Title: PSI 2018/2019 - Quantum Information Review - Lecture 1

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Collection: PSI 2018/2019 - Quantum Information Review (Gottesman)

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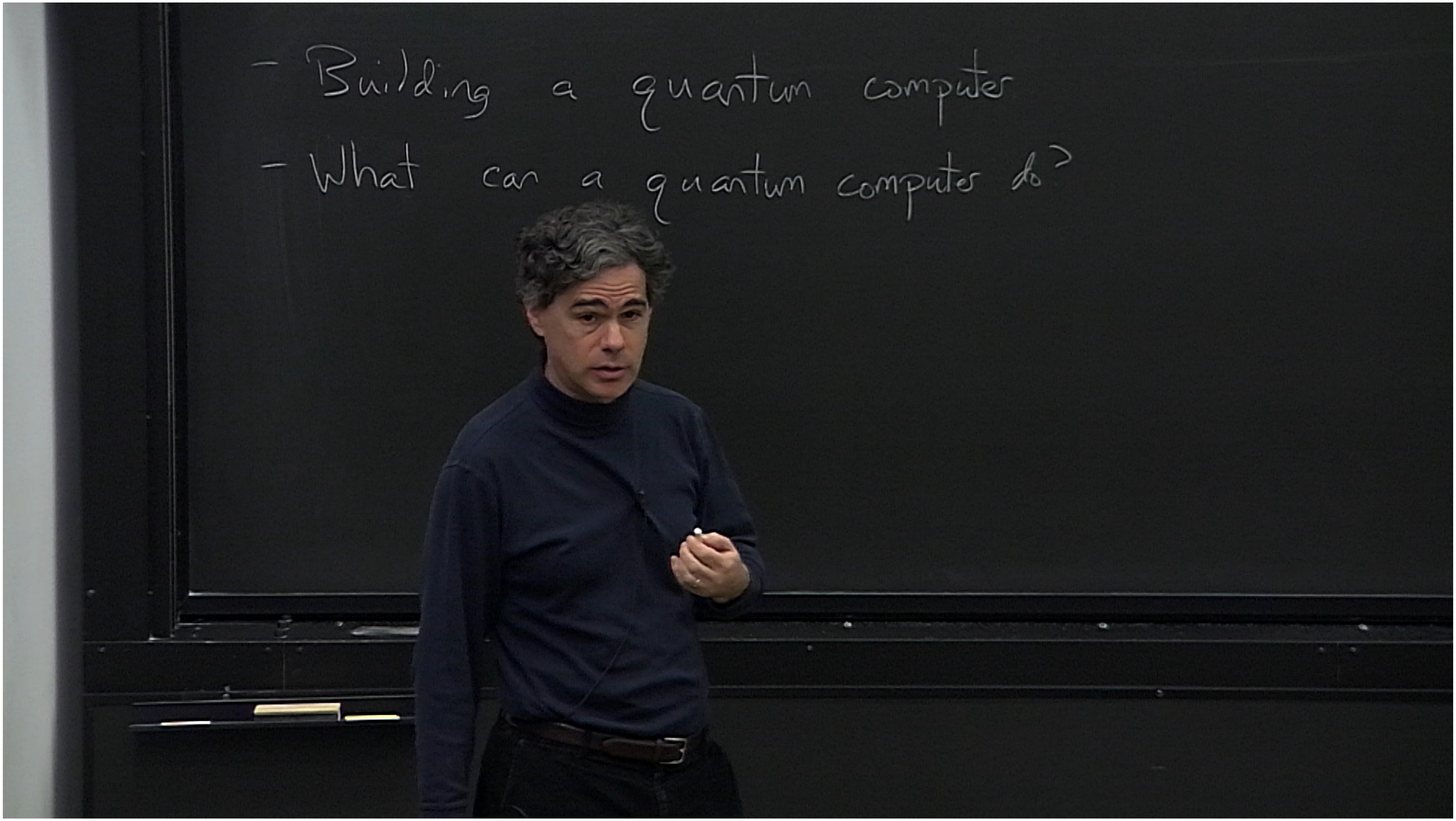
- Building a quantum computer
- What can a quantum computer do?

