Title: PSI 2018/2019 - Strong Field Gravity - Lecture 9

Speakers: William East

Collection: PSI 2018/2019 - Strong Field Gravity (East)

Date: March 14, 2019 - 9:00 AM

URL: http://pirsa.org/19030016

Generalized Harmonic formulation

$$\Box x^a = H^a \quad \text{(Source functions)}$$

Promote H_a to independent variables

$R_{ab} = 4\pi \left(2 T_{ab} - g_{ab} T \right)$ EFEs trace-reversed

$$R_{ab} = -\frac{1}{2} g^{cd} \partial_c \partial_d g_{ab} + \nabla_{(a} \Gamma_{b)} + F(g_{ab}, \partial_c g_{ab})$$

$$\Gamma_c = g^{ab} \Gamma_{abc} = g_{cd} \nabla_a \nabla^a x^d$$

$$-8\pi\left(2T_{ab} - g_{ab}T\right) = \underbrace{g^{cd}\partial_c\partial_d}_{\text{principal part}} g_{ab} + \partial$$

$$\partial_a H_b + \partial_b H_a + 2 H_d$$

$$T) = \underbrace{g^{cd}\partial_c\partial_d}_{\text{principal part}} g_{ab} + \partial_b g^{cd}\partial_c g_{ad} + \partial_a g^{cd}\partial_c g_{bd} +$$

$$\partial_a H_b + \partial_b H_a + 2 H_d \Gamma^d{}_{ab} + 2\Gamma^c{}_{db}\Gamma^d{}_{ca}$$

$$L(H_a) = 0 \quad (**)$$

Some function

$$H_a = F_a(g_{ab})$$

$$H_a = 0$$

Har

$$\ldots_{b} + \partial_b g^{cd} \partial_c g_{ad} + \partial_a g^{cd} \partial_c g_{bd} + \ldots$$

$(*)$

$$\ldots + 2 H_d \Gamma^d{}_{ab} + 2 \Gamma^c{}_{db} \Gamma^d{}_{ca}$$

$(**)$

$\swarrow$ Some function

$$H_a = F_a(g_{ab})$$

(Simple choice)

$$H_a = 0$$

Harmonic gauge

Evolve ($g_{ab}, \partial_t g_{ab}, H_a, \partial_t H_a$) according to (*) + (**)

$$C^a = \Box x^a - H^a$$

Gauranteed to have $C^a = 0$ for all time?

