

Title: PSI 2018/2019 - Quantum Information Review - Lecture 10

Speakers: Daniel Gottesman

Collection: PSI 2018/2019 - Quantum Information Review (Gottesman)

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URL: <http://pirsa.org/19030047>

Database Search

Unstructured search:

Given an oracle $O: \mathbb{Z}_N \rightarrow \mathbb{Z}_2$.

Find if $\exists x_0$ s.t. $O(x_0) = 1$.

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Classically this takes $\Omega(N)$ queries.

$$\Theta(N)$$

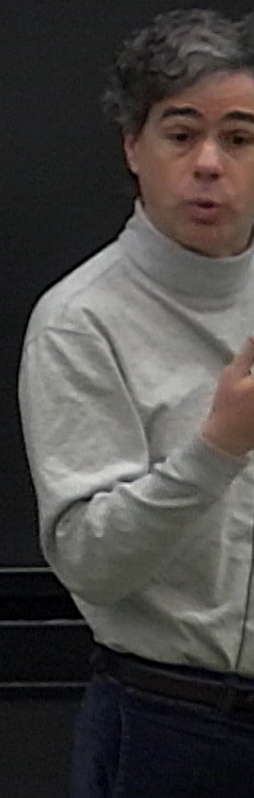
Grover's algorithm:
uses $\sim \sqrt{N}$ queries

Lower bound $\Omega(\sqrt{N})$.

$\rightarrow \mathbb{R}_2$.

$\rangle = 1$.

$\rangle$ queries.



Database Search

Unstructured Search:

Given an oracle $O: \mathbb{Z}_N \rightarrow \mathbb{Z}_2$.

Find if $\exists x_0$ s.t. $O(x_0) = 1$.
 x_0 is "marked"

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Only 1 (or 0) marked element

$$|S\rangle = |x_0\rangle$$

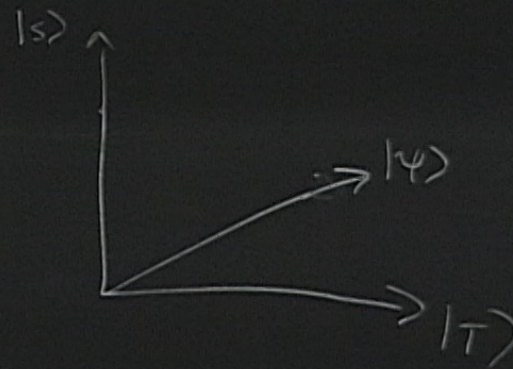
$$|T\rangle = \sum_{x \neq x_0} |x\rangle$$

Only 1 (or 0) marked element

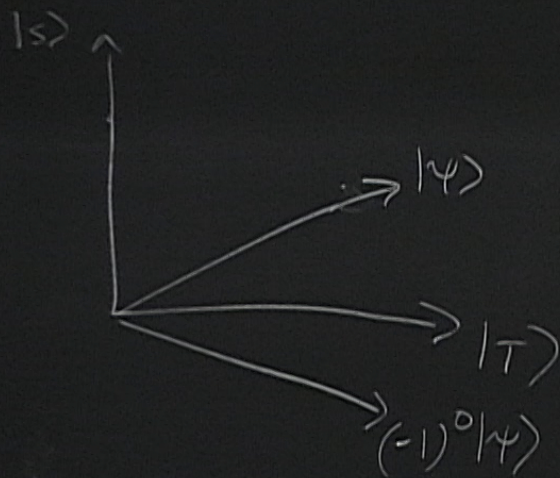
$$|S\rangle = |x_0\rangle$$

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2-dimensional Hilbert space
spanned by $|S\rangle$ & $|T\rangle$

