Title: PSI 2018/2019 - Strong Field Gravity - Lecture 14

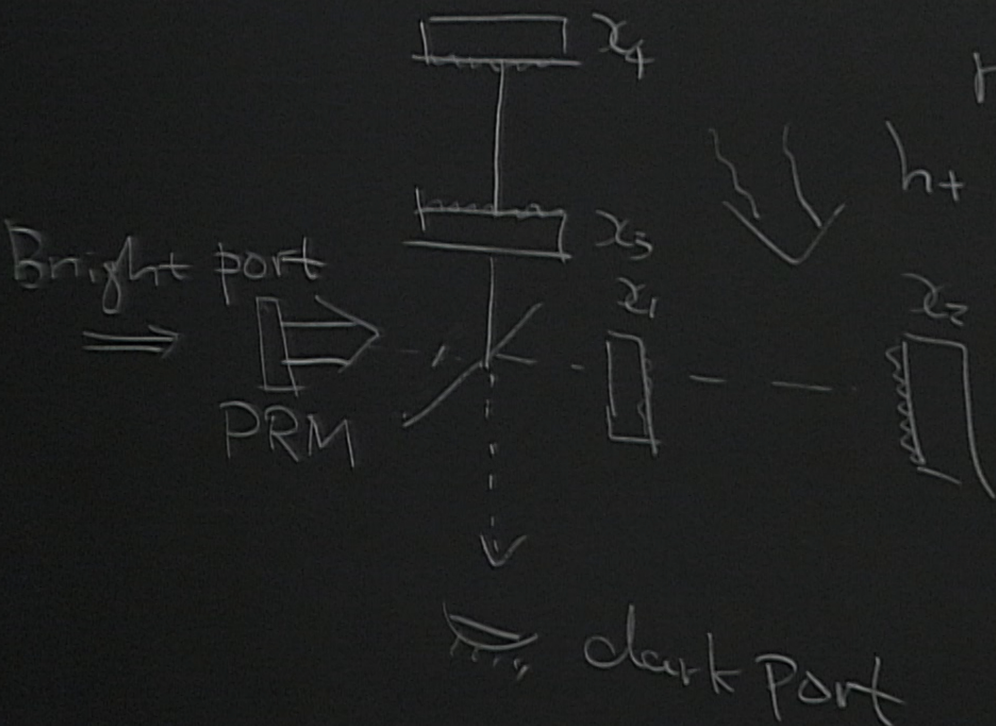
Speakers: William East

Collection: PSI 2018/2019 - Strong Field Gravity (East)

Date: March 21, 2019 - 9:00 AM

URL: http://pirsa.org/19030021

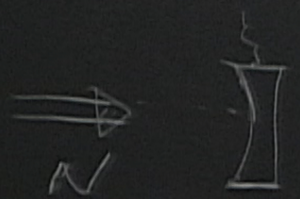
LIGO Interferometer



$$\text{readout} \propto \left[(x_2 - x_1) - (x_3 - x_4)\right]$$

$$\propto h_+ + n_c + n_q$$

Quantum noise:



$$\frac{\delta x_0}{\sqrt{N}} = \delta x_{\text{shot}}$$: Quantum Shot noise

$$\delta P_0 \sim h\nu \,, \quad \delta N \sim \sqrt{N} \,, \quad \delta P = \delta P_0 \cdot \delta N \sim h\nu\sqrt{N}$$

$$\delta x_{\text{rad}} \sim \frac{\delta P}{M\Omega^2}$$: Quantum Radiation Pressure noise

$$S_{hq} = \frac{h_{SQL}^2}{2}\left(\frac{1}{K} + K\right) \qquad K \propto N$$

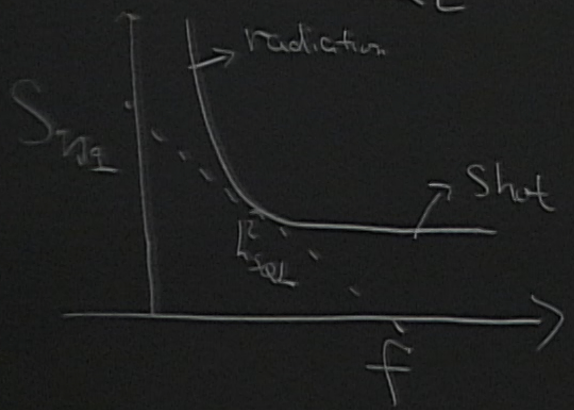
$K \to N \to$ rad

$\frac{1}{K} \propto \frac{1}{N}$ shot

$\geq h_{SQL}^2$ iff $K = 1$

$$h_{SQL} = \sqrt{\frac{8\hbar}{M\Omega^2 L^2}}$$

$$= 2\times10^{-25}\,\mathrm{Hz}^{-1/2}\left(\frac{200\,\mathrm{kg}}{M}\right)\left(\frac{100\,\mathrm{Hz}}{f}\right)\left(\frac{20\,\mathrm{km}}{L}\right)$$



ω

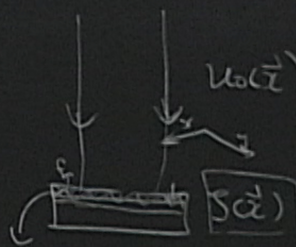
Ground state.

