Title: PSI 2018/2019 - Strong Field Gravity - Lecture 10

Speakers: William East

Collection: PSI 2018/2019 - Strong Field Gravity (East)

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Problem in GR (see also Chap. 8 arxiv: g

1) Choose γ_{ij} & K_{ij} at $t=0$

$$ {}^{(3)}R + K^2 - K_{ij}K^{ij} = 16\pi E \leftarrow T^{ab} n_a n_b $$

$$ D_j K^j_i - D_i K = 8\pi p_i \leftarrow T_{ab} n^a \gamma^b_i $$

Initial Data Problem in GR

Two goals:

1) Choose γ_{ij} & K_{ij}

$${}^{(3)}R + K^2 - K_{ij}K^{ij} =$$

$$D_j K^j_i - D_i K =$$

oblem in GR (see also Chap. 8 an

1) Choose γ_{ij} + K_{ij} at $t=0$

$$ {}^{(3)}R + K^2 - K_{ij}K^{ij} = 16\pi E \leftarrow T^{ab} n_a n_b $$

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arXiv: gr-qc/0703035)

2) Choose γ_{ij}, K_{ij} s.t. represent physical system of interest, as closely as possible

Divide DOFs in <u>free</u> & <u>constrained</u> data

Constraint Eqn's: 4

DOFs in γ_{ij}, K_{ij} = 6+6 = 12

Specify 8 DOFs (plus matter)

Conformal Transformation

$$g_{ab} = \omega^2 \tilde{g}_{ab}$$

n-dim metric

$$C^c_{ab} = \Gamma^c_{ab} - \tilde{\Gamma}^c_{ab} = \omega^{-1}\left(\delta^c_a \tilde{\nabla}_b \omega + \delta^c_b \tilde{\nabla}_a \omega - \tilde{g}_{ab} \tilde{\nabla}_c \omega\right)$$

Scalar deriv $\nabla_a \phi = \partial_a \phi = \tilde{\nabla}_a \phi$

Vector deriv: $\nabla_a V_b = \tilde{\nabla}_a V_b - \left(\delta^c_a \delta^d_b + \delta^d_a \delta^c_b - \tilde{g}_{ab} \tilde{g}^{cd}\right) \omega^{-1} (\tilde{\nabla}_c \omega) V_d$

$$R = \omega^{-2}\tilde{R} - 2(n-1)\tilde{g}^{ab}\omega^{-3}(\tilde{\nabla}_a\tilde{\nabla}_b\omega) - (n-1)(n-4)\tilde{g}^{ab}\omega^{-4}(\tilde{\nabla}_a\omega)(\tilde{\nabla}_b\omega)$$

Conformal Transerve Traceless Decomposition

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$$

constrained

free data

$$^{(3)}R = \tilde{R}\psi^{-4} - 8\psi^{-5}\tilde{D}_i\tilde{D}_i\psi$$

$$K_{ij} = A^{ij} + \frac{1}{3}K\gamma_{ij}$$

trace-free

$$A^{ij} = \psi^{-10}\hat{A}^{ij}$$

Ham. Const. $\Rightarrow$ $\tilde{D}_i \tilde{D}_i \psi - \frac{1}{8}\tilde{R}\psi + \frac{1}{8}\hat{A}^{ij}\hat{A}_{ij}\psi^{-7} - \frac{1}{12}K^{2}$

$$D_j K^j{}_i = D_j A^{ij} + \frac{1}{3} D_i K$$

$$D_j A^{ij} = \tilde{D}_j A^{ij} + C^i{}_{jk} A^{jk} + C^j{}_{jk} A^{ik}$$

$$= \tilde{D}_j A^{ij} + 10 A^{ij} \tilde{D}_j (\ln\psi) - 2(\tilde{D}^i \ln\psi)\cancelto{0}{\tilde{\gamma}_{jk} A^{jk}}$$

$$= \psi^{-10}\tilde{D}_j(\psi^{10} A^{ij}) = \tilde{D}_j \hat{A}^{ij}$$

Mom. Const. $\Rightarrow$ $\tilde{D}_j \hat{A}^{ij} - \frac{2}{3}\psi^6 \tilde{D}^i K = 8\pi \psi^{10} \rho^i$

