Title: PSI 2018/2019 - Strong Field Gravity - Lecture 5

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Collection: PSI 2018/2019 - Strong Field Gravity (East)

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Generalized Penrose processes

$$0 \leq \Delta E_{BH} - \Omega_H \Delta J_{BH}$$

Superradiance

Scalar field:

Massless Klein-Gordon Eqn: $\Box \psi = \nabla_a \nabla^a \psi = 0$

Stress-energy tensor:

$$T_{ab} = \nabla_a \psi \nabla_b \psi - \frac{1}{2} g_{ab} \nabla_c \psi \nabla^c \psi$$

$$\psi = \Psi(r,\theta)\, \mathrm{Re}\left[e^{-i(\omega t - m\phi)} \right]$$

frequency

azimuthal number

$$\nabla^a \psi = 0$$

$$\Delta E_{BH} = \int dt \int_{r=r_+} \chi^a T_{ab} \hat{t}^b \, dS_{BH}$$

$$\chi^a = \hat{t}^a + \Omega_H \hat{\phi}^a$$

$$\chi^a \hat{t}^a = \chi^a (\chi^a - \Omega_H \hat{\phi}^a)$$

$$= \int dt \int_{r=r_+} \left[\chi^a (\nabla_a \psi)(\nabla_b \psi) \hat{t}^b - \frac{1}{2} \chi_a \hat{t}^a \nabla_c \psi \nabla^c \psi \right] dS_{BH}$$

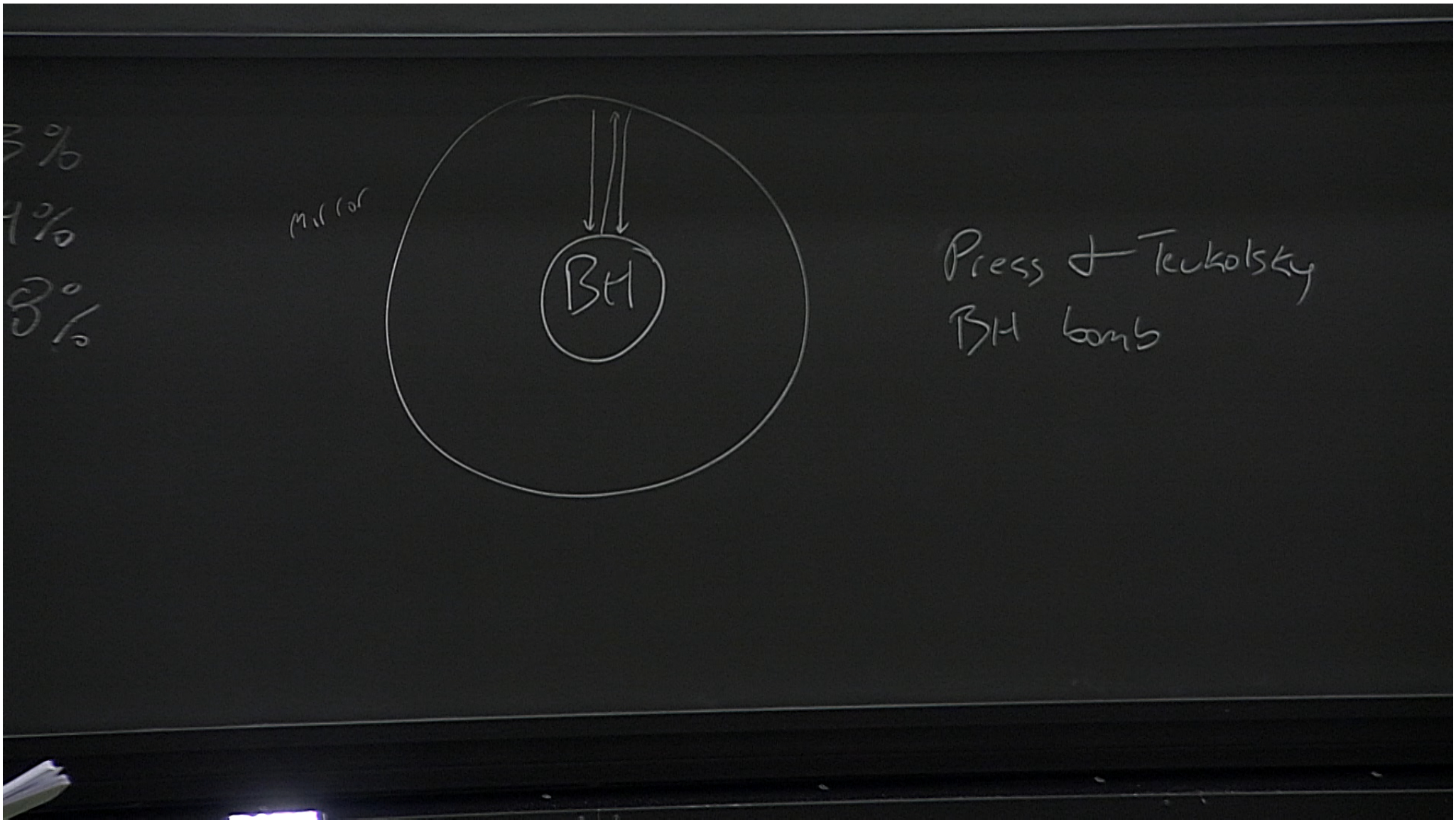
$$= \int dt \int_{r=r_+} dS_{BH} (\partial_t \psi + \Omega_H \partial_\phi \psi) \partial_t \psi$$