$x_Q = \sqrt{\frac{\hbar}{M\omega}}$: zero point fluctuation

$= \frac{h_{SQL} \cdot L}{\sqrt{8}}$

Coating thermal noise

$u_0(\vec{x})$ $u_{out}(\vec{x})$



$Q_{coat} \ll Q_{sub}$

$S_F(f) = 4R(f)\, k_B T$

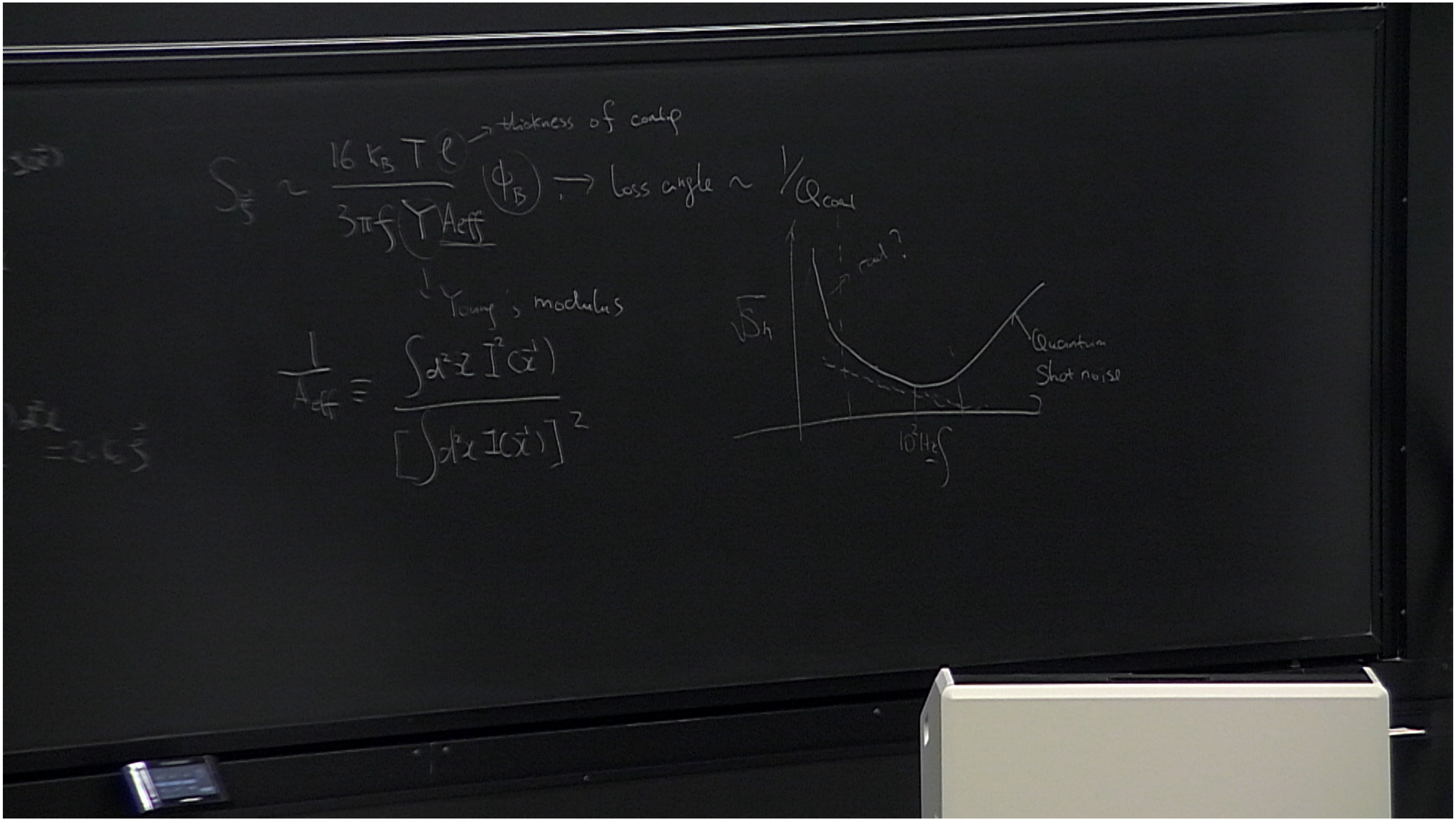
fluctuation loss

$u_{out}(\vec{x}) = u_0(\vec{x})\, e^{i 2 k_0 \xi(\vec{x})}$

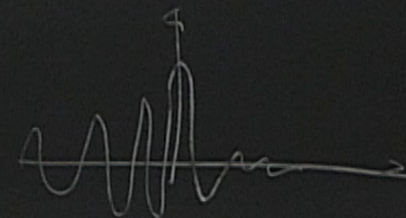
$$\rho = \frac{\int u_{out}(\vec{x})\, u_0^*(\vec{x})\, d^2x}{\int u_0^*(\vec{x})\, u_0(\vec{x})\, d^2x}$$

$$\delta\rho = 2ik_0 \frac{\int \xi(\vec{x})\, I(\vec{x})\, d^2x}{\int I(\vec{x})\, d^2x} = 2ik_0 \vec{\xi}$$

$I(x) = |u_0(x)|^2$


thickness of coating
$S_\xi \sim \frac{16 k_B T \ell}{3\pi f Y A_{eff}} \phi_B \rightarrow$ loss angle $\sim 1/Q_{coat}$
Young's modulus
$\frac{1}{A_{eff}} \equiv \frac{\int d^2x\, I^2(\vec{x})}{\left[\int d^2x\, I(\vec{x})\right]^2}$
$\sqrt{S_h}$
coat?
Quantum
Shot noise
10^2 Hz
f

Matched Filter



Readout $x(t) = n(t) + h(t - t_a)$

arriving time

$$C(\tau) = \int_{-\infty}^{\infty} x(t)\, \underline{g}(t+\tau)\, dt$$

$$= \int_{-\infty}^{\infty} \tilde{x}(f)\, \underline{g}^*(f)\, e^{-i2\pi f \tau}\, df$$

$$C = \int_{-\infty}^{\infty} \tilde{n}(f)\,\tilde{g}^*(f)\,e^{-i2\pi f\tau}\,df \;\rightarrow \text{average} = 0$$

$$+ \int_{-\infty}^{\infty} \tilde{h}(f)\,\tilde{g}^*(f)\,e^{-i2\pi f\tau}\,df = \bar{C}$$

$$\langle \tilde{n}(f)\,\tilde{n}^*(f') \rangle = S_n(f)\,\delta(f-f')$$

Signal to no

