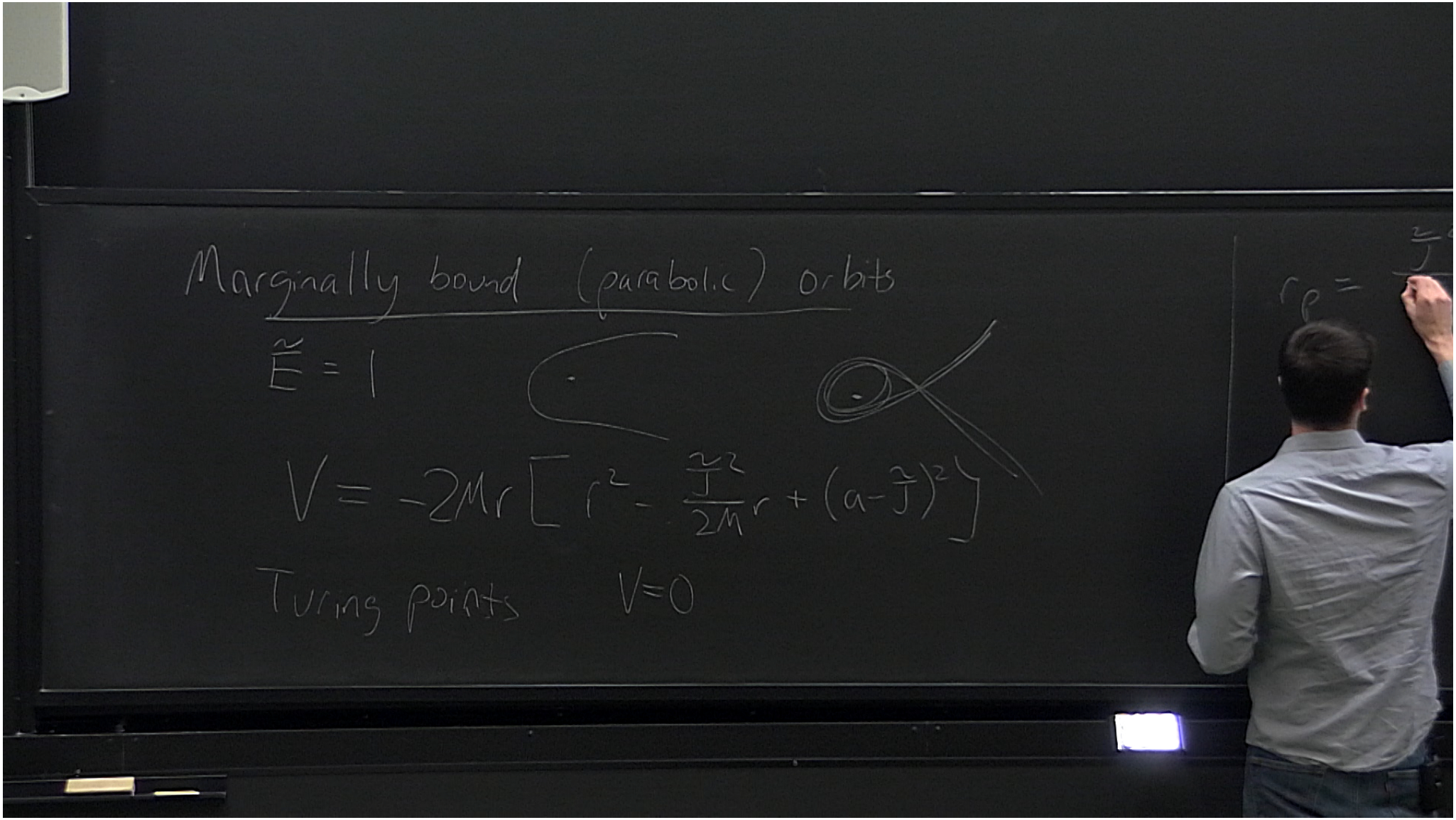
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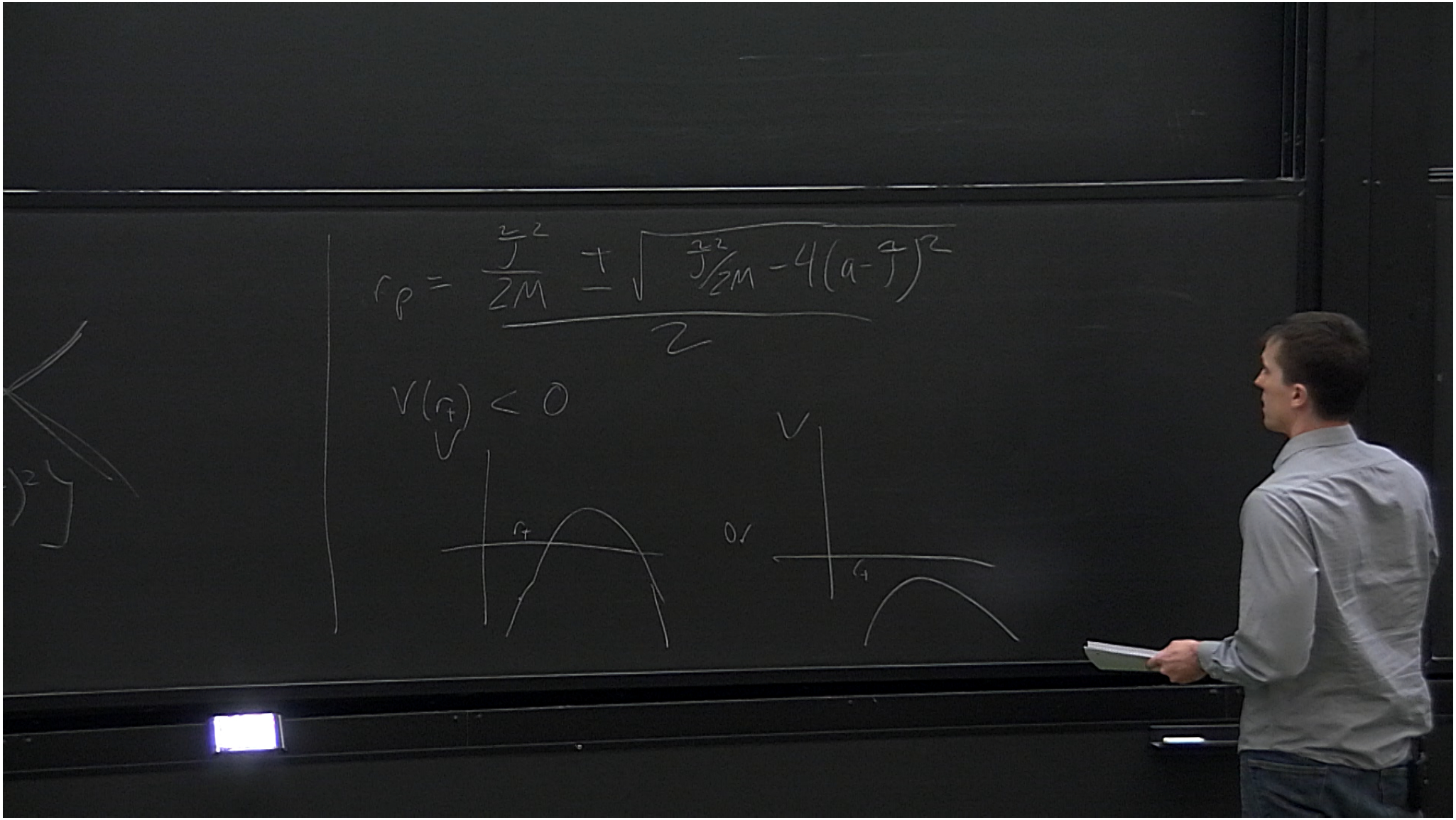
Speakers: William East

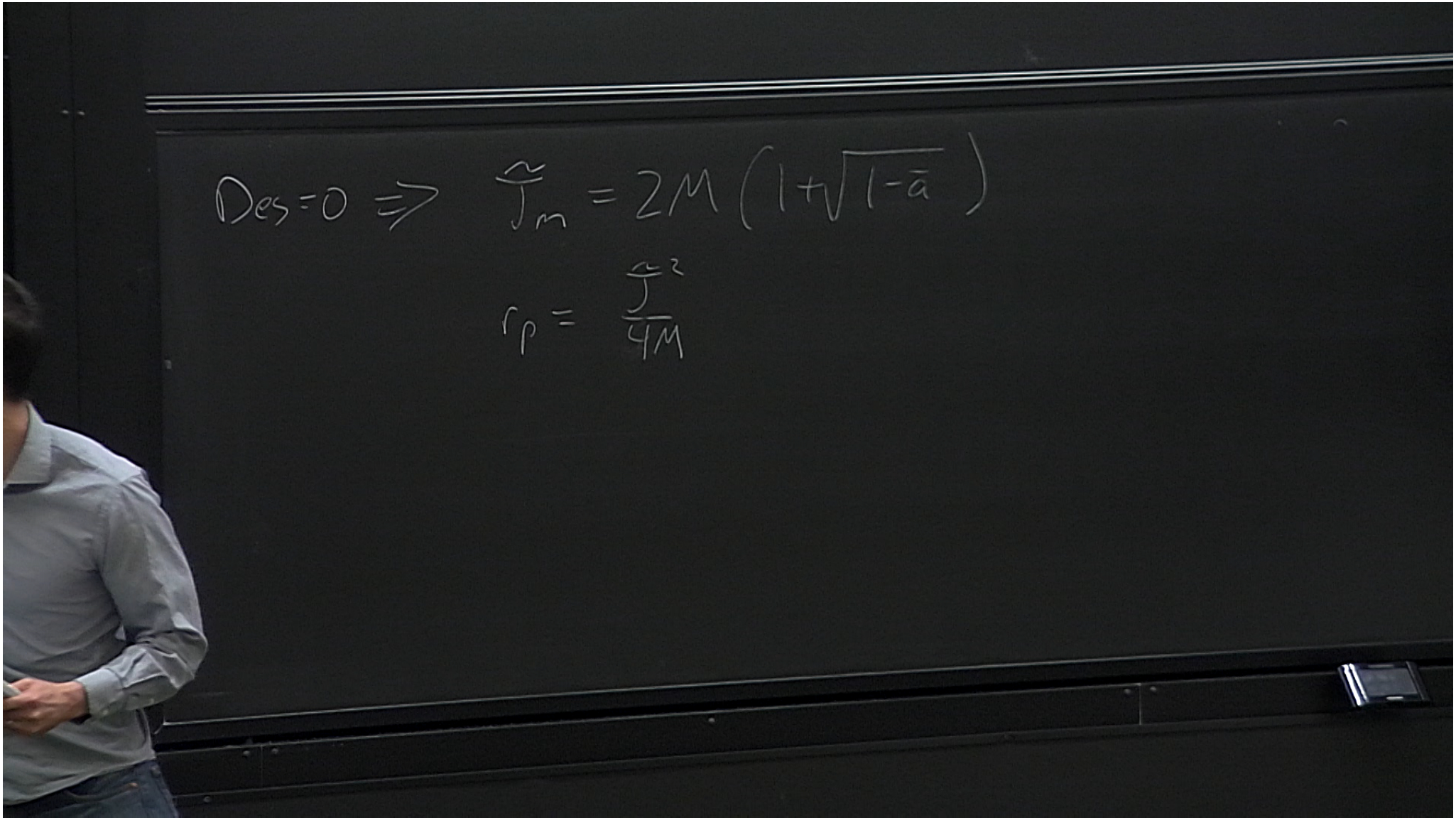
Collection: PSI 2018/2019 - Strong Field Gravity (East)

