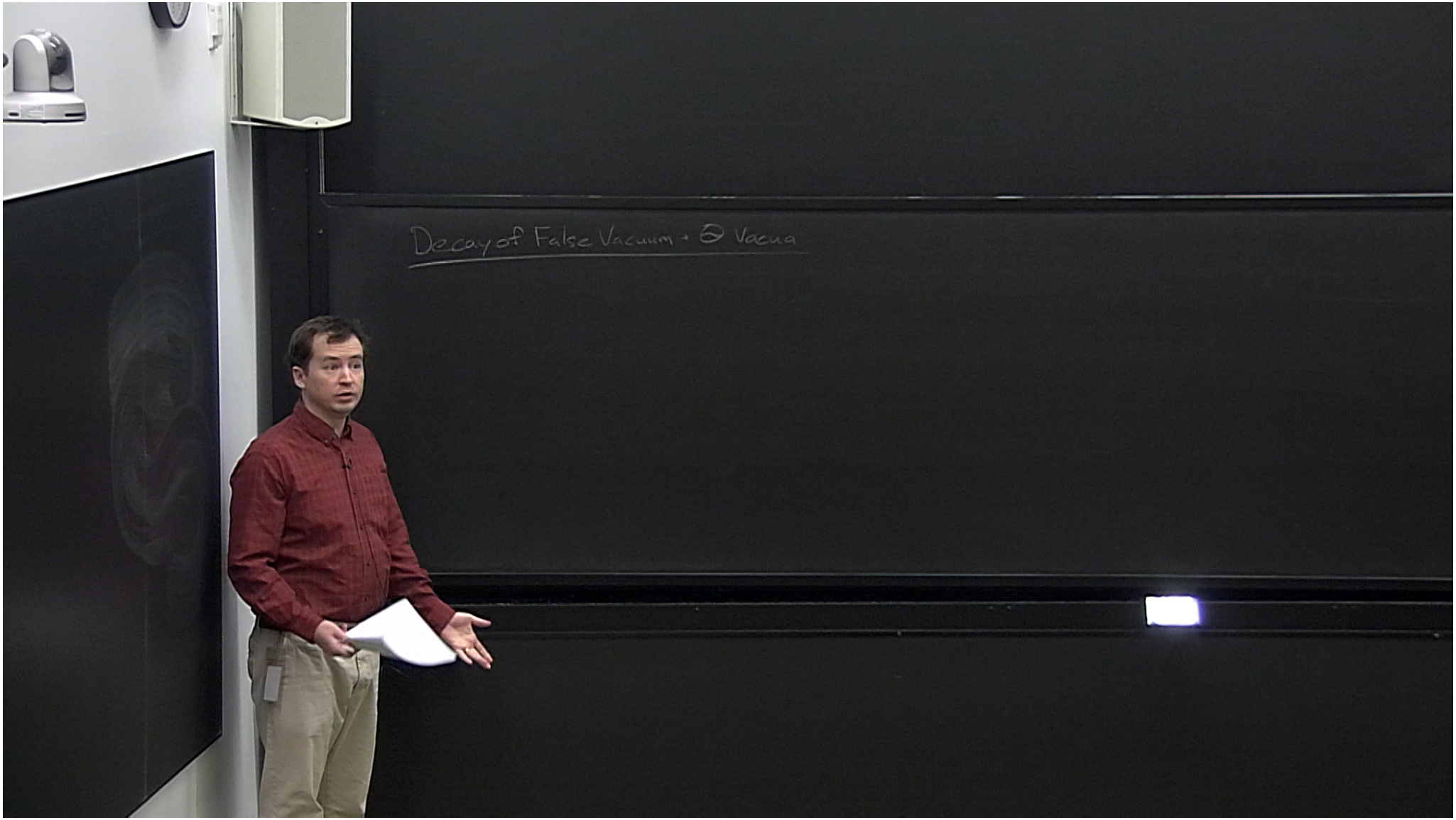
Title: PSI 2018/2019 - Quantum Field Theory III - Lecture 12

Speakers: Dan Wohns

Collection: PSI 2018/2019 - Quantum Field Theory III (Gomis, Wohns and Ali)

