Title: PSI 2018/2019 - Quantum Information Review - Lecture 2

Speakers: Daniel Gottesman

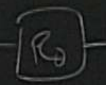
Collection: PSI 2018/2019 - Quantum Information Review (Gottesman)

Date: March 05, 2019 - 11:30 AM

URL: http://pirsa.org/19030039

Important: QI class tomorrow at 9 AM

Hadamard $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Phase rotation $R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$ 

CNOT $|a,b\rangle \longmapsto |a, a \oplus b\rangle$

Toffoli $|a,b,c\rangle \longmapsto |a,b, c \oplus ab\rangle$

Universal gate sets

Classical: Compute arbitrary functions $f(x_0, x_1, \ldots, x_{n-1})$ $\mathbb{Z}_2^n \to \mathbb{Z}_2$

Polynomials of degree $\leq n$, each term is product of a subset of $\{x_0, x_1, \ldots, x_{n-1}\}$

Irreversible set of universal gates:

{NOT, AND}

a AND b = ab

NOT a = a⊕1

NOT((NOT a) AND (NOT b)) = a OR b

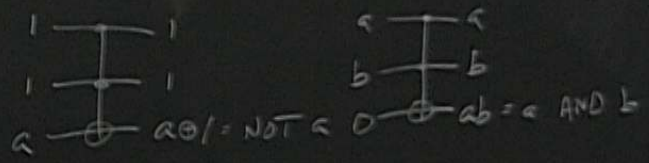
= ab ⊕ a ⊕ b

a XOR b = a ⊕ b

= (a AND NOT b) OR (NOT a AND b)

Reversible:

{Toffoli, 0 & 1 ancillas}



Universal set of quantum gates:

Arbitrary unitary $U(2^n)$ or $SU(2^n)$

Exact or arbitrarily close approximate

set of gates dense in $U(2^n)$

2^n)

$U(2^n)$

Exact:

{CNOT, single-qubit unitaries}

Approximate:

$\{H, R_{\pi/2}, \text{Toff}\}$, $\{H, R_{\pi/4}, \text{CNOT}\}$

No-Cloning Theorem

Not possible: $|\psi\rangle \longmapsto |\psi\rangle \otimes |\psi\rangle$

Proof:

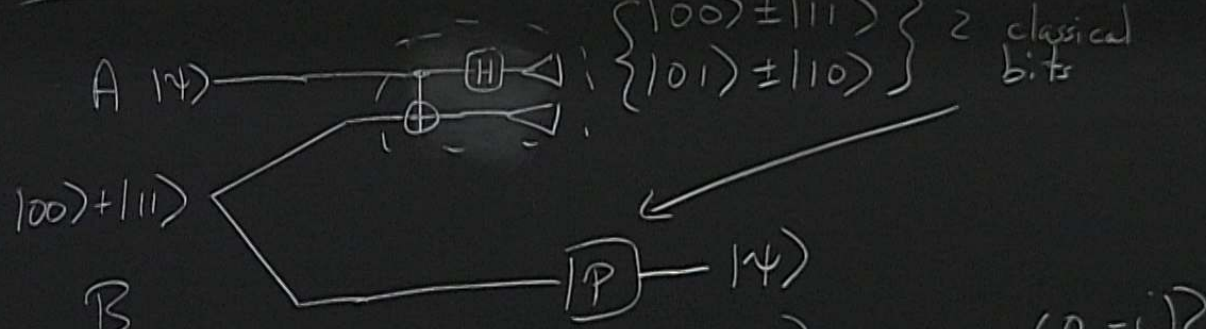
$|\psi\rangle \to |\psi\rangle \otimes |\psi\rangle$

$|\phi\rangle \to |\phi\rangle \otimes |\phi\rangle$

$|\psi\rangle + |\phi\rangle \to (|\psi\rangle + |\phi\rangle) \otimes (|\psi\rangle + |\phi\rangle)$