$$= \int dt \int_{r=r_+} dS_{BH} (\omega - m\Omega_H) \omega \underbrace{|\Psi|^2 \sin^2(\omega t - m\phi)}_{\geq 0}$$

$$0 < \omega < m\Omega_H \Rightarrow \Delta E_{BH} < 0$$

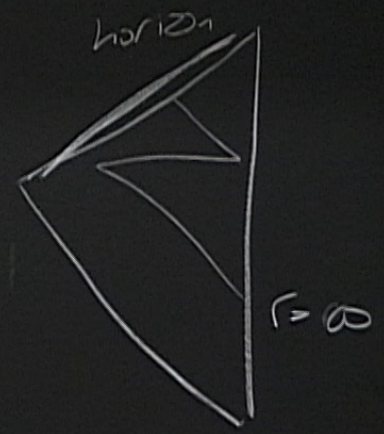
$$\frac{\delta E}{\delta J} = \frac{\omega}{m}$$

Maximum Amplification factor	$s=0$	scalar	0.3%
	$s=1$	E-M	4.4%
	$s=2$	GWs	138%


3%
4%
8%
mirror
BH
Press & Teukolsky
BH bomb

Press & Teukolsky

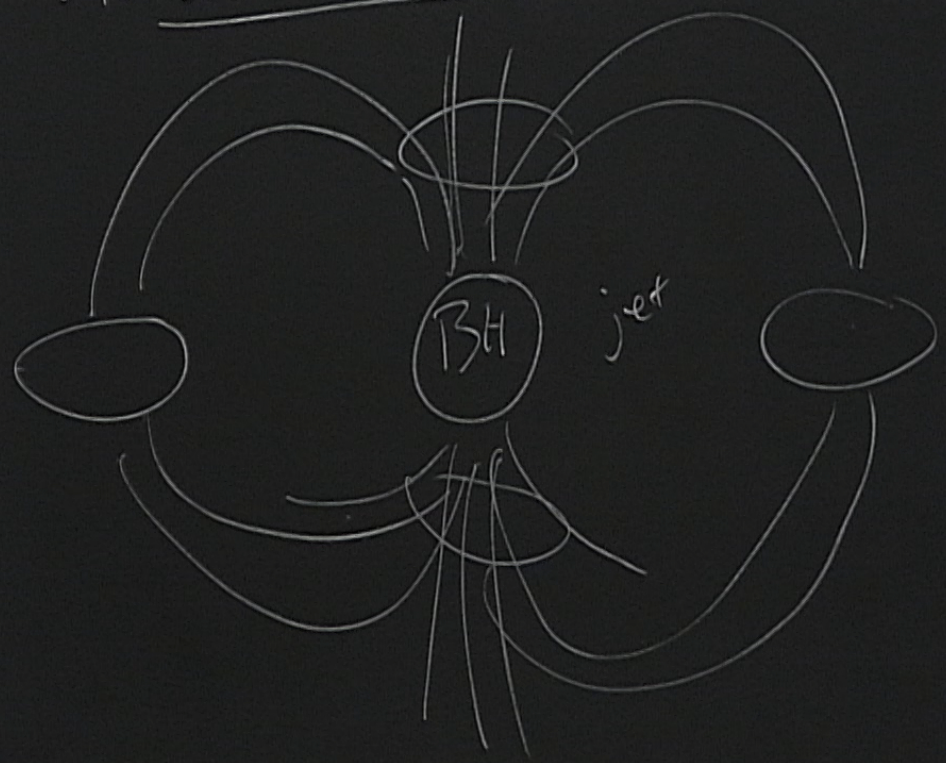
BH bomb



Generalized Ergoregion for Charged BHs

$$E_K = -\hat{t}^a P_a = -m\hat{t}^a U_a - q\hat{t}^a A_a$$

Blandford Znajek



Massive

K-G Eqn.

$\mu^{-1} >> \Lambda$

$\Box\phi = \nabla$

Massive scalar field

K-G Eqn. $\Box\phi = \mu^2\phi$

$\mu^{-1} \gg M$, $r \gg M$, non-relativistic limit

$$\Box\phi = \nabla_a\left(g^{ab}\partial_b\phi\right) = \frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}\,g^{ab}\partial_b\phi\right)$$

For Schw. $\sqrt{-g} = r\sin\theta$