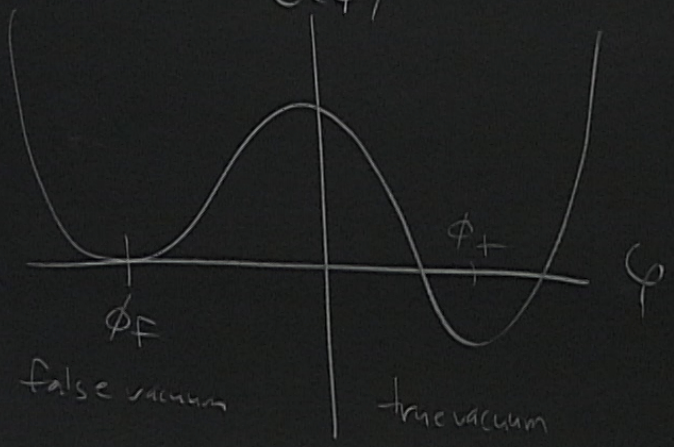
Date: March 11, 2019 - 2:00 PM

URL: http://pirsa.org/19030004


Decay of False Vacuum + Vacua

Decay of False Vacuum + Θ Vacua

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - U(\varphi)$$



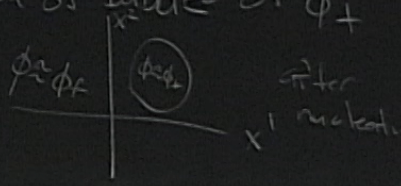
instanton method works in general D

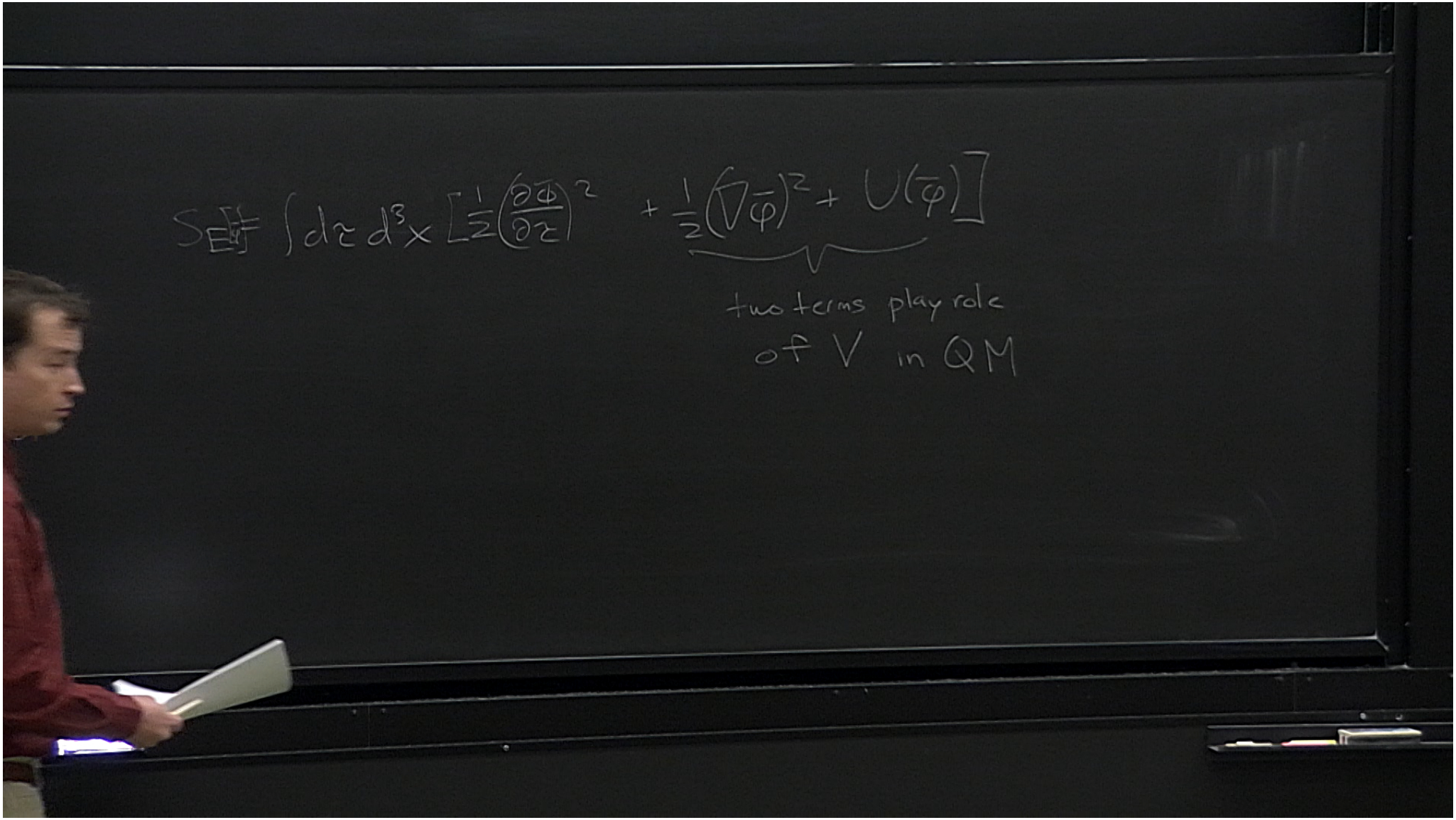
$$\frac{\Gamma}{vol} = A\, e^{-S_E[\bar{\phi}]}$$

