Title: PSI 2018/2019 - Quantum Field Theory III - Lecture 14

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Collection: PSI 2018/2019 - Quantum Field Theory III (Gomis, Wohns and Ali)

Date: March 18, 2019 - 2:00 PM

URL: http://pirsa.org/19030006

Chiral Anomaly in 2D Massless QED

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi$$

$U(1)_V$ (V ↑ vector) $\quad \psi \to e^{i\alpha}\psi \qquad J^\mu_V = J^\mu = \bar{\psi}\gamma^\mu\psi$

$U(1)_A$ (A ↑ axial) $\quad \psi \to e^{i\alpha\gamma^5}\psi \qquad J^\mu_A = J^\mu_5 = \bar{\psi}\gamma^\mu\gamma^5\psi$

In massless QED
we have same symmetries (+currents)

$$A_\mu \to A_\mu$$

In 2D $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ $\eta^{\mu\nu} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

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$$\gamma^0 = \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma^1 = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\gamma^5 = -\gamma^0\gamma^1 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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$$A_\mu \to A_\mu + \partial_\mu \alpha$$

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$\gamma^5 P_\pm \psi = \pm P_\pm \psi$ positive + negative chirality components

$P_\pm = \frac{1 \pm \gamma^5}{2} = P_{R,L}$

$\bar{\psi} \not{D} \psi = \psi_+^\dagger i(D_0 + D_1)\psi_+ + \psi_-^\dagger i(D_0 - D_-)\psi_-$

no mixing of $\psi_+ + \psi_-$

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$\partial_\mu \bar{\psi}_\pm \gamma^\mu \psi_\pm = 0$

↑ separate conservation of number currents in $\pm$ sectors

$A_\mu \to A_\mu + \partial_\mu \alpha$

$$\alpha = -\sum_{n \neq 0} \frac{L}{2\pi n} a_1(t) \sin\left(\frac{2\pi n}{L} x\right)$$

$A_1(x,t) \to A_1(t)$

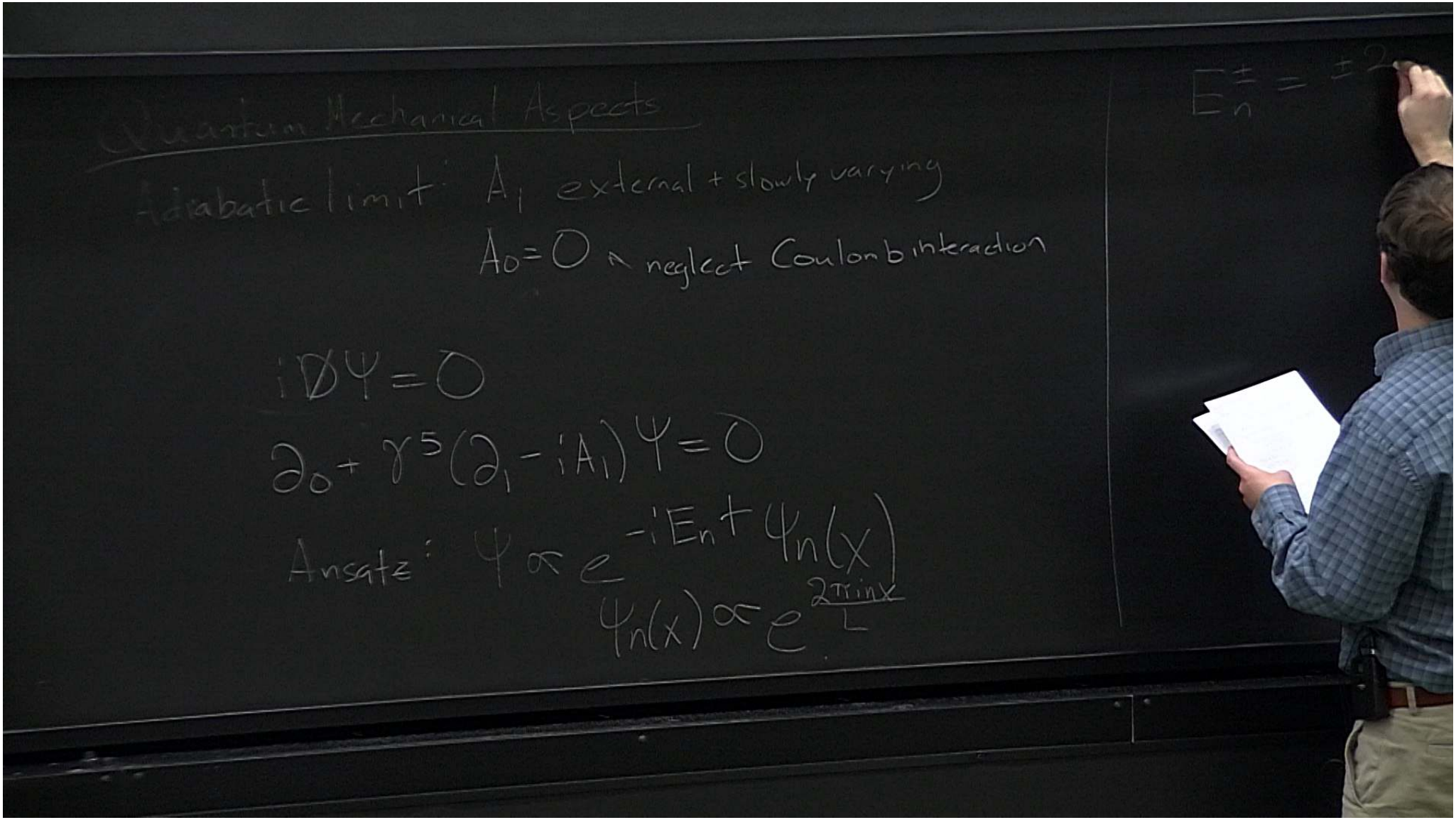
$\alpha = \frac{2\pi n x}{L}$ respects periodicity of A_μ