$$g^{tt}\partial_t^2\phi + \partial_i\partial^i\phi - \mu^2\phi = 0 \qquad g^{tt} \approx -\left(1+\frac{2M}{r}\right)$$

Use ansatz: $\phi = \frac{1}{\sqrt{2\mu}}\left[\psi(x)e^{-i\omega t} + \psi(x)e^{i\omega t}\right]$

$$\omega \approx \mu$$

$$\left[-\left(1+\frac{2M}{r}\right)(-i\omega)^2 + \partial_i\partial^i - \mu^2\right]\psi = 0$$

$$(\omega^2-\mu^2)\psi = \left(-\partial_i\partial^i - \frac{2M\omega^2}{r}\right)\psi$$

$$(u-\mu)\psi = \left(-\frac{1}{2\mu}\partial_i\partial^i - \frac{\mu M}{r}\right)\psi$$

$$\alpha = \mu M$$

Energy $= u - \mu$

Bohr radius $\sim \mu^{-1}\alpha$

(n, l, m)

$l = m = 1$

$$\phi \sim e^{-r/2r_s} \, Y^l_m(\theta, \phi) \, e^{-i(\omega t)}$$

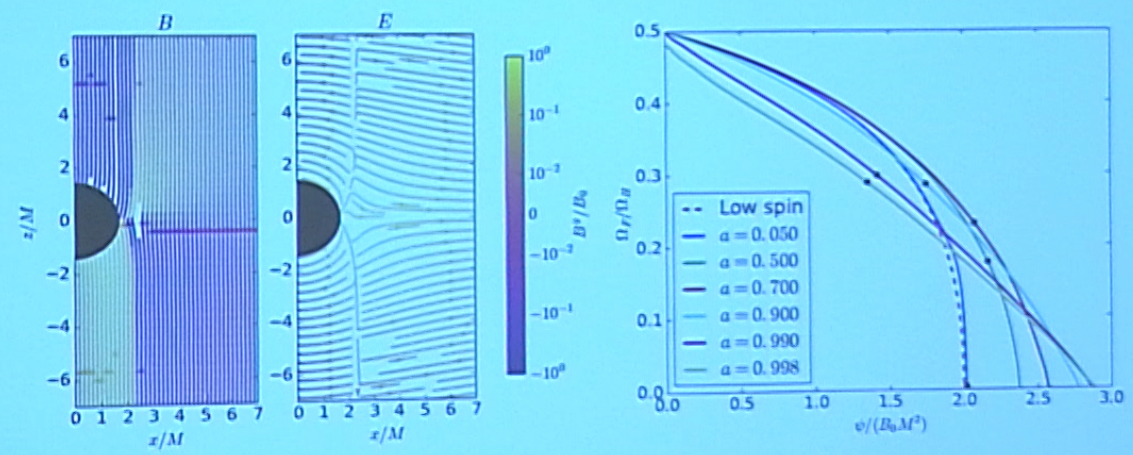
complex

$$\mathrm{Re}(\omega) = \mu\left(1 - \frac{\alpha^2}{2n^2}\right)$$

$$|\mathrm{Im}(\omega)| \ll |\mathrm{Re}(\omega)|$$

$$\alpha = 0.1 \left(\frac{\mu}{10^{-12}\,\mathrm{eV}} \right) \left(\frac{M_{BH}}{10\,M_\odot} \right)$$