bounce solution

Γ/vol ← nucleation per unit vol of bubbles of ϕ_+

$\propto \det' S''$




$S_E = \int d\tau\, d^3x \left[\frac{1}{2}\left(\frac{\partial\bar{\varphi}}{\partial\tau}\right)^2 + \frac{1}{2}(\nabla\bar{\varphi})^2 + U(\bar{\varphi}) \right]$
two terms play role
of V in QM

$$S_E[\bar{\phi}] = \int d\tau\, d^3x \left[\frac{1}{2}\left(\frac{\partial\bar{\phi}}{\partial\tau}\right)^2 + \frac{1}{2}(\vec{\nabla}\bar{\phi})^2 + U(\bar{\phi})\right]$$

two terms play role of V in QM

boundary conditions: $\lim_{\tau\to\pm\infty} \bar{\phi} = \phi_f$ analog of b.cs. in QM

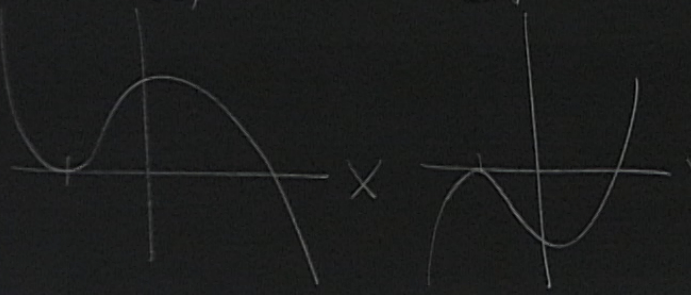
$\lim_{|\vec{x}|\to\infty} \bar{\phi} = \phi_f$ for $S_E[\bar{\phi}] < \infty$

$$S_E[\bar{\phi}] = \int d\tau\, d^3x \left[\frac{1}{2}\left(\frac{\partial \bar{\phi}}{\partial \tau}\right)^2 + \frac{1}{2}(\vec{\nabla}\bar{\phi})^2 + U(\bar{\phi}) \right]$$

two terms play role of V in QM

boundary conditions: $\lim_{\tau \to \pm\infty} \bar{\phi} = \phi_f$ analog of b.cs. in QM

$\lim_{|\vec{x}| \to \infty} \bar{\phi} = \phi_f$ for $S_E[\bar{\phi}] < \infty$



$$E_E = \int d\tau\, d^3x \left[\frac{1}{2}\left(\frac{\partial \bar{\phi}}{\partial \tau}\right)^2 + \frac{1}{2}\left(\nabla \bar{\phi}\right)^2 + U(\bar{\phi}) \right]$$