From *) $R_{ab} - 4\pi(2T_{ab} - g_{ab}T) = \nabla_{(a}C_{b)}$

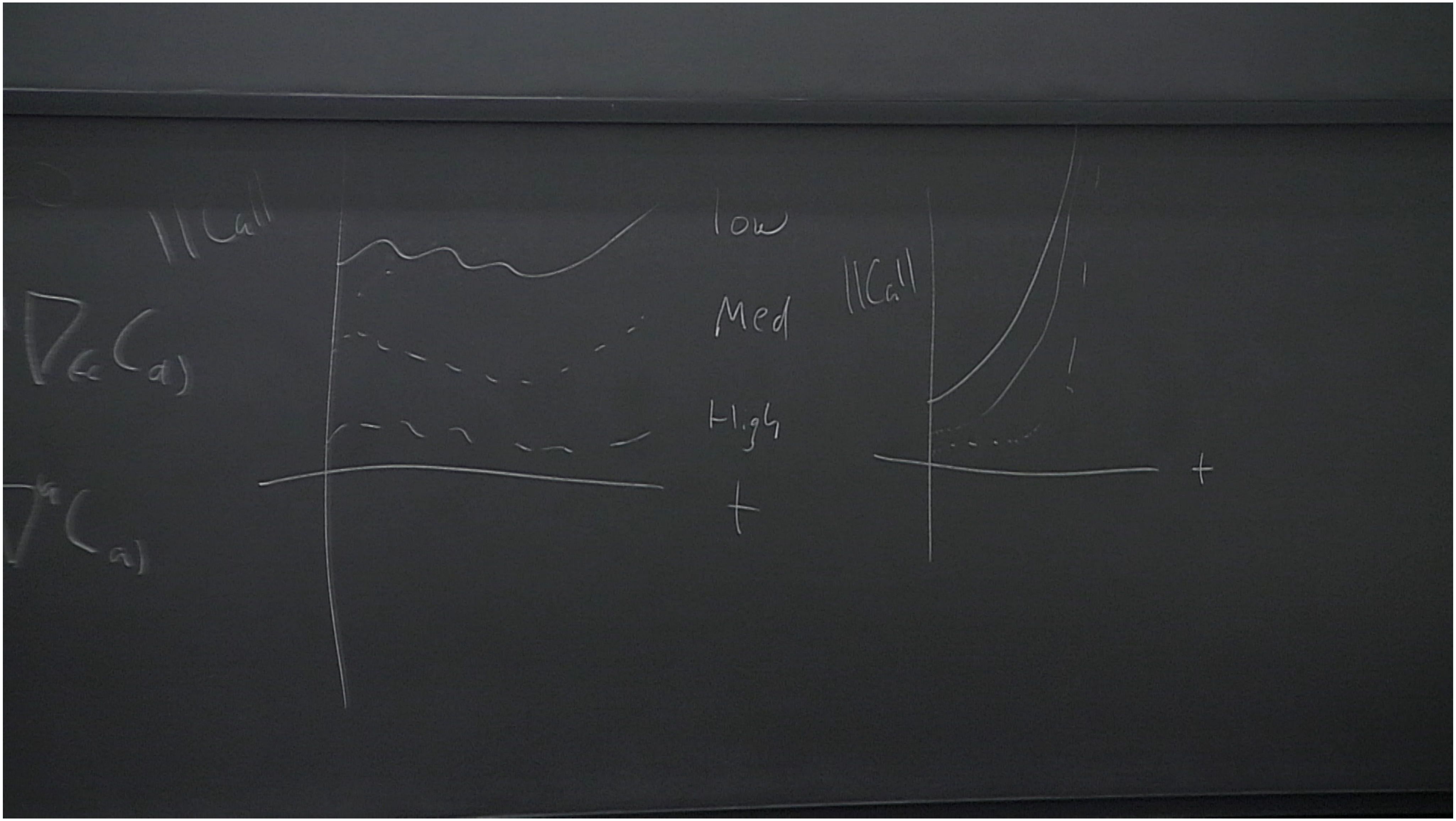
$n^a n^b \to$ Ham $\quad n^a \gamma^a_b \to$ Mom Cnst.

If $C_a = 0$ at $t=0$, $\partial_t C_a = 0$ at $t=0$

$$\left(R_{ab} - \frac{1}{2} R g_{ab}\right) - 8\pi T_{ab} = \nabla_{(a} C_{b)} - \frac{1}{2} g_{ab} g^{cd} \nabla_{(c} C_{d)}$$

