Title: PSI 2018/2019 - Quantum Information Review - Lecture 11

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Collection: PSI 2018/2019 - Quantum Information Review (Gottesman)

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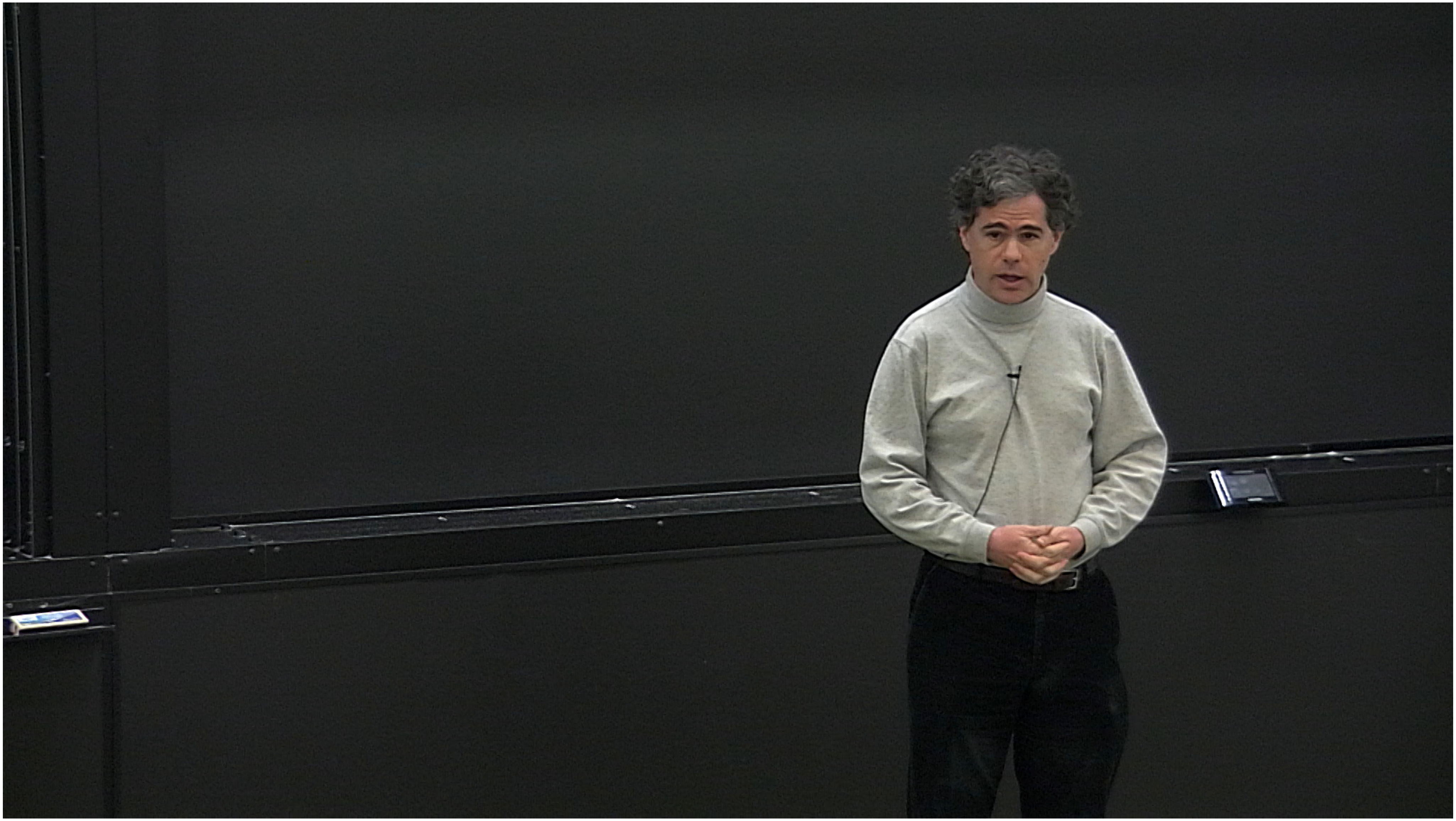
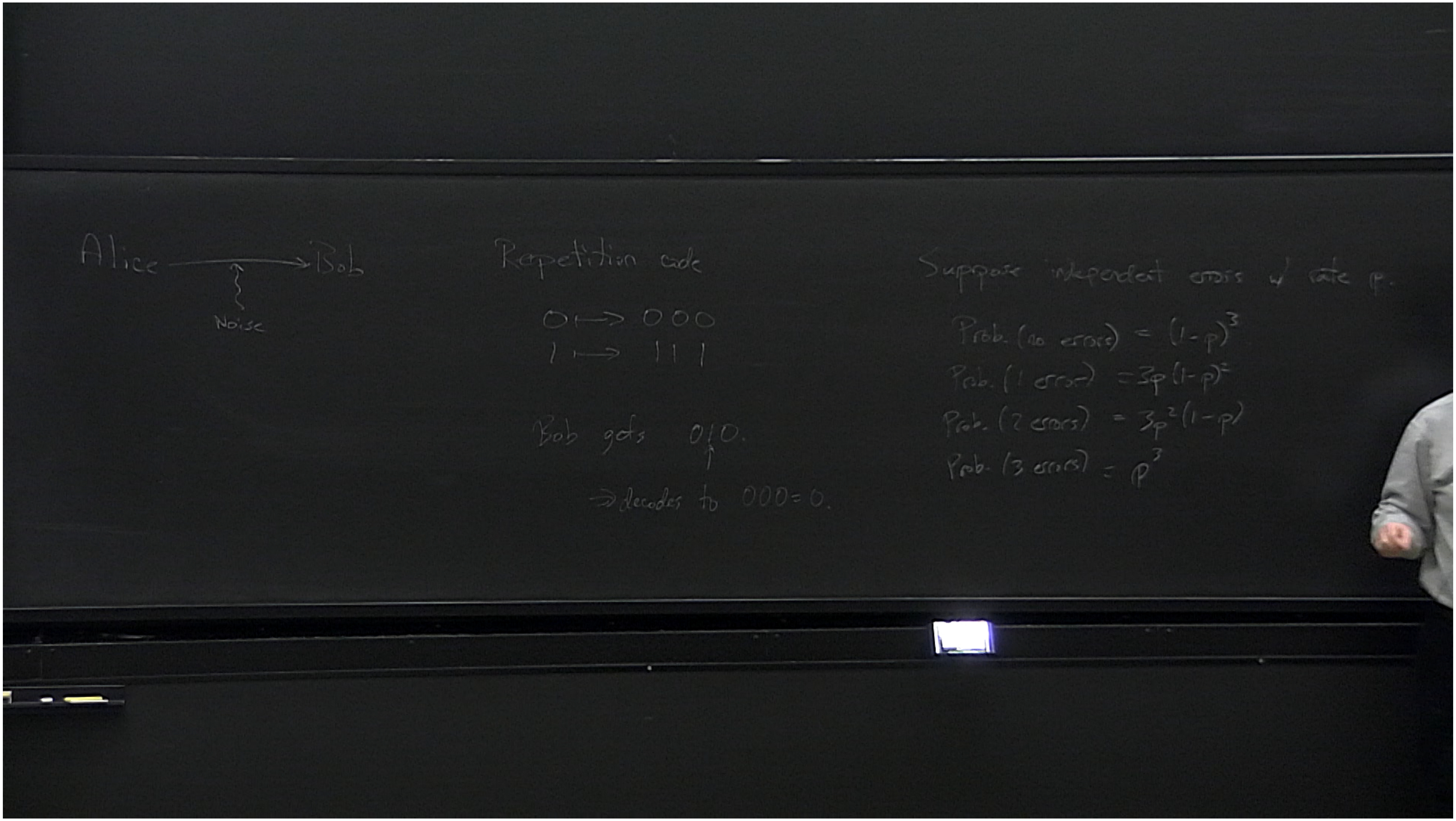