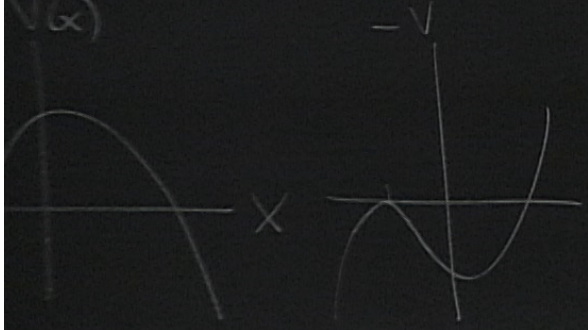
two terms play role of V in QM

Boundary conditions: $\lim_{\tau \to \pm\infty} \bar{\phi} = \phi_f$ analog of b.cs. in QM

$\lim_{|\vec{x}| \to \infty} \bar{\phi} = \phi_f$ for $S_E[\bar{\phi}] < \infty$

$\Big\}$ $\lim_{\rho \to \infty} \bar{\phi} = \phi_f$

$\rho^2 = \tau^2 + \vec{x}^2$ Euclidean radius



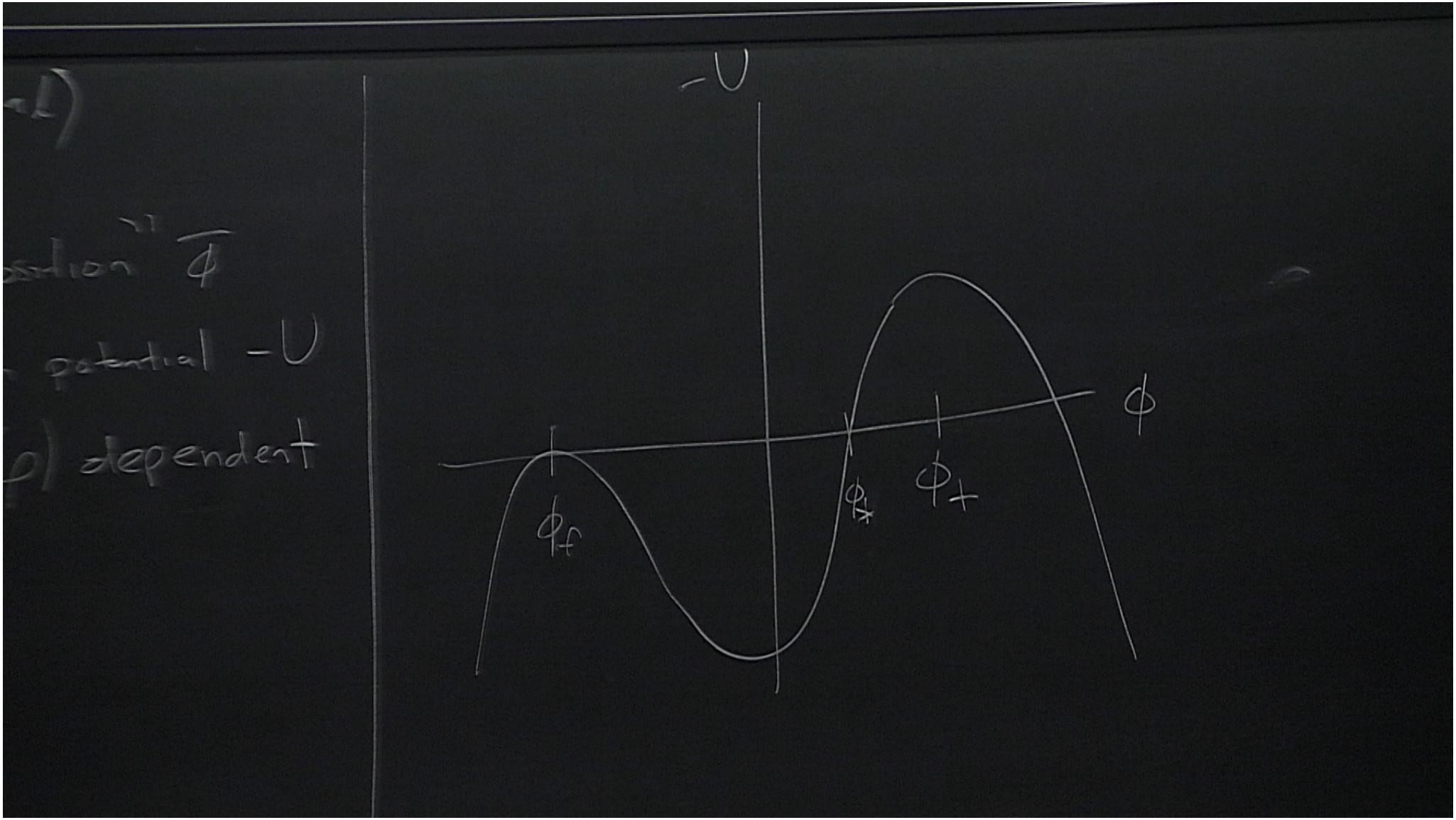
min value of S_E is for a bounce with O(4) symmetry (Coleman et. al)