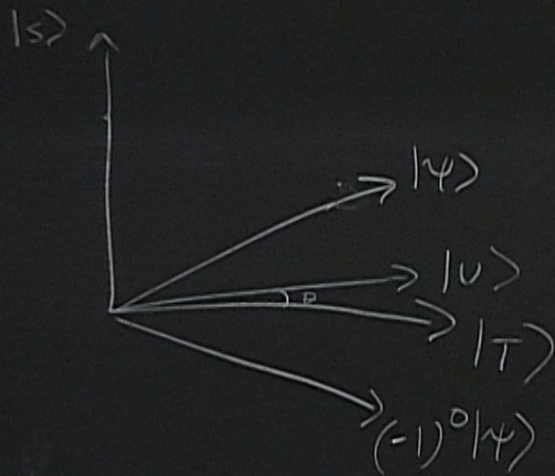


$$|\psi\rangle = \alpha|S\rangle + \beta|T\rangle$$



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$$|U\rangle = H^{\otimes n} |0 \dots 0\rangle = |S\rangle + |T\rangle$$

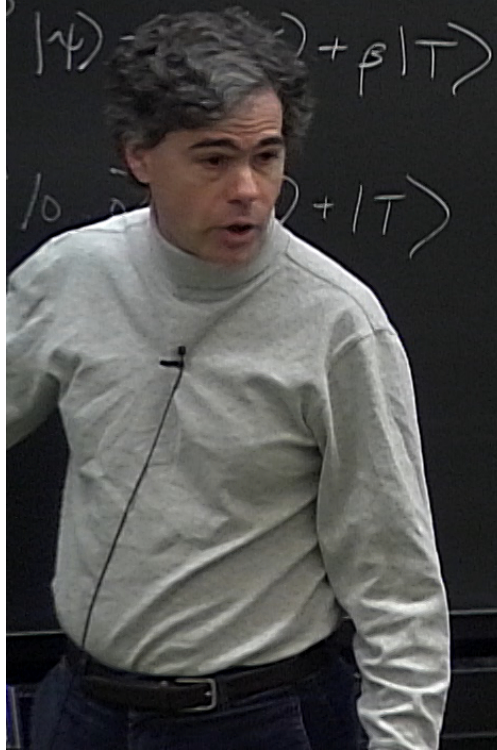
$$+ \beta |T\rangle$$

$$|\psi\rangle = -\alpha |S\rangle + \beta |T\rangle$$

$$|0\rangle = |S\rangle + |T\rangle$$

$$F_0 |0\rangle = |0\rangle$$

$$F_0 |y\rangle = -|y\rangle \quad y \neq 0$$



$$F_0 |00\rangle = |00\rangle$$

$$F_0 |y\rangle = -|y\rangle \quad y \neq 0, 0$$

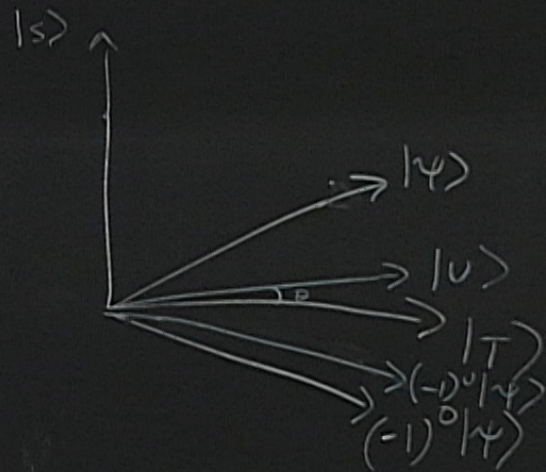
$$(-1)^U = H^{\otimes n} F_0 H^{\otimes n} \Rightarrow \text{Reflection over } |U\rangle$$

