Title: PSI 2018/2019 - Quantum Field Theory III - Lecture 14

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Collection: PSI 2018/2019 - Quantum Field Theory III (Gomis, Wohns and Ali)

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Chiral Anomaly in 2D Massless QED

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi$$

$U(1)_V$ — vector

$$\psi \to e^{i\alpha}\psi \qquad J^\mu_V = J^\mu = \bar{\Psi}\gamma^\mu\psi$$

$U(1)_A$ — axial

$$\psi \to e^{i\alpha\gamma^5}\psi \qquad J^\mu_A = J^\mu_5 = \bar{\Psi}\gamma^\mu\gamma^5\psi$$

In massless QED

we have same symmetries (+currents)

$$A_\mu \rightarrow A_\mu$$

In 2D $\quad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \eta^{\mu\nu} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

In massless QED

we have same symmetries (+currents)

$$A_\mu \to A_\mu$$

In 2D $\quad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \eta^{\mu\nu} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$$\gamma^0 = \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma^1 = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\gamma^5 = -\gamma^0\gamma^1 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In massless QED

we have same symmetries (+currents)

$$A_\mu \to A_\mu \left(+\partial_\mu \alpha\right)$$

In 2D $\quad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \eta^{\mu\nu} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

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$$\gamma^5 P_\pm \psi = \pm P_\pm \psi$$

positive + negative chirality componts

$$P_\pm = \frac{1 \pm \gamma^5}{2} = P_{R,L}$$

$$i \bar{\psi} \not{D} \psi = \psi_+^\dagger \, i (D_0 + D_1) \psi_+ + \psi_-^\dagger \, i (D_0 - D_1) \psi_-$$

no mixing of ψ_+ + ψ_-

$$\gamma^5 P_\pm \psi = \pm P_\pm \psi$$

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$$P_\pm = \frac{1 \pm \gamma^5}{2} = P_{R,L}$$

$$i \overline{\Psi} \not{D} \psi = \psi_+^\dagger \, i (D_0 + D_1) \psi_+ + \psi_-^\dagger \, i (D_0 - D_1) \psi_-$$

no mixing of ψ_+ + ψ_-

$$\partial_\mu \overline{\Psi}_\pm \gamma^\mu \psi_\pm = 0$$

↑ separate conservation of number currents in ± sectors axial

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

$$\alpha = -\sum_{n\neq 0} \frac{L}{2\pi n}\, a_1(t) \sin\left(\frac{2\pi n}{L} x\right)$$