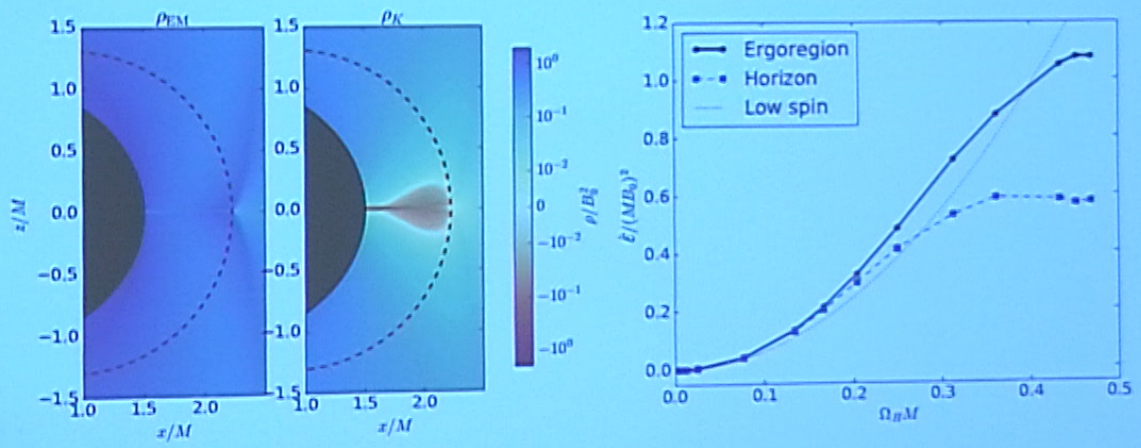
Black hole rotation powering jets



WE & Yang (2018)

Uniform field force-free jet solution (see also Komissarov 2004, Palenzuela et al. 2010, others)

Dissipating negative energy



WE & Yang (2018)

In ergosphere Killing energy density can be negative.