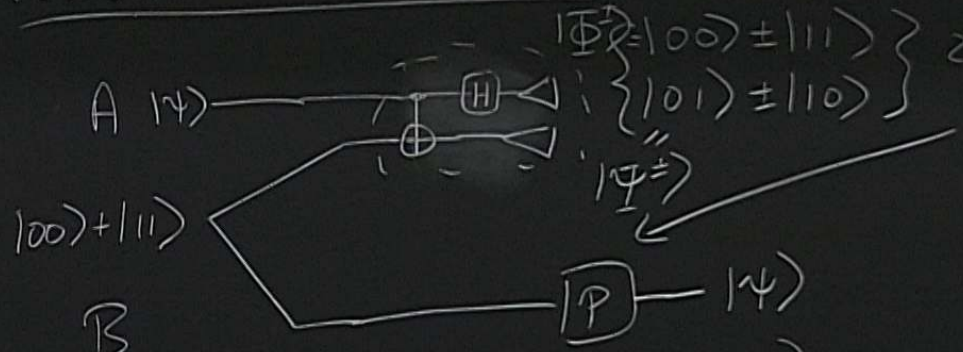
$= (|\psi\rangle|\psi\rangle + |\phi\rangle|\phi\rangle) + |\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle$

Quantum Teleportation:



$P \in \{I, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\}$

Quantum Teleportation:



$P \in \{I, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\}$

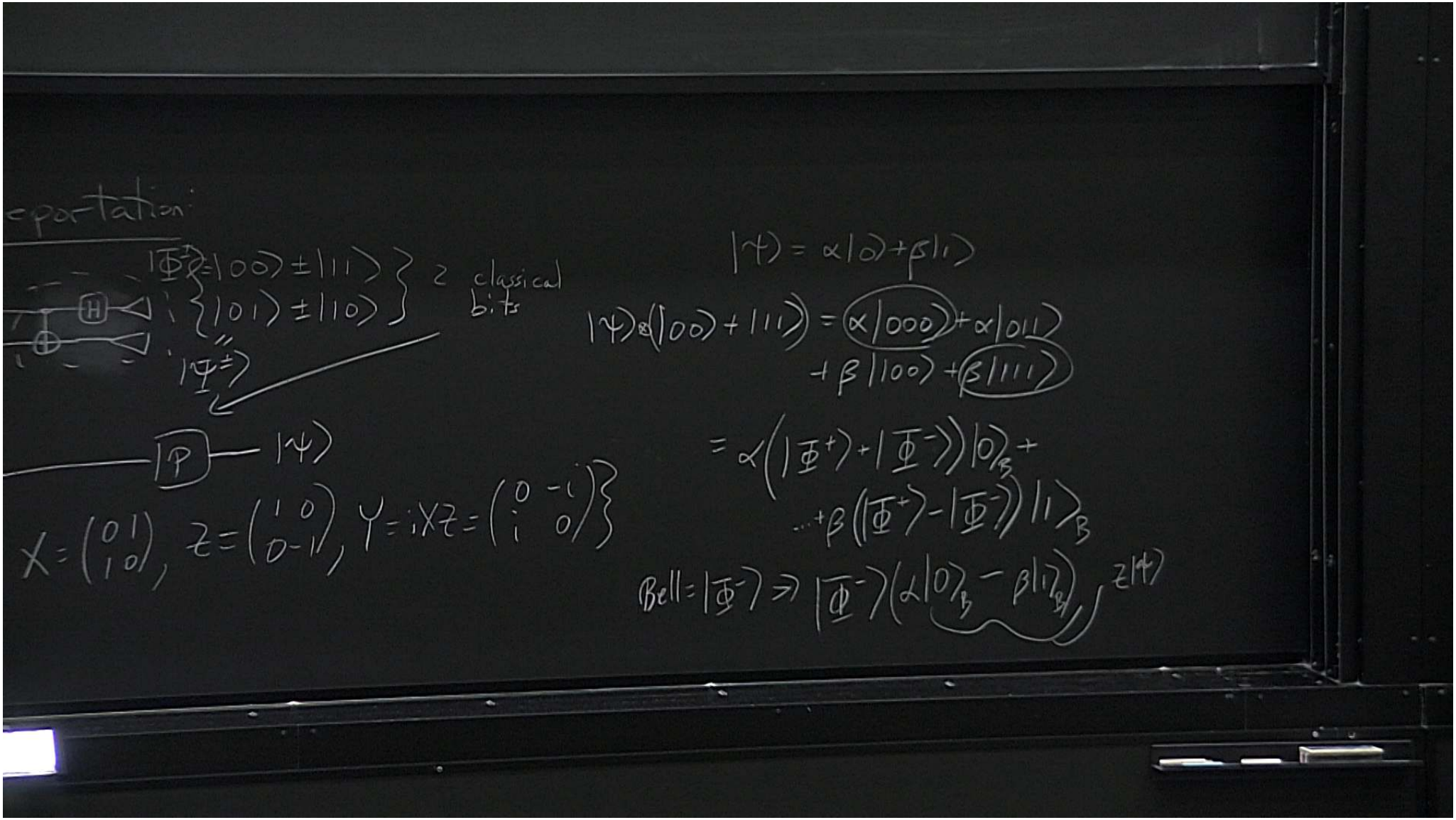
