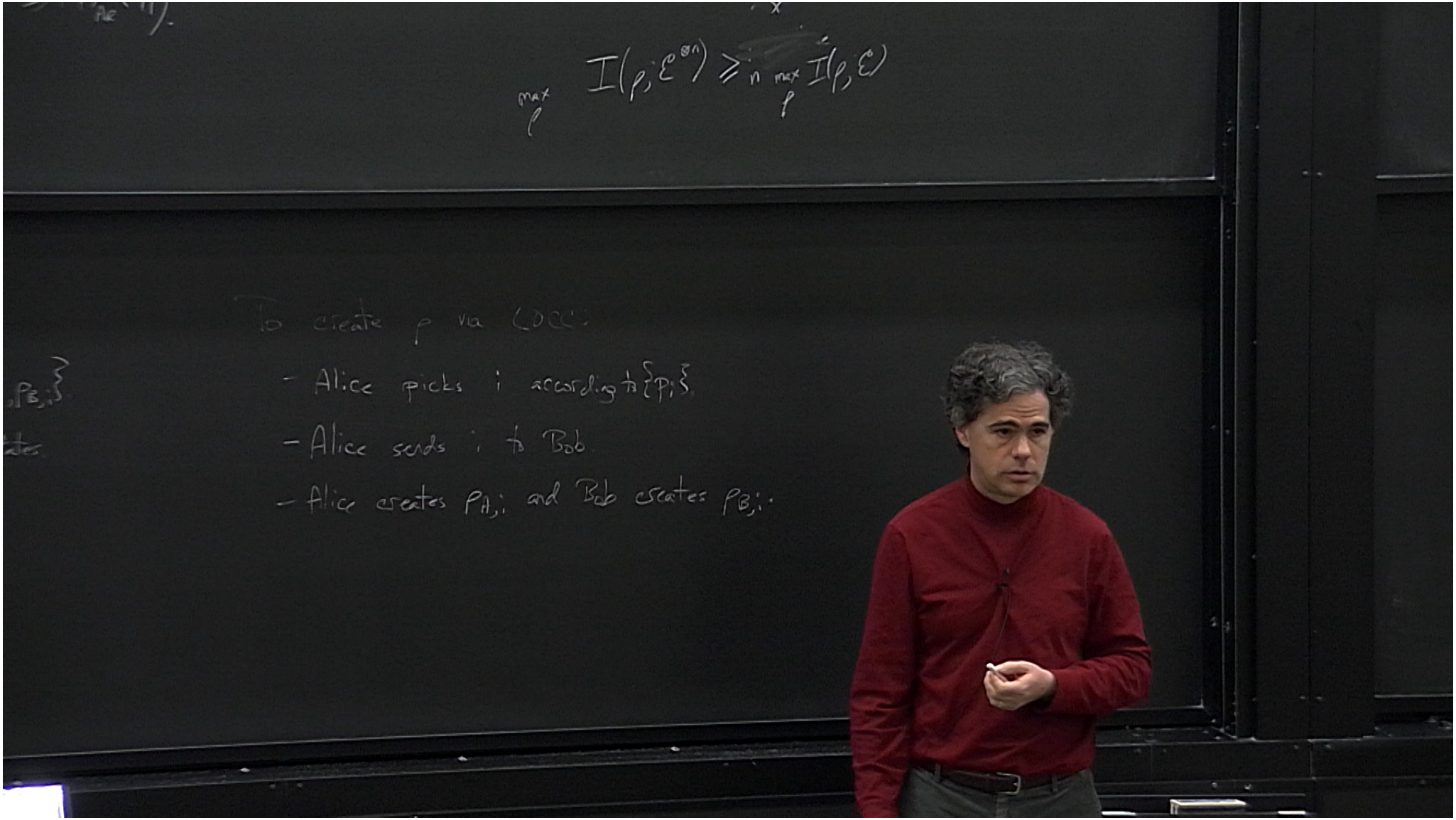
$$\max_{\rho} I(\rho;\mathcal{E}^{\otimes n}) \geq n \max_{\rho} I(\rho;\mathcal{E})$$

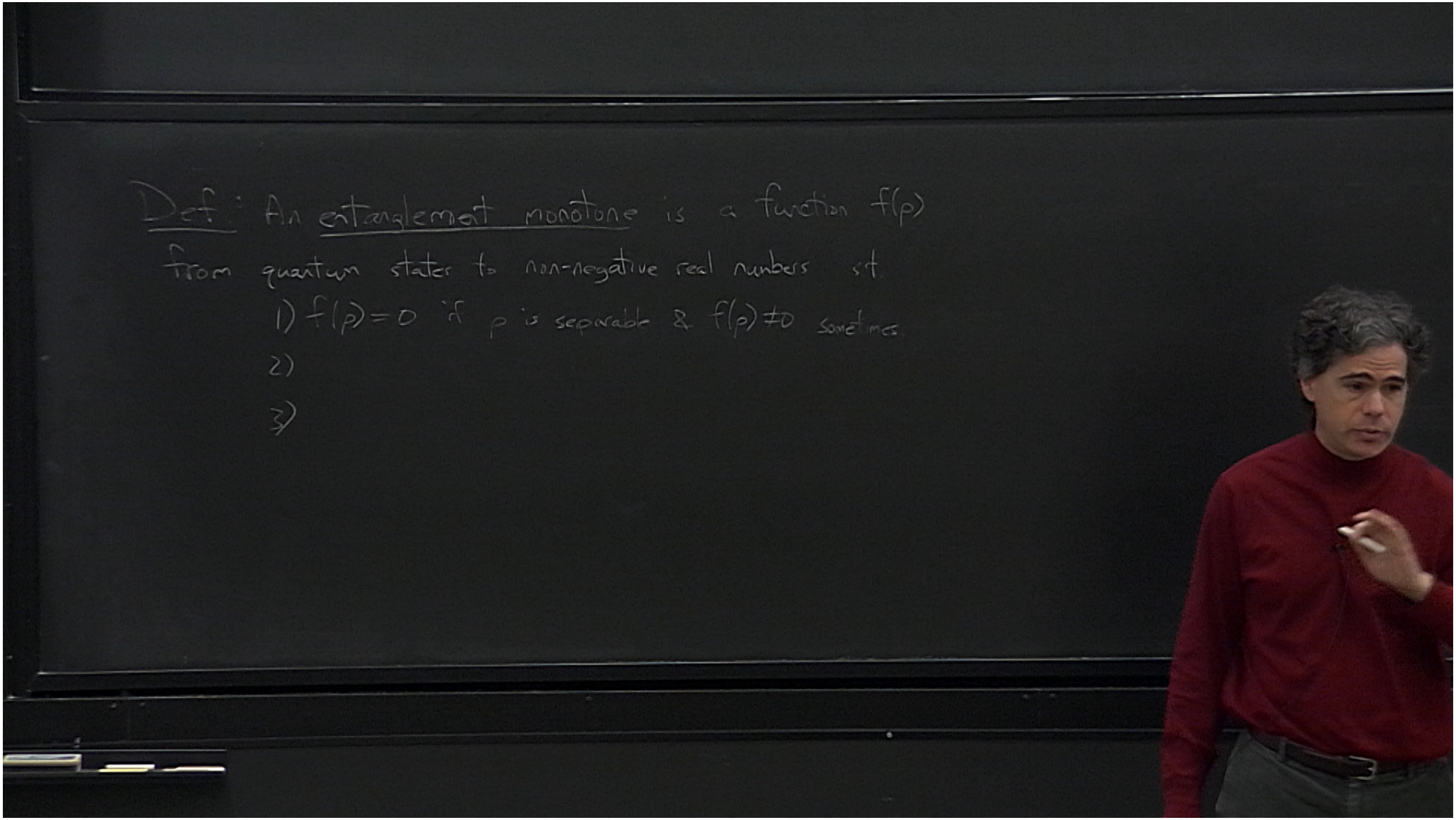
LOCC protocol:

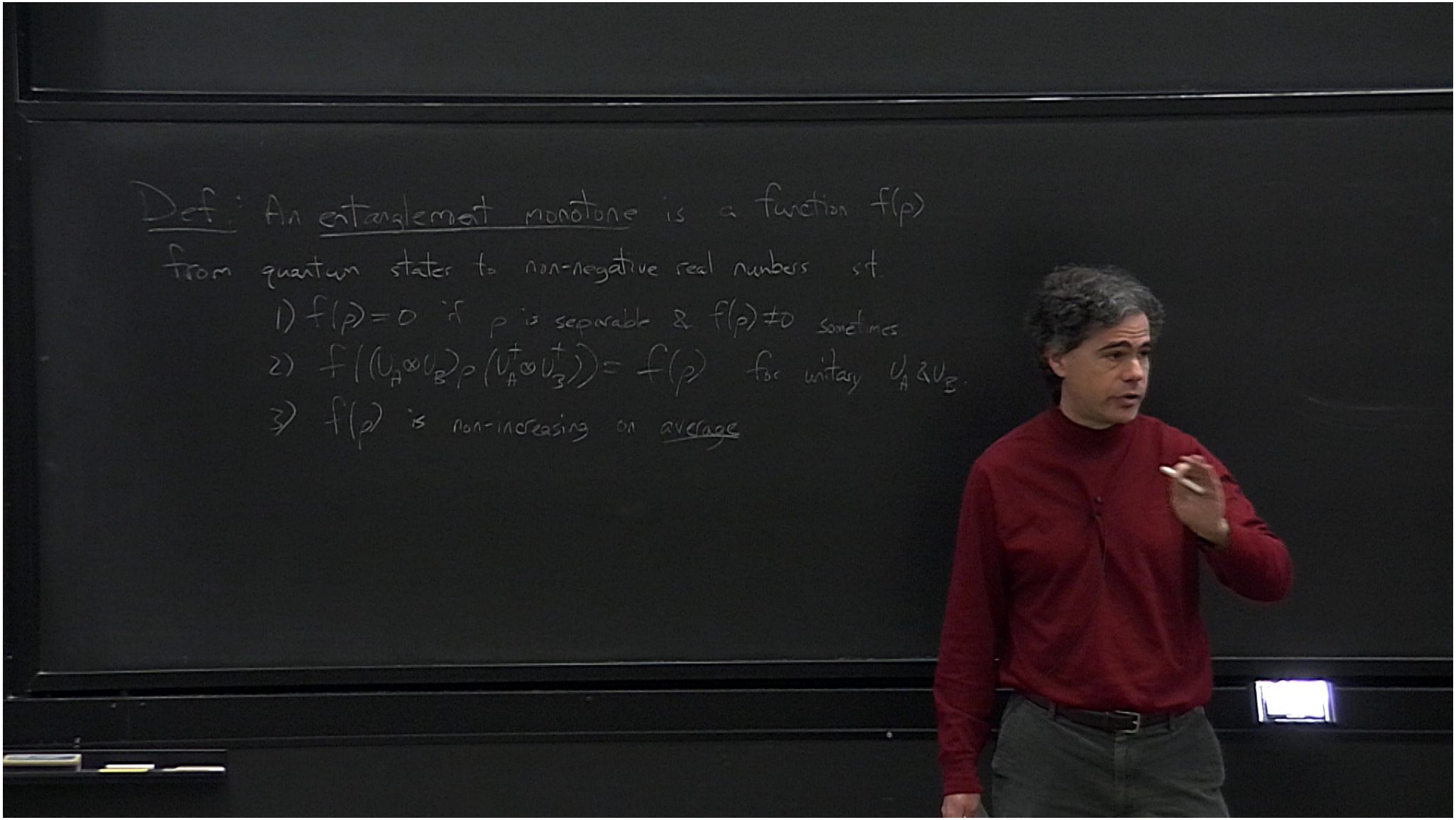
Def:




$\max_\rho I(\rho; \mathcal{E}^{\otimes n}) \geq$
Def: A mixed state ρ is separable
if $\rho = \sum_i p_i \rho_{A,i} \otimes \rho_{B,i}$ for some $\{p_i, \rho_{A,i}, \rho_{B,i}\}$
p_i are probabilities $\rho_{A,i}$ & $\rho_{B,i}$ are quantum states