$$l = m = 1$$

$$\phi \sim e^{-r/2r_s} \, Y^l_m(\theta, \phi) \, e^{-i(\omega t)}$$

complex

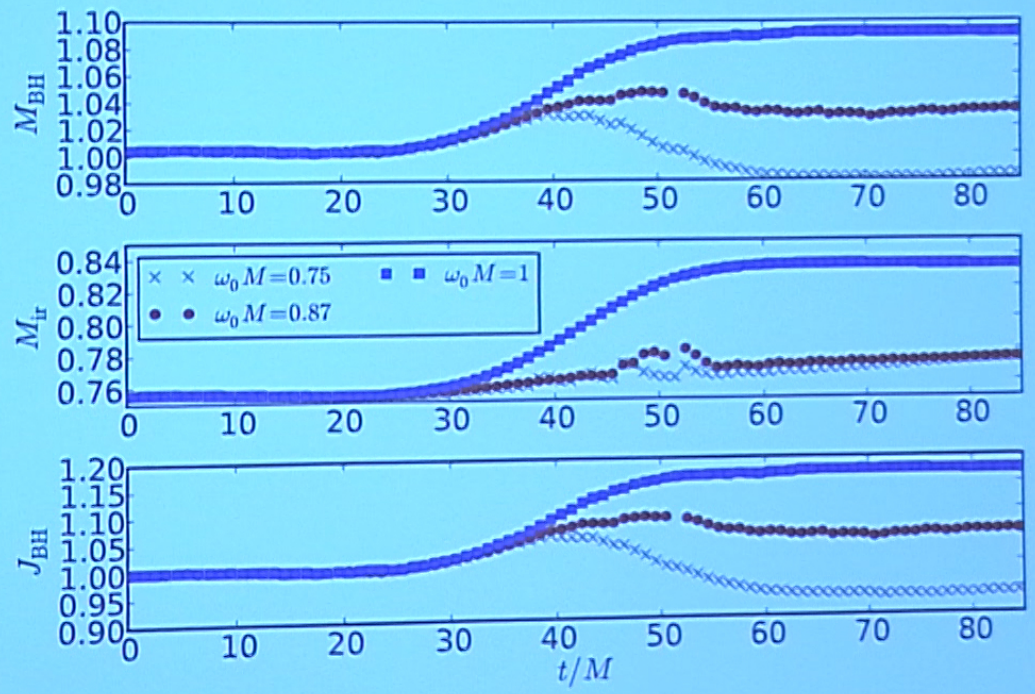
$$n_a n_b T^{ab}$$

$$n^b T^a{}_b \hat{T}_a$$

$$\mathrm{Re}(\omega) = \mu \left(1 - \frac{\alpha^2}{2n^2}\right)$$

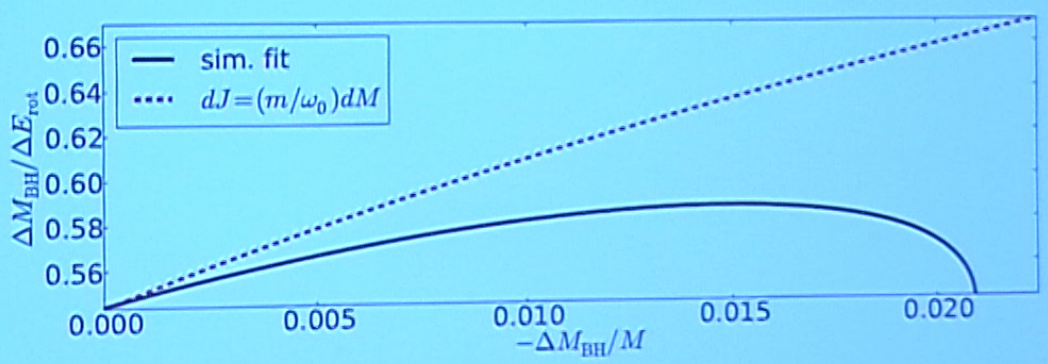
$$|\mathrm{Im}(\omega)| \ll |\mathrm{Re}(\omega)|$$

Nonlinear gravitational wave superradiance



WE, Ramazanoglu & Pretorius (2013)

Liberating energy/angular momentum from a black hole

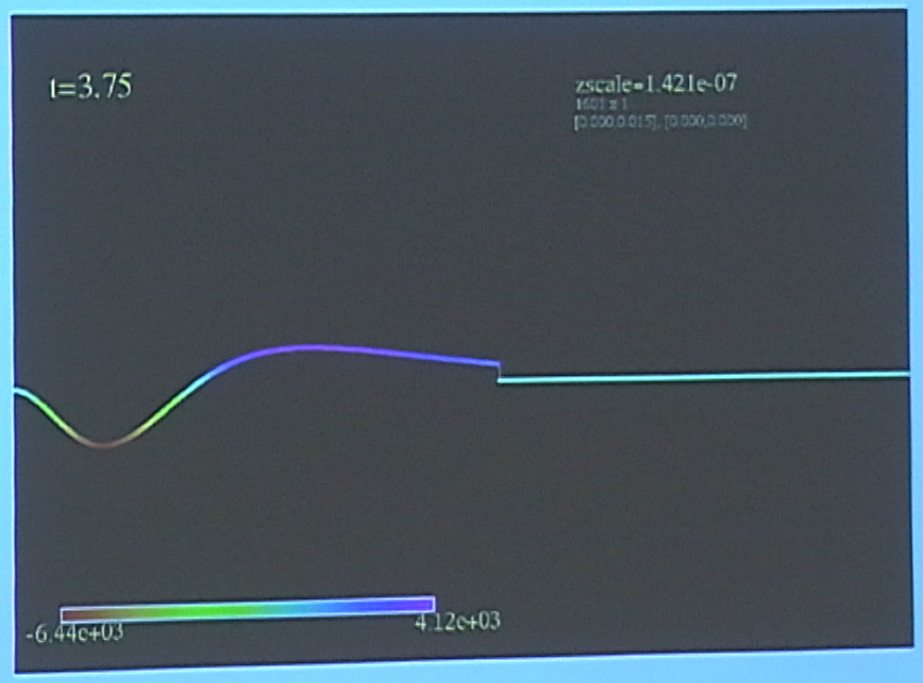


WE, Ramazanoglu & Pretorius (2013)