$$0 = \nabla^a \nabla_{(a} C_{b)} - \frac{1}{2} \nabla_b \nabla^a C_{a)}$$

$$\nabla_a \nabla^a C_b = -R^a_b C_a$$


low
Med
High
t
t

Add to $(*)$ $\quad K\left(n_{(a}C_{b)} - \frac{1}{2} g_{ab} n^d C_d\right)$

$$\Box C^a = -R^a_b C^b + K \nabla_b [n^{(b} C^{a)}]$$

$$K \sim \frac{1}{[L]}$$

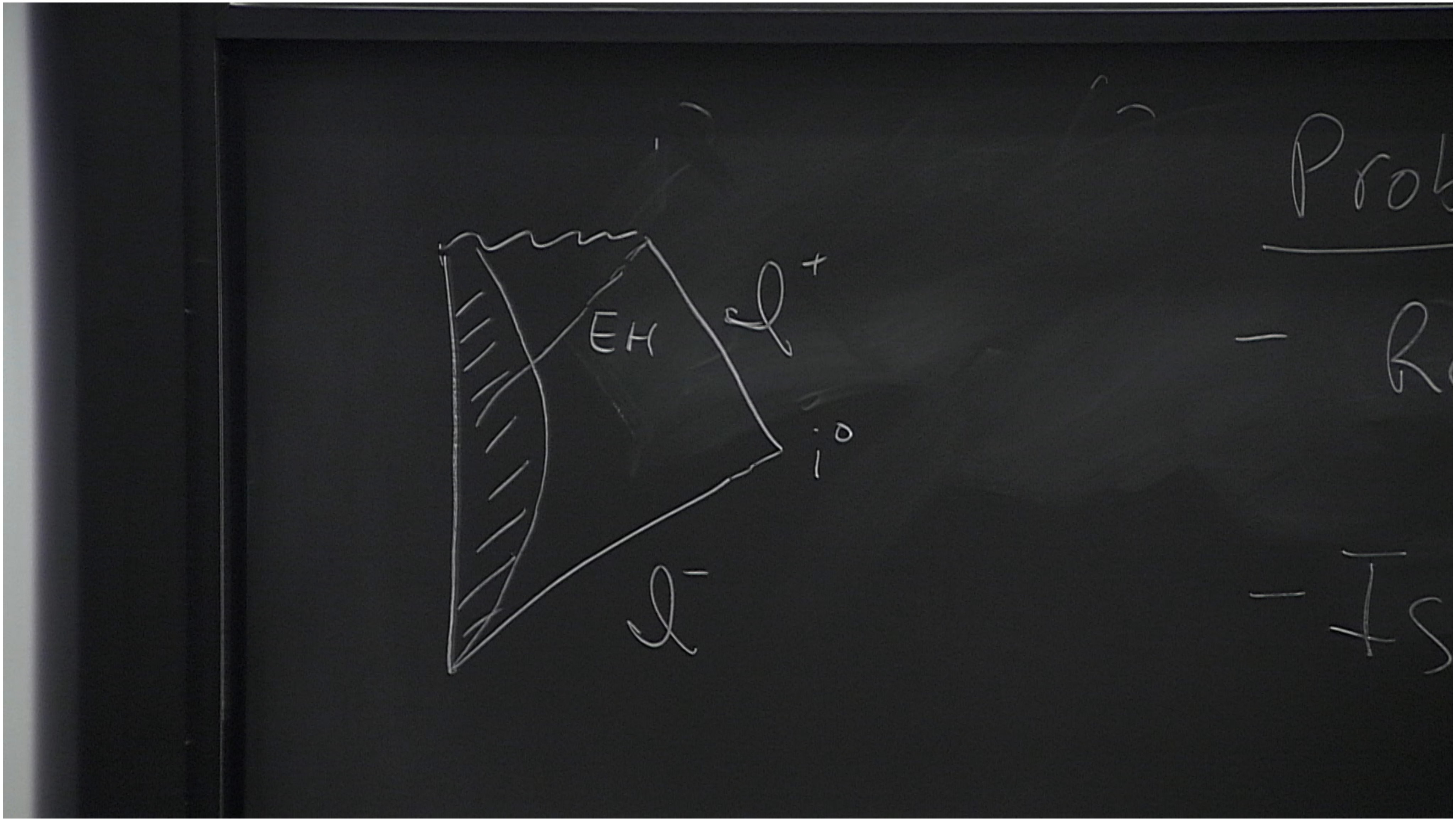
Event horizon – the boundary of the causal past

BH is $B = M - J^-(\mathscr{I}^+)$

EH is ∂B

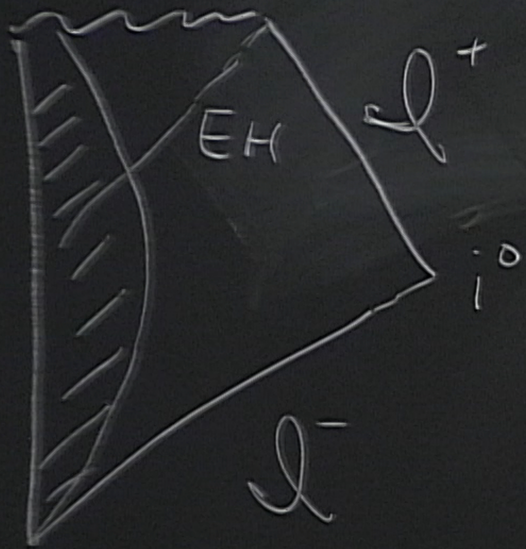
dary of the causal past of future null infinity

$J^-(\mathscr{I}^+)$


EH
$\mathscr{I}^+$
i^0
$\mathscr{I}^-$
Prob
– Re
– Is

Problems with Event horizons

- Requires knowledge of whole spacetime out to $\mathscr{I}^+$
- Is telelogical, i.e. "anticipates" future