Date: March 06, 2019 - 9:00 AM

URL: http://pirsa.org/19030010

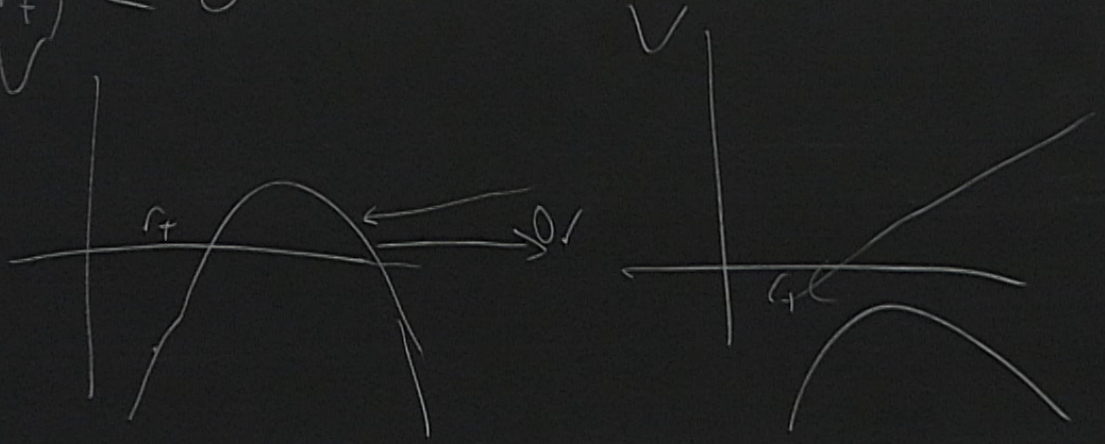

Marginally bound (parabolic) orbits
$\tilde{E} = 1$
$V = -2Mr\left[r^2 - \frac{\tilde{J}^2}{2M}r + (a-\tilde{J})^2\right]$
Turning points $V=0$
$r_p =$


$r_p = \frac{\frac{J^2}{2M} \pm \sqrt{\frac{J^2}{2M} - 4(a-\bar{J})^2}}{2}$
$V(r) < 0$
r_+
or
V
r_+


$D_{es} = 0 \Rightarrow \tilde{r}_m = 2M\left(1+\sqrt{1-\tilde{a}}\right)$
$r_p = \frac{\tilde{J}^2}{4M}$

$$r_\pm = \frac{\frac{J^2}{2M} \pm \sqrt{\frac{J^2}{2M} - 4(a - \frac{J}{2})^2}}{2}$$

$V(r_+) < 0$



$D_{eS}=0 \Rightarrow \tilde{J}_m = 2M\left(1+\sqrt{1-\bar{a}}\right) \qquad (\tilde{J}_m > 0)$

$r_p = \frac{\tilde{J}^2}{4M} = M\left(2-\bar{a}+2\sqrt{1-\bar{a}}\right)$

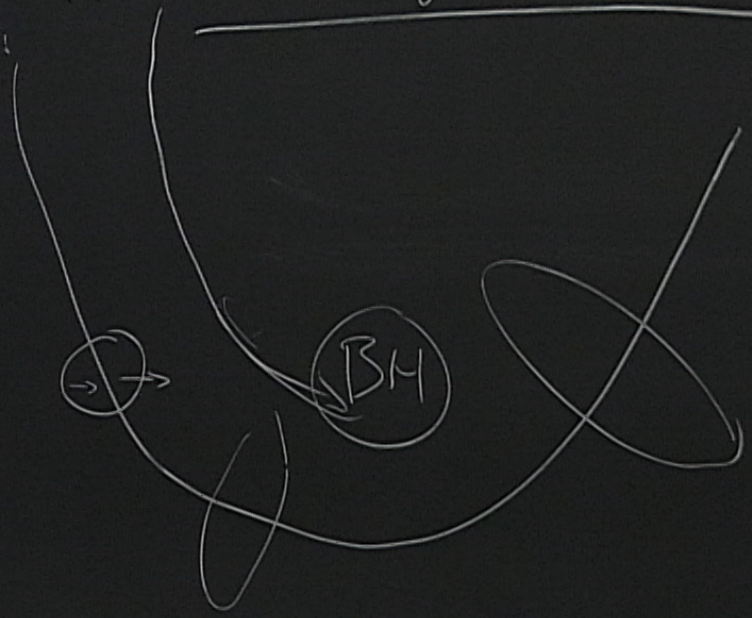
$\therefore r < r_p$

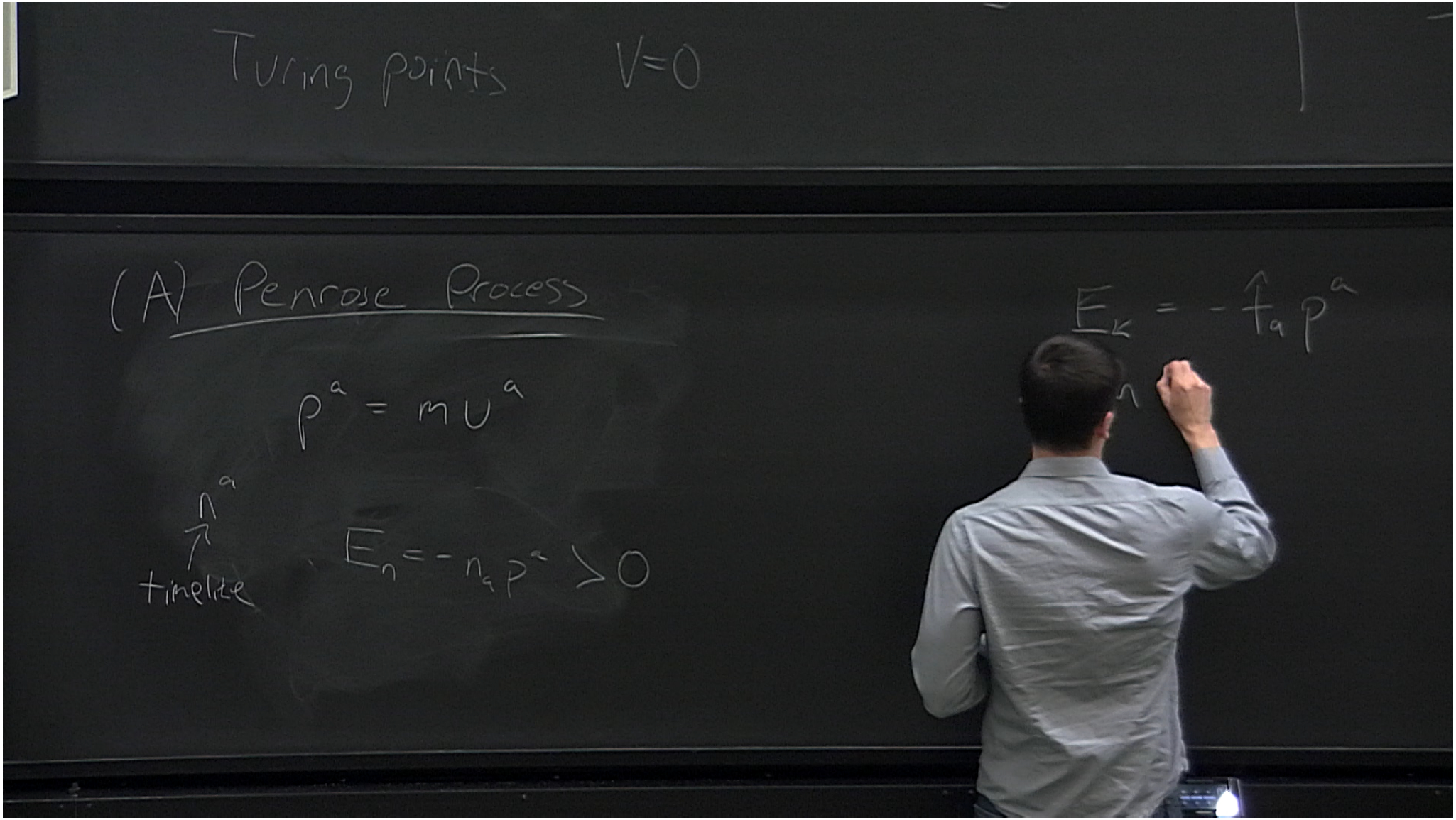
For $\bar{a}=-1 \quad r_p = M(3+2\sqrt{2}) \simeq 5.8M$