Further decompose: $\hat{A}^{ij} = (\tilde{\mathbb{L}} X)^{ij} + \hat{A}^{ij}_{TT}$

logitudinal, constrained

transverse, free data

TT $\tilde{D}_j(\hat{A}^{ij}_{TT}) = 0 = \tilde{\gamma}_{ij}\hat{A}^{ij}$

Conformal Killing operator: $(\tilde{\mathbb{L}} X)^{ij} = \tilde{D}^i X^j + \tilde{D}^j X^i - \frac{2}{3}\left(\tilde{D}_k X^k\right)\tilde{\gamma}^{ij}$

Conformal vector Laplacian

$$\tilde{\Delta}_L X^i = \tilde{D}_j (\tilde{\mathcal{L}} X)^{ij} = \tilde{D}_j \hat{A}^{ij}$$

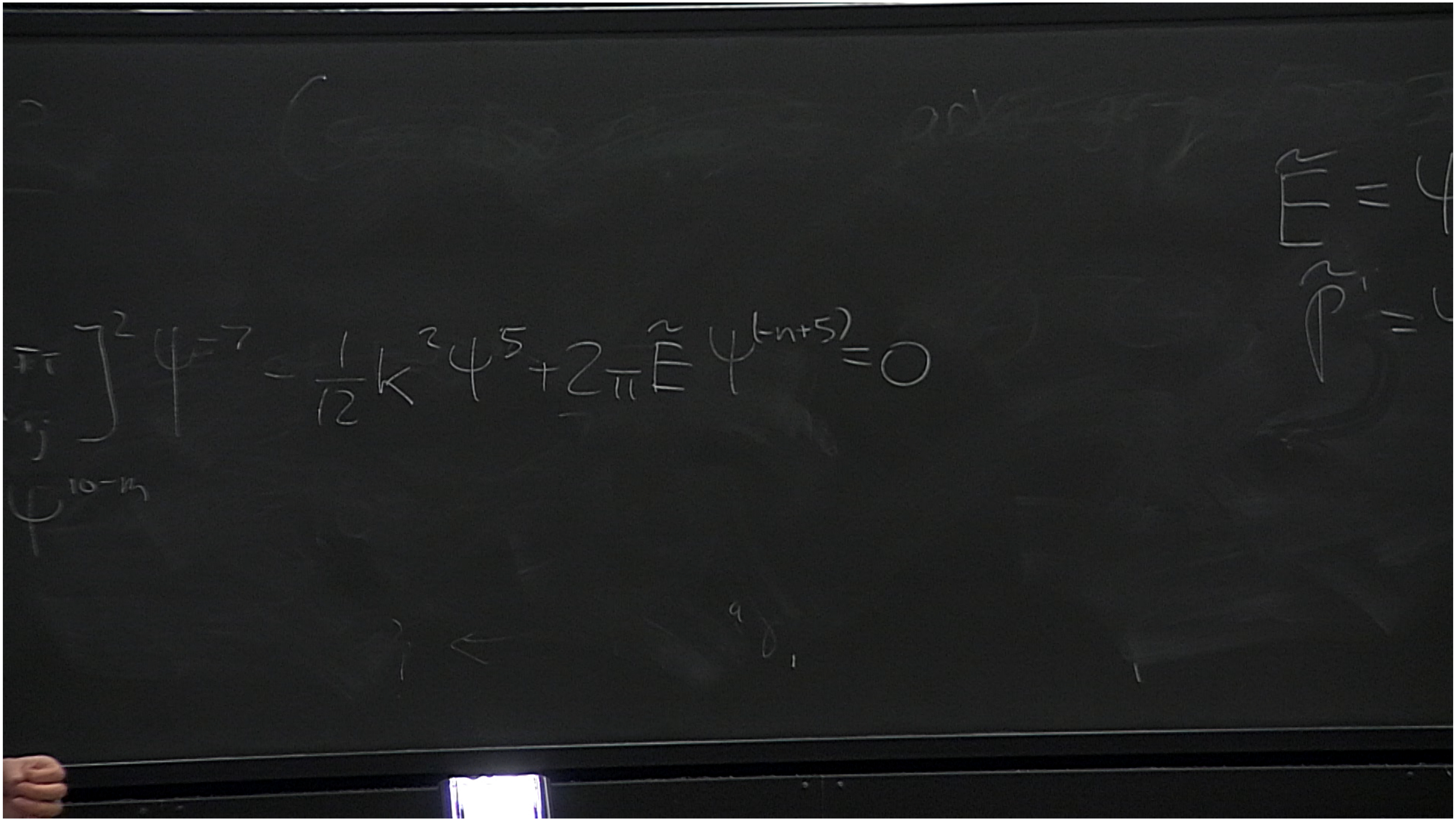
CTT Ham: $\tilde{D}_i \tilde{D}^i \psi - \frac{1}{8}\tilde{R} + \frac{1}{8}\left[(\tilde{\mathcal{L}} X)_{ij} + \hat{A}^{TT}_{ij}\right]^2 \psi^{-7}$