Alice → Bob
Noise
Repetition code
$0 \mapsto 000$
$1 \mapsto 111$
Bob gets 010.
⇒ decodes to 000 = 0.
Suppose independent errors w/ rate p.
Prob. (no errors) = $(1-p)^3$
Prob. (1 error) = $3p(1-p)^2$
Prob. (2 errors) = $3p^2(1-p)$
Prob. (3 errors) = p^3

$$\text{Alice} \xrightarrow{\quad} \text{Bob}$$

Noise

Repetition code

$$0 \longmapsto 000$$
$$1 \longmapsto 111$$

Bob gets 010.

$\Rightarrow$ decodes to $000 = 0$.

Suppose independent errors w/ rate p.

$$\text{Prob.}(\text{no errors}) = (1-p)^3$$

$$\text{Prob.}(1 \text{ error}) = 3p(1-p)^2$$

$\rightarrow$ Bob is correct.

$$\text{Prob.}(2 \text{ errors}) = 3p^2(1-p)$$

$$\text{Prob.}(3 \text{ errors}) = p^3$$

$\rightarrow$ Bob is wrong

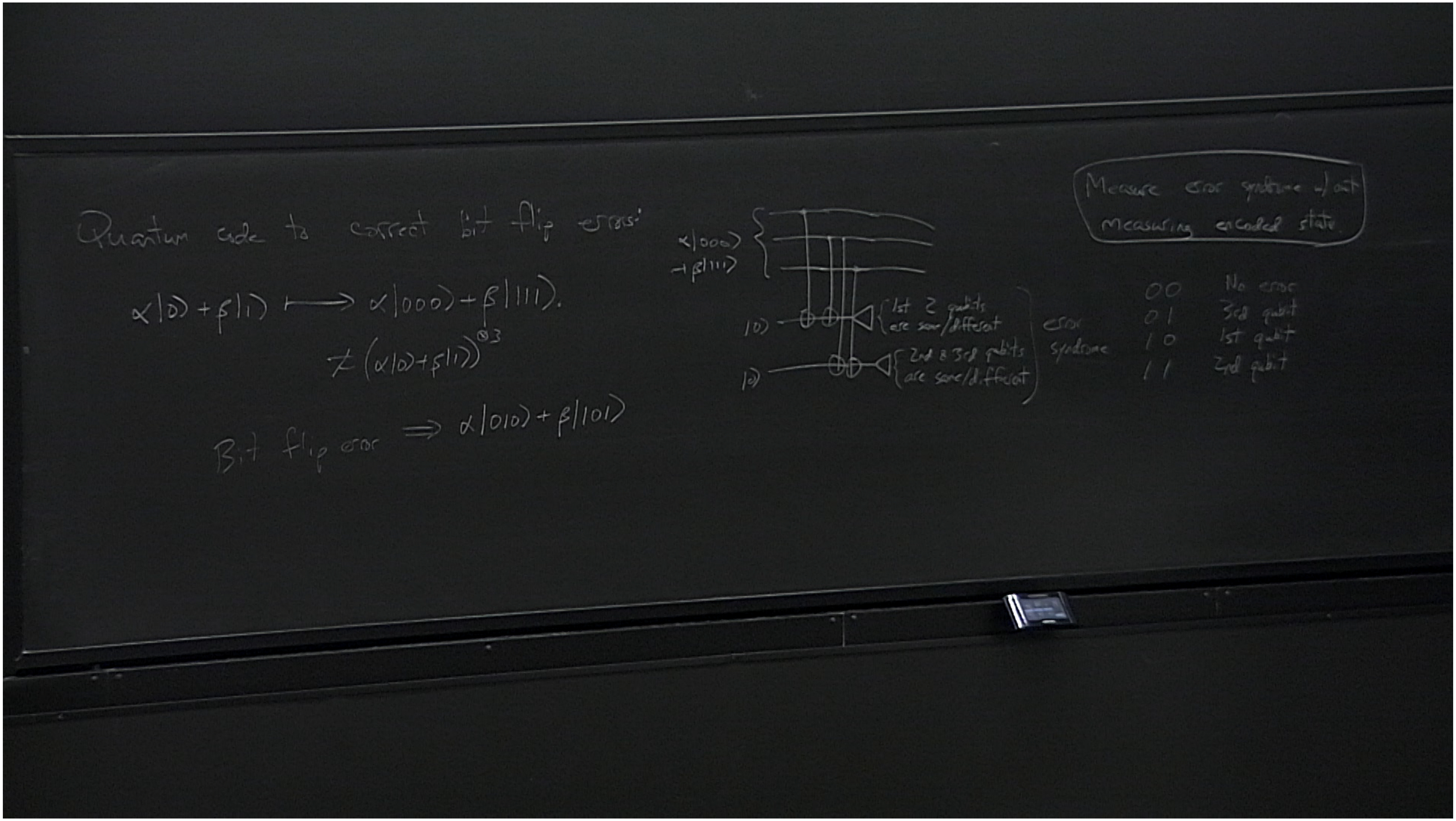
$$O(p^2) < p$$

$$|\psi\rangle \rightarrow |\psi\rangle|\psi\rangle|\psi\rangle.$$

$\ldots\rangle.$

Barriers to Quantum Error Correction:

1. No-Cloning prohibits repeating information
2. Measuring qubits to look for errors collapses superpositions.
3. Need to correct phase errors as well as bit flips.
4. Need to correct continuous rotations, decoherence, ...