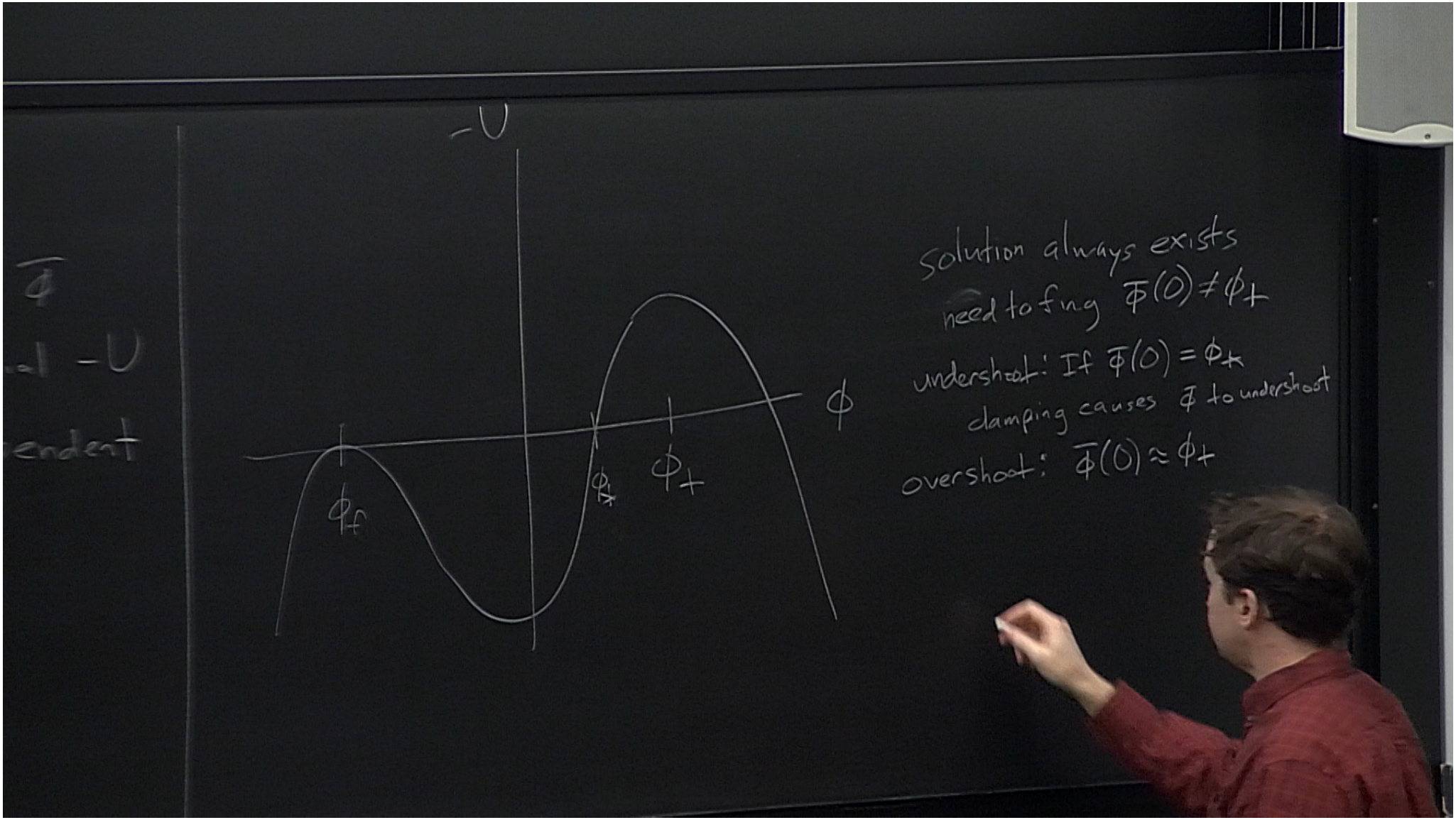
EOM: $\frac{d^2\bar{\phi}}{d\rho^2} + \underbrace{\frac{3}{\rho}\frac{d\bar{\phi}}{d\rho}}_{\text{damping term}} = U'(\bar{\phi})$ ← like EOM for "position" $\bar{\phi}$ of a particle moving in potential $-U$ with "time" (ρ) dependent damping