$$A_1(x,t) \to A_1(t)$$

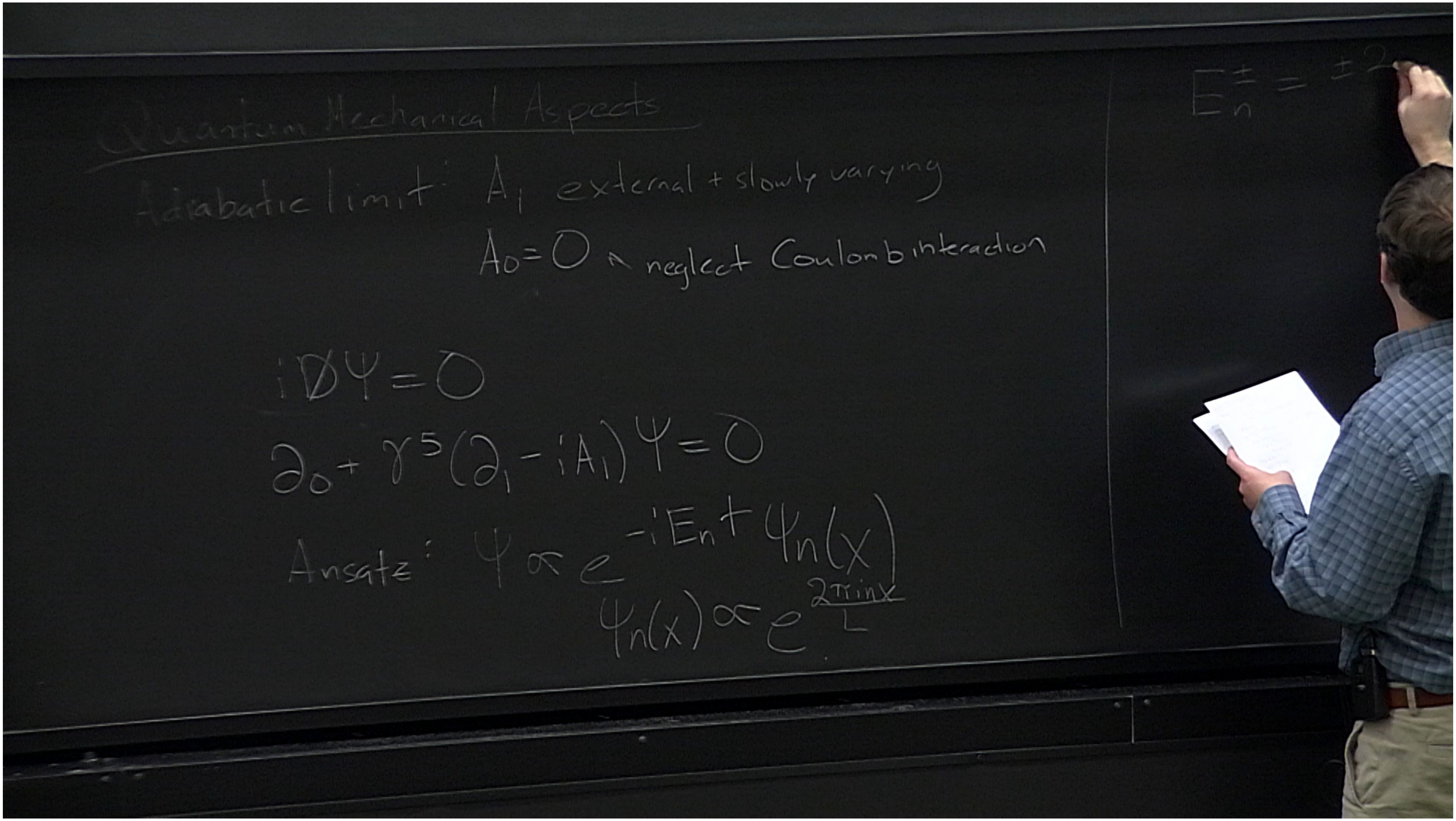
$$\alpha = \frac{2\pi n x}{L}$$ respects periodicity of A_μ

