Title: PSI 2018/2019 - Quantum Information Review - Lecture 2

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Collection: PSI 2018/2019 - Quantum Information Review (Gottesman)

Date: March 05, 2019 - 11:30 AM

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Important: QI class tomorrow at 9 AM

Hadamard $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $-\boxed{H}-$

Phase rotation $R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$ $-\boxed{R_\theta}-$

CNOT $|a,b\rangle \longmapsto |a, a\oplus b\rangle$

Toffoli $|a,b,c\rangle \longmapsto |a,b,c\oplus ab\rangle$

Universal gate sets

Classical: Compute arbitrary functions $f(x_0, x_1, \ldots, x_{n-1})$ $\mathbb{Z}_2^n \to \mathbb{Z}_2$

Polynomials of degree $\leq n$, each term is product of a subset of $\{x_0, x_1, \ldots, x_{n-1}\}$

Irreversible set of universal gates:

$\{\text{NOT}, \text{AND}\}$

$a \text{ AND } b = ab$

$\text{NOT } a = a \oplus 1$

$\text{NOT}((\text{NOT } a) \text{ AND } (\text{NOT } b)) = a \text{ OR } b$

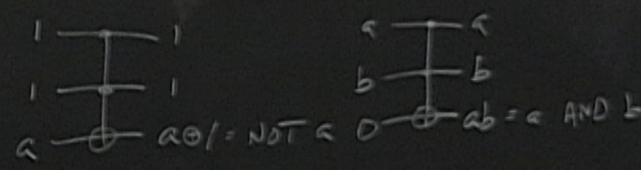
$= ab \oplus a \oplus b$

$a \text{ XOR } b = a \oplus b$

$= (a \text{ AND NOT } b) \text{ OR } (\text{NOT } a \text{ AND } b)$

Reversible:

$\{\text{Tof}, 0 \,\&\, 1 \text{ ancillas}\}$



Universal set of quantum gates:

Arbitrary unitary $U(2^n)$ or $SU(2^n)$

<u>Exact</u> or arbitrarily close

<u>approximate</u>

set of gates dense in $U(2^n)$

(2^n)

$U(2^n)$

Exact:

$\{\text{CNOT}, \text{ single-qubit unitaries}\}$

Approximate:

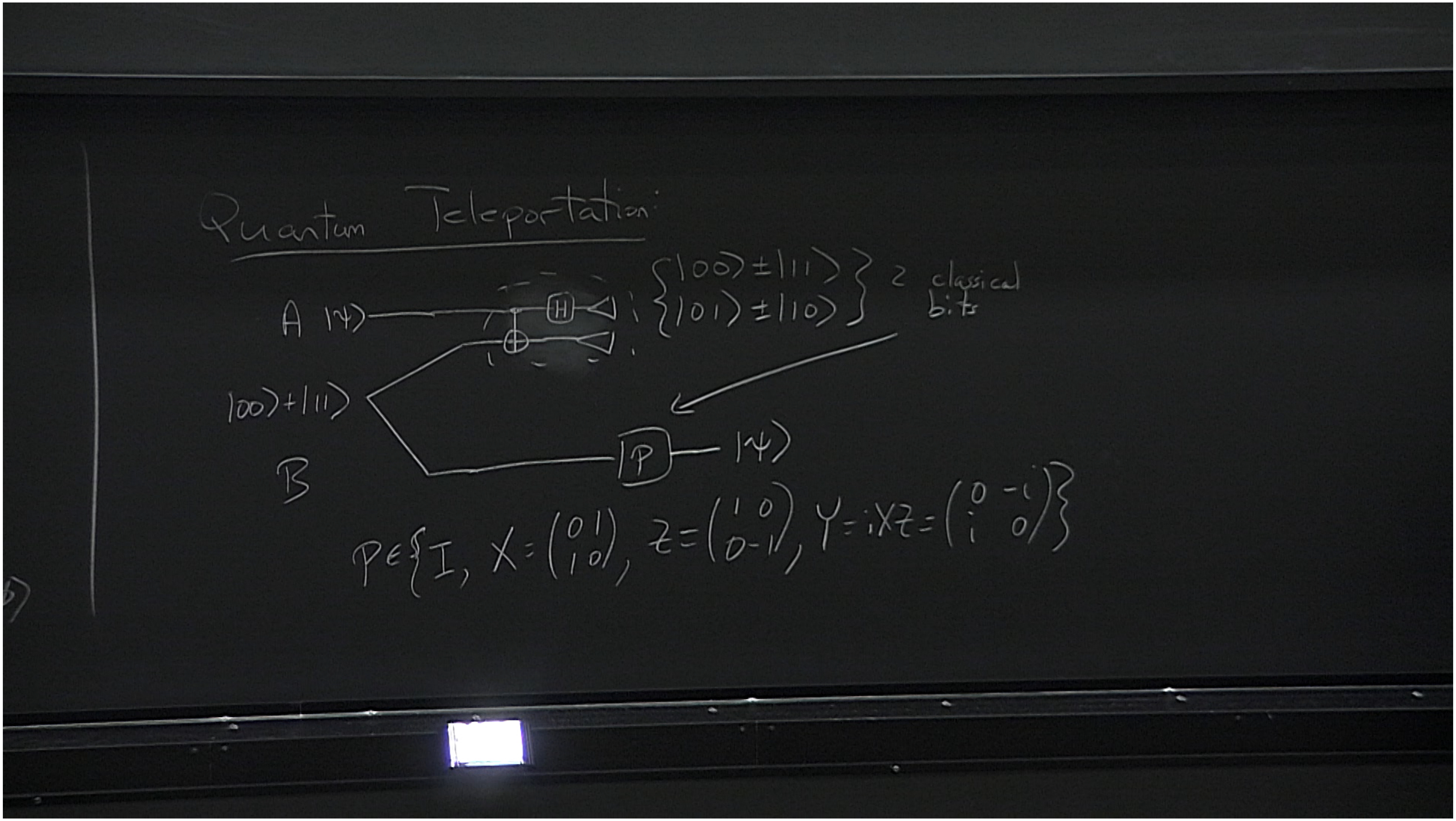
$\{H, R_{\pi/2}, \text{Tof}\}, \ \{H, R_{\pi/4}, \text{CNOT}\}$