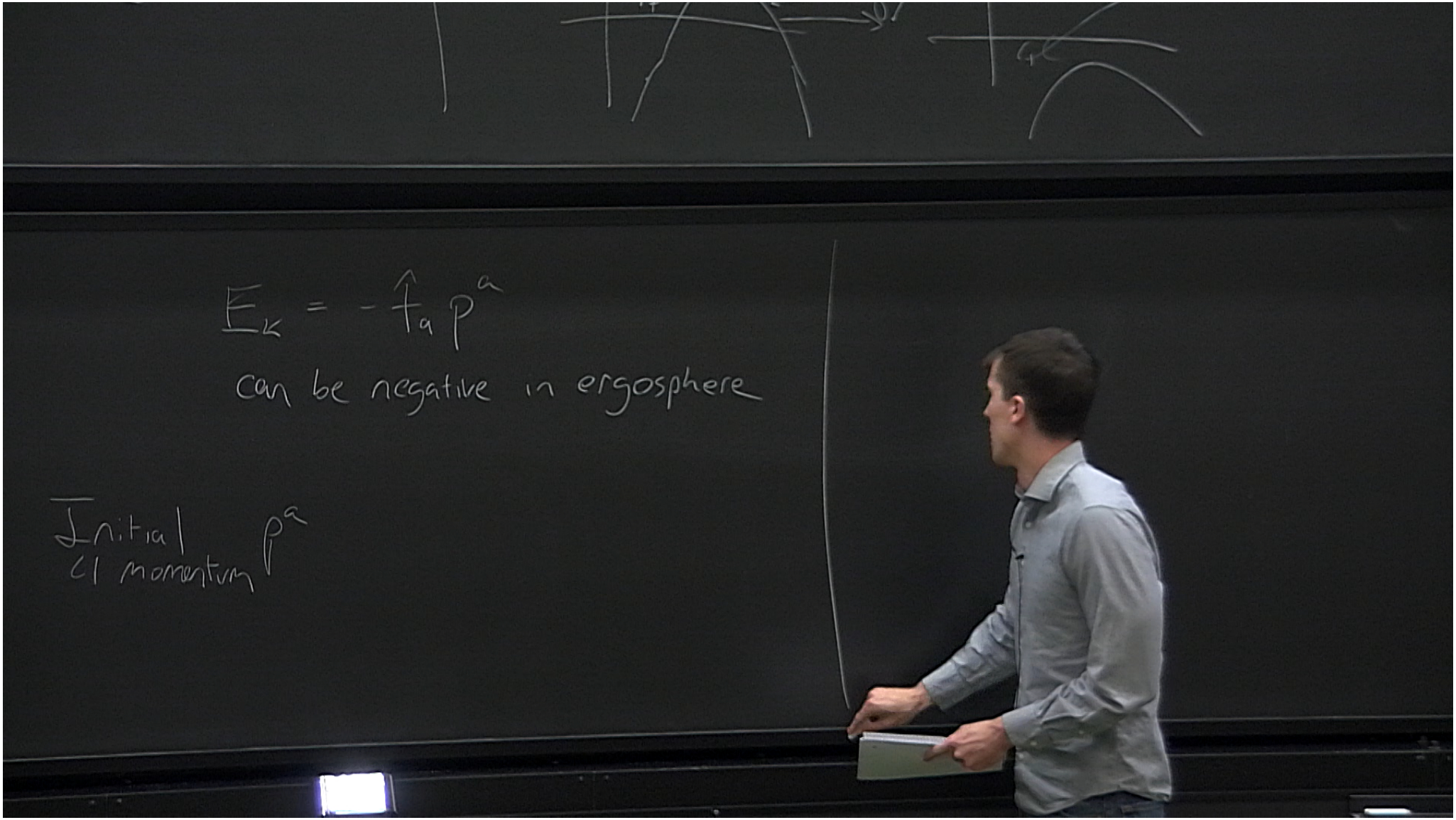
$\bar{a}=0 \quad r_p = 4M$

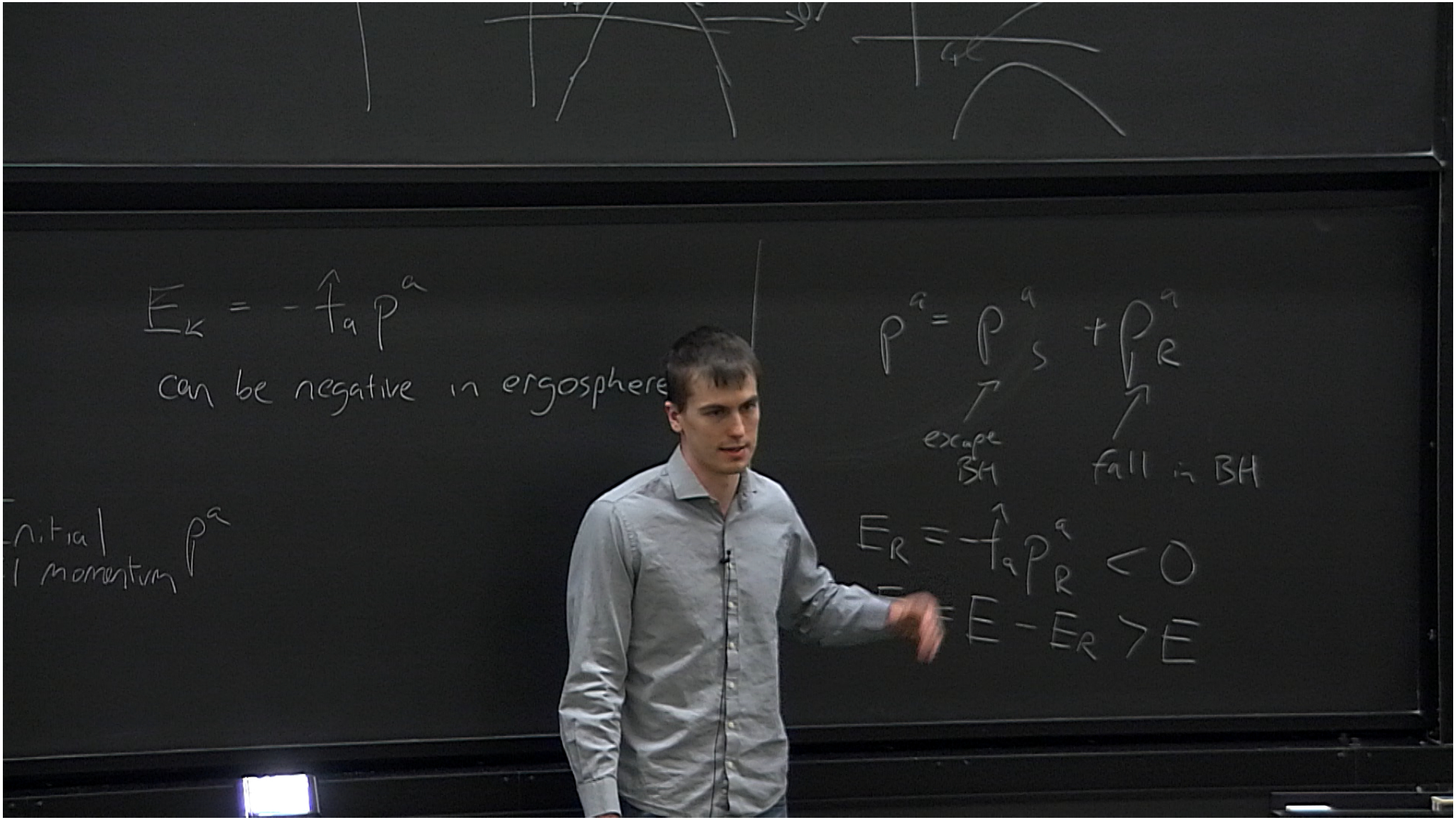
$\bar{a}=1 \quad r_p = M$

Tidal disruption events




Turing points V=0
(A) Penrose Process
$p^a = mU^a$
n^a
timelike
$E_n = -n_a p^a > 0$
$E_K = -t_a p^a$


$E_K = -\hat{t}_a p^a$
can be negative in ergosphere
Initial CI momentum p^a


$E_K = -\hat{t}_a p^a$
can be negative in ergosphere
$p^a = p^a_S + p^a_R$
escape BH
fall in BH
$E_R = -\hat{t}_a p^a_R < 0$
$E - E_R > E$