Efficiency of energy extraction reduced in nonlinear regime

Superradiant instability in Anti-de Sitter Space

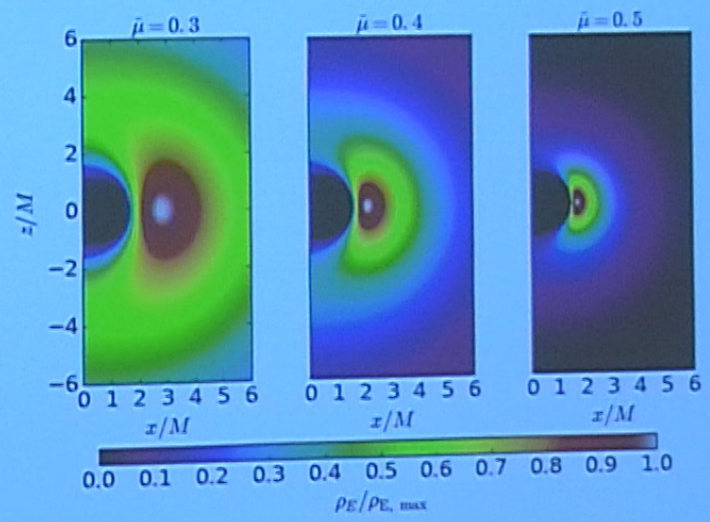
Charged BH/scalar field in AdS



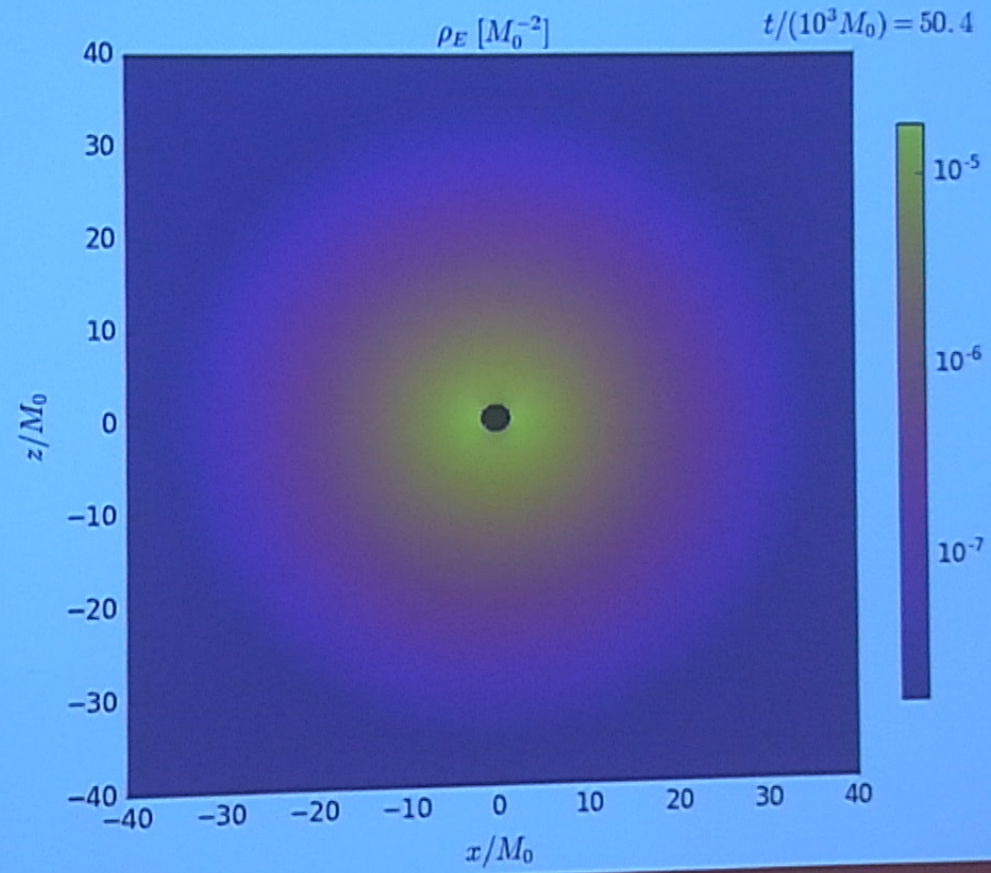
Bosch, Green, & Lehner (2016)

Superradiant instability of massive bosons

- **Massive** bosons with frequency $\omega < m\Omega_H$ grow exponentially in time.
- Significant when Compton wavelength comparable to black hole radius.