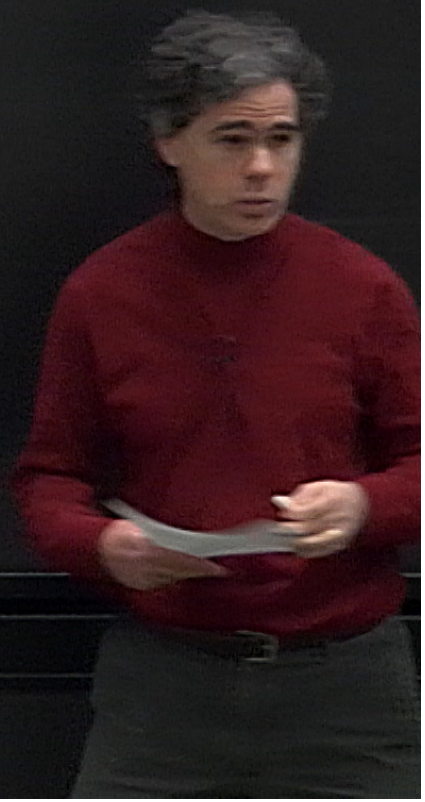

$\max_\rho I(\rho, \mathcal{E}^{\otimes n}) \geq_n \max_\rho I(\rho, \mathcal{E})$
To create ρ via LOCC:
- Alice picks i according to $\{p_i\}$
- Alice sends i to Bob
- Alice creates $\rho_{A,i}$ and Bob creates $\rho_{B,i}$.

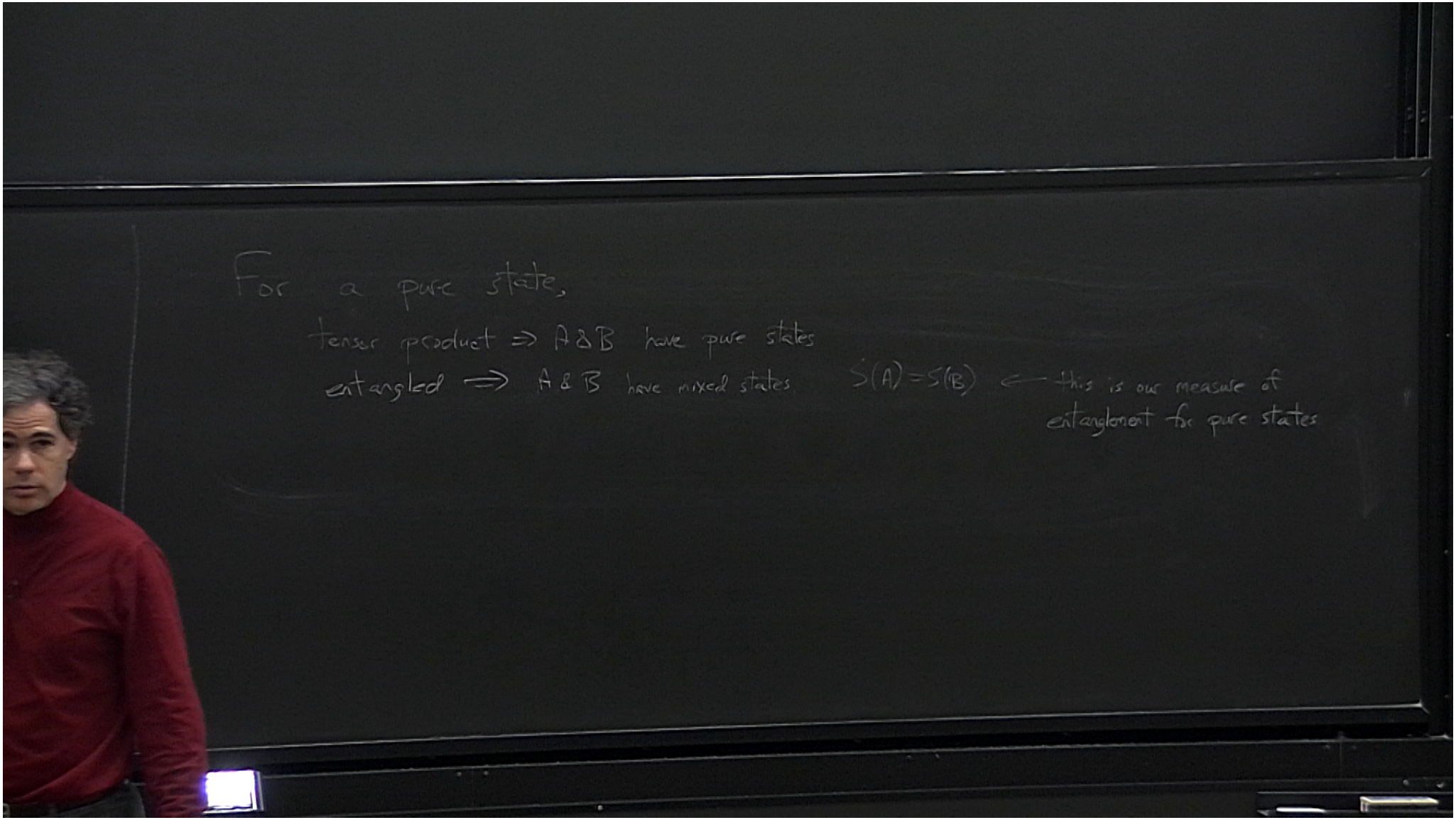

Def: An entanglement monotone is a function f(ρ)
from quantum states to non-negative real numbers s.t.
1) f(ρ) = 0 if ρ is separable & f(ρ) ≠ 0 sometimes
2)
3)

