Title: PSI 2018/2019 - Strong Field Gravity - Lecture 4

Speakers: William East

Collection: PSI 2018/2019 - Strong Field Gravity (East)

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$\delta J \Omega_H < \delta E$

BH Area Thm + Thermodynamics

Hawking (1971): BH Area: $\delta A_{BH} \geq 0$

Assumes: NEC, Cosmic Censorship

BH Area

$$A_{BH} = \int \left| \sqrt{g_{\theta\theta}\, g_{\phi\phi}} \right|_{r=r_+} d\theta\, d\phi$$

$$g_{\theta\theta} g_{\phi\phi} = (r_+^2 + a^2)\sin^2\theta$$

$$A_{BH} = 4\pi\left(r_+^2 + a^2\right) \qquad r_+ = M + \sqrt{M^2 - a^2}$$

$$= 4\pi\left(\left(M + \sqrt{M^2 - a^2}\right)^2 + a^2\right)$$

$$= 8\pi\left(M^2 + \sqrt{M^4 - J^2}\right)$$

BH Angular Mom

$$\frac{\delta A_{BH}}{8\pi} = 2M\delta M + \frac{1}{2}(M^4-J^2)^{-1/2}(4M^3\delta M - 2J\delta J)$$

$$= \frac{J}{\sqrt{M^4-J^2}}\Bigg(\underbrace{\left(\frac{2M\sqrt{M^4-J^2}}{J} + \frac{2M^3}{J}\right)}_{=\frac{2M}{J}\left(M^2+\sqrt{M^4-J^2}\right)}\delta M - \delta J\Bigg)$$

$$\frac{2M}{J}\left(M^2+\sqrt{M^4-J^2}\right) = \frac{2r_+}{\bar a} = \Omega_H^{-1}$$

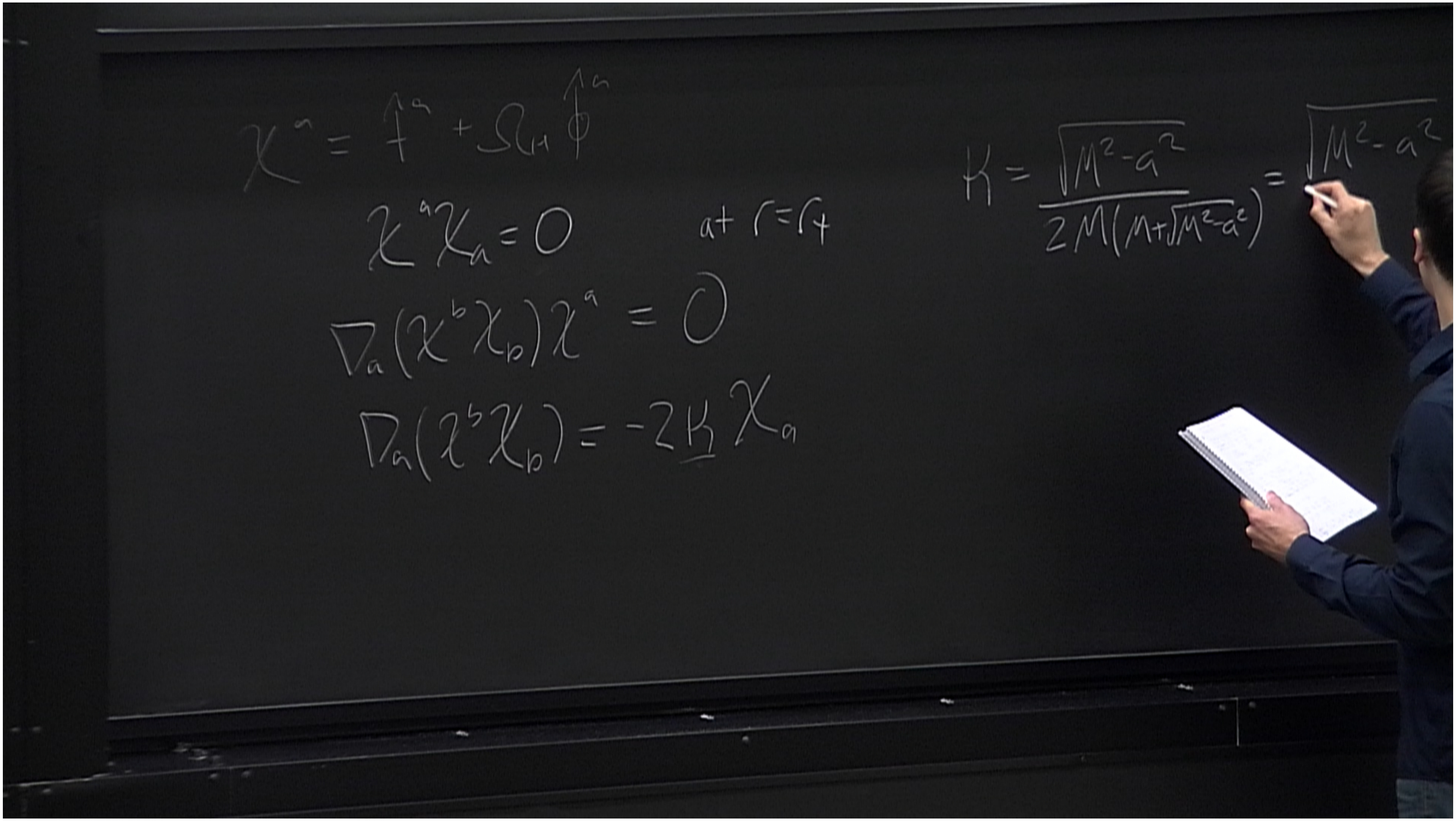
$$\boxed{\frac{\delta A_{BH}}{8\pi} = \frac{\bar a}{\sqrt{1-\bar a^2}}\left(\Omega_H^{-1}\delta M - \delta J\right)} \geq 0 \qquad \delta J\,\Omega_H$$