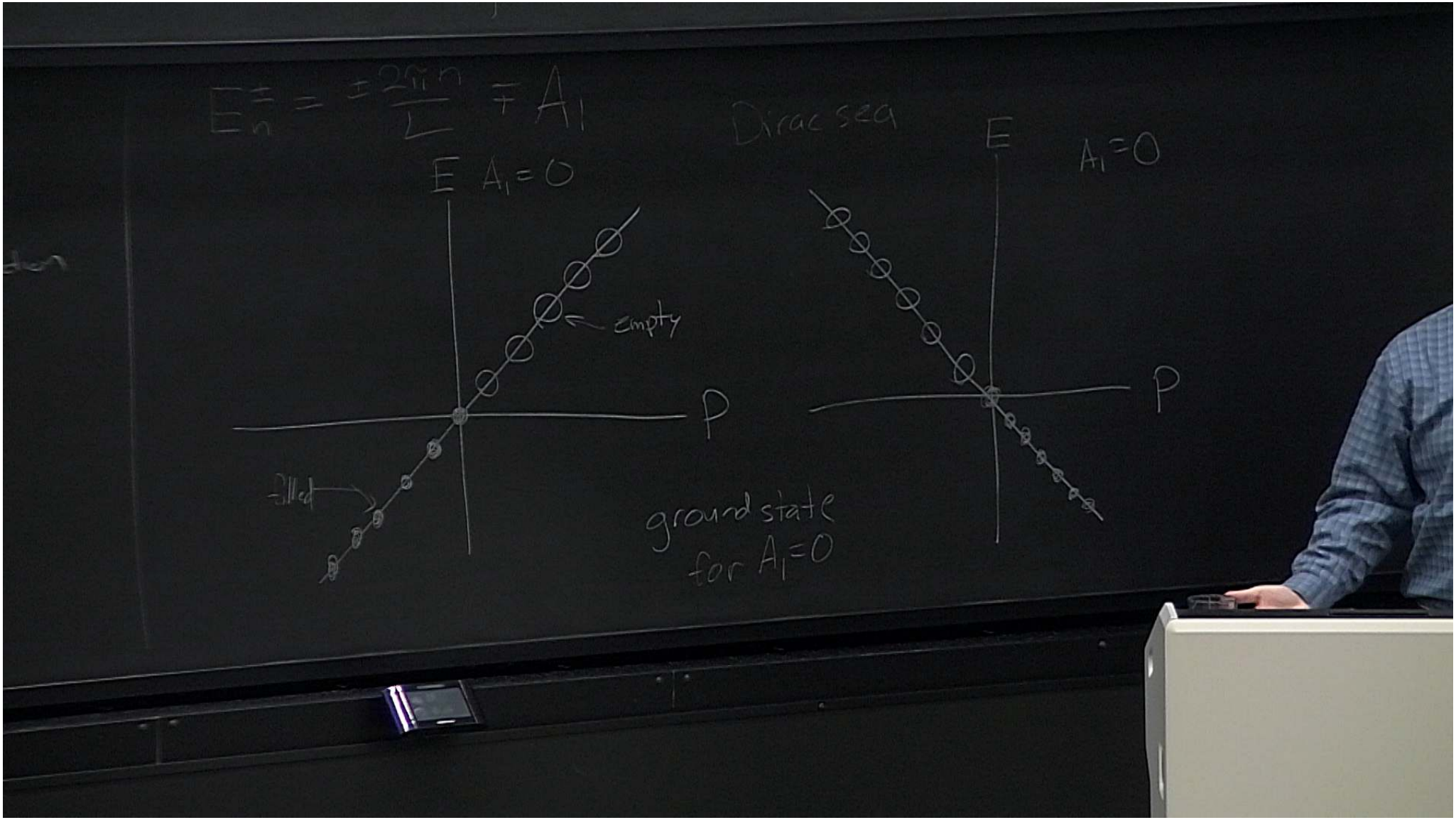
A_1 and $A_1 + \frac{2\pi n}{L}$ are gauge equivalent for $n \in \mathbb{Z}$