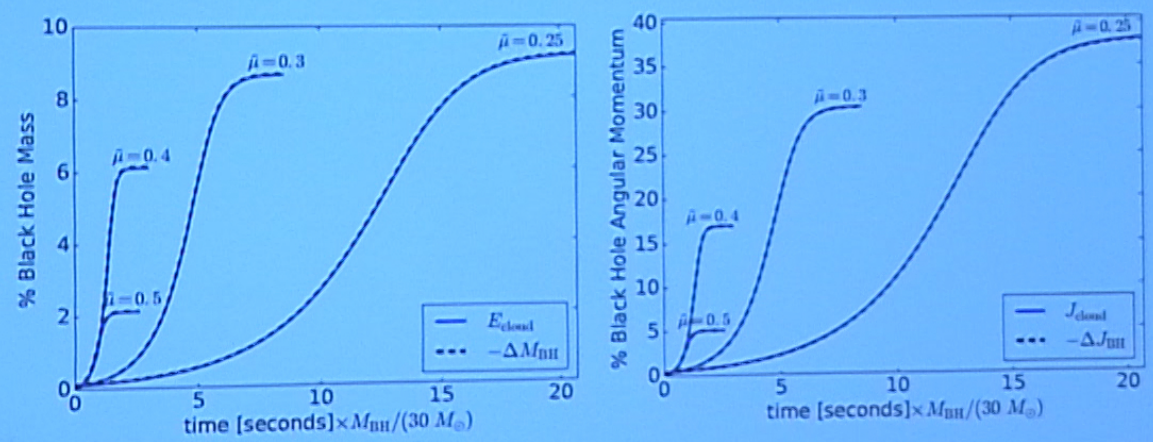


Massive vector field energy density



Superradiant instability: spinning down a black hole

Black hole with initial spin $a = 0.99$.



WE & Pretorius (2017)

Vector boson mass: $\approx 10^{-12}$ eV $(\tilde{\mu}/0.25)(30M_\odot/M_{BH})$

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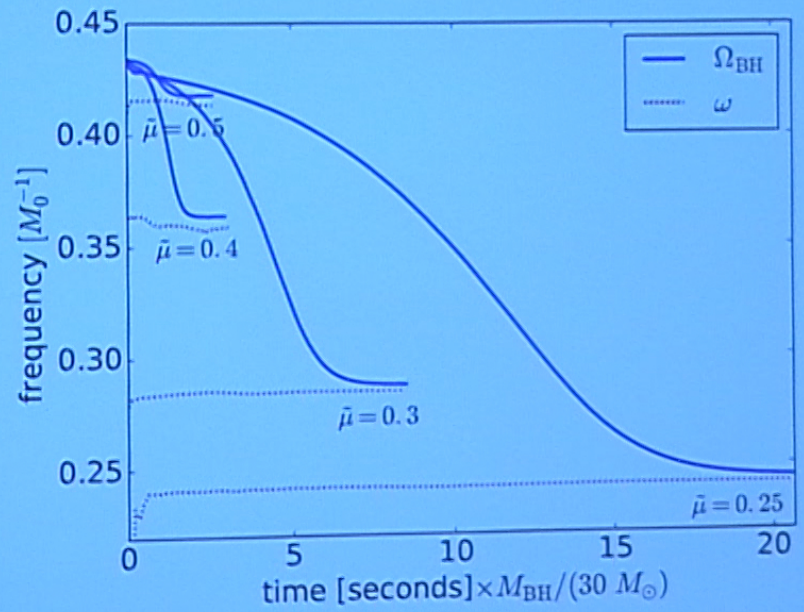