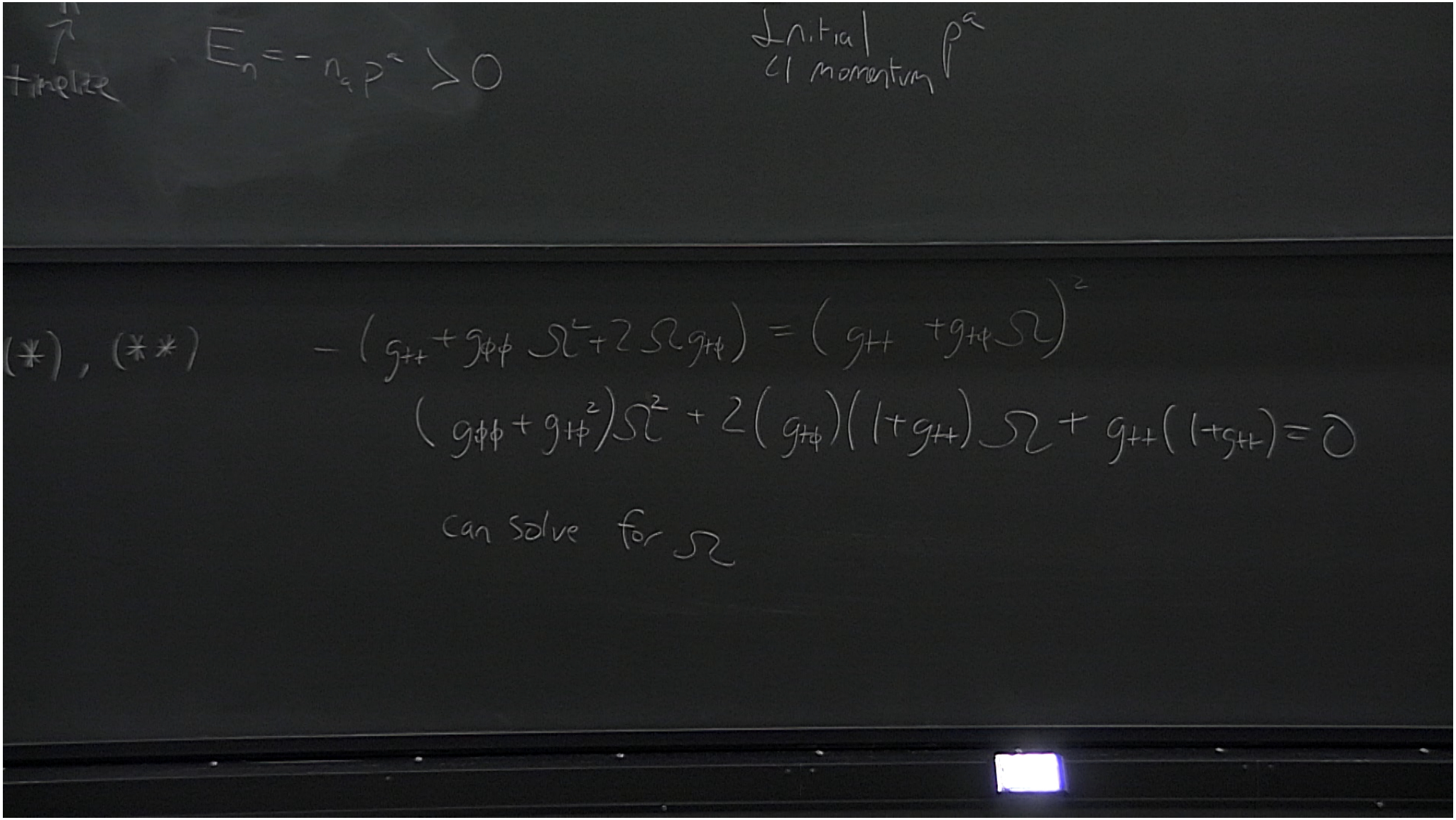
$\tilde{a} = 1 \qquad r_c = M$

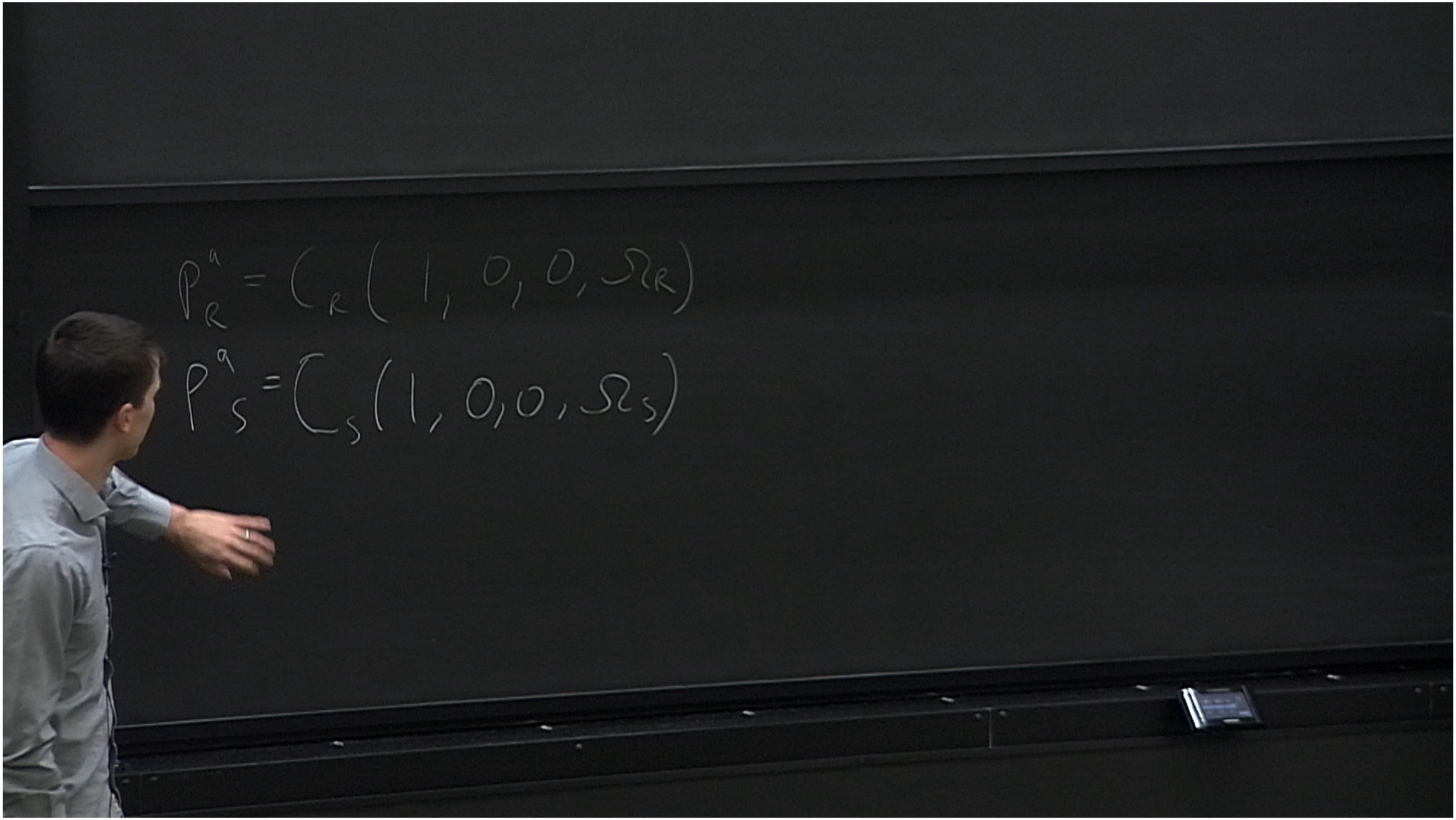
Assume $\tilde{E} = 1$, $m = 1$ at $r = 0$

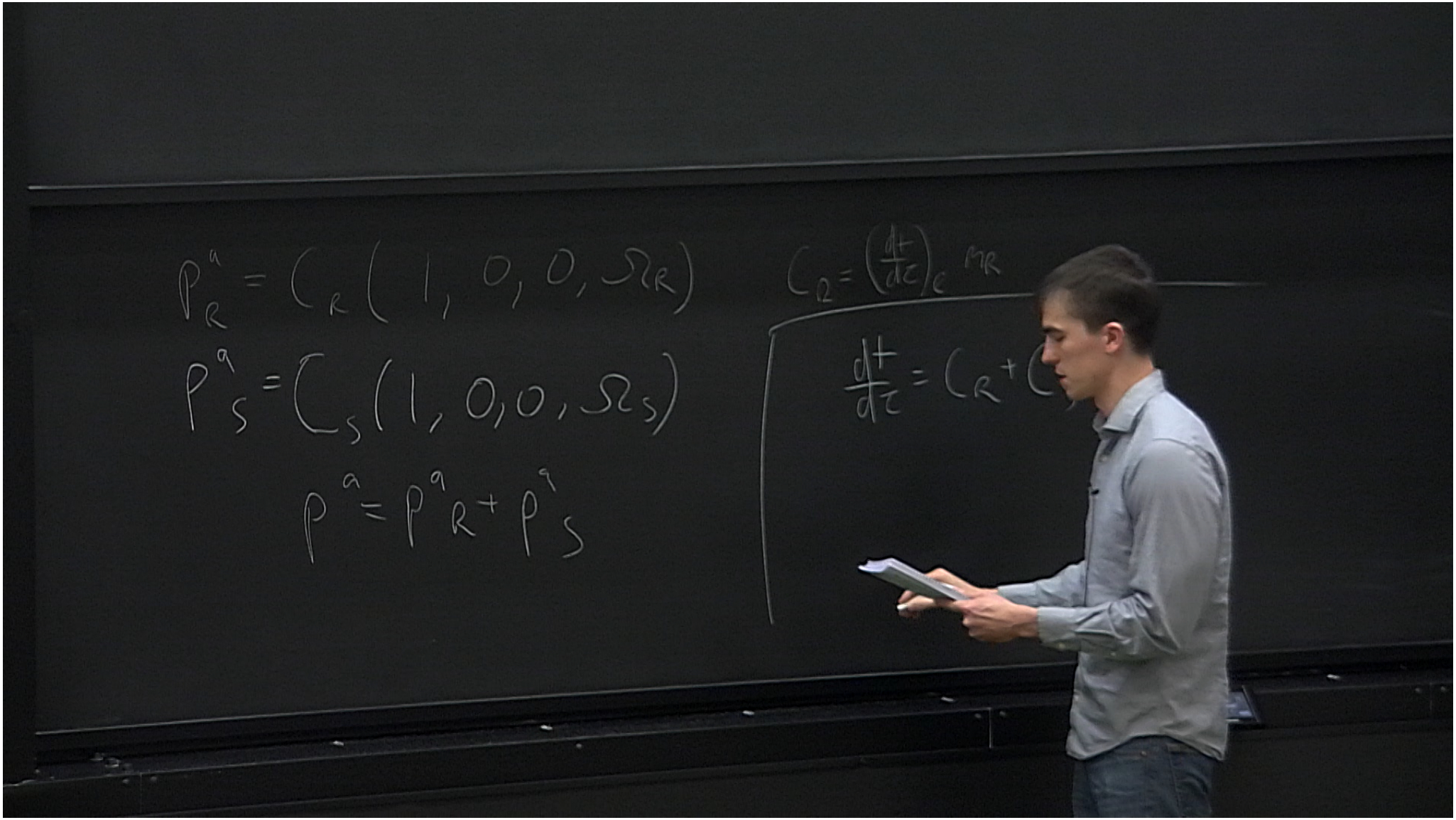
$$p^a = \frac{dt}{d\tau}(1, 0, 0, \Omega)$$