Problems with Ev

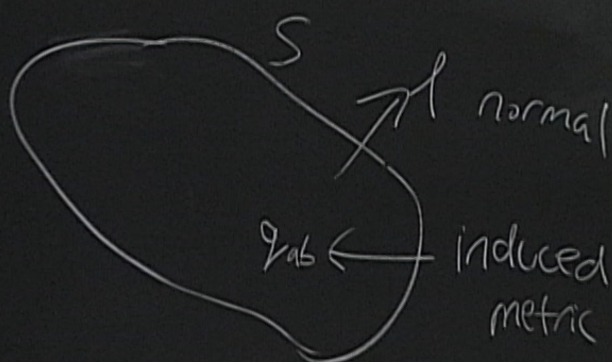
- Requires knowledge

- Is telelogical

Introduce Apparent Horizon

Timeslice Σ_t

S is closed 2-dim spacelike surface $\subset \Sigma_t$



$$\theta := \mathcal{L}_\ell \left(\ln \left(\sqrt{q} \right) \right) = q^{ab} \nabla_a \ell_b$$

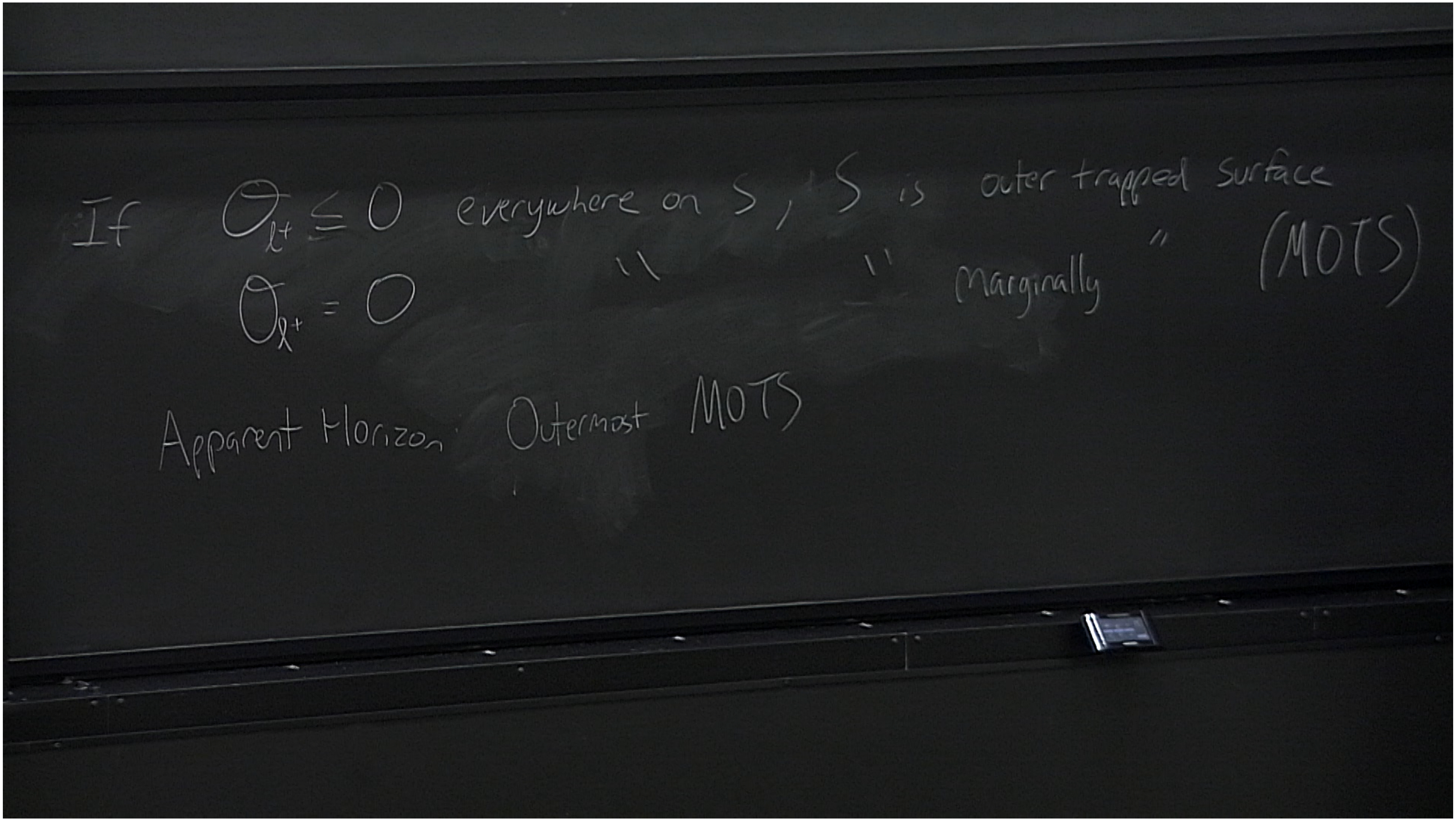
Take ℓ to be null

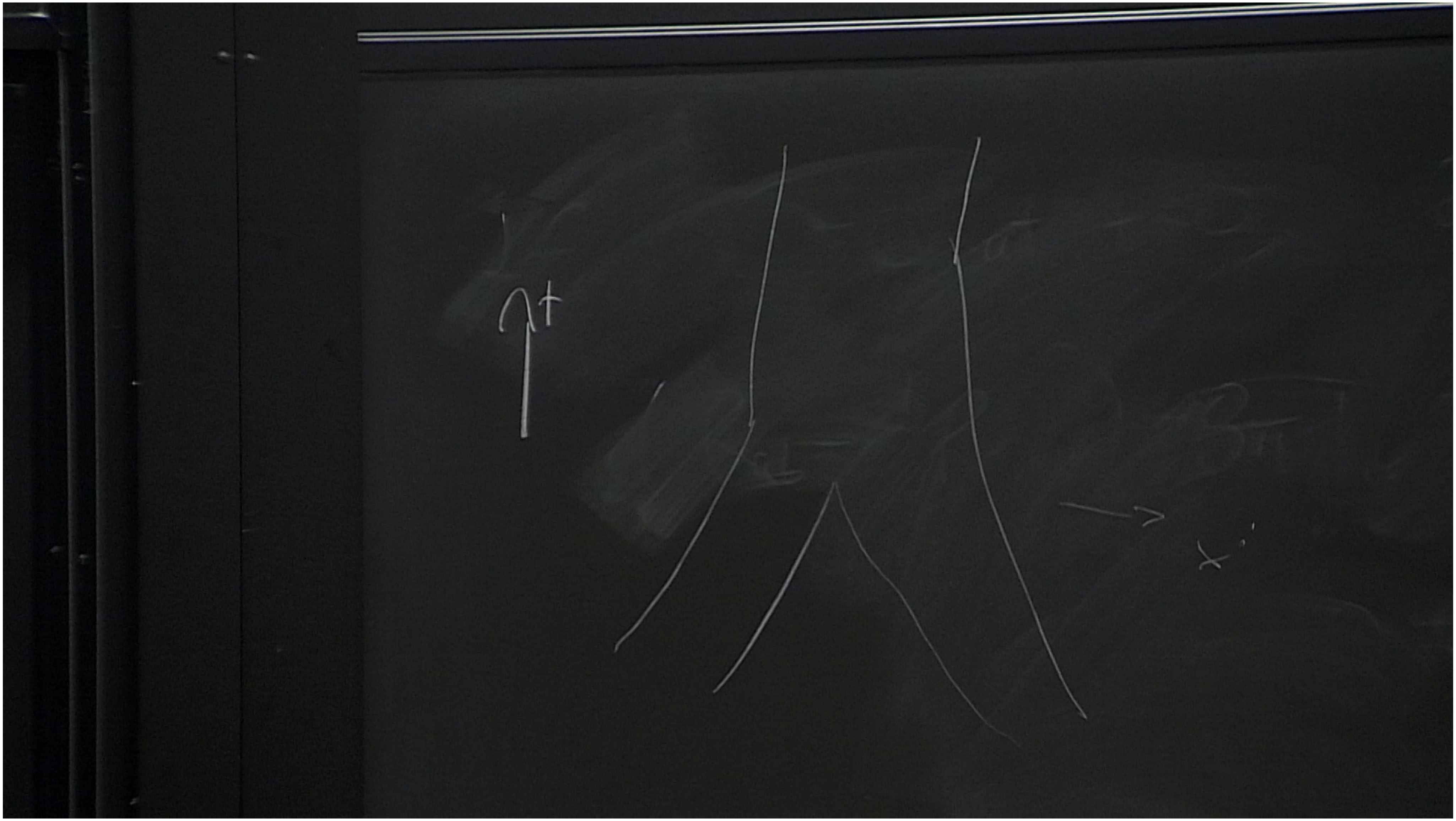
Outward/Inward $\ell_{\pm}^a = n^a \pm s^a$