$|\psi\rangle = \alpha|0$

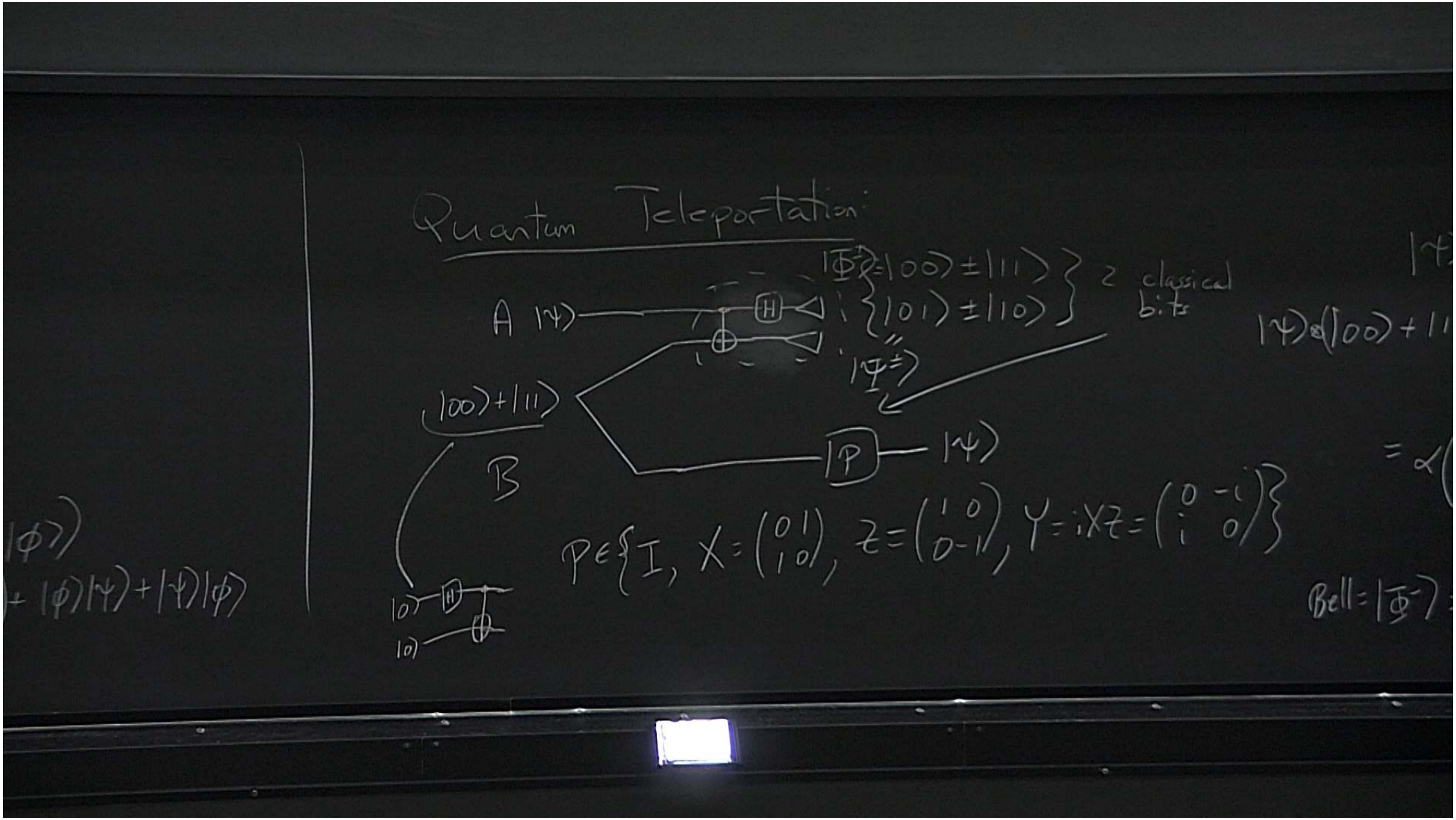
$|\psi\rangle \otimes (|00\rangle + |11\rangle) = (\alpha$