$$M_{ir} = \sqrt{\frac{A_{BH}}{16\pi}}$$

$$a=0 \,, \quad M_{ir} = \sqrt{\frac{4\pi(2M)^2}{16\pi}} = M$$

$$a=M \quad M_{ir} = \sqrt{\frac{4\pi(M^2+M^2)}{16\pi}} = \frac{1}{\sqrt{2}}M$$

$$E_{rot} = M - M_{ir} = \left(1-\frac{1}{\sqrt{2}}\right)M \approx 0.29\,M$$



$\chi^a = t^a + \Omega_H \phi^a$
$\chi^a \chi_a = 0$
at $r = r_+$
$\nabla_a(\chi^b \chi_b)\chi^a = 0$
$\nabla_a(\chi^b \chi_b) = -2\kappa \chi_a$
$\kappa = \frac{\sqrt{M^2 - a^2}}{2M(M + \sqrt{M^2 - a^2})} = \sqrt{M^2 - a^2}$

$\Omega_H \phi^a$

$_{a} = 0$ at $r = r_+$

$\chi_b)\chi^a = 0$

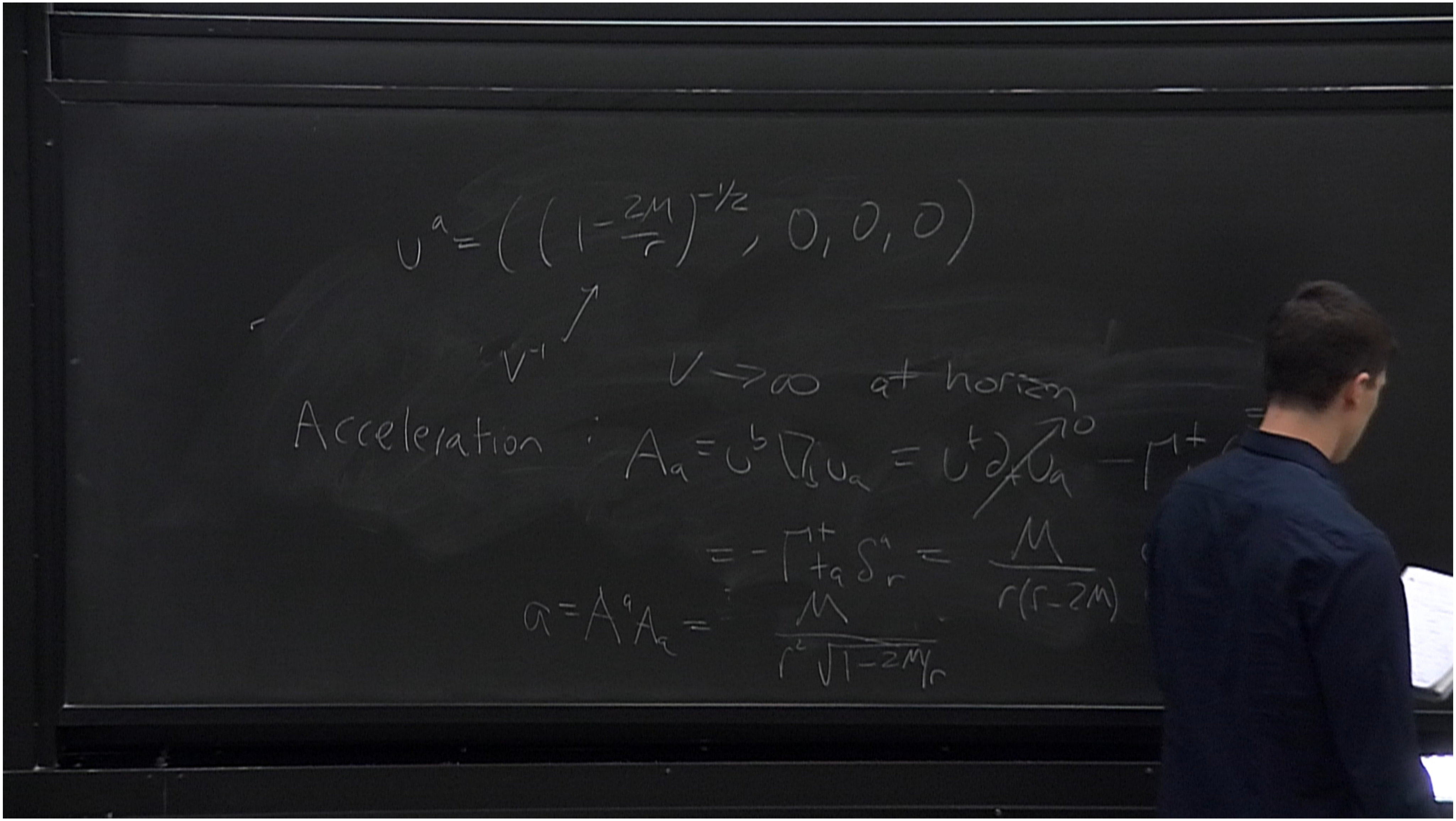
$\chi_b) = -2\underline{\kappa}\, \chi_a$