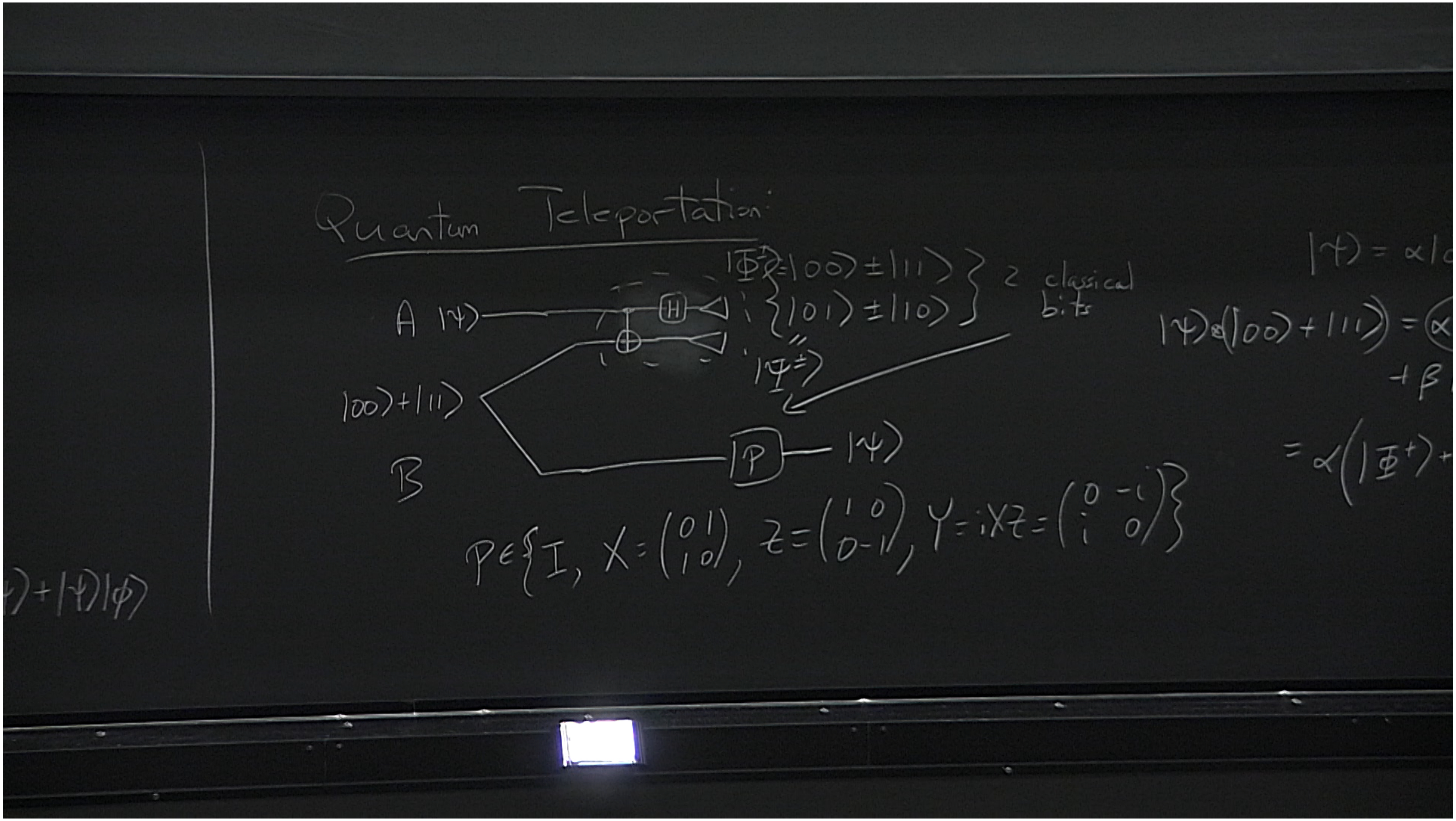
No-Cloning Theorem

Not possible: $|\psi\rangle \longmapsto |\psi\rangle \otimes |\psi\rangle$

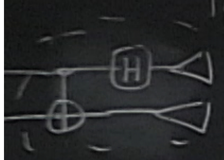
Proof:

$$|\psi\rangle \to |\psi\rangle \otimes |\psi\rangle$$
$$|\phi\rangle \to |\phi\rangle \otimes |\phi\rangle$$
$$|\psi\rangle + |\phi\rangle \longrightarrow \left(|\psi\rangle + |\phi\rangle\right) \otimes \left(|\psi\rangle + |\phi\rangle\right)$$
$$= \left(|\psi\rangle|\psi\rangle + |\phi\rangle|\phi\rangle\right) + |\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle$$


Quantum Teleportation:
A |ψ⟩
H
|00⟩ ± |11⟩
|01⟩ ± |10⟩
2 classical bits
|00⟩+|11⟩
B
P
|ψ⟩
$P \in \{I, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\}$


Quantum Teleportation:
A $|\psi\rangle$
B
$|00\rangle+|11\rangle$
$|\Phi^{\pm}\rangle = |00\rangle \pm |11\rangle$
$|\Psi^{\pm}\rangle = |01\rangle \pm |10\rangle$
2 classical bits
H
P
$|\psi\rangle$
$P \in \{I, X=\begin{pmatrix}0&1\\1&0\end{pmatrix}, Z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}, Y=iXZ=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\}$
$|\psi\rangle = \alpha|$
$|\psi\rangle\otimes(|00\rangle+|11\rangle) =$
$+\beta$
$= \alpha(|\Phi^+\rangle +$
$\rangle + |\psi\rangle|\phi\rangle$

eportation:



$|\Phi^\pm\rangle = |00\rangle \pm |11\rangle$

$|\Psi^\pm\rangle = |01\rangle \pm |10\rangle$

} 2 classical bits

$|\Psi^\pm\rangle$

$\boxed{P}$ — $|\psi\rangle$

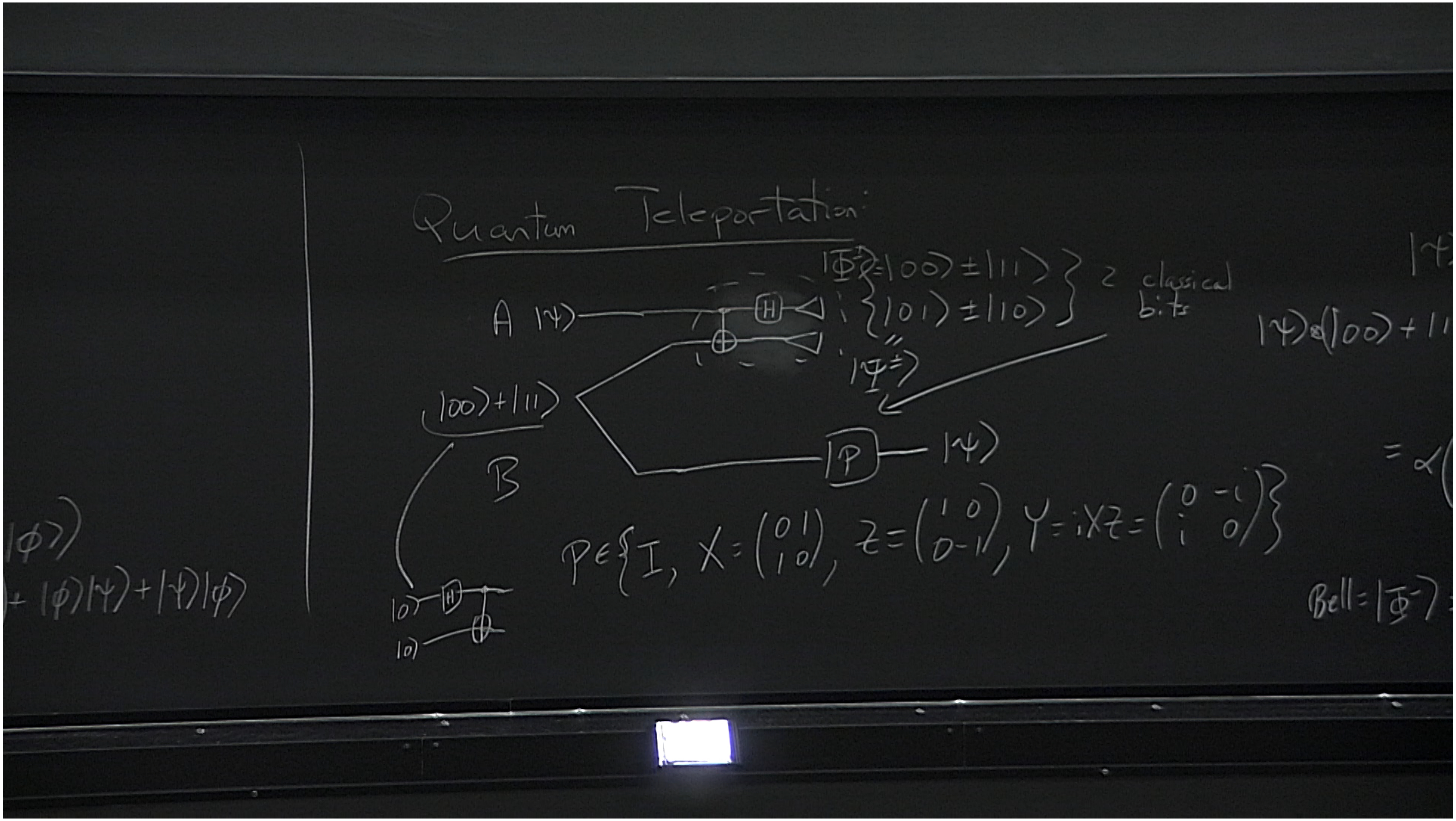
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ }

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\psi\rangle \otimes (|00\rangle + |11\rangle) = \alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle$

$= \alpha\big(|\Phi^+\rangle + |\Phi^-\rangle\big)|0\rangle_B + \dots + \beta\big(|\Phi^+\rangle - |\Phi^-\rangle\big)|1\rangle_B$

Bell $= |\Phi^-\rangle \Rightarrow |\Phi^-\rangle\underbrace{\big(\alpha|0\rangle_B - \beta|1\rangle_B\big)}$, $Z|\psi\rangle$


Quantum Teleportation:
2 classical bits
A
B
P ∈ {I, X, Z, Y = iXZ}
Bell

$\dots\rangle \pm |11\rangle \Big\}$ 2 classical bits
$\dots\rangle \pm |10\rangle \Big\}$