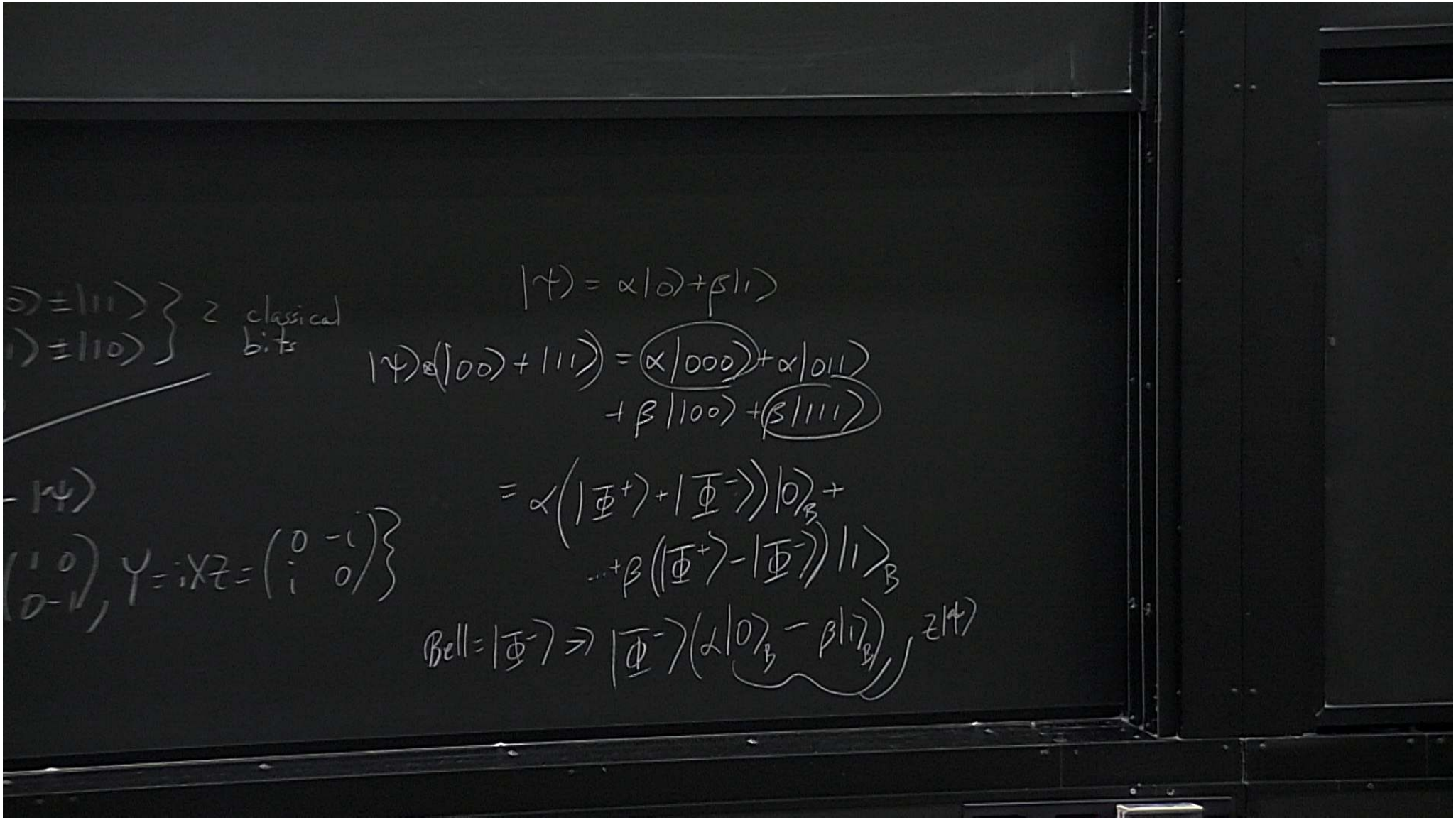
$+ \beta$

$= \alpha(|\Phi^+\rangle +$

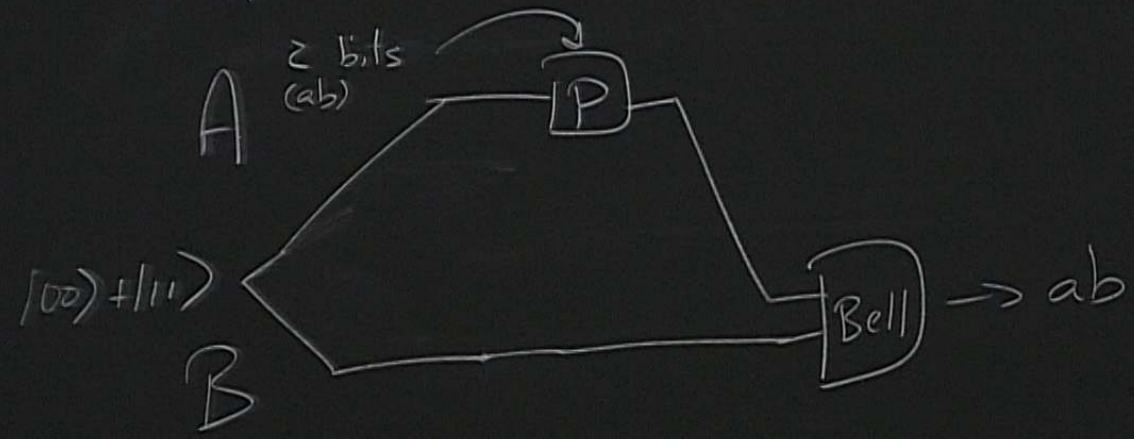
$+|\psi\rangle|\phi\rangle$


eportation:
$|\Phi^\pm\rangle = |00\rangle \pm |11\rangle$
$|\Psi^\pm\rangle = |01\rangle \pm |10\rangle$
2 classical bits
H
$|\Psi\rangle$
P
$|\psi\rangle$
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$|\psi\rangle \otimes (|00\rangle + |11\rangle) = \alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle$
$= \alpha(|\Phi^+\rangle + |\Phi^-\rangle)|0\rangle_B + \ldots + \beta(|\Phi^+\rangle - |\Phi^-\rangle)|1\rangle_B$
Bell: $|\Phi^-\rangle \Rightarrow |\Phi^-\rangle(\alpha|0\rangle_B - \beta|1\rangle_B)$ $= |\psi\rangle$


Quantum Teleportation:
A
B
2 classical bits
P
P ∈ {I, X = (0 1 / 1 0), Z = (1 0 / 0 -1), Y = iXZ = (0 -i / i 0)}
Bell = |Φ⁻⟩


2 classical bits
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$|\psi\rangle \otimes (|00\rangle + |11\rangle) = \alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle$
$= \alpha(|\Phi^+\rangle + |\Phi^-\rangle)|0\rangle_B + \beta(|\Phi^+\rangle - |\Phi^-\rangle)|1\rangle_B$
Bell $= |\Phi^-\rangle \Rightarrow |\Phi^-\rangle(\alpha|0\rangle_B - \beta|1\rangle_B)$
$Z|\psi\rangle$

Superdense Coding



$\{p_i : |\psi_i\rangle\}$

$D \times D$ matrix, positive matrix w/ trace 1.

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = \sum_{ab} \rho_{ab} |a\rangle\langle b|$$

Unitary $U : \rho \mapsto U \rho U^\dagger$

$$\rho_{AB} = \sum_{a,a',b,b'} \rho_{aba'b'} |a\rangle_A |b\rangle_B \langle a'|_A \langle b'|_B$$

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_{a,a'} \left(\sum_c \rho_{aca'c} \right) |a\rangle_A \langle a'|_A$$

$$\forall \rho_A \quad \exists |\psi\rangle_{AR} \quad \text{s.t.}$$

$$\rho_A = \mathrm{tr}_R |\psi\rangle_{AR} \langle \psi|_{AR}$$