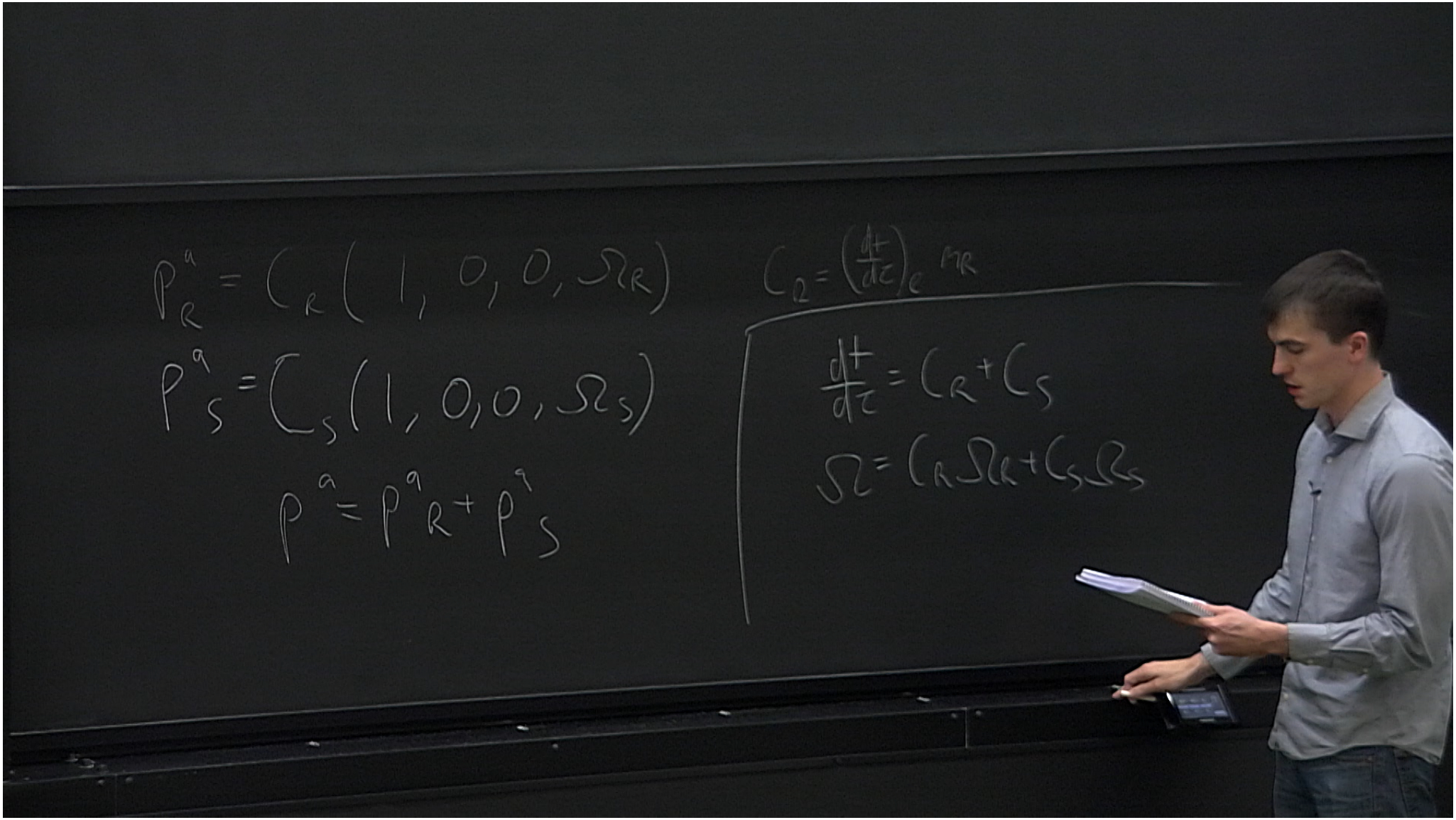
$$-1 = p^a p^a = \left(\frac{dt}{d\tau}\right)^2 \left(g_{tt} + g_{\phi\phi}\Omega^2 + 2\Omega g_{t\phi} \right) \quad (*)$$

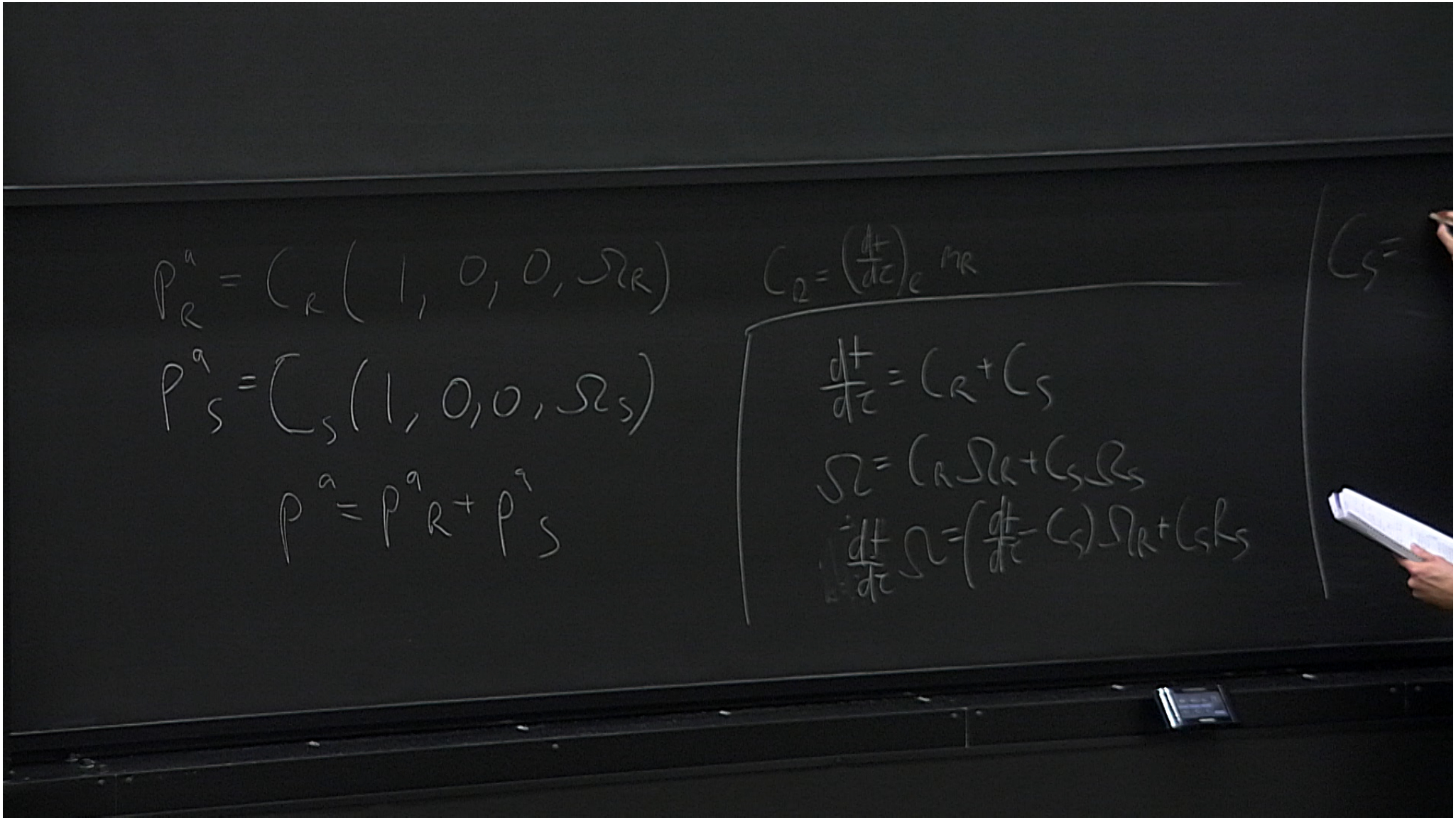
$$\Omega = \frac{d\phi/d\tau}{dt/d\tau} = \frac{d\phi}{dt} \qquad E = \tilde{E} = 1 \qquad \frac{d\tau}{dt} = -\left(g_{tt} + g_{t\phi}\Omega\right) \quad (**)$$