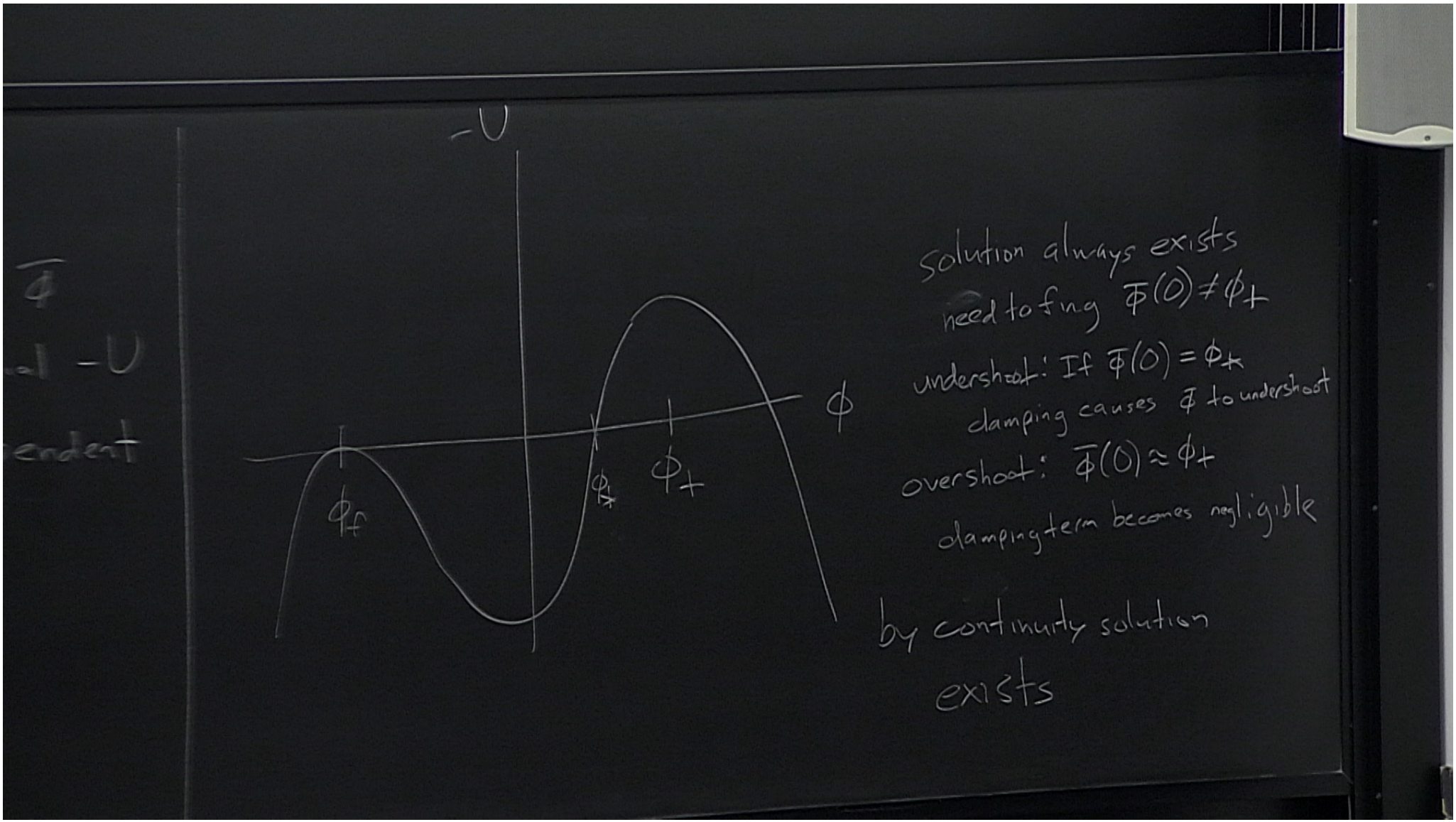
$U' = \frac{dU}{d\phi}$

other b.c. regularity at origin

$$\left.\frac{d\bar{\phi}}{d\rho}\right|_{\rho=0} = 0$$

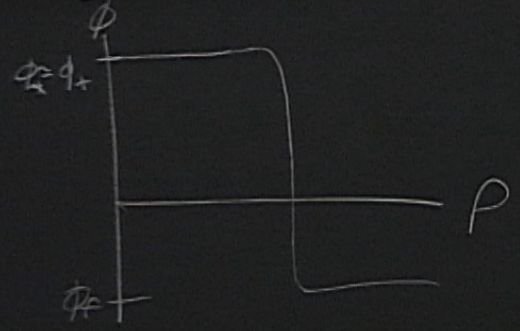

-U
ϕ
ϕf
ϕ+
ϕ+


-U
ϕ
ϕ_f
ϕ_*
ϕ_+
solution always exists
need to find $\bar{\phi}(0) \neq \phi_+$
undershoot: If $\bar{\phi}(0) = \phi_*$
clamping causes $\bar{\phi}$ to undershoot
overshoot: $\bar{\phi}(0) \approx \phi_+$


-U
ϕ
ϕ_f
ϕ_*
ϕ_+
solution always exists
need to find $\bar{\phi}(0) \neq \phi_+$
undershoot: If $\bar{\phi}(0) = \phi_*$
damping causes $\bar{\phi}$ to undershoot