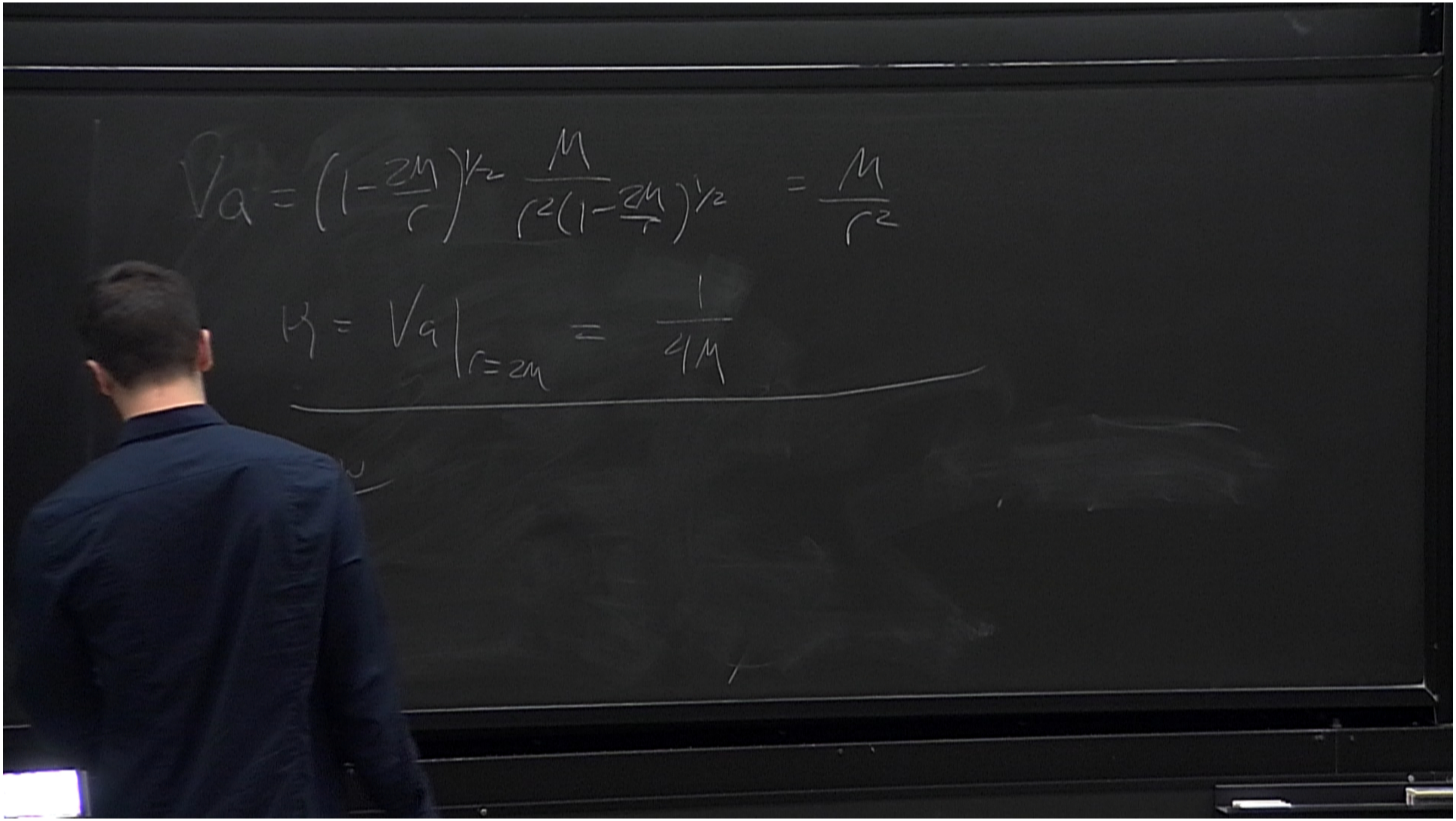
$$\kappa = \frac{\sqrt{M^2-a^2}}{2M(M+\sqrt{M^2-a^2})} = \frac{\sqrt{M^2-a^2}}{2Mr_+} = (\Omega_H/\bar{a})\sqrt{1-\bar{a}^2}$$

For $a=0$ $\quad \kappa = \frac{1}{4M}$

0th law BH Thermodynamics

Surface gravity κ is constant on stationary BH horizon


$u^a = ((1-\frac{2M}{r})^{-1/2}, 0, 0, 0)$
V^{-1}
$V \to \infty$ at horizon
Acceleration: $A_a = u^b \nabla_b u_a = u^t \partial_t u_a - \Gamma^t$
$= -\Gamma^t{}_{ta}\delta^a_r = \frac{M}{r(r-2M)}$
$a = A^a A_a = \frac{M}{r^2\sqrt{1-2M/r}}$