$$SNR^2 = \frac{\bar{C}^2}{N^2}$$

$$N^2 = \text{variance of } c = \overline{(c-\bar{c})^2} = \left\langle \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{n}(f)\,\tilde{n}^*(f')\,\tilde{g}^*(f)\,\tilde{g}(f')\,e^{-2\pi i (f-f')\tau}\,df\,df' \right\rangle = \int_{-\infty}^{\infty} S_n(f)\,|\tilde{g}(f)|^2\,df$$

$$C = \int_{-\infty}^{\infty} \tilde{n}(f)\,\tilde{q}^*(f)\,e^{-i2\pi f\tau}\,df \rightarrow \text{average} = 0$$

$$+ \int_{-\infty}^{\infty} \tilde{h}(f)\,\tilde{q}^*(f)\,e^{-i2\pi f\tau}\,df = \bar{C}$$

$$\langle \tilde{n}(f)\,\tilde{n}^*(f') \rangle = S_n(f)\,\delta(f-f')$$

Signal to noise ratio

$$\boxed{SNR^2 = \frac{\bar{C}^2}{N^2}}$$

$$N^2 = \text{Variance of } C = \overline{(C-\bar{C})^2} = \left\langle \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{n}(f)\,\tilde{n}^*(f')\,\tilde{q}^*(f)\,\tilde{q}(f')\,e^{-2\pi i(f-f')\tau}\,df\,df' \right\rangle = \int_{-\infty}^{\infty} S_n(f)\,|\tilde{q}(f)|^2\,df$$

Inner product : $\langle a, b \rangle = 2 \int_0^\infty \frac{df}{S_n(f)} \left[\hat{a}(f) \hat{b}^*(f) + \hat{a}^*(f) b(f) \right]$

$$SNR = \frac{\langle h e^{2\pi i f(\tau - t_a)}, S_n g \rangle}{\langle S_n g, S_n g \rangle} \Rightarrow \hat{g}(f) = \gamma \frac{\hat{h}(f)}{S_n(f)} e^{i2\pi f(\tau - t_a)}$$

$$\frac{\delta(SNR)}{\delta(g_f)} = 0$$

$$\mathrm{SNR}_{opt} = \langle h , h \rangle$$

$$= 2 \sqrt{\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)}}$$