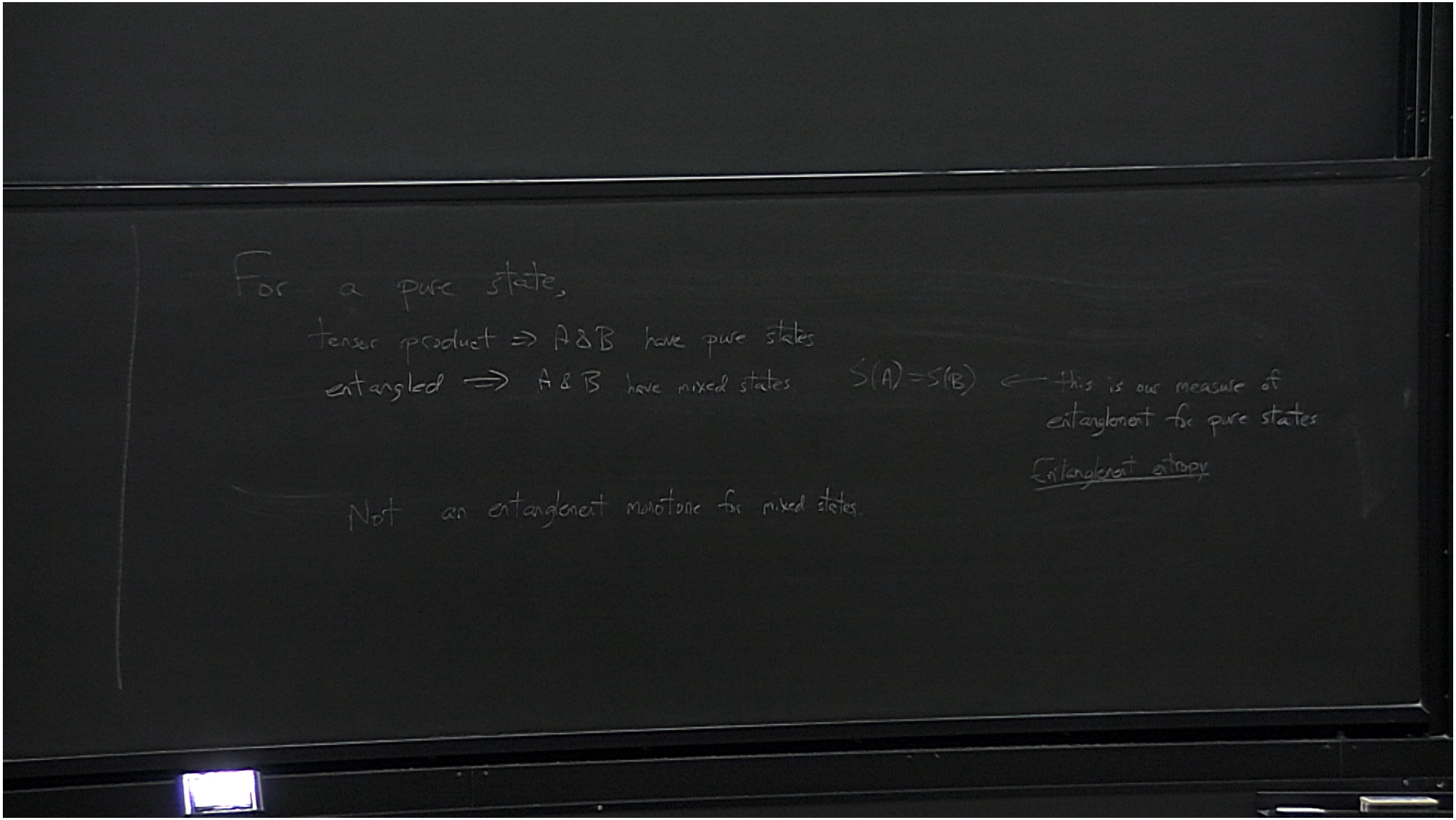

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from quantum states to non-negative real numbers s.t.
1) f(ρ) = 0 if ρ is separable & f(ρ) ≠ 0 sometimes
2) f((U_A ⊗ U_B) ρ (U_A† ⊗ U_B†)) = f(ρ) for unitary U_A & U_B.
3) f(ρ) is non-increasing on average

Def: An entanglement monotone is a function $f(\rho)$ from quantum states to non-negative real numbers s.t.

1) $f(\rho) = 0$ if ρ is separable & $f(\rho) \neq 0$ sometimes.
2) $f((U_A \otimes U_B)\rho(V_A^\dagger \otimes V_B^\dagger)) = f(\rho)$ for unitary U_A & U_B.
3) $f(\rho)$ is non-increasing on average under LOCC.




For a pure state,
tensor product ⇒ A & B have pure states
entangled ⇒ A & B have mixed states
S(A) = S(B) ← this is our measure of entanglement for pure states


For a pure state,
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S(A) = S(B) ← this is our measure of entanglement for pure states
Entanglement entropy
Not an entanglement monotone for mixed states.

Entanglement of formation

$$E_f(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i S(\mathrm{tr}_B |\psi_i\rangle\langle\psi_i|)$$

$\rho \xrightarrow{\text{LOCC}} \sigma$

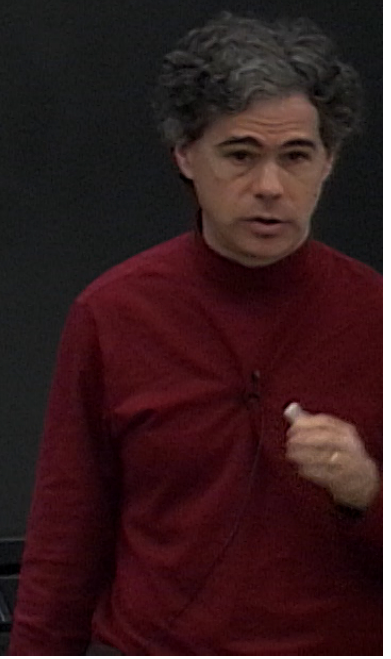
$\rho \leftarrow \{p_i, |\psi_i\rangle\}$