$Va = \left(1-\frac{2M}{r}\right)^{1/2} \frac{M}{r^2\left(1-\frac{2M}{r}\right)^{1/2}} = \frac{M}{r^2}$
$\kappa = Va\big|_{r=2M} = \frac{1}{4M}$

$$Va = \left(1-\frac{2M}{r}\right)^{1/2} \frac{M}{r^2\left(1-\frac{2M}{r}\right)^{1/2}} = \frac{M}{r^2}$$

$$\kappa = Va\Big|_{r=2M} = \frac{1}{4M}$$

1st Law: $\delta M = \frac{\kappa}{8\pi}\,\delta A_{BH} + \Omega_H\,\delta J$

cf. $dE = T\,dS - P\,dV$

$$M_{ir} = \sqrt{\frac{A_{BH}}{16\pi}}$$

$$a=0\,, \quad M_{ir} = \sqrt{\frac{4\pi(2M)^2}{16\pi}} = M$$

$$a=M \quad M_{ir} = \sqrt{\frac{4\pi(M^2+M^2)}{16\pi}} = \frac{1}{\sqrt{2}}M$$

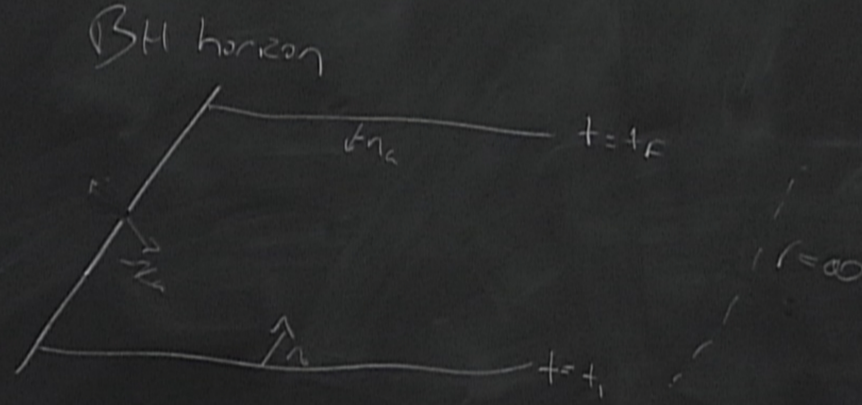
$$E_{rot} = M - M_{ir} = \left(1-\frac{1}{\sqrt{2}}\right)M \approx 0.29\,M$$