overshoot: $\bar{\phi}(0) \approx \phi_+$
damping term becomes negligible
by continuity solution
exists

Thin-wall limit

If $U(\phi_+) \simeq U(\phi_-)$



$$S_E = -\frac{1}{2}\pi^2 R_0^4 \epsilon + 2\pi^2 R_0^3 \sigma$$

volume term: $U(\phi_-) - U(\phi_+)$

surface term: tension of bubble wall $= \int_{\phi_E}^{\phi_-} d\phi \sqrt{2V}$

R_0: radius of bubble

$$\frac{dS_E}{dR_0} = 0 \Rightarrow R_0 = \frac{3\sigma}{\epsilon}$$

$$S_E = \frac{27\pi^2\sigma^4}{2\epsilon^3}$$

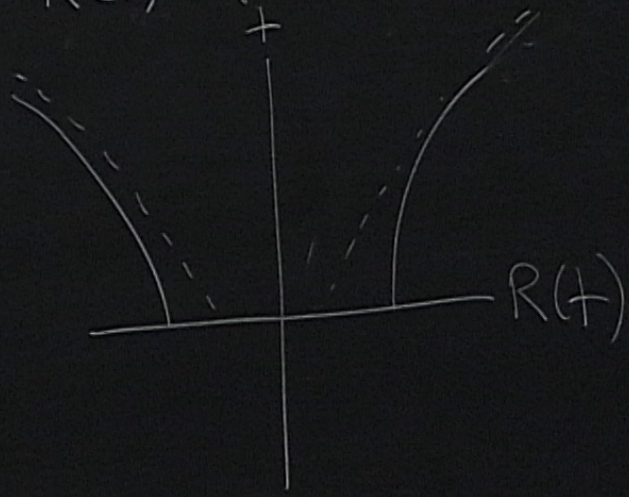
$$\frac{27\pi^2}{2} \simeq 100 \qquad \left(\frac{1}{l_p H_0}\right)^4 \simeq e^{560}$$