Quantum code to correct bit flip errors:
$\alpha|0\rangle + \beta|1\rangle \longmapsto \alpha|000\rangle + \beta|111\rangle$.
$\neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$
Bit flip error $\Longrightarrow \alpha|010\rangle + \beta|101\rangle$
$\alpha|000\rangle + \beta|111\rangle$
$|0\rangle$
$|0\rangle$
1st 2 qubits are same/different
2nd & 3rd qubits are same/different
Measure error syndrome w/out measuring encoded state
error syndrome
00 No error
01 3rd qubit
10 1st qubit
11 2nd qubit

$$H: \quad |0\rangle \longleftrightarrow |+\rangle = |0\rangle + |1\rangle$$

$$|1\rangle \longleftrightarrow |-\rangle = |0\rangle - |1\rangle.$$

$$Z \longleftrightarrow X$$

$$H: |0\rangle \longleftrightarrow |+\rangle = |0\rangle + |1\rangle$$

$$|1\rangle \longleftrightarrow |-\rangle = |0\rangle - |1\rangle.$$

$$Z \longleftrightarrow X$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|+++\rangle + \beta|---\rangle.$$

$$Z \text{ error} \rightarrow \alpha|-++\rangle + \beta|+--\rangle.$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

3-qubit phase error correcting code.

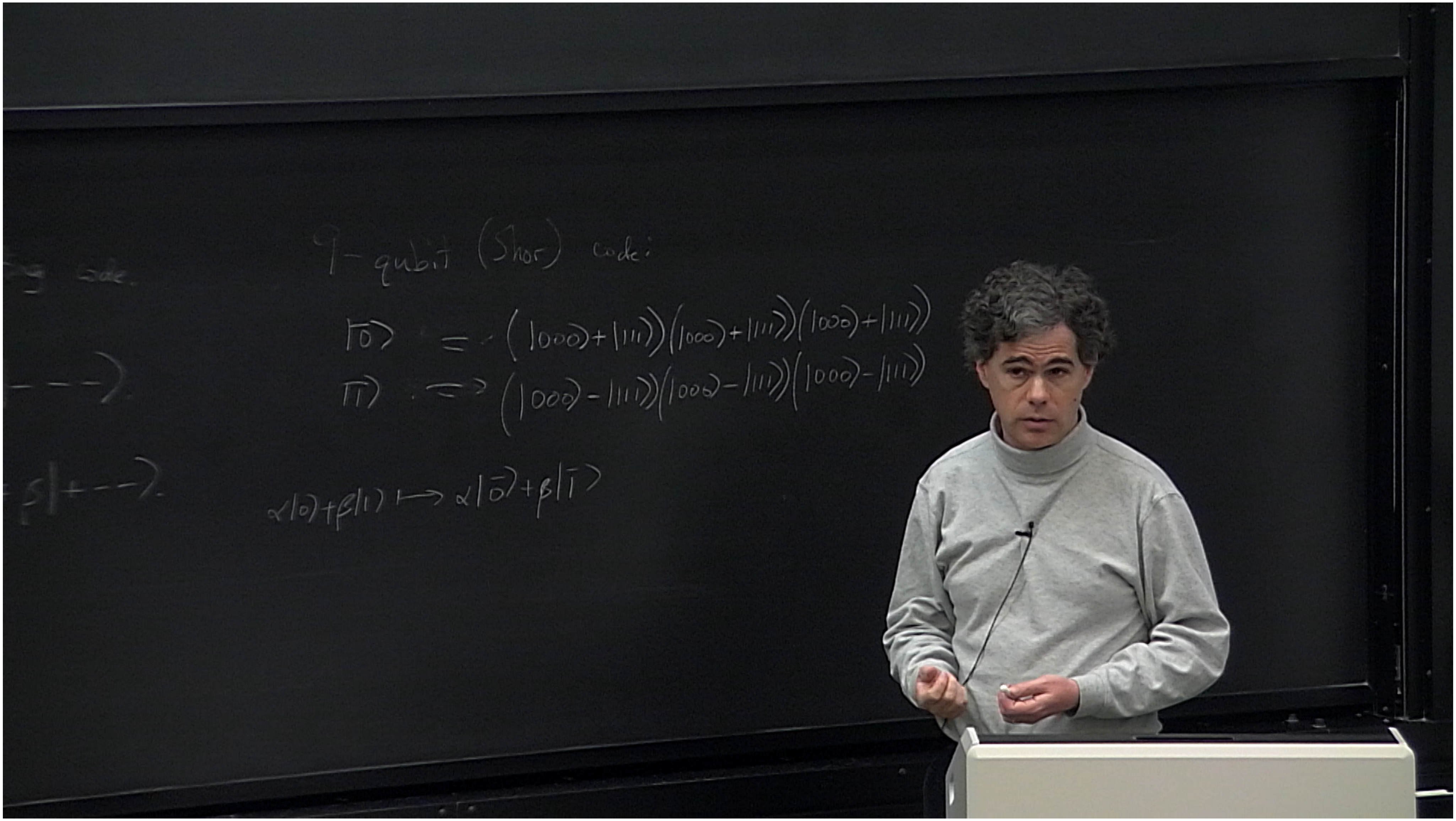
$H: |0\rangle \longleftrightarrow |+\rangle = |0\rangle + |1\rangle$