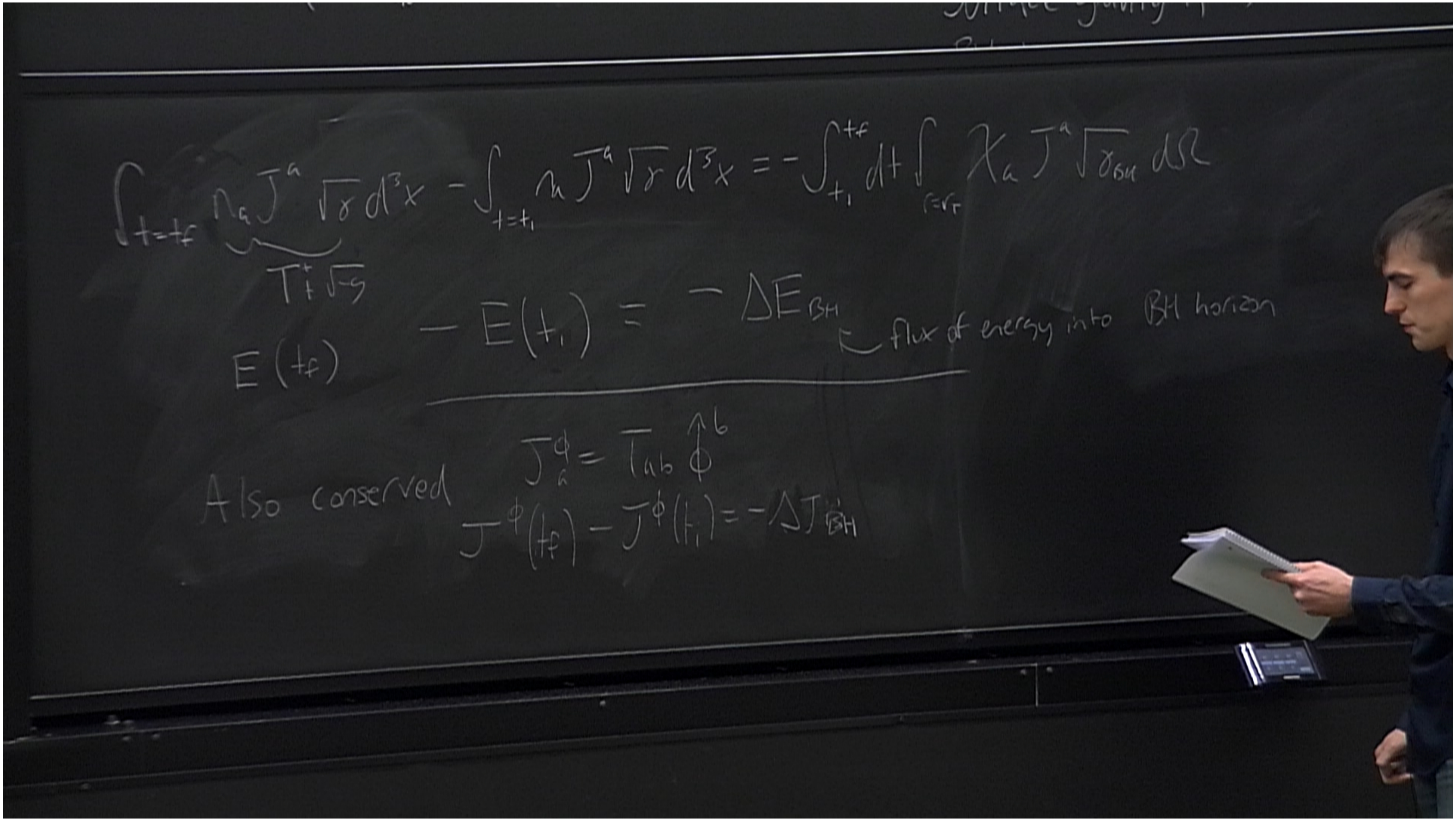
2nd law

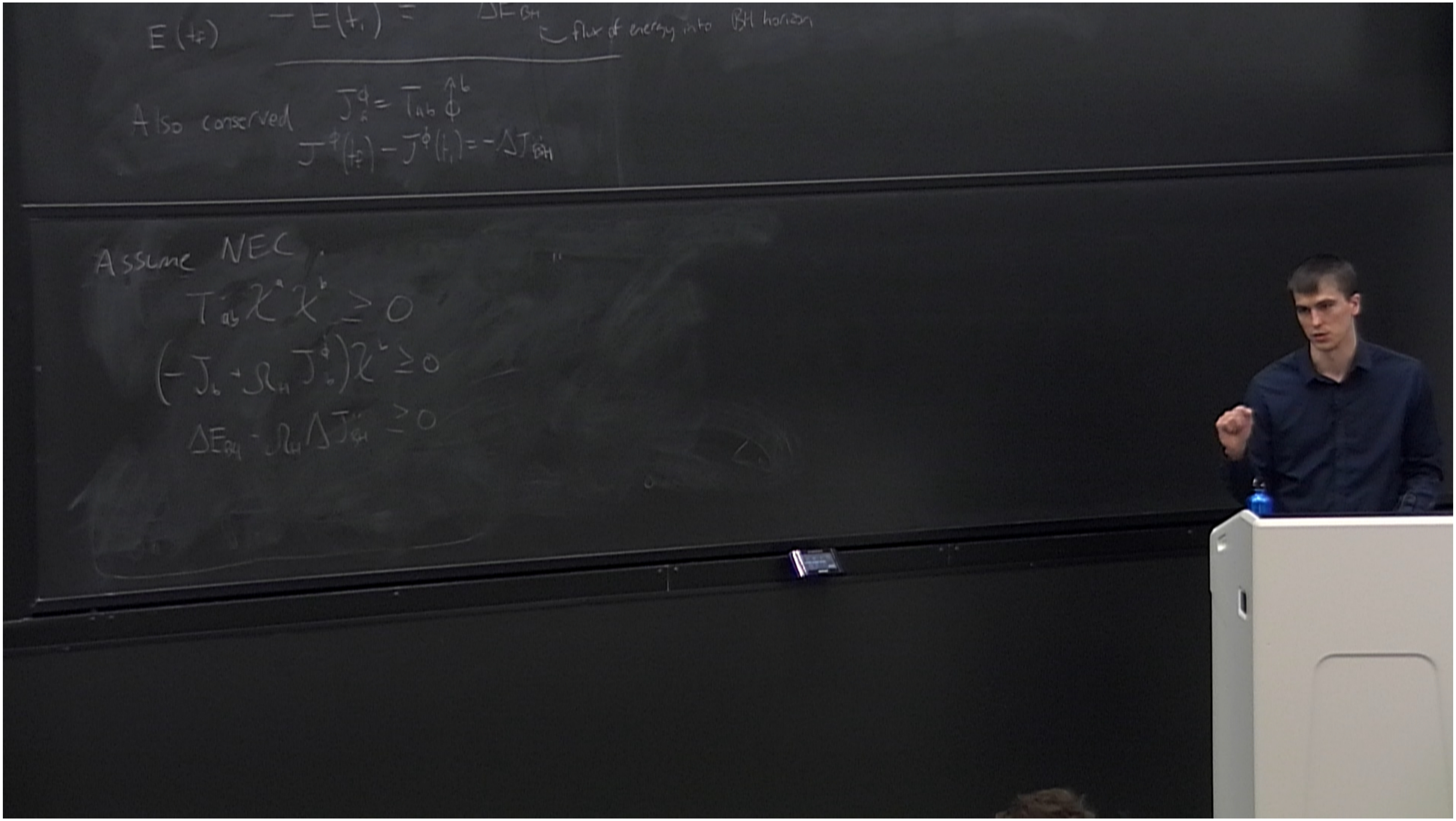
3rd law

$M_1 =$

Stoke's Thm: $\int_M \underbrace{\nabla_a J^a}_{=0} \sqrt{-g}\, d^n x = \int_{\partial M} n_a J^a \sqrt{\gamma}\, \underbrace{d^{n-1}x}_{=0}$




$\int_{t=t_f} n_a J^a \sqrt{\gamma}\, d^3x - \int_{t=t_i} n_a J^a \sqrt{\gamma}\, d^3x = -\int_{t_i}^{t_f} dt \int_{r=r_+} \chi_a J^a \sqrt{\gamma_{\partial H}}\, d\Omega$
$T^t{}_t \sqrt{-g}$
$E(t_f) - E(t_i) = -\Delta E_{BH}$
flux of energy into BH horizon
Also conserved
$J^\phi_a = T_{ab}\phi^b$
$J^\phi(t_f) - J^\phi(t_i) = -\Delta J^\phi_{BH}$


$E(t_f) - E(t_i) = \Delta E_{BH}$
flux of energy into BH horizon
Also conserved $J^{\phi}_a = T_{ab}\phi^b$
$J^{\phi}(t_f) - J^{\phi}(t_i) = -\Delta J_{BH}$
Assume NEC
$T_{ab}\chi^a\chi^b \geq 0$
$(-J_b + \Omega_H J^{\phi}_b)\chi^b \geq 0$
$\Delta E_{BH} - \Omega_H \Delta J_{BH} \geq 0$