$$(-1)^U |U\rangle = |U\rangle$$

$$(-1)^U |\perp U\rangle = -|\perp U\rangle$$

marked element

$|x\rangle$



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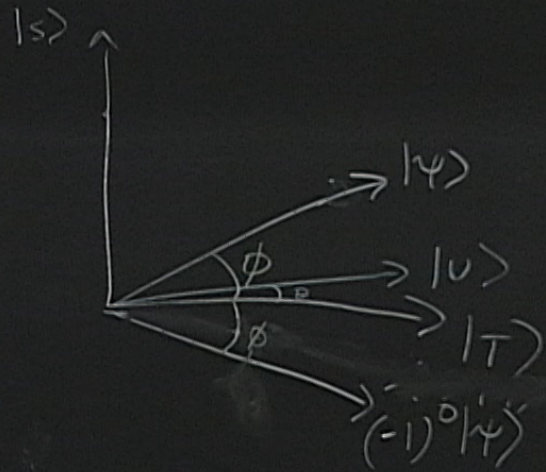
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marked element

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1 bit space

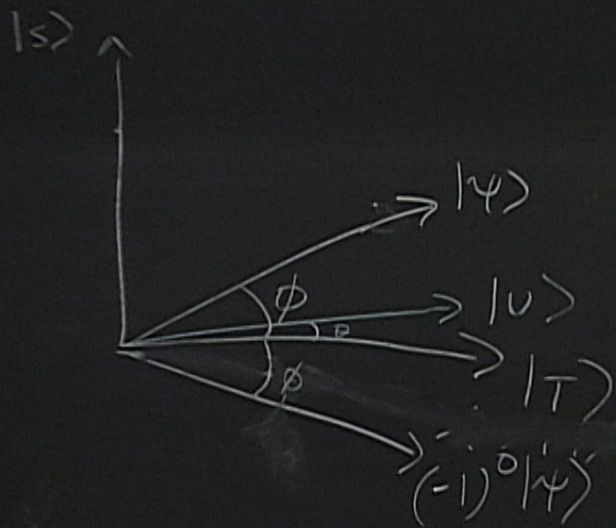
$|S\rangle$ & $|T\rangle$



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$$G = (-1)^U (-1)^0$$

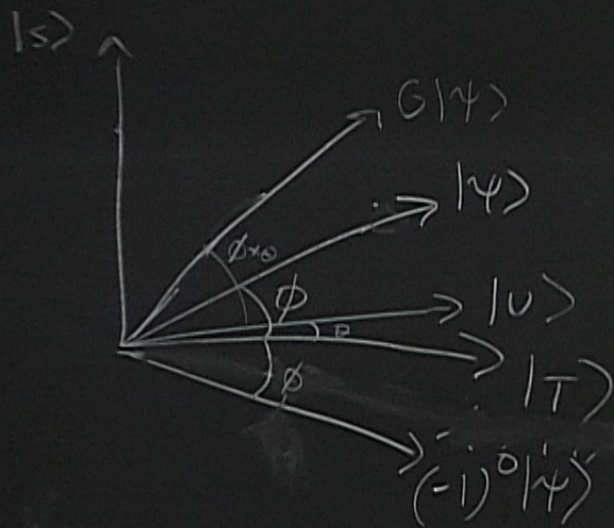
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$$F_0 |y\rangle$$

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$$(-1)^U |U\rangle$$

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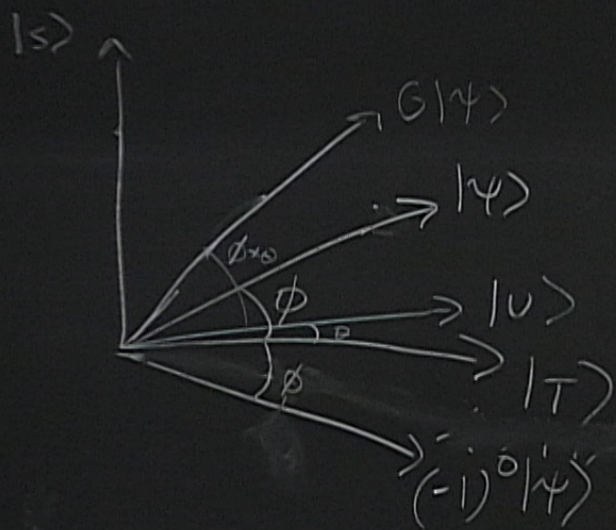
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$$(-1)^U$$