$\gamma^5 = -\gamma^0\gamma^1 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

→ A_1 lives on a circle of circumference $\frac{2\pi}{L}$

integral of A_1 around loop is - invariant


Quantum Mechanical Aspects
-adiabatic limit: A_1 external + slowly varying
$A_0 = 0$ ← neglect Coulomb interaction
$iD\Psi = 0$
$\partial_0 + \gamma^5(\partial_1 - iA_1)\Psi = 0$
Ansatz: $\Psi \propto e^{-iE_n t}\psi_n(x)$
$\psi_n(x) \propto e^{\frac{2\pi i n x}{L}}$
$E_n^{\pm} = \pm$


$E_n = \frac{2\pi n}{L} + A_1$
E $A_1=0$
empty
P
filled
ground state
for $A_1=0$
Dirac sea
E $A_1=0$
P

are gauge equivalent for $n \in \mathbb{Z}$

$$\frac{\Delta Q_5}{\Delta t} = -\frac{L}{\tilde{\pi}} \frac{\Delta A_1}{\Delta t}$$

$$\dot{Q}_5 = -\frac{L}{\tilde{\pi}} \dot{A}_1$$

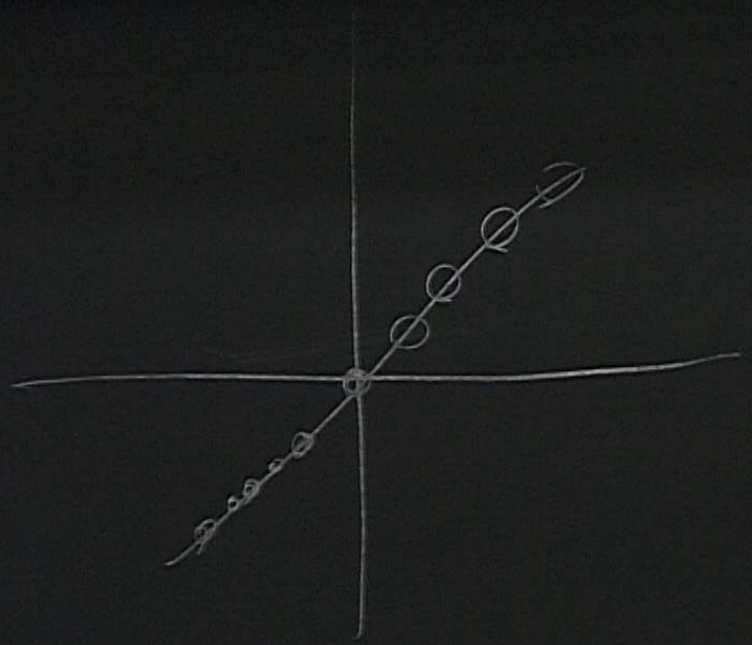
$$Q_5 = \int dx\, J_5^0$$

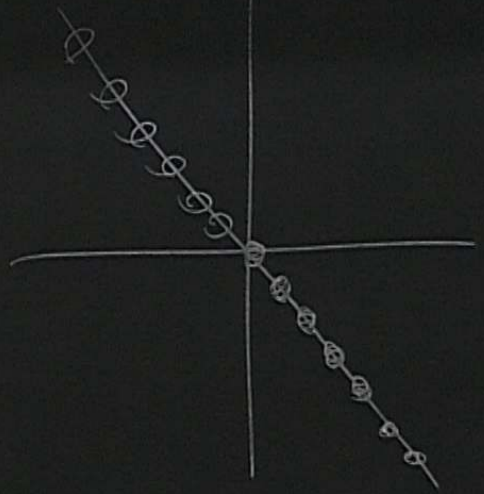
$$L A_1 = \int \epsilon^{0\mu} A_\mu dx$$