$\quad |1\rangle \longleftrightarrow |-\rangle = |0\rangle - |1\rangle.$

$Z \longleftrightarrow X$

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|+++\rangle + \beta|---\rangle.$

$Z \text{ error} \rightarrow \alpha|-++\rangle + \beta|+--\rangle.$


9-qubit (Shor) code:
$|\bar{0}\rangle = (|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)$
$|\bar{1}\rangle = (|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$
$\alpha|0\rangle+\beta|1\rangle \mapsto \alpha|\bar{0}\rangle+\beta|\bar{1}\rangle$

$$R_{2\theta} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\theta} \end{pmatrix} = e^{i\theta} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = \cos\theta\, I - i \sin\theta\, Z$$

$$R_{2\theta}^{(j)} |\psi\rangle |0\rangle_{syn} = \cos\theta\, |\psi\rangle |0\rangle_{syn} - i \sin\theta\, Z^{(j)} |\psi\rangle |0\rangle_{syn}$$

$$\xrightarrow{\text{w/ error}} \cos\theta\, |\psi\rangle |\text{no error}\rangle_{syn} - i \sin\theta\, Z^{(j)} |\psi\rangle | Z^{(j)}\rangle_{syn}$$

Measure syndrome:

"No error" w/ prob $\cos^2\theta$, remaining state $|\psi\rangle$

"$Z^{(j)}$" w/ prob $\sin^2\theta$, " " $Z^{(j)}|\psi\rangle$

Thm: If a QECC corrects errors A & B, then it also corrects $\alpha A + \beta B$.

Cor: A QECC that corrects I, X, Y, Z therefore corrects arbitrary single-qubit errors.

If it corrects t-qubit Paulis, then it corrects arbitrary t-qubit errors.

$$|\psi\rangle\langle\psi| \longmapsto \sum_k A_k |\psi\rangle\langle\psi| A_k^\dagger$$

w/ some prob. $|\psi\rangle \to A_k|\psi\rangle \xhookrightarrow[\text{QECC}]{} |\psi\rangle$