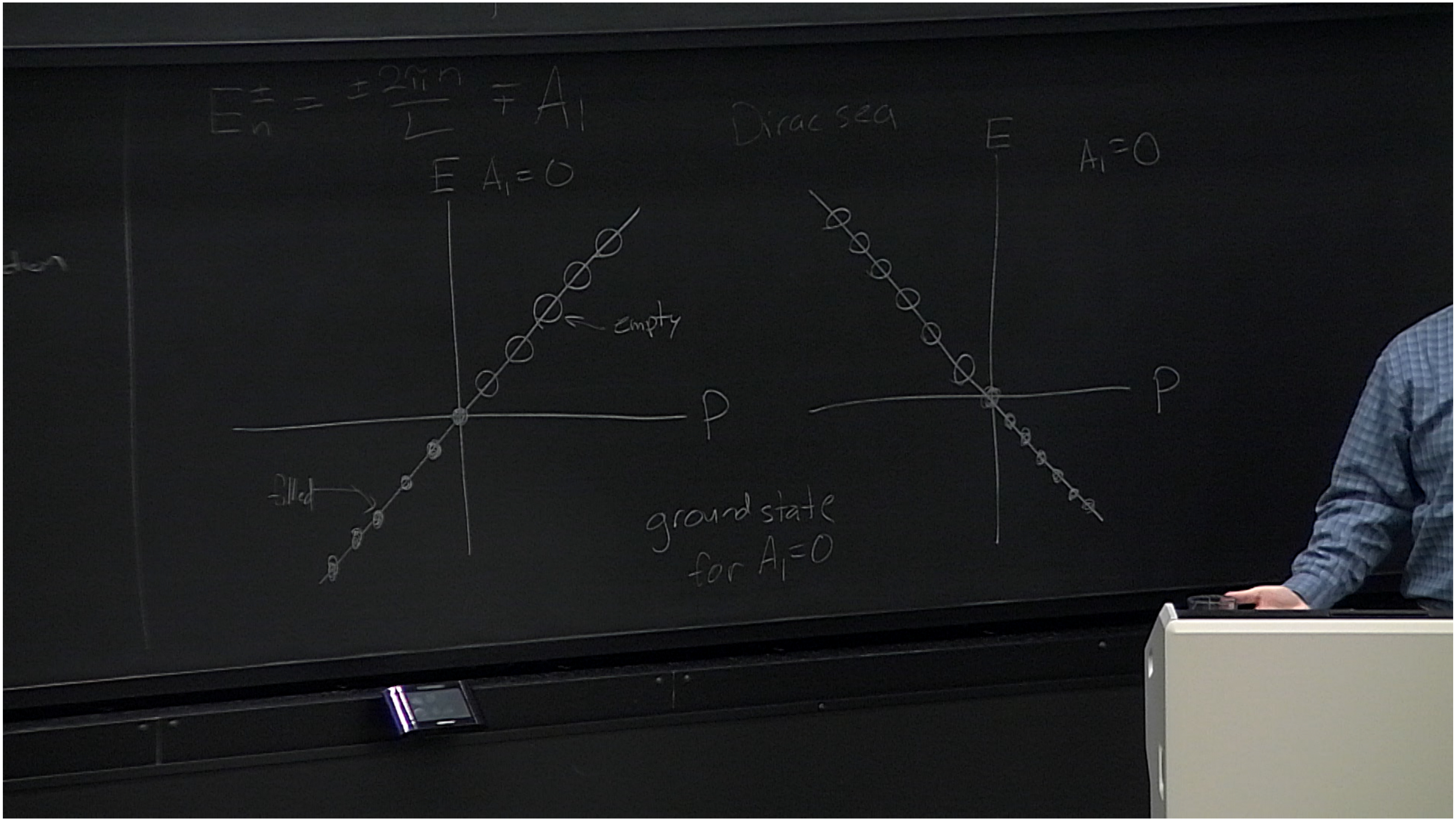
A_1 and $A_1 + \frac{2\pi n}{L}$ are gauge equivalent for $n \in \mathbb{Z}$

$$\gamma^5 = -\gamma^0\gamma^1 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

→ A_1 lives on a circle of circumference $\frac{2\pi}{L}$

integral of A_1 around loop is invariant


Quantum Mechanical Aspects
Adiabatic limit: A_1 external + slowly varying
$A_0 = 0$ ← neglect Coulomb interaction
$i\not{D}\Psi = 0$
$\partial_0 + \gamma^5(\partial_1 - iA_1)\Psi = 0$
Ansatz: $\Psi \propto e^{-iE_n t}\,\varphi_n(x)$
$\varphi_n(x) \propto e^{\frac{2\pi i n x}{L}}$
$E_n^{\pm} = \pm 2$


$E_n^{\pm} = \pm\frac{2\pi n}{L} \mp A_1$
Dirac sea
E $A_1=0$
empty
filled
p
E
$A_1=0$
p
ground state
for $A_1=0$

are gauge equivalent for $n \in \mathbb{Z}$

$$\frac{\Delta Q_5}{\Delta t} = -\frac{L}{\tilde{\pi}} \frac{\Delta A_1}{\Delta t}$$