$\dots |\psi\rangle$

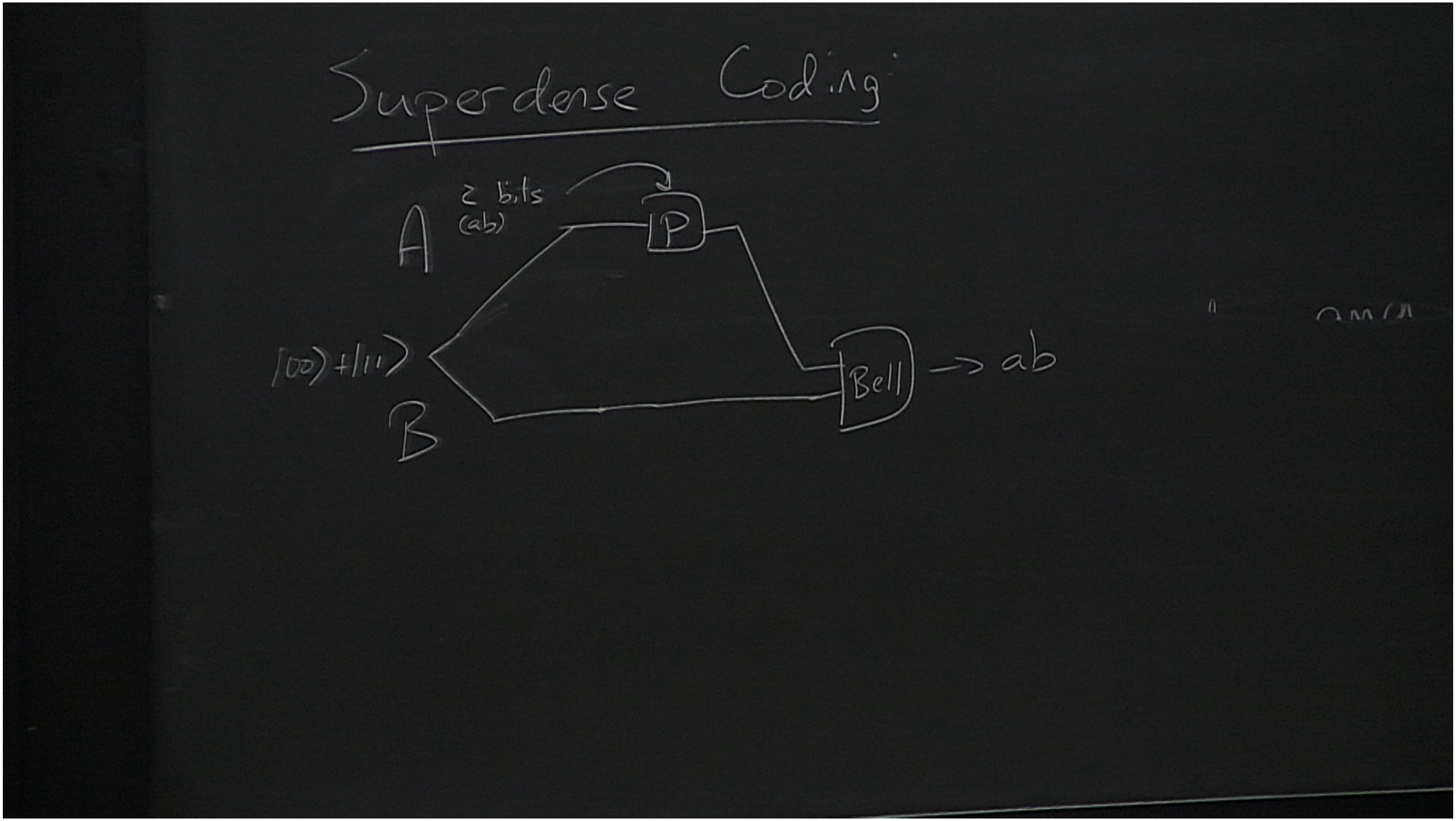
$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \; Y = iXZ = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \Big\}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle \otimes (|00\rangle + |11\rangle) = \alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle$$

$$= \alpha\left(|\Phi^+\rangle + |\Phi^-\rangle\right)|0\rangle_B + \dots + \beta\left(|\Phi^+\rangle - |\Phi^-\rangle\right)|1\rangle_B$$

$$\text{Bell} = |\Phi^-\rangle \Rightarrow |\Phi^-\rangle\big(\underbrace{\alpha|0\rangle_B - \beta|1\rangle_B}_{Z|\psi\rangle}\big)$$


Superdense Coding
A
2 bits
(ab)
P
|00⟩+|11⟩
B
Bell
→ ab

$$\{p_i : |\psi_i\rangle\}$$

$D \times D$ matrix, positive matrix w/ trace 1.

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = \sum_{ab} \rho_{ab} |a\rangle\langle b|$$

Unitary U: $\rho \mapsto U \rho U^\dagger$

$$\rho_{AB} = \sum_{a,a',b,b'} \rho_{aba'b'} \, |a\rangle_A |b\rangle_B \langle a'|_A \langle b'|_B$$

$$\rho_A = \mathrm{tr}_B \, \rho_{AB} = \sum_{a,a'} \Big(\sum_c \rho_{aca'c} \Big) |a\rangle_A \langle a'|_A$$

$\forall \rho_A \quad \exists \, |\psi\rangle_{AR} \quad$ s.t.

$$\rho_A = \mathrm{tr}_R \, |\psi\rangle_{AR} \langle\psi|_{AR}$$