CTT Mom Cnst: $(\tilde{\Delta}_L X)^i - \frac{2}{3}\psi^6 \tilde{D}^i K = 8\pi \tilde{p}^i \psi^{10-n}$

$$(\tilde{\mathcal{L}}X)^{ij} = \tilde{D}_j \hat{A}^{ij}$$

$$\frac{1}{8}\tilde{R} + \frac{1}{8}\left[(\tilde{\mathcal{L}}X)_{ij} + \hat{A}_{ij}^{TT}\right]^2 \psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi\tilde{E}\psi^{-n+5} = 0$$

$$-\frac{2}{3}\psi^6 \tilde{D}^i K = 8\pi\tilde{p}^i \psi^{10-n}$$


$]^2 \psi^{-7} - \frac{1}{12} k^2 \psi^5 + 2\pi \tilde{E} \psi^{(-n+5)} = 0$
ψ^{10-n}
$\tilde{E} = \psi$
$\tilde{P}' = \psi$

$$\tilde{E} = \psi^n E$$

$$\tilde{p}^i = \psi^m p^i$$

$$\tilde{E}\,\psi^{(-n+5)} = 0$$

Chose $m = 10$

Mom. const indep of ψ when K = constant

$$\Psi = \bar{\Psi} + \varepsilon \qquad |\varepsilon| << |\bar{\Psi}|$$

solutions to Ham.

$$\tilde{D}_i \tilde{D}^i \varepsilon = \underbrace{\left[\frac{1}{8} \tilde{R} + \frac{7}{8} \hat{A}^{ij} \hat{A}_{ij} + \frac{5}{2} K^2 + 2\pi (n-5) \tilde{E} \right]}_{C} \varepsilon$$

$C \geq 0$ have (local) uniqueness by maximum principle

Hence want $n \geq 5$

$n = 8$

Dominant EC: $-T^{a}{}_{b}\, n^{b}$ is future pointing causal

$$-E^2 + p_i p^i \leq 0 \quad (3+1)$$

If $\tilde{E}^2 \geq \tilde{\gamma}^{ij}\tilde{P}_i\tilde{P}_j$ then

$$E^2 = \psi^{-16}\tilde{E}^2 \geq \psi^{-16}\tilde{\gamma}_{ij}\tilde{P}^i\tilde{P}^j = \gamma$$

Recap of CTT Eqns

Free data: $\tilde{E}, \tilde{P}^i, \tilde{\gamma}_{ij}, K, \hat{A}^{TT}_{ij}$

Constrained data: X^i, ψ

then

$$E^2 = \psi^{-16}\tilde{E}^2 \geq \psi^{-16}\tilde{\gamma}_{ij}\tilde{p}^i\tilde{p}^j = \gamma_{ij}p^i p^j$$

Free data : $\tilde{E}, \tilde{p}^i, \tilde{\gamma}_{ij}, K, \hat{A}^{TT}_{ij}$; DOFs : $5+1+2$