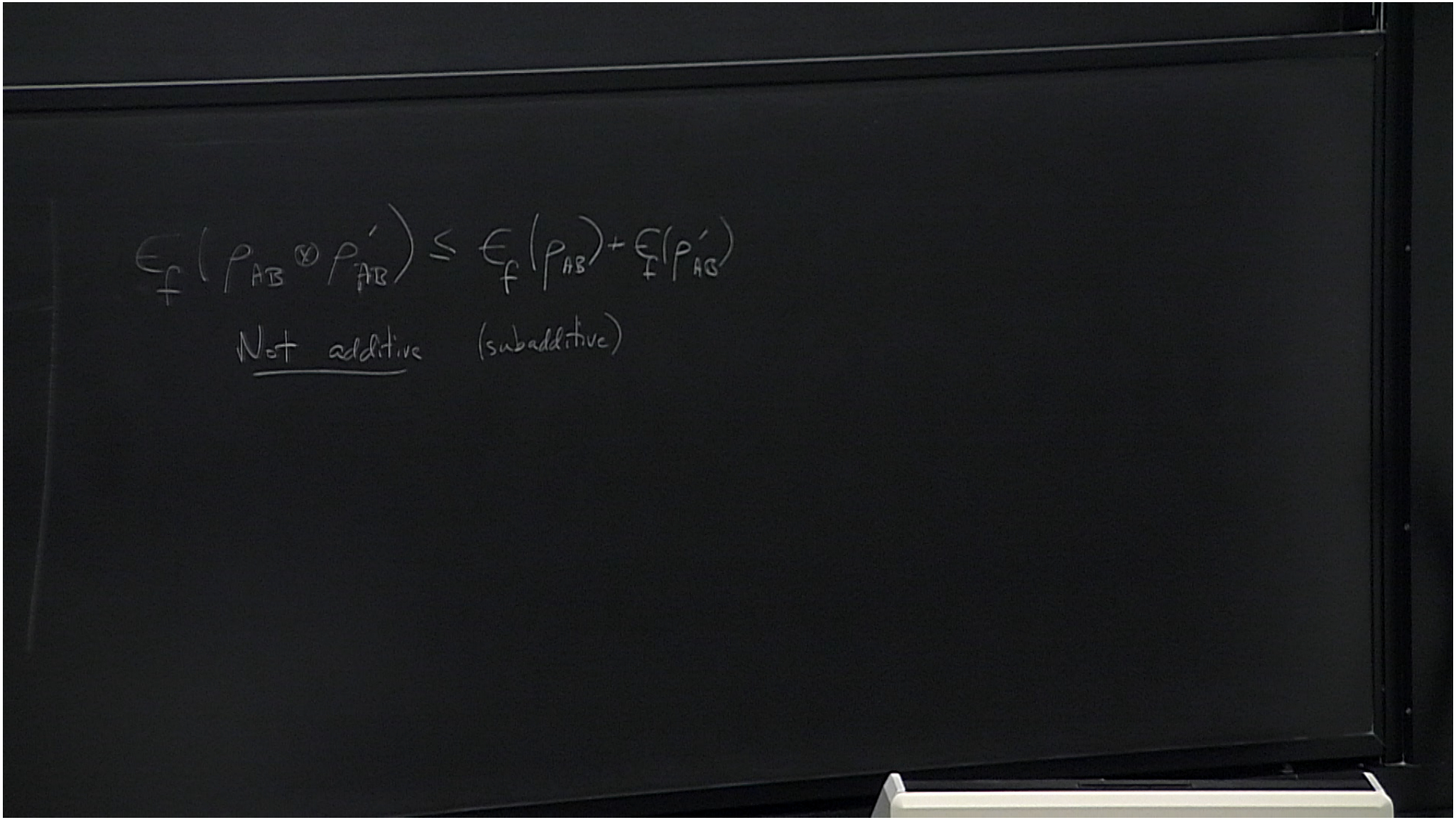
Taken over ensembles $\{p_i, |\psi_i\rangle\}$ s.t. $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

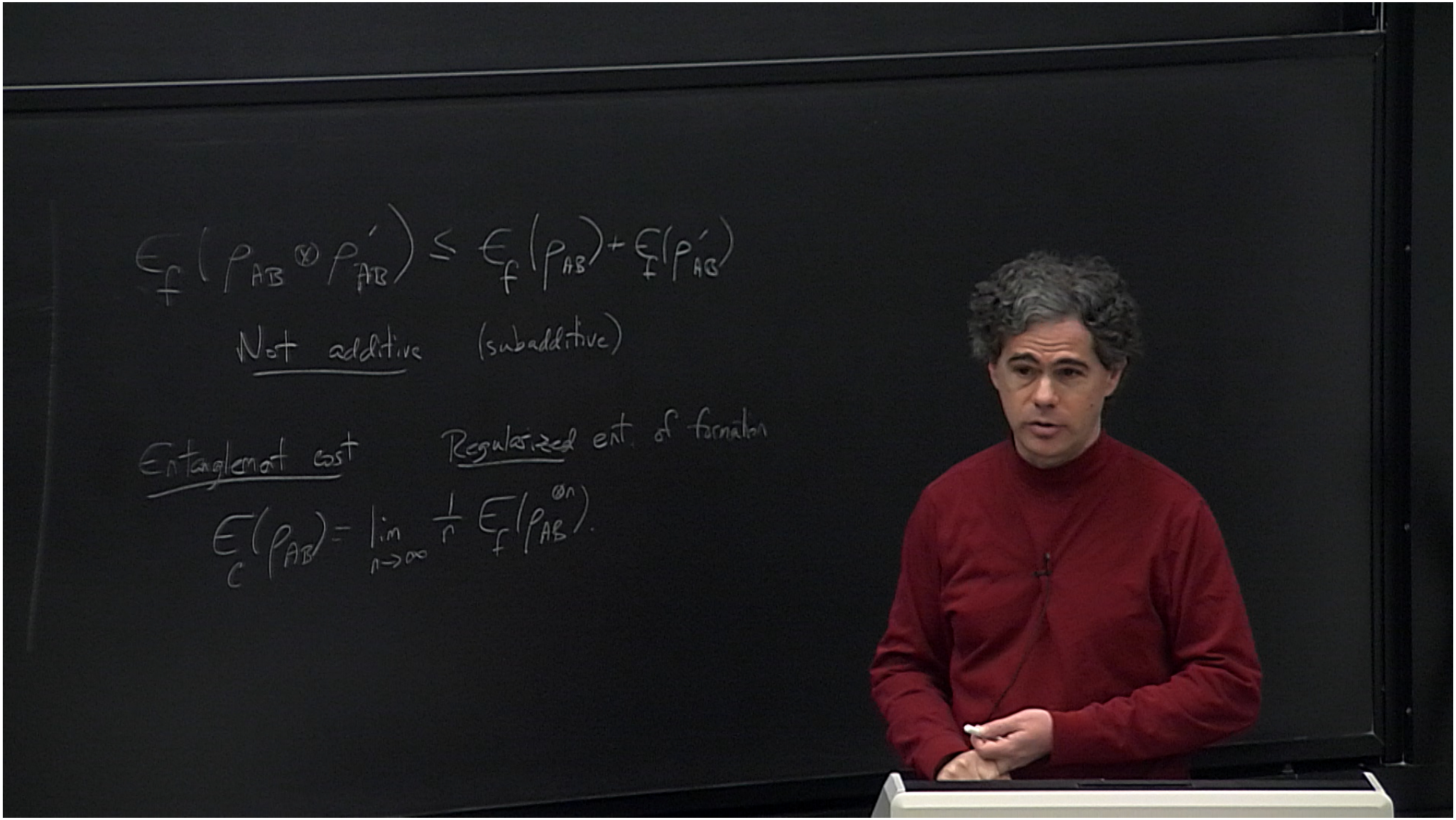
1) 0 for separable states. Reduces to entanglement entropy for pure states