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- What can a quantum computer do?
- Quantum information vs. classical information
- Applications to physics
- Applications to classical computer science

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Qubit: 2-D Hilbert space

$$\alpha|0\rangle + \beta|1\rangle$$

n-qubit system: 2^n-dimensional Hilbert space

$$\sum_x \alpha_x |x\rangle$$

(Tensor) product states:

$$(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$$

Entangled states:

$$|00\rangle + |11\rangle$$

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$$(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$$

Entangled states:

$$|00\rangle + |11\rangle$$

$$= |0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle$$

Computation:

Unitary operation built up out of <u>quantum gates</u>

One-qubit gate acts on a single qubit out of n.

Two-qubit gate acts on 2 qubits.

Unitary operations: $|\psi\rangle \longmapsto U|\psi\rangle$

Computation:

Unitary operation built up out of quantum gates

One-qubit gate acts on a single qubit out of n.

Two-qubit gate acts on 2 qubits.

Unitary operations: $|\psi\rangle \longmapsto U|\psi\rangle$; $U^\dagger(U|\psi\rangle) = |\psi\rangle$

Reversible classical computation:

NOT
1-bit gate
Reversible

in	out
0	1
1	0

XOR
2-bit gate
Reversible

in a	in b	out a	out $a \oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

AND
2-bit gate
Not reversible

in a	in b	out a	out ab
0	0	0	0
0	1	0	0
1	0	1	0
1	1	1	1

To

AND

2-bit gate

~~Not~~ reversible

in a	in b		out a	out ab
0	0	→	0	0
0	1	→	0	0
1	0	→	1	0
1	1	→	1	1

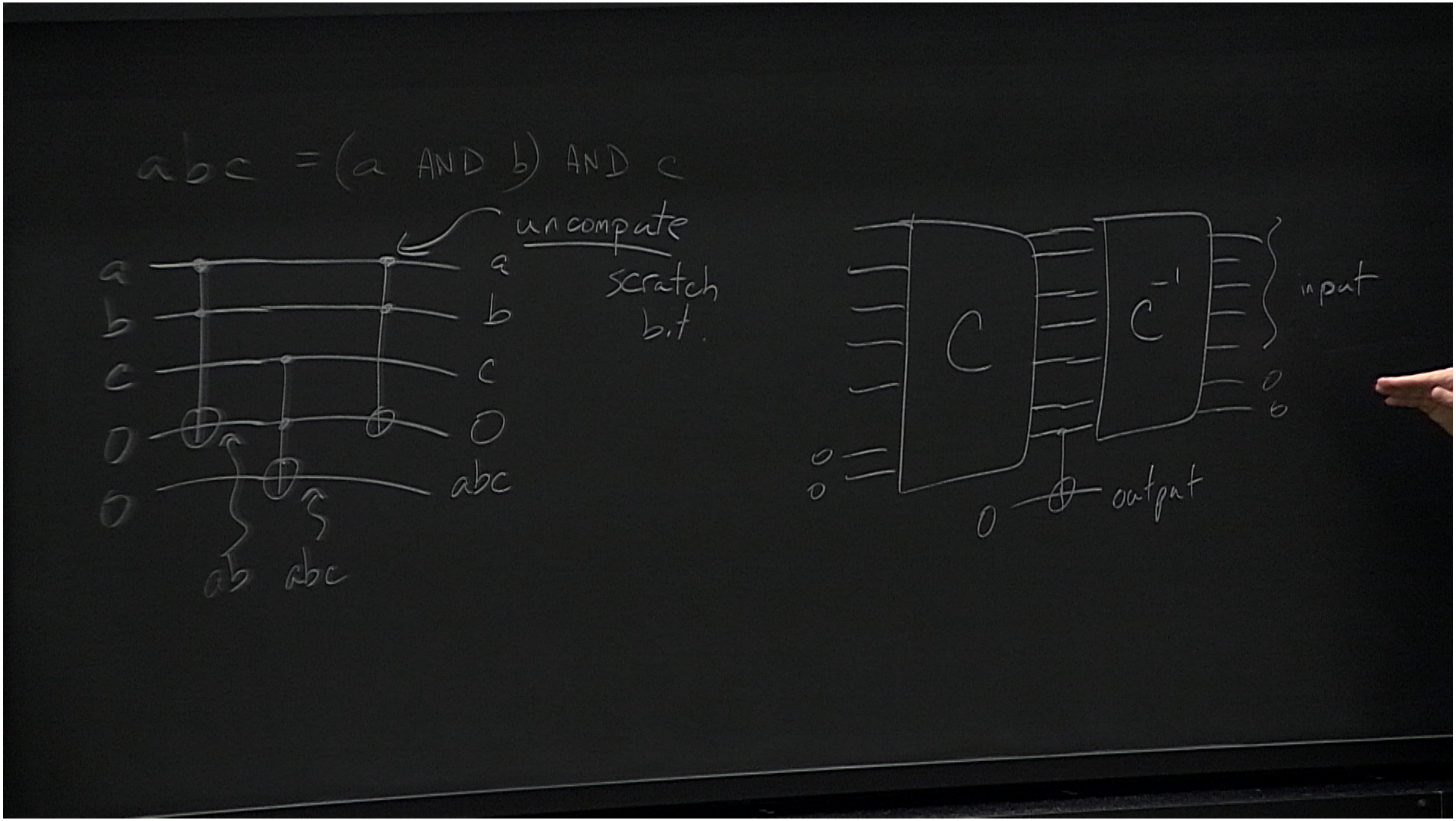
Toffoli gate

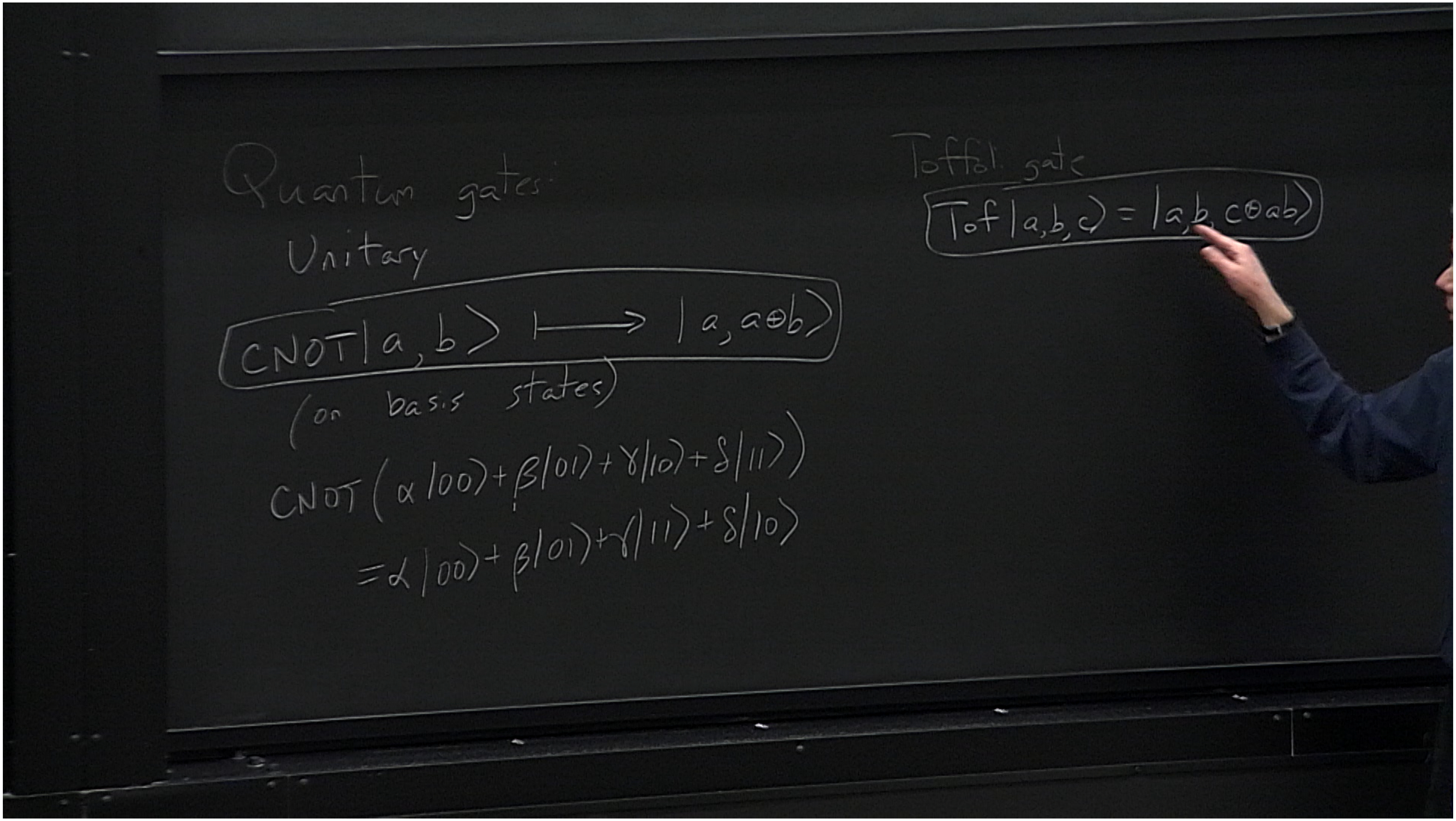
3-bit gate

reversible

$a,b,0 \xrightarrow{Tof} a,b,ab$

in a	in b	in c		out a	out b	out $c \oplus ab$
0	0	0	→	0	0	0
0	0	1	→	0	0	1
0	1	0	→	0	1	0
0	1	1	→	0	1	1
1	0	0	→	1	0	0
1	0	1	→	1	0	1
1	1	0	→	1	1	1
1	1	1	→	1	1	0


abc = (a AND b) AND c
uncompute
scratch bit.
a
b
c
0
0
a
b
c
0
abc
ab
abc
C
C^{-1}
input
0
0
0
0
0
output


Quantum gates:
Unitary
$\text{CNOT}|a,b\rangle \longmapsto |a, a\oplus b\rangle$
(on basis states)
$\text{CNOT}(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle)$
$=\alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle+\delta|10\rangle$
Toffoli gate
$\text{Tof}|a,b,c\rangle = |a,b,c\oplus ab\rangle$

Quantum gates:

Unitary

$$\boxed{CNOT|a,b\rangle \longmapsto |a, a\oplus b\rangle}$$

(on basis states)

$$CNOT\left(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle\right)$$

$$=\alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle+\delta|10\rangle$$

Toffoli gate

$$\boxed{Tof|a,b,c\rangle = |a,b,c\oplus ab\rangle}$$

Hadamard transform 1-qubit

$$\boxed{H=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \qquad \begin{array}{l}|0\rangle \to |0\rangle+|1\rangle\\ |1\rangle \to |0\rangle-|1\rangle\end{array}}$$

$$H^2 = I$$

Phase rotation $-\boxed{R_\theta}-$

$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \qquad \begin{aligned} |0\rangle &\rightarrow |0\rangle \\ |1\rangle &\rightarrow e^{i\theta}|1\rangle \end{aligned}$$