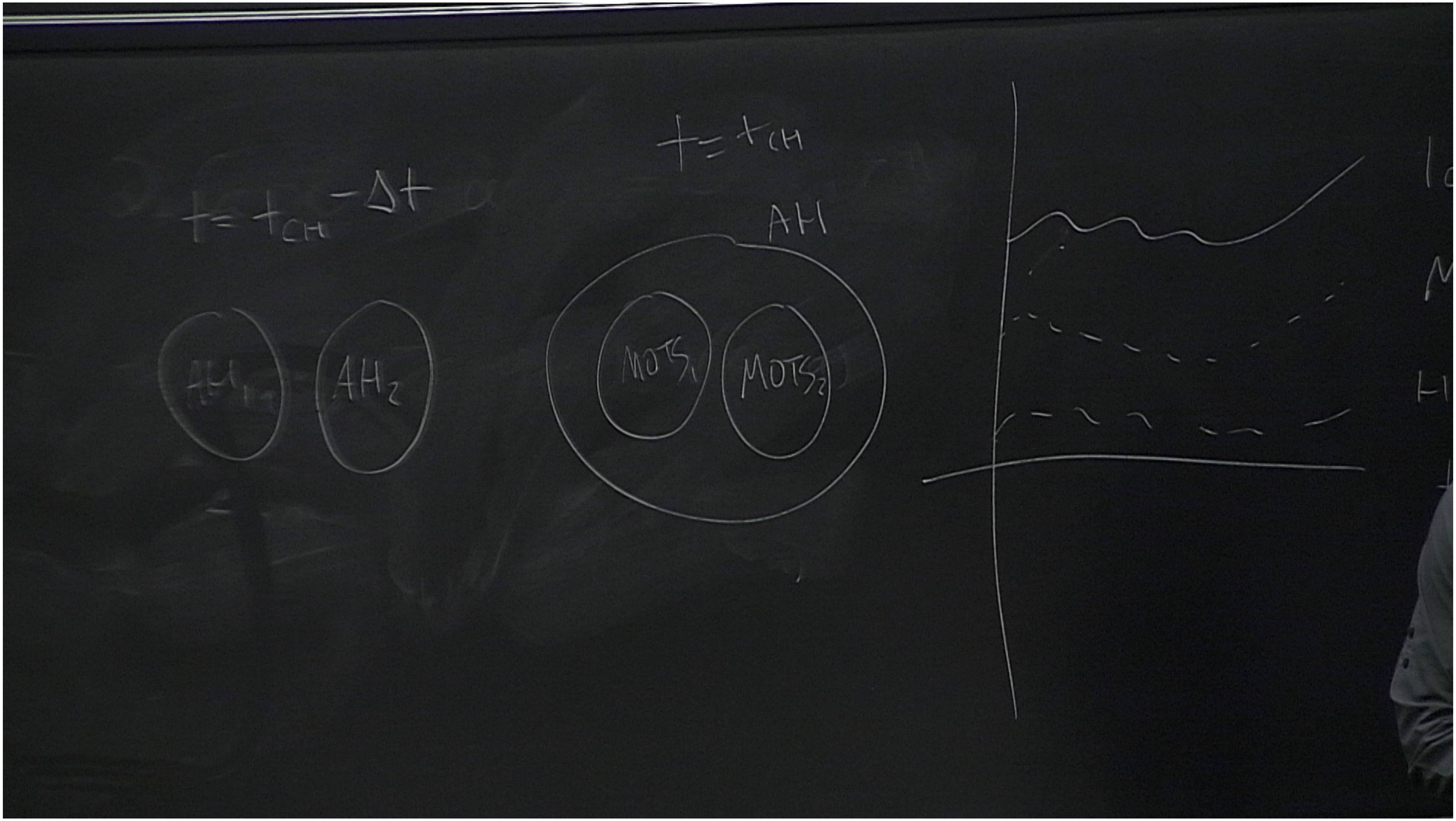
$n^a n_a = -1$, $s^a s_a = 1$, $\ell_{\pm}^a \ell_a^{\pm} = 0$, $\ell_a^+ \ell_-^a = -2$

Outward null expansion

$\Theta_{\ell_+} = q^{ab} \nabla_a \ell_b^+ = D_i s^i + K_{ij} s^i s^j - K$ in 3+1 vars


If $\theta_{l^+} \leq 0$ everywhere on S, S is outer trapped surface
$\theta_{l^+} = 0$ " " marginally " (MOTS)
Apparent Horizon: Outermost MOTS


λ^+


t = t_CH − Δt
AH_1
AH_2
t = t_CH
AH
MOTS_1
MOTS_2