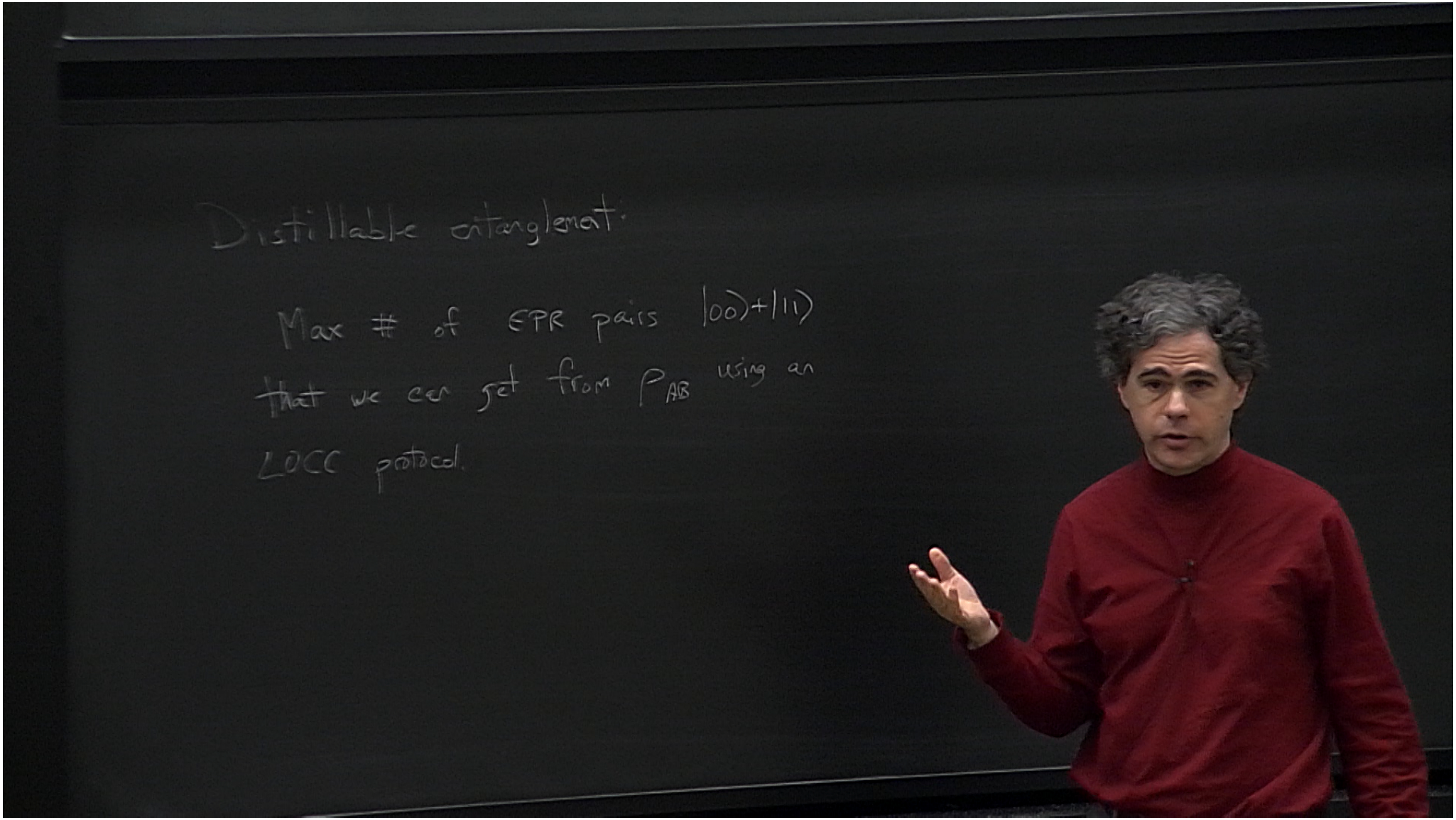
2) ✓

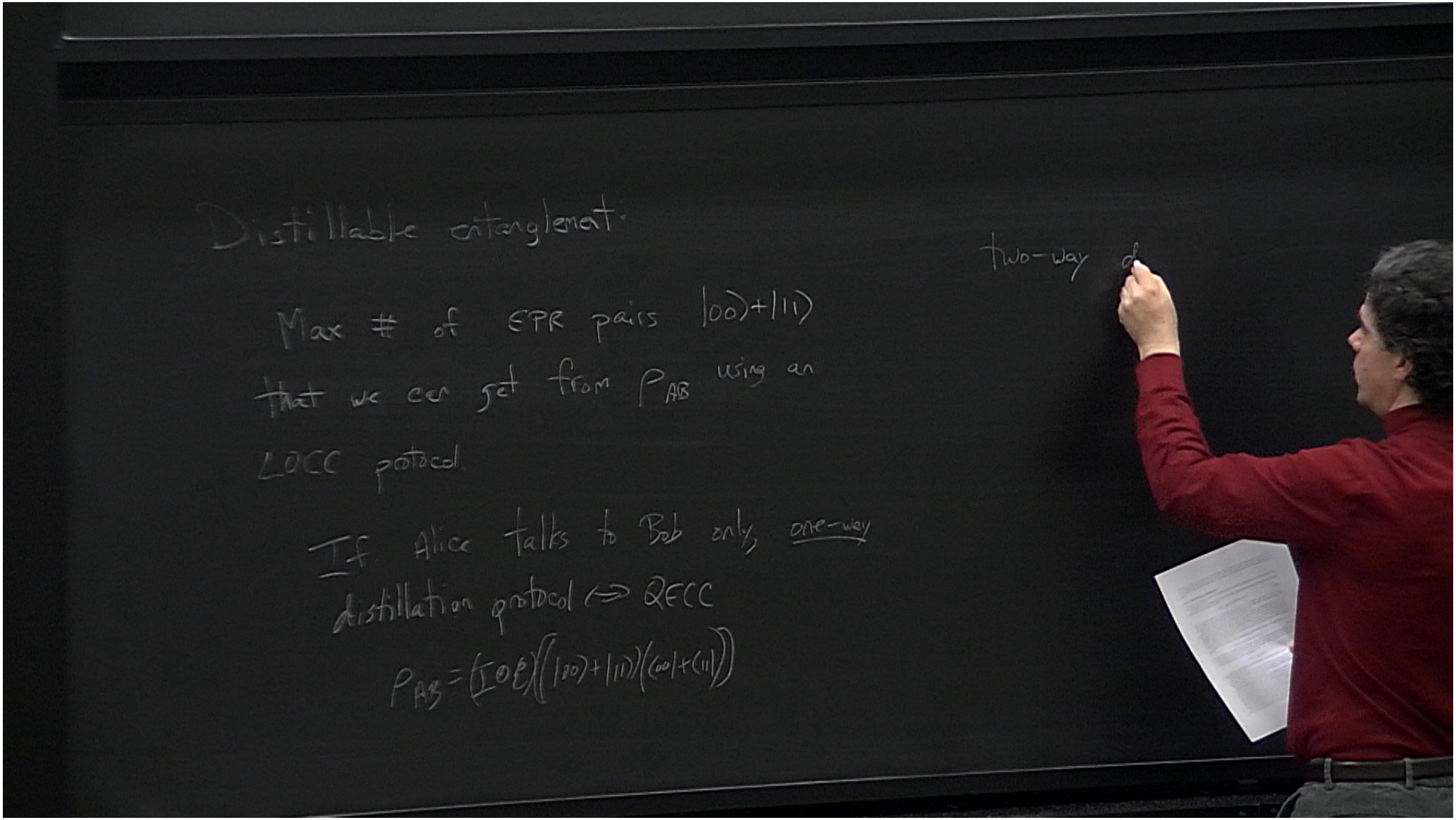
3) ✓

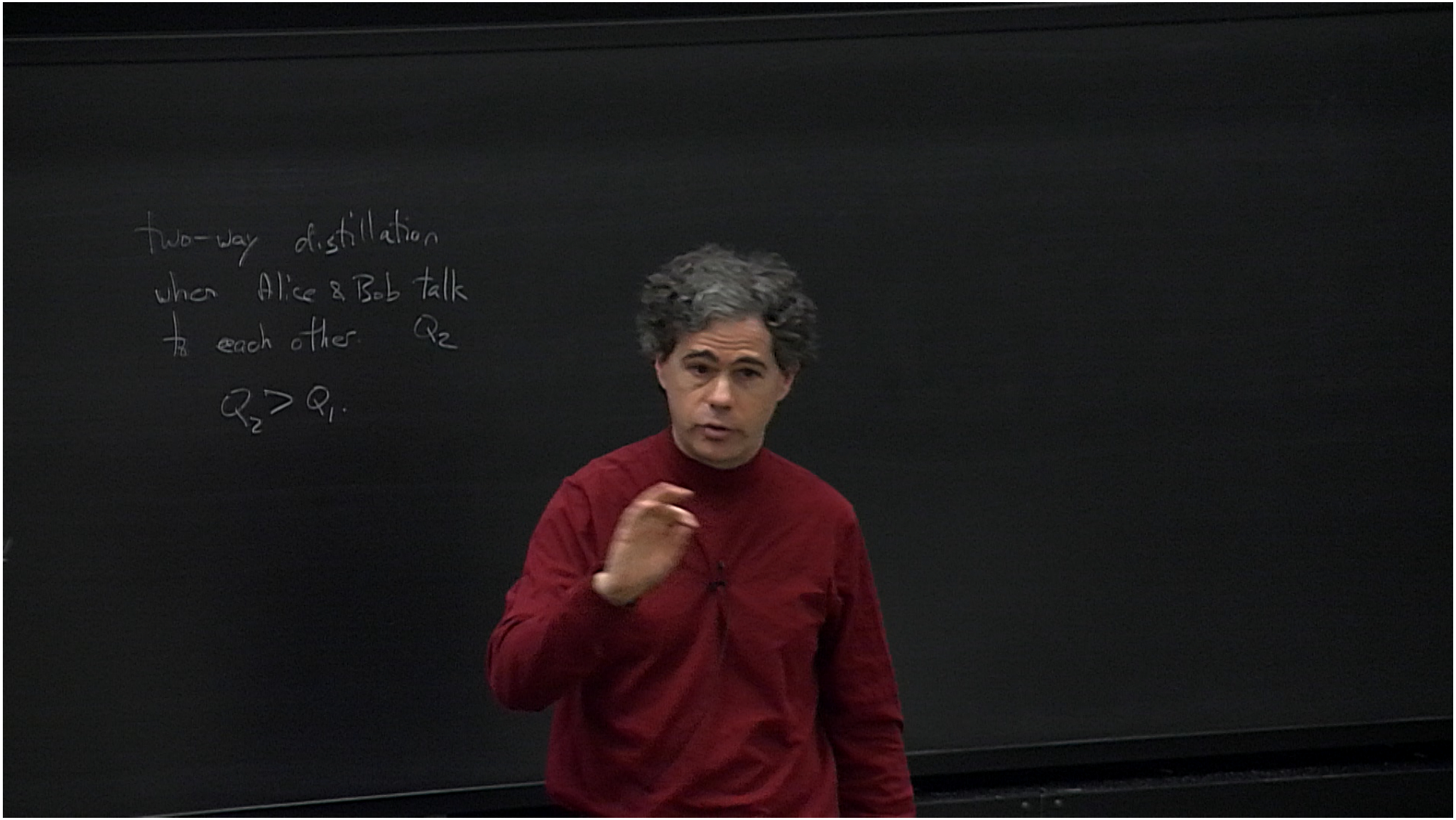


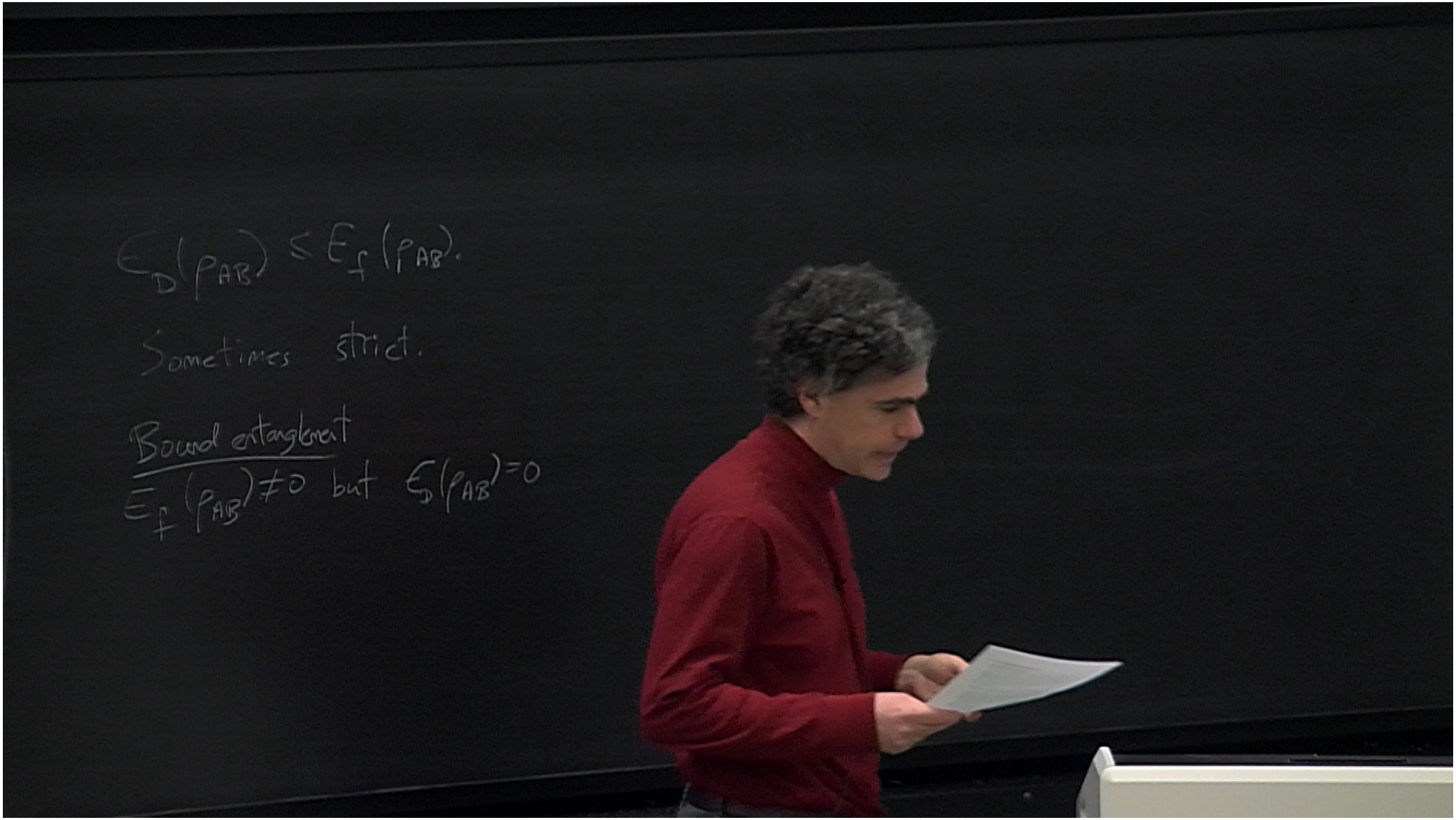

$E_f(\rho_{AB} \otimes \rho'_{AB}) \leq E_f(\rho_{AB}) + E_f(\rho'_{AB})$
Not additive (subadditive)


$E_f(\rho_{AB} \otimes \rho'_{AB}) \leq E_f(\rho_{AB}) + E_f(\rho'_{AB})$
Not additive (subadditive)
Entanglement cost
Regularized ent. of formation
$E_c(\rho_{AB}) = \lim_{n\to\infty} \frac{1}{n} E_f(\rho_{AB}^{\otimes n})$.


Distillable entanglement:
Max # of EPR pairs $|00\rangle+|11\rangle$ that we can get from ρ_{AB} using an LOCC protocol.


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Max # of EPR pairs $|00\rangle+|11\rangle$
that we can get from ρ_{AB} using an
LOCC protocol.
If Alice talks to Bob only, one-way
distillation protocol $\leftrightarrow$ QECC
$\rho_{AB} = (\mathbb{I}\otimes\mathcal{E})(|\Phi\rangle+|11\rangle)(\langle\Phi|+\langle 11|)$
two-way


two-way distillation
when Alice & Bob talk
to each other. Q_2
$Q_2 > Q_1$.


$E_D(\rho_{AB}) \leq E_f(\rho_{AB})$.
Sometimes strict.
Bound entanglement
$E_f(\rho_{AB}) \neq 0$ but $E_D(\rho_{AB}) = 0$

Negativity:

ρ^{T_A} Partial transpose.

$(\rho_A \otimes \rho_B)^{T_A} = \rho_A^T \otimes \rho_B = \rho_A^* \otimes \rho_B$ still a quantum state

On an entangled state, might not be quantum state.
Can have negative eigenvalues.

Negativity is sum of absolute values of negative eigenstates of $\rho_{AB}^{T_A}$.