onstrained data : X^i, ψ $\quad 4$

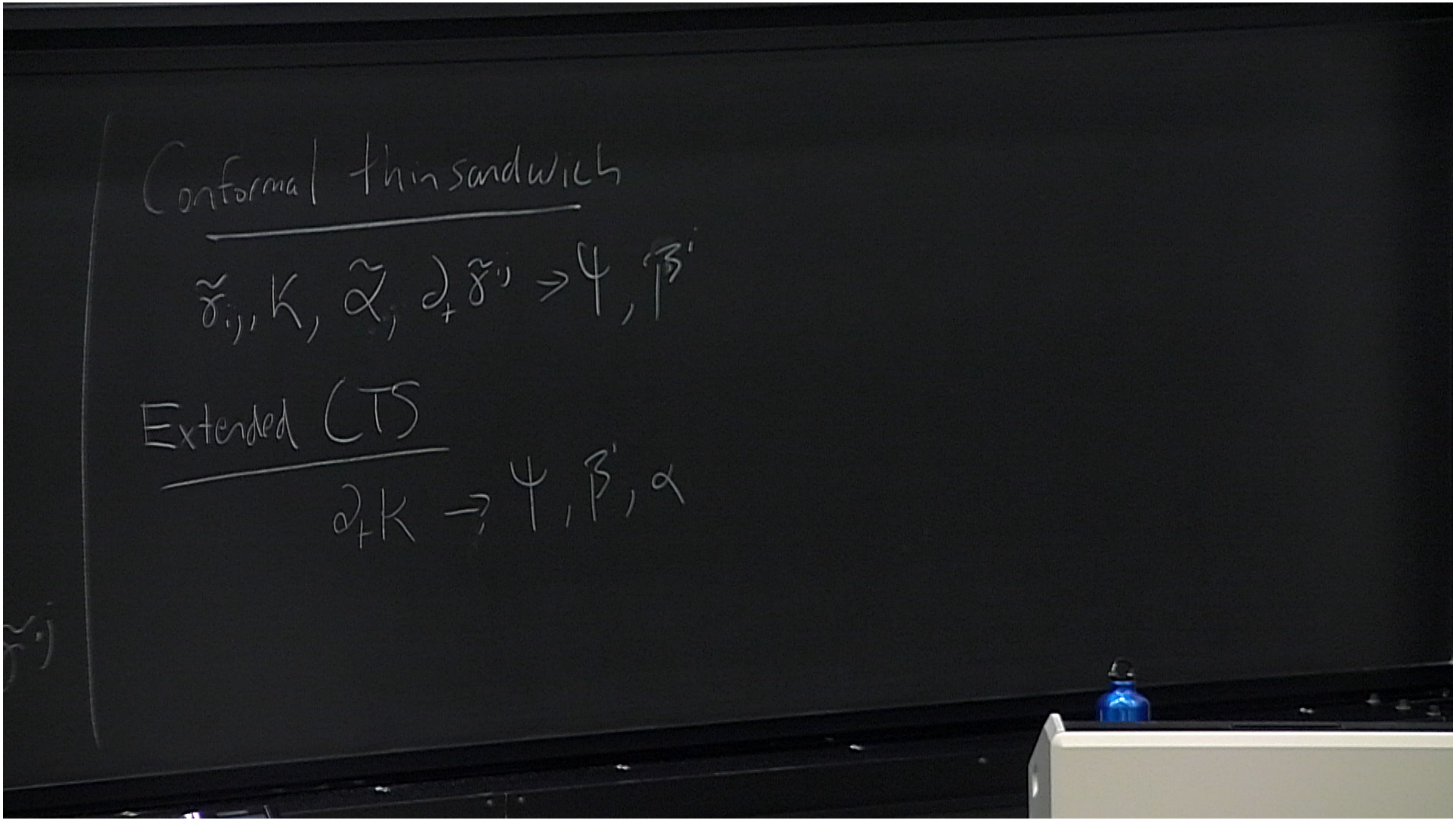
$$s : 5 + 1 + 2$$

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$$

$$K_{ij} = \frac{1}{3} K \gamma_{ij} + \psi^{-10} \left(\hat{A}^{ij}_{TT} + (\tilde{L} X)^{ij} \right)$$

$$E = \psi^{-8} \tilde{E}$$

$$\rho^i = \tilde{\rho}^i \psi^{-10}$$


Conformal thin sandwich
$\tilde{\gamma}_{ij}, K, \tilde{\alpha}, \partial_t \tilde{\gamma}^{ij} \Rightarrow \psi, \beta^i$
Extended CTS
$\partial_t K \Rightarrow \psi, \beta^i, \alpha$

Choosing free data

Strategies: Use superposition $\tilde{\gamma}_{ij} =$

Quasi-circular, assume approx. Killing ve

$$\xi^a = \hat{t}^a + \Omega_{orb} \hat{\phi}^a$$

$$\tilde{\gamma}_{ij} = \gamma_{ij}^{(1)} + \gamma_{ij}^{(2)} - f_{ij}$$

$$\mathcal{L}_\xi \, g_{ab} \approx 0$$

Killing vector

$$+ \int R_{ab} \, \phi^a$$