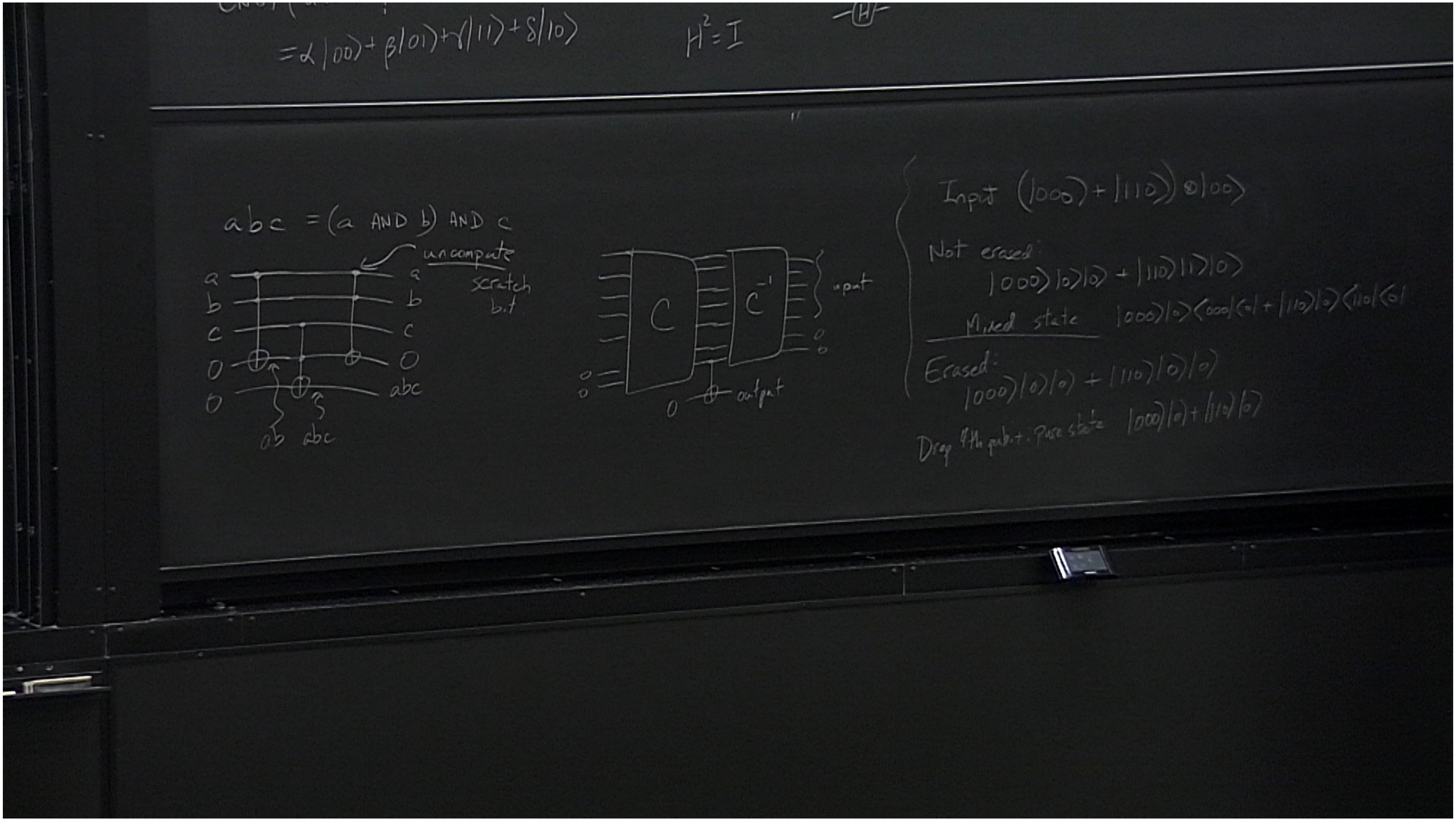
$$|0\rangle - \boxed{H} - \boxed{R_\pi} - \boxed{H} - |1\rangle$$

State preparation

- Make $|0\rangle$ states

Measurement

- Say in standard basis


=α|00⟩+β|01⟩+γ|11⟩+δ|10⟩
H²=I
abc =(a AND b) AND c
uncompute
scratch bit
a b c 0 0
a b c 0 abc
ab abc
C
C⁻¹
input
output
Input (|000⟩+|110⟩)⊗|00⟩
Not erased:
|000⟩|0⟩|0⟩ + |110⟩|1⟩|0⟩
Mixed state |000⟩|0⟩⟨000|⟨0| + |110⟩|0⟩⟨110|⟨0|
Erased:
|000⟩|0⟩|0⟩ + |110⟩|0⟩|0⟩
Drop 4th qubit: Pure state |000⟩|0⟩+|110⟩|0⟩