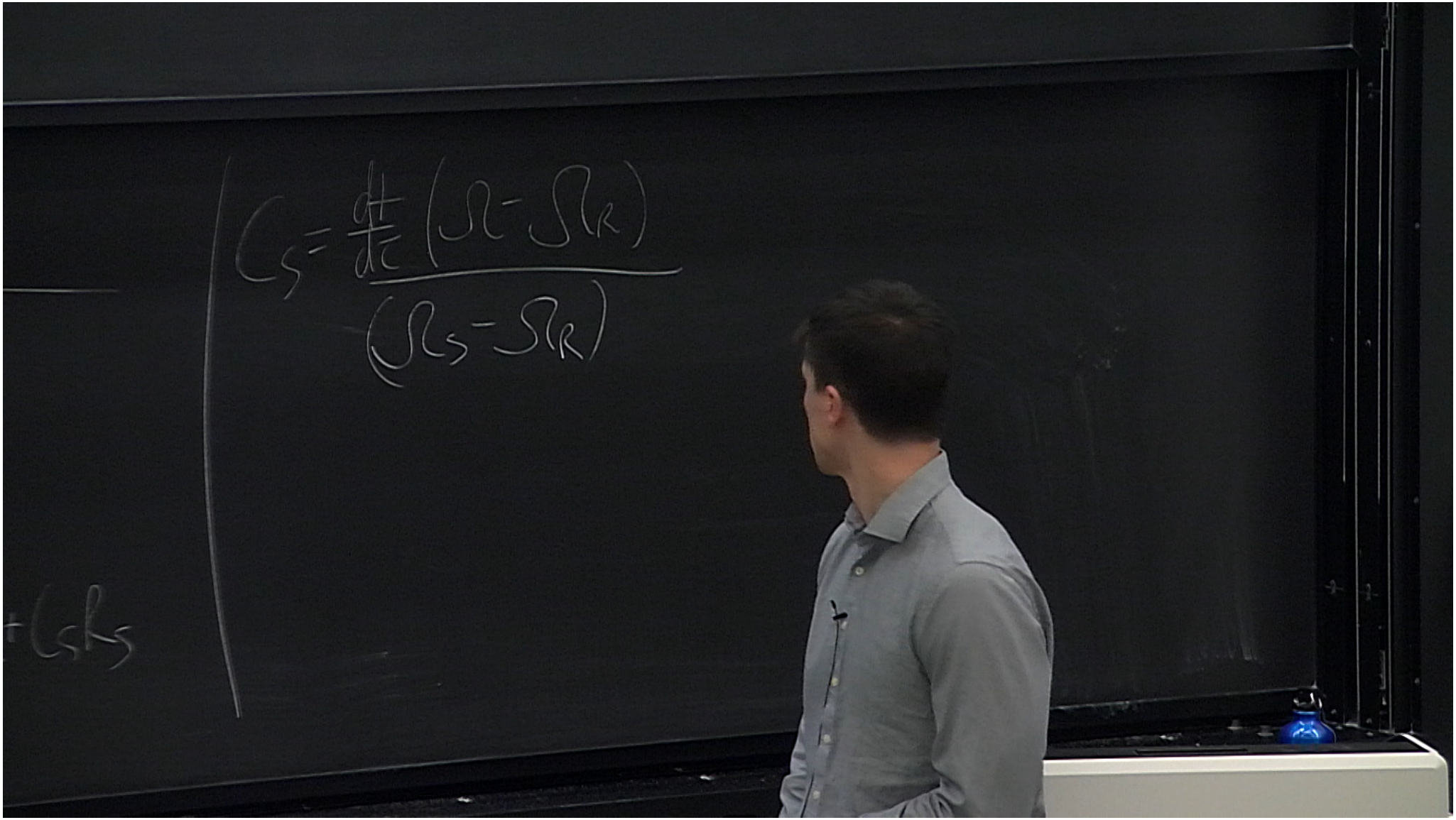

timelike
$E_n = -n_a p^a > 0$
Initial 4-momentum p^a
(*), (**)
$-(g_{tt} + g_{\phi\phi}\Omega^2 + 2\Omega g_{t\phi}) = (g_{tt} + g_{t\phi}\Omega)^2$
$(g_{\phi\phi} + g_{t\phi}^2)\Omega^2 + 2(g_{t\phi})(1+g_{tt})\Omega + g_{tt}(1+g_{tt}) = 0$
Can solve for Ω

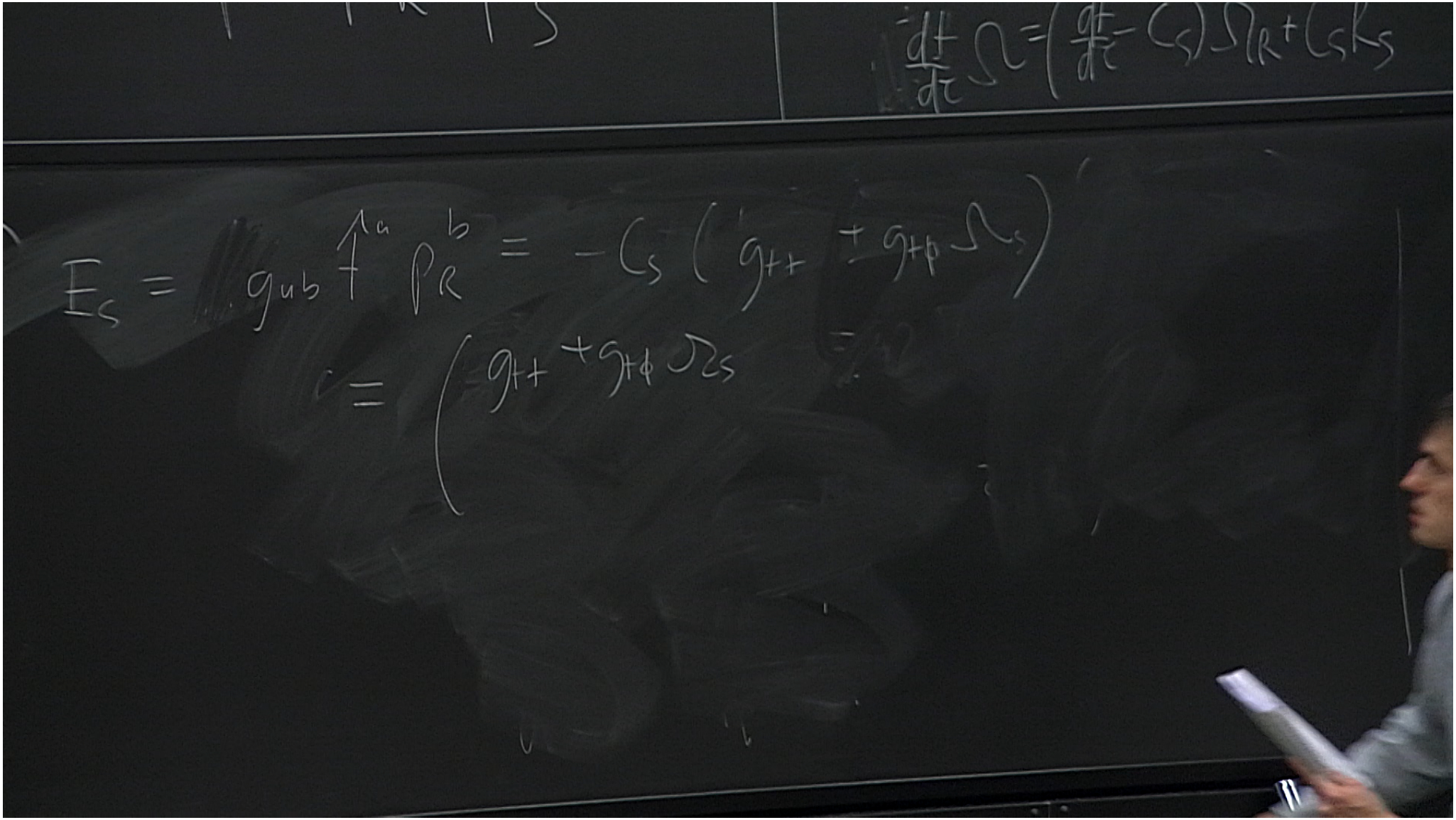

$p_R^a = C_R(1, 0, 0, \Omega_R)$
$p_S^a = C_S(1, 0, 0, \Omega_S)$

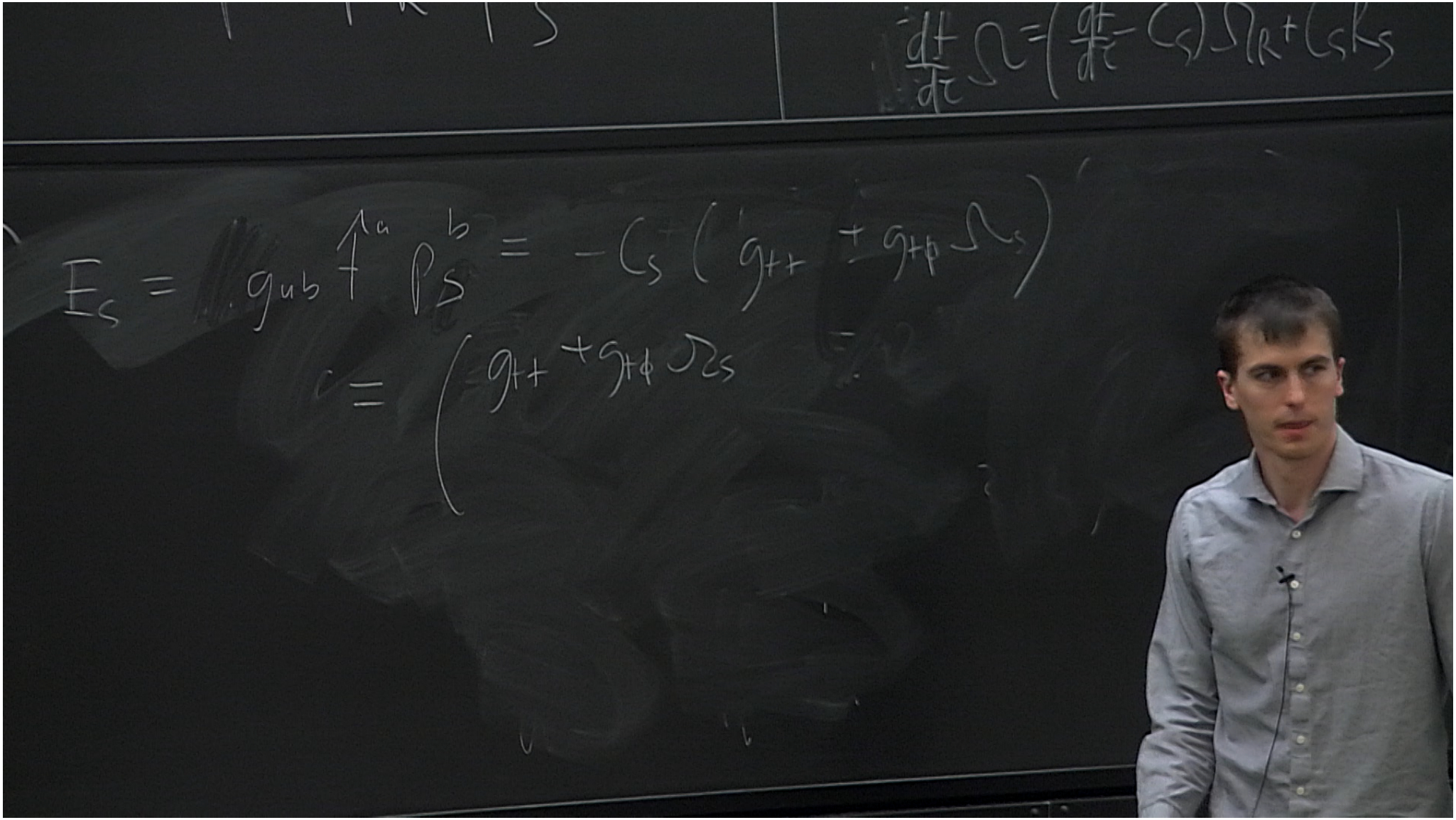

$p^a_R = C_R(1, 0, 0, \Omega_R)$
$p^a_S = C_S(1, 0, 0, \Omega_S)$
$p^a = p^a_R + p^a_S$
$C_2 = \left(\frac{dt}{d\tau}\right)_c$ MR
$\frac{dt}{d\tau} = C_R +$