$\rangle \longrightarrow A_k|\psi\rangle$

$\hookrightarrow |\psi\rangle$

QECC

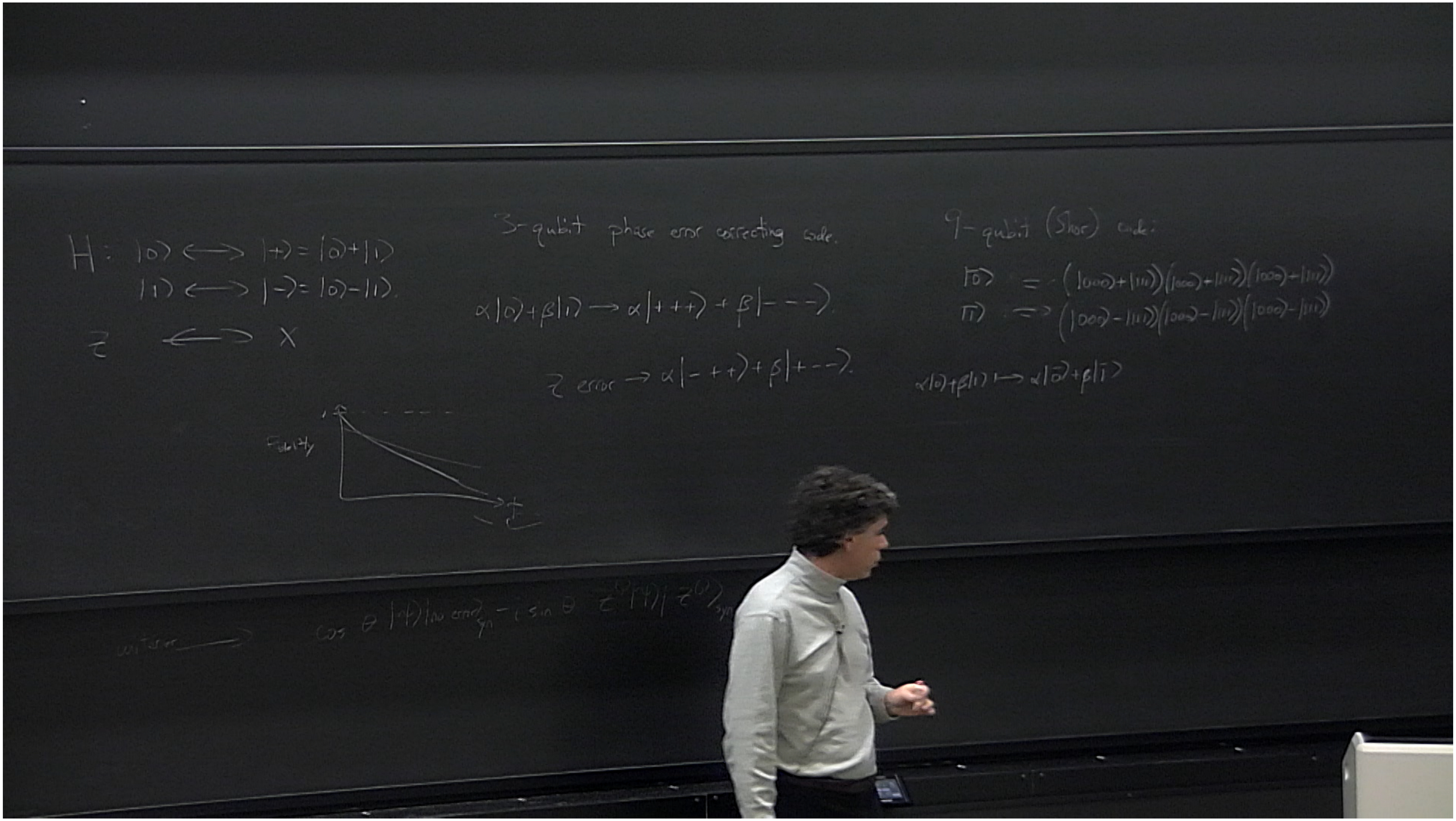
$U = I + \varepsilon E$ on every qubit, ε small

$$U^{\otimes n}|\psi\rangle = |\psi\rangle + \varepsilon\left(E^{(1)} + E^{(2)} + E^{(3)} + \dots + E^{(n)}\right)|\psi\rangle$$

$$+ \varepsilon^2\left(E^{(1)} \otimes E^{(2)} + E^{(1)} \otimes E^{(3)} + \dots\right)|\psi\rangle + \varepsilon^3 \dots$$

QECC corrects single-qubit errors

$$\longrightarrow |\psi\rangle + O(\varepsilon^2)$$


H: |0⟩ ⟷ |+⟩ = |0⟩+|1⟩
|1⟩ ⟷ |−⟩ = |0⟩−|1⟩.
Z ⟷ X
3-qubit phase error correcting code.
α|0⟩+β|1⟩ → α|+++⟩ + β|−−−⟩.
Z error → α|−++⟩+β|+−−⟩.
9-qubit (Shor) code:
|0̄⟩ = (|000⟩+|111⟩)(|000⟩+|111⟩)(|000⟩+|111⟩)
|1̄⟩ = (|000⟩−|111⟩)(|000⟩−|111⟩)(|000⟩−|111⟩)
α|0⟩+β|1⟩ ↦ α|0̄⟩+β|1̄⟩
Fidelity