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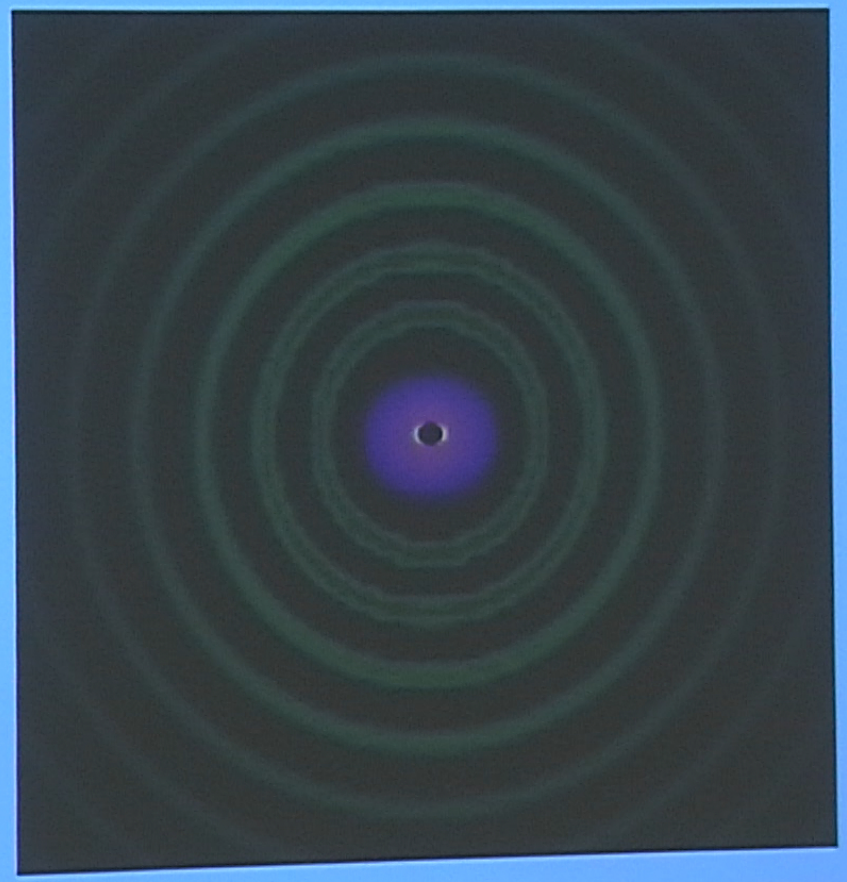
Vector boson mass: $\approx 10^{-12}$ eV $(\tilde{\mu}/0.25)(30 M_\odot / M_{\rm BH})$

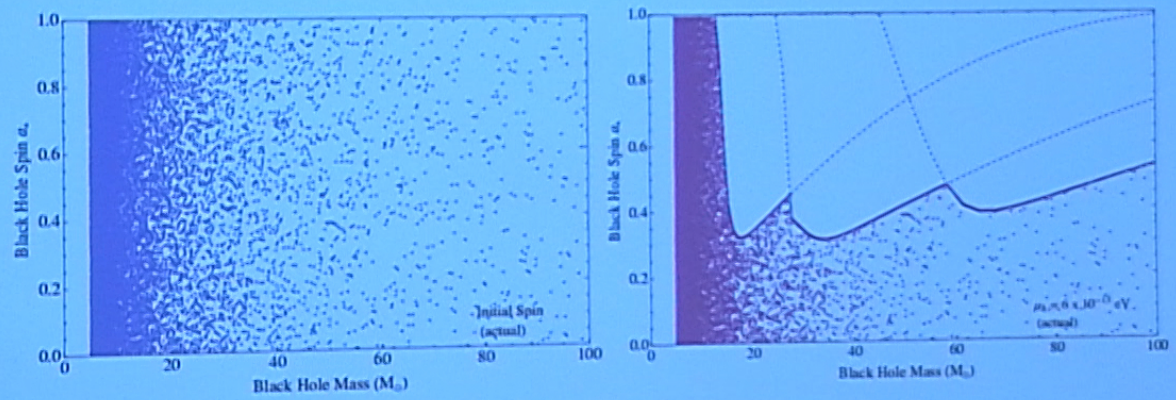
Massive vector field energy density and GWs



WE (2018)

Observing the superradiant instability

Indirect probe: Use black hole spin measurements from GWs or accretion disks (Arvanitaki et al. 2015, 2017)



Need reliable spin measurements