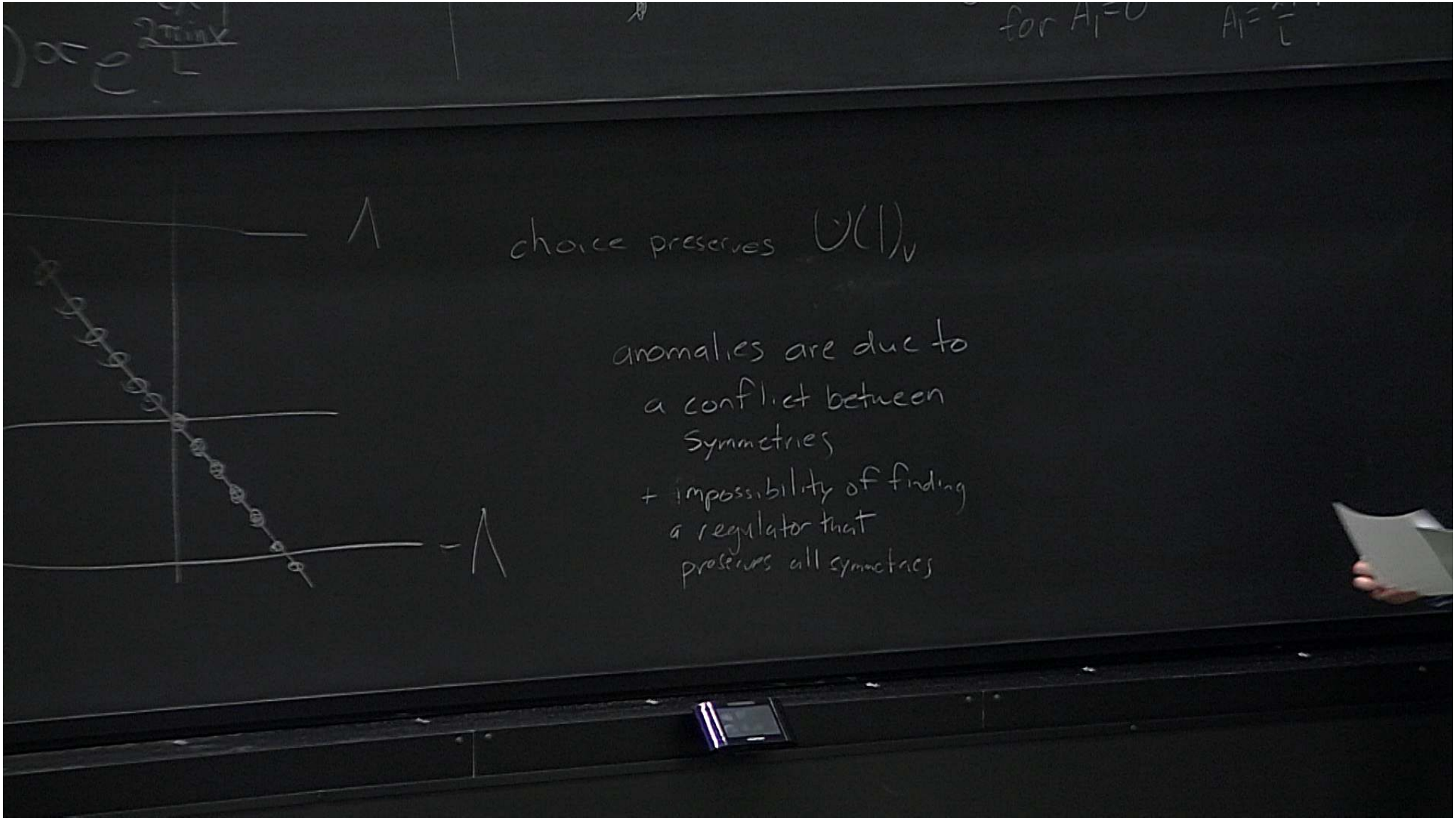
$$\epsilon^{1\mu} A_\mu = 0 \quad (A_0 = 0)$$

$$\partial_\mu J_5^\mu = \frac{1}{\tilde{\pi}} \epsilon^{\mu\nu} F_{\mu\nu}$$

$\psi_n(x) \propto e^{\frac{2\pi i n x}{L}}$






$\propto e^{\frac{2\pi i n x}{L}}$
for $A_1 = 0$
$A_1 = \frac{2\pi}{L}$
Λ
$-\Lambda$
choice preserves $U(1)_V$
anomalies are due to
a conflict between
symmetries
+ impossibility of finding
a regulator that
preserves all symmetries