$$\phi \xrightarrow{G} \phi + 2\theta$$

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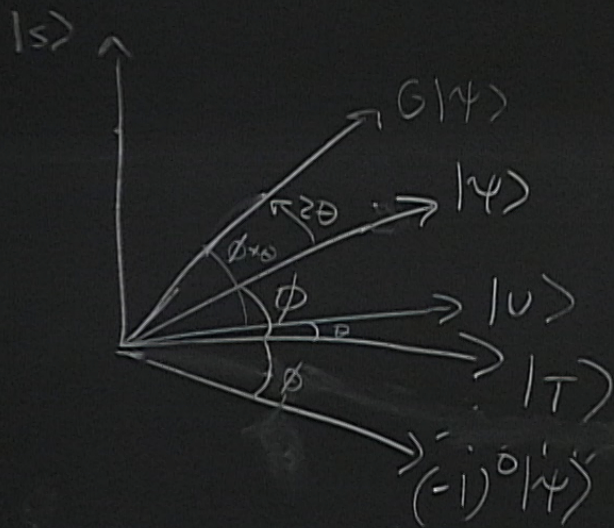
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Grover's algorithm:

- 1) Initialize to state $|\psi_0\rangle = |0\rangle = H^{\otimes n} |0\rangle \rightarrow \phi_0 = \theta$
 - 2) Apply $G = (-1)^U (-1)^O$ M times. $\rightarrow |\psi_k\rangle = G|\psi_{k-1}\rangle \quad \phi_k = \phi_{k-1} + 2\theta = (2k+1)\theta$
 - 3) Measure to get a candidate for x_0 .
 - 4) Use classical oracle to check $O(x_0)$, repeat if $O(x_0) \neq 1$.
- $M+1$ queries.

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$M+1$ queries.

$$\begin{aligned} |\psi_k\rangle &= \cos \phi_k |T\rangle + \sin \phi_k |S\rangle \\ &= \cos((2k+1)\theta) |T\rangle + \sin((2k+1)\theta) |S\rangle \end{aligned}$$

$$\text{Want } (2M+1)\theta \approx \frac{\pi}{2}.$$

$$M \sim \frac{\pi}{4} \frac{1}{\theta} \sim \frac{\pi}{4} \sqrt{N}.$$

$$\text{Normalized } |U\rangle \rightarrow \frac{1}{\sqrt{N}} |U\rangle$$

$$\text{Normalized } |T\rangle \rightarrow \frac{1}{\sqrt{N-1}} |T\rangle$$

$$\cos \theta = \langle U|T \rangle \frac{1}{\sqrt{N(N-1)}} = \sqrt{\frac{N-1}{N}}$$

$$\sin \theta = \frac{1}{\sqrt{N}} \approx \theta$$

+ marked elements:

$$|S\rangle = \sum_{x|D(x)=1} |x\rangle$$

$$|T\rangle = \sum_{x|D(x)=0} |x\rangle$$

$$|U\rangle = |S\rangle + |T\rangle = \sum_x |x\rangle$$

$$\text{Normalized } |T\rangle \rightarrow \frac{1}{\sqrt{N-t}} |T\rangle$$

$$\cos \theta = \frac{1}{\sqrt{N(N-t)}} \langle U | T \rangle = \sqrt{\frac{N-t}{N}}$$