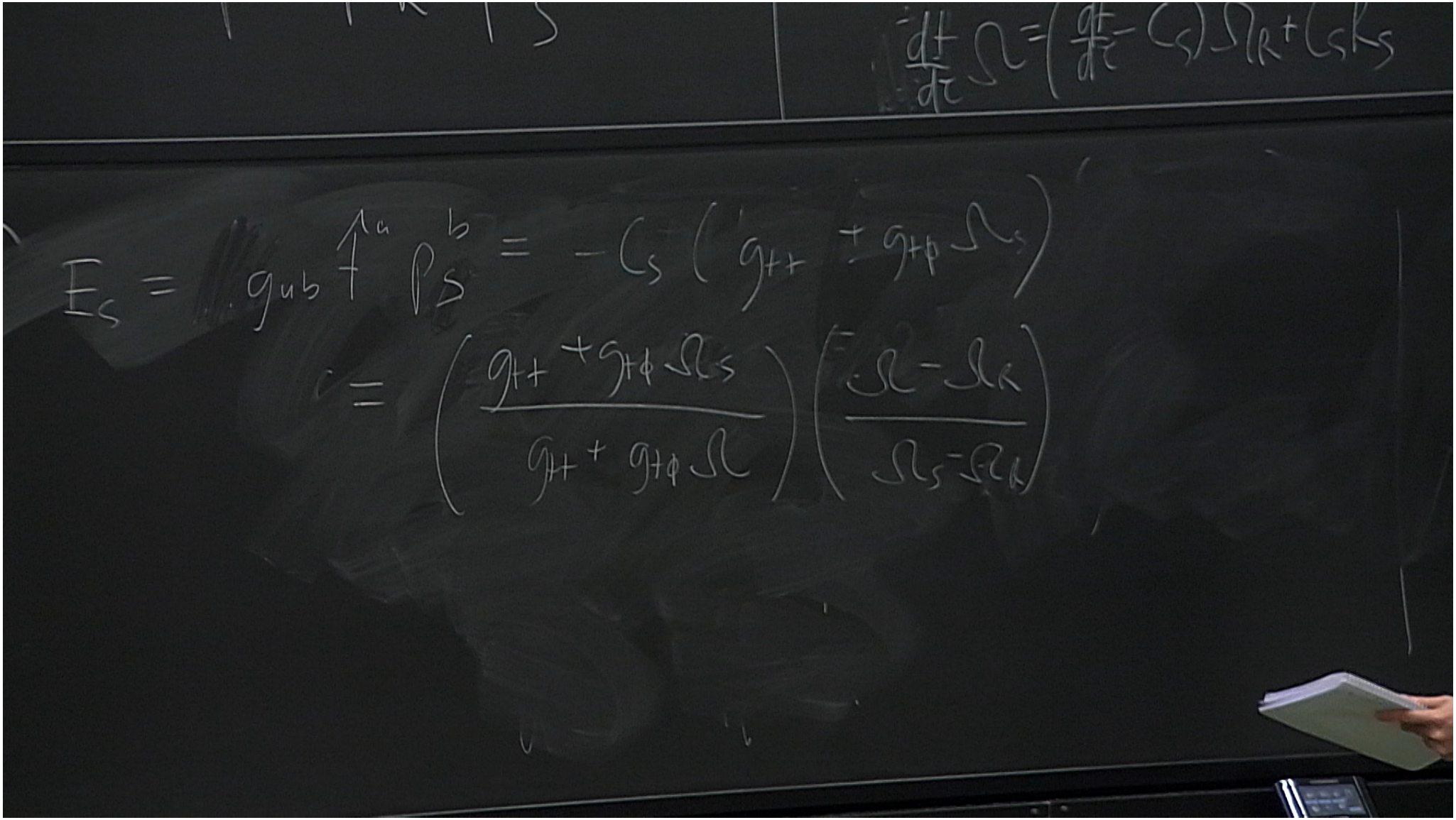

$P_R^a = C_R(1, 0, 0, \Omega_R)$
$P_S^a = C_S(1, 0, 0, \Omega_S)$
$P^a = P_R^a + P_S^a$
$C_2 = \left(\frac{dt}{dc}\right)_e$ MR
$\frac{dt}{dc} = C_R + C_S$
$\Omega = C_R\Omega_R + C_S\Omega_S$


$p_R^a = C_R(1, 0, 0, \Omega_R)$
$p_S^a = C_S(1, 0, 0, \Omega_S)$
$p^a = p_R^a + p_S^a$
$C_2 = \left(\frac{dt}{dc}\right)_c$ MR
$\frac{dt}{dc} = C_R + C_S$
$\Omega = C_R\Omega_R + C_S\Omega_S$
$\frac{dt}{dc}\Omega = \left(\frac{dt}{dc} - C_S\right)\Omega_R + C_S\Omega_S$
$C_S =$


$C_S = \frac{d}{dc}(S_L - S_R)$
$(S_{LS} - S_R)$
$+ C_S k_S$


$\frac{dt}{d\tau}\Omega = \left(\frac{g_{tt}}{g_{\tau\tau}} - \zeta_s\right)\Omega_R + \zeta_s K_s$
$E_s = g_{ub} f^u P_R^b = -\zeta_s \left(g_{tt} + g_{t\phi}\Omega_s\right)$
$= \left(g_{tt} + g_{t\phi}\Omega_s\right)$

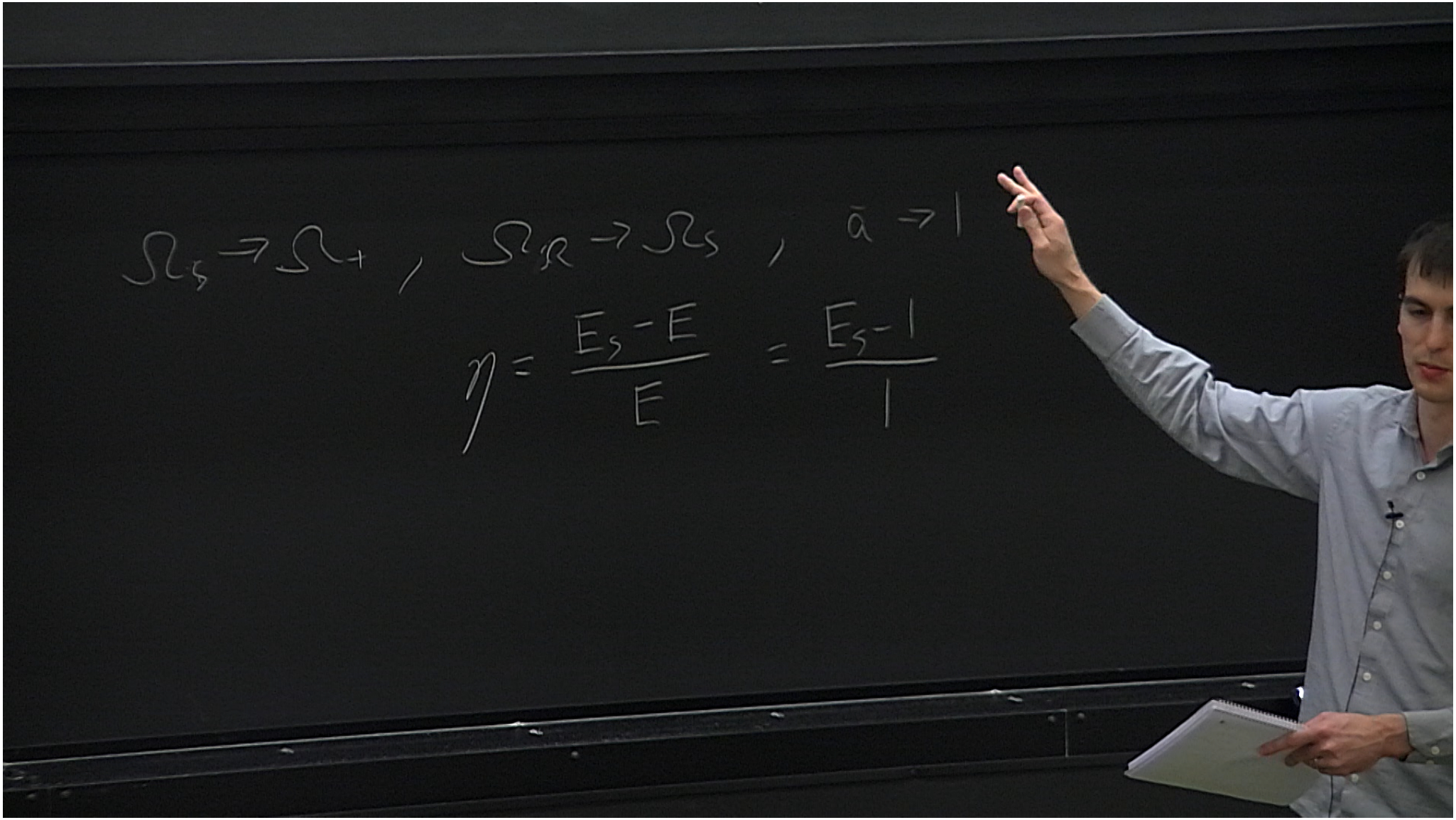

$\frac{dt}{d\tau}\Omega = \left(\frac{g_{tt}}{g_{t\phi}} - C_s\right)\Omega_R + C_s K_s$
$E_s = g_{ab} f^a p_s^b = -C_s\left(g_{tt} + g_{t\phi}\Omega_s\right)$
$= \left(g_{tt} + g_{t\phi}\Omega_s\right)$