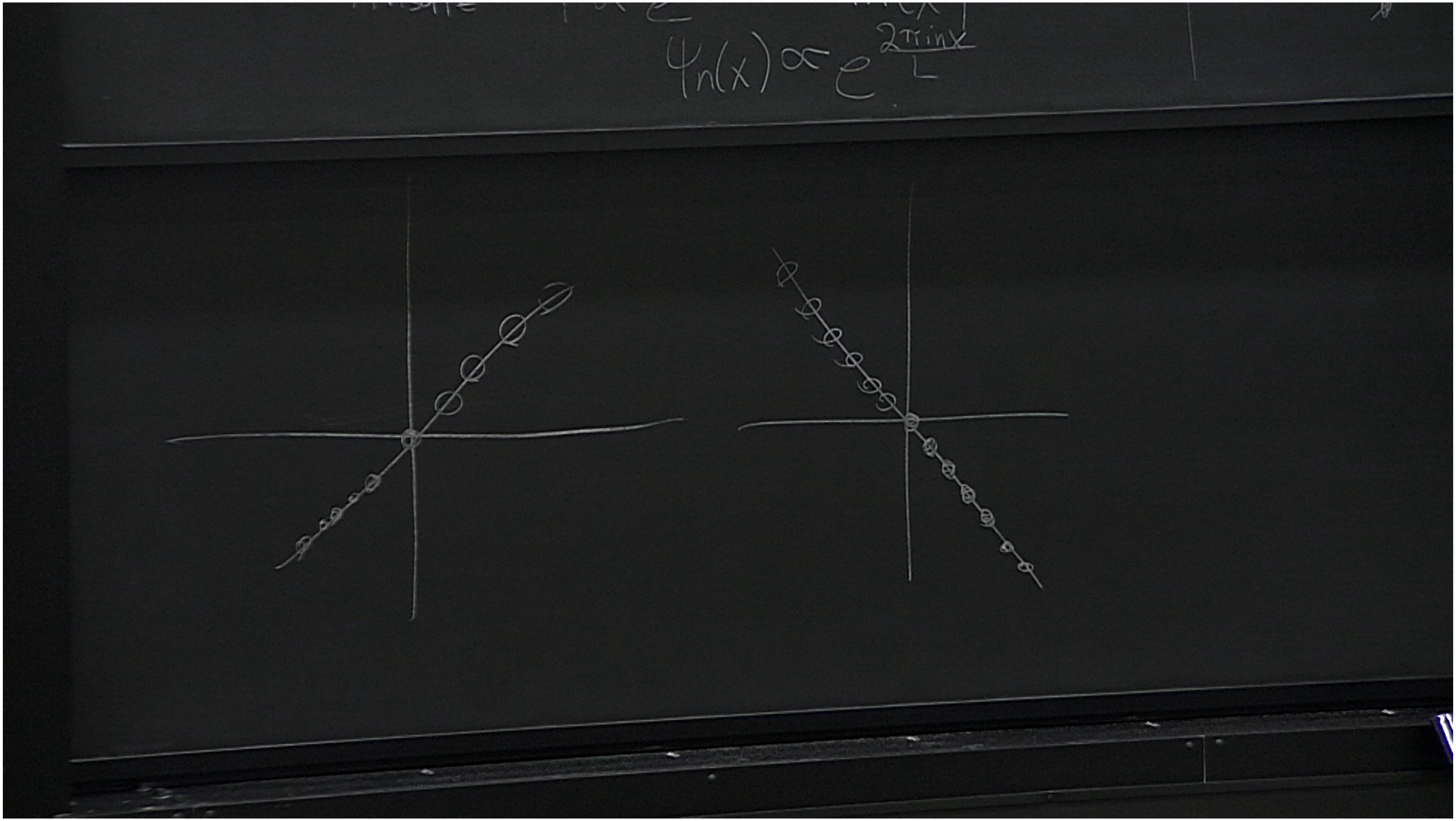
$$\dot{Q}_5 = -\frac{L}{\tilde{\pi}} \dot{A}_1$$

