Title: PSI 2018/2019 - Quantum Information Review - Lecture 3

Speakers: Daniel Gottesman

Collection: PSI 2018/2019 - Quantum Information Review (Gottesman)

Date: March 06, 2019 - 11:30 AM

URL: http://pirsa.org/19030040

Density matrix is positive, trace 1 matrix

General dynamic operation is a CPTP map:

1) Linear

2)

Density matrix is positive, trace 1 matrix

General dynamic operation is a <u>CPTP map</u>:

<u>quantum operations</u>

<u>quantum channels</u>

1) Linear
2) Trace-preserving
3) Positive
4) Completely positive $(\mathcal{E}\otimes I)(\rho_{AR}) \geq 0$ if $\rho_{AR} \geq 0$.

ositive, trace 1 matrix

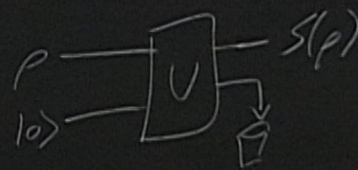
n is a <u>CPTP map</u>:
<u>quantum operations</u>
<u>quantum channels</u>

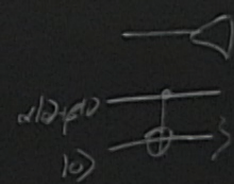
$\mathcal{I})(\rho_{AR}) \geq 0$ if $\rho_{AR} \geq 0$

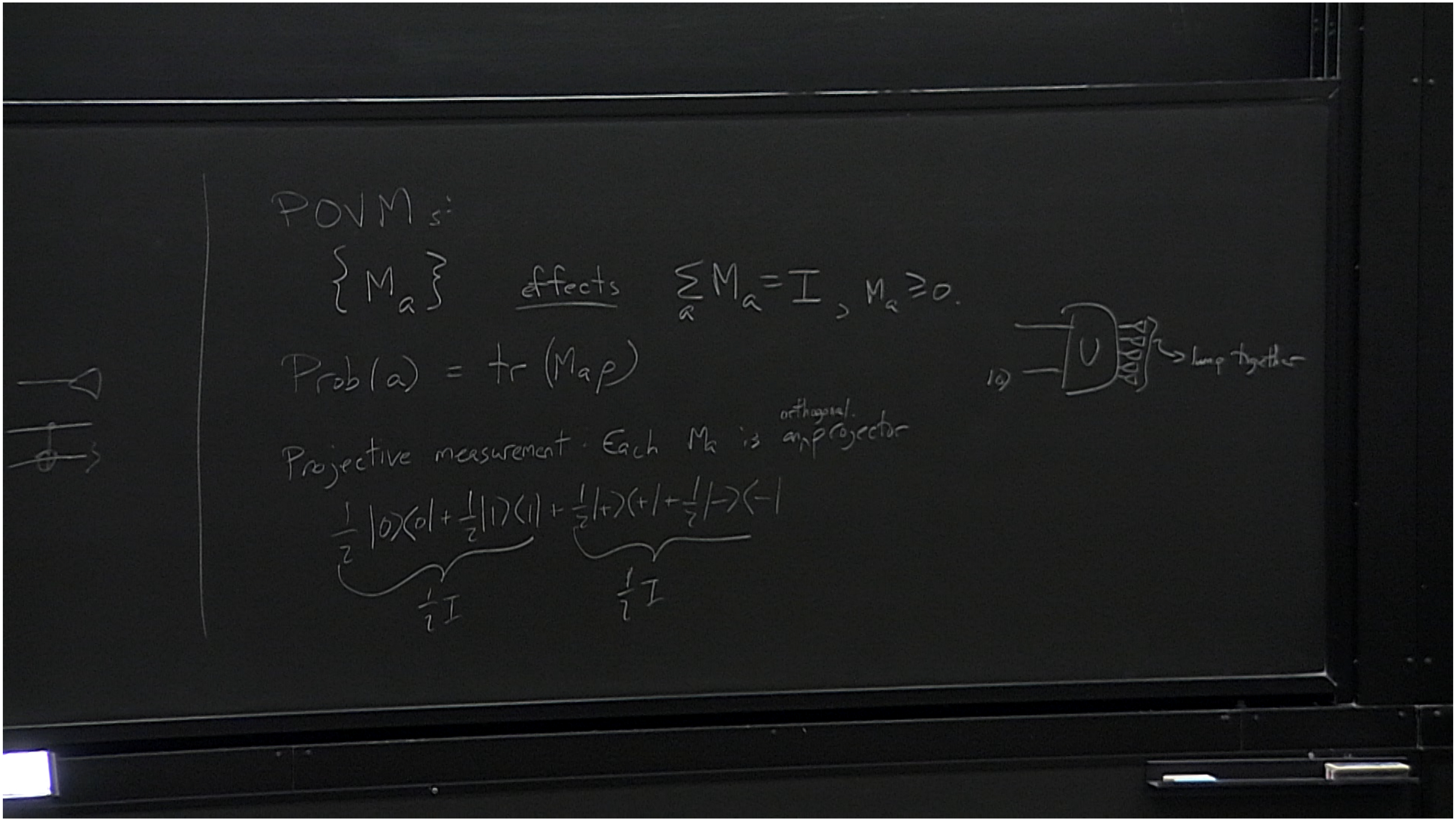
Kraus form:

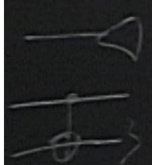
$$S(\rho) = \sum_k A_k \rho A_k^\dagger$$

$$\sum_k A_k^\dagger A_k = I$$



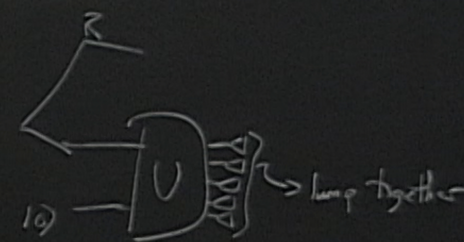



POVMs:
$\{M_a\}$ effects $\sum_a M_a = I$, $M_a \geq 0$.
$\mathrm{Prob}(a) = \mathrm{tr}(M_a \rho)$
Projective measurement: Each M_a is an orthogonal projector
$\underbrace{\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|}_{\frac{1}{2}I} + \underbrace{\frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|}_{\frac{1}{2}I}$
$|0\rangle$
U
lump together



POVMs:

$\{M_a\}$ effects $\sum_a M_a = I$, $M_a \geq 0$.



$$\text{Prob}(a) = \text{tr}(M_a \rho)$$

Projective measurement: Each M_a is an orthogonal projector

$$\underbrace{\tfrac{1}{2}|0\rangle\langle 0| + \tfrac{1}{2}|1\rangle\langle 1|}_{\frac{1}{2}I} + \underbrace{\tfrac{1}{2}|+\rangle\langle +| + \tfrac{1}{2}|-\rangle\langle -|}_{\frac{1}{2}I}$$

Fidelity:

$F(|\phi\rangle, |\psi\rangle) = |\langle\phi|\psi\rangle|$

0 iff $\perp$

1 iff same

Measure $\{ |\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi| \}$

If $|\phi\rangle$, get $|\psi\rangle\langle\psi|$ outcome

with $F(|\phi\rangle, |\psi\rangle)^2$.

$F(|\psi\rangle, \sigma)^2 = \mathrm{tr}(|\psi\rangle\langle\psi| \sigma)$

$F(\rho, \sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$

Properties

1. $0 \leq F($

2. $\rho = \sigma \Leftrightarrow$

Properties of fidelity:

1. $0 \leq F(\rho,\sigma) \leq 1$
2. $\rho = \sigma \Leftrightarrow F(\rho,\sigma) = 1$
3. $F(\rho,\sigma) = 0$ iff ρ & σ have orthogonal support.
4. $F(\rho,\sigma) = F(\sigma,\rho)$
5. $F(U\rho U^\dagger, U\sigma U^\dagger) = F(\rho,\sigma)$ for unitary U
6. $F(S(\rho), S(\sigma)) \geq F(\rho,\sigma)$
7. Uhlmann's thm: $F(\rho,\sigma) = \max_{|\psi\rangle,|\phi\rangle} F(|\phi\rangle, |\psi\rangle)$, $|\phi\rangle_{AR}$ & $|\psi\rangle_{AR}$ are purifications of ρ & σ
   $\mathrm{tr}_R |\phi\rangle = \rho$, $\mathrm{tr}_R |\psi\rangle = \sigma$

Trace distance:

$$D(\rho,\sigma) = \frac{1}{2}\operatorname{tr}|\rho-\sigma|$$

Statistical distance

$$D(\{p_i\},\{q_i\}) = \frac{1}{2}\sum_i |p_i - q_i|$$

Thm: $D(\rho,\sigma)$ is maximum over POVMs $\{E_m\}$ of classical statistical distance between $\operatorname{tr}(\rho E_m)$ & $\operatorname{tr}(\sigma E_m)$. Best guess of ρ vs σ with 50% prior probabilities is

$$\frac{1}{2}(1+D(\rho,\sigma))$$

Proof: Optimal POVM: Subspace of positive eigenvalues of $\rho-\sigma$
vs " " negative " of $\rho-\sigma$

...VMs $\{E_m\}$

...zen

...ness of ρ vs. σ

...genvalues of $\rho-\sigma$

... of $\rho-\sigma$

Properties of $D(\rho,\sigma)$:

1. $0 \leq D(\rho,\sigma) \leq 1$
2. $D(\rho,\sigma)=0 \Leftrightarrow \rho=\sigma$
3. $D(\rho,\sigma)=D(\sigma,\rho)$
4. Triangle inequality $D(\rho,\sigma) \leq D(\rho,\eta)+D(\eta,\sigma)$
5. $D(U\rho U^\dagger, U\sigma U^\dagger)=D(\rho,\sigma)$ for unitary U
6. $D(S(\rho),S(\sigma)) \leq D(\rho,\sigma)$ for CPTP $S(\cdot)$