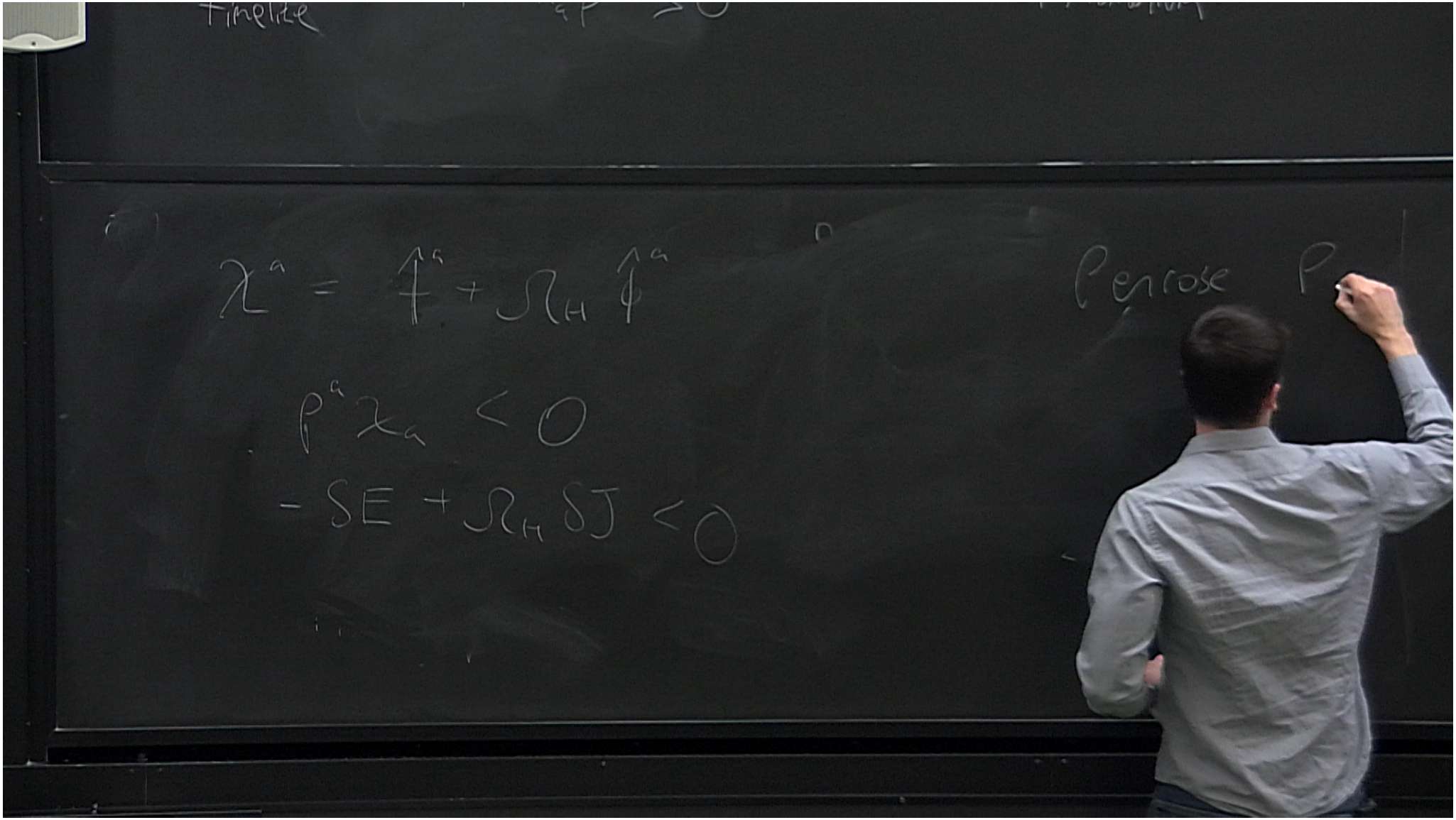

$\frac{d\Omega}{dt} = \left(\frac{g_{+\phi}}{g_{++}} - C_S\right)\Omega_R + C_S\Omega_S$
$E_S = g_{ab} f^a p_S^b = -C_S\left(g_{++} + g_{+\phi}\Omega_S\right)$
$C = \left(\frac{g_{++} + g_{+\phi}\Omega_S}{g_{++} + g_{+\phi}\Omega}\right)\left(\frac{\Omega - \Omega_R}{\Omega_S - \Omega_R}\right)$

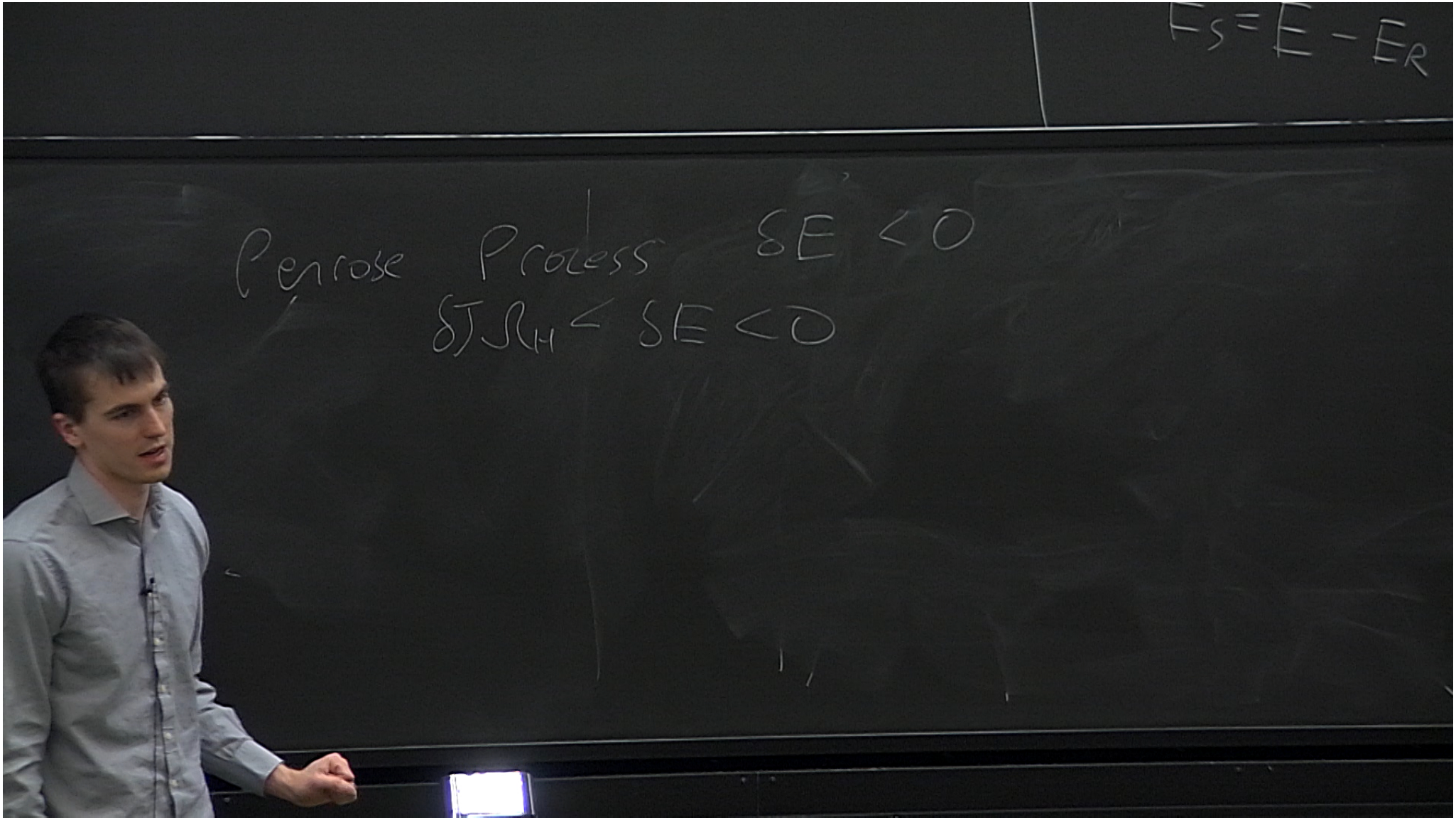
$$\Omega_{\pm} = \frac{-2g_{+\phi} \pm \sqrt{4g_{+\phi}^2 - 4g_{++}}}{2g_{\phi\phi}}$$

$$\Omega_- < \Omega_R < \Omega_S < \Omega_+$$




$\Omega_S \to \Omega_+$, $\Omega_R \to \Omega_S$, $\bar{a} \to 1$
$\eta = \frac{E_S - E}{E} = \frac{E_S - 1}{1}$


timelike
$\chi^a = \hat{t}^a + \Omega_H \hat{\phi}^a$
$p^a \chi_a < 0$
$-\delta E + \Omega_H \delta J < 0$
Penrose


$E_S = E - E_R$
Penrose Process $\delta E < 0$
$\delta J \Omega_H < \delta E < 0$