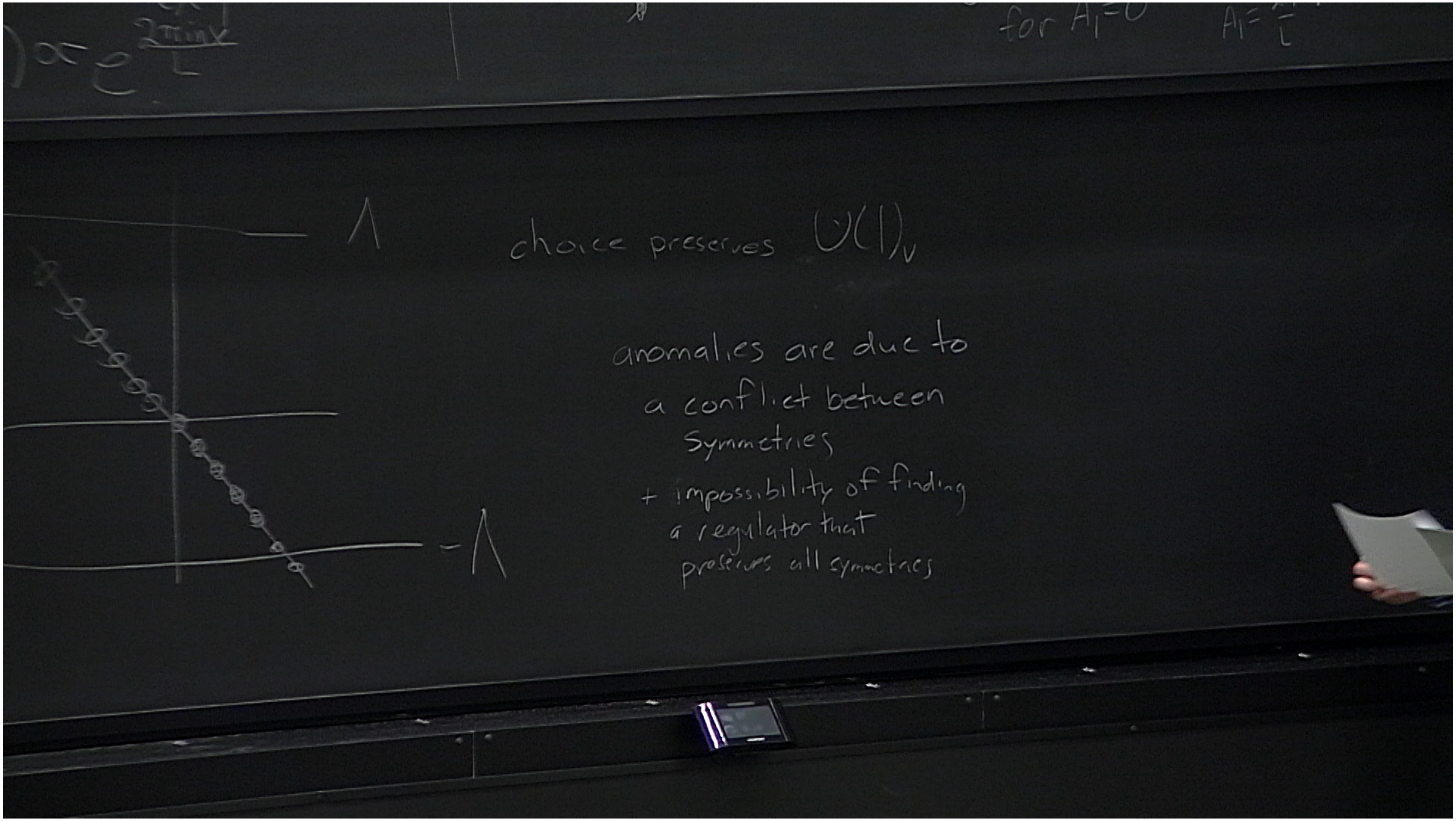
$$Q_5 = \int dx\, J_5^0$$

$$L A_1 = \int \epsilon^{0\mu} A_\mu dx$$

$$\epsilon^{1\mu} A_\mu = 0 \quad (A_0 = 0)$$

$$\boxed{\partial_\mu J_5^\mu = \frac{1}{\tilde{\pi}} \epsilon^{\mu\nu} F_{\mu\nu}}$$


$\psi_n(x) \propto e^{\frac{2\pi i n x}{L}}$


for $A_1=0$
Λ
choice preserves $U(1)_V$
anomalies are due to
a conflict between
symmetries
+ impossibility of finding
a regulator that
preserves all symmetries
$-\Lambda$