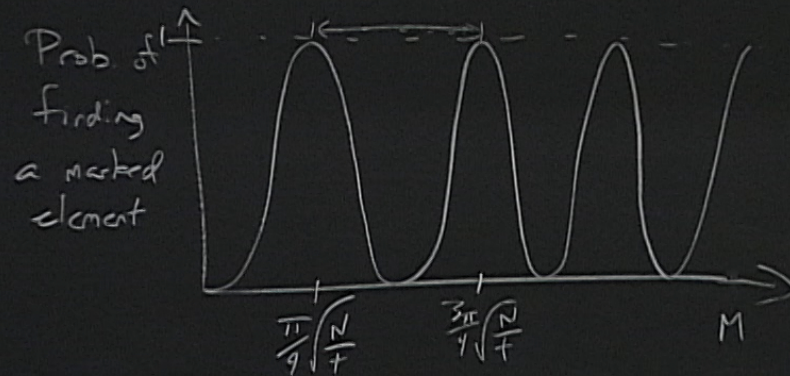
$$\sin \theta = \sqrt{\frac{t}{N}} \approx \theta$$

$$\Rightarrow M \in \frac{\pi}{4} \sqrt{\frac{N}{t}}.$$

L -dimensional Hilbert space
spanned by $|s\rangle$ & $|t\rangle$

$$\phi \xrightarrow{G} \phi + 2\theta$$

$$G = (-1)$$



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$$G = (-1)(-1)$$

Run Grover's algorithm for t iterations,
where k is a superposition

$\sum_k |k\rangle$, do Fourier transform
to determine t .



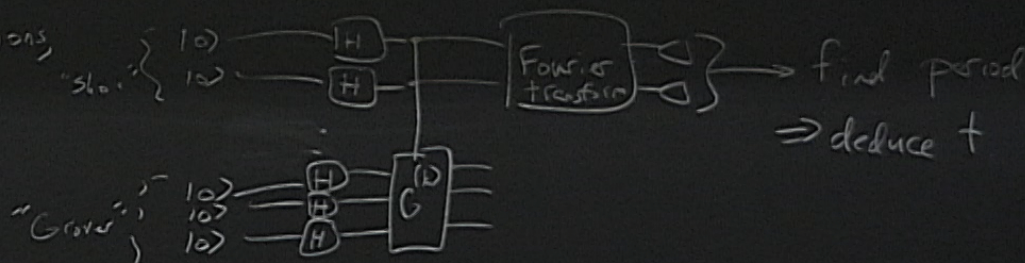
$$\phi \xrightarrow{G} \phi + 2\theta$$

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$$(-1) \quad 1 \quad 0 \quad 1 \quad 0$$

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Amplitude Amplification:

$$U |0\rangle = \sqrt{1-\varepsilon} |\psi\rangle + \sqrt{\varepsilon} |\phi\rangle$$

Want $|\phi\rangle$

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Purify measurement to get $(-1)^0$: $|\psi\rangle \rightarrow |\psi\rangle$
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Amplitude Amplification:

$$(-1)^0:$$

$$U |0\rangle = \sqrt{1-\varepsilon} |\psi\rangle + \sqrt{\varepsilon} |\phi\rangle = |u\rangle$$

Want $|\phi\rangle$

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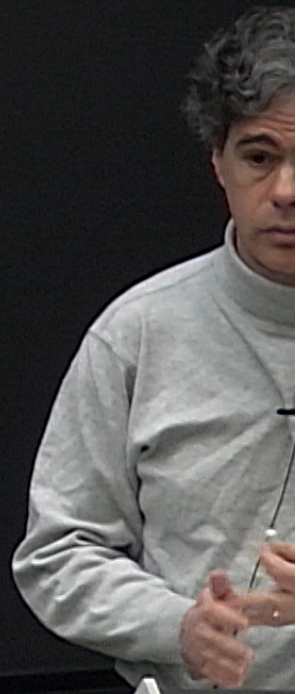
$$(-1)^U = U F_0 U^\dagger$$

$$G = (-1)^U (-1)^0$$

Apply G to $|u\rangle$ M times

Get $\approx |\phi\rangle$ if $M \sim \frac{\pi}{4} \frac{1}{\sqrt{\epsilon}}$

$|u\rangle$
 $|u\rangle \rightarrow |u\rangle$
 $|\phi\rangle \rightarrow -|\phi\rangle$



$$U^0 = U E U^\dagger$$

$$= (-1)^0 (-1)^0$$

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