tension of bubble wall $= \int_{\phi_f}^{\phi_+} d\phi \sqrt{2V}$

$\tau^2 + R^2(\tau) = R_0^2$

Real time evolution $\tau \to -it$ $O(4) \to O(3,1)$

$R(t) = \sqrt{t^2 + R_0^2}$ $t \gg R_0$ $\frac{dR}{dt} \simeq 1$



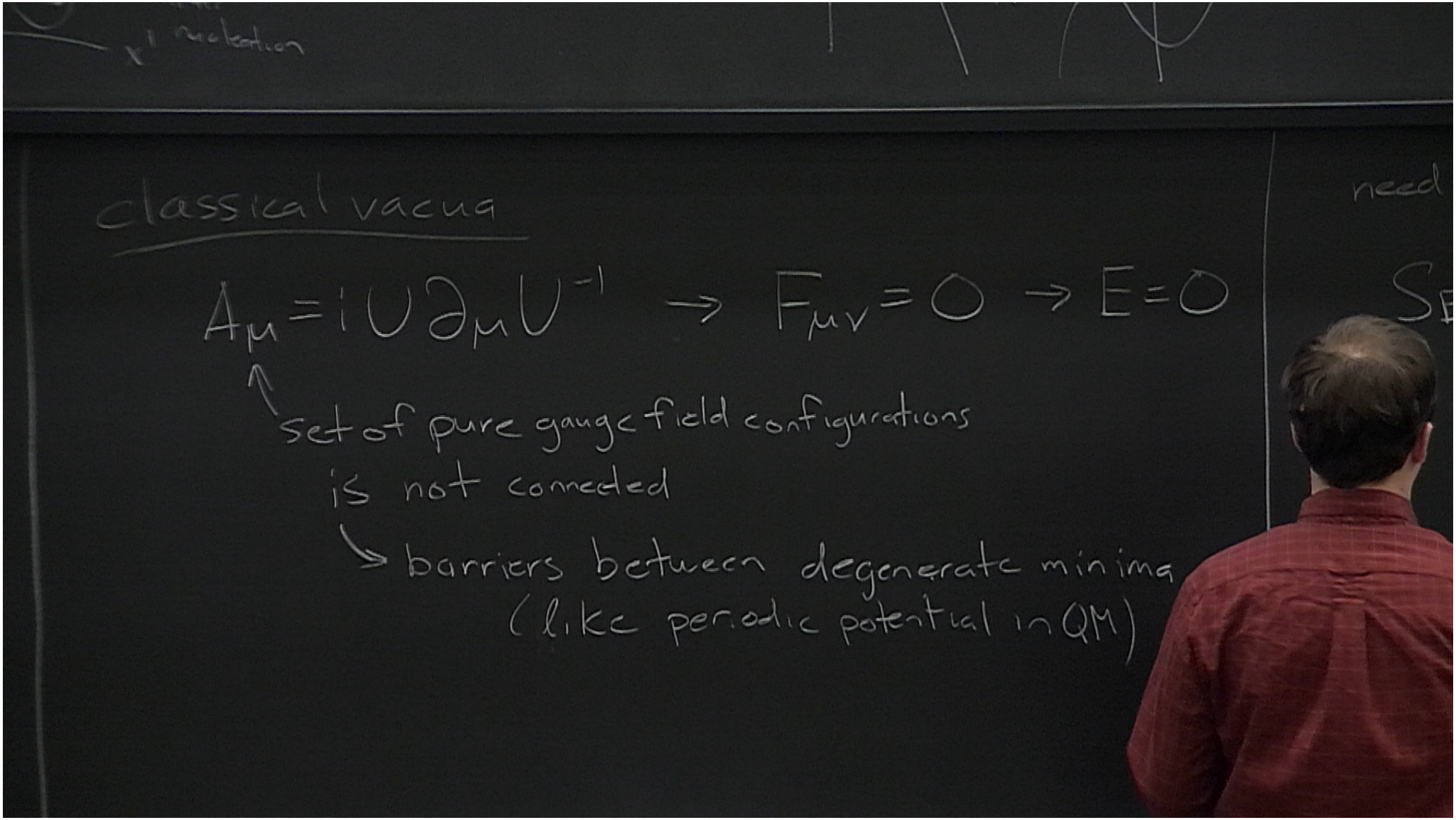
$\frac{3\pi^2}{2} \simeq 100$ $\left(\frac{1}{\beta_0 H_0}\right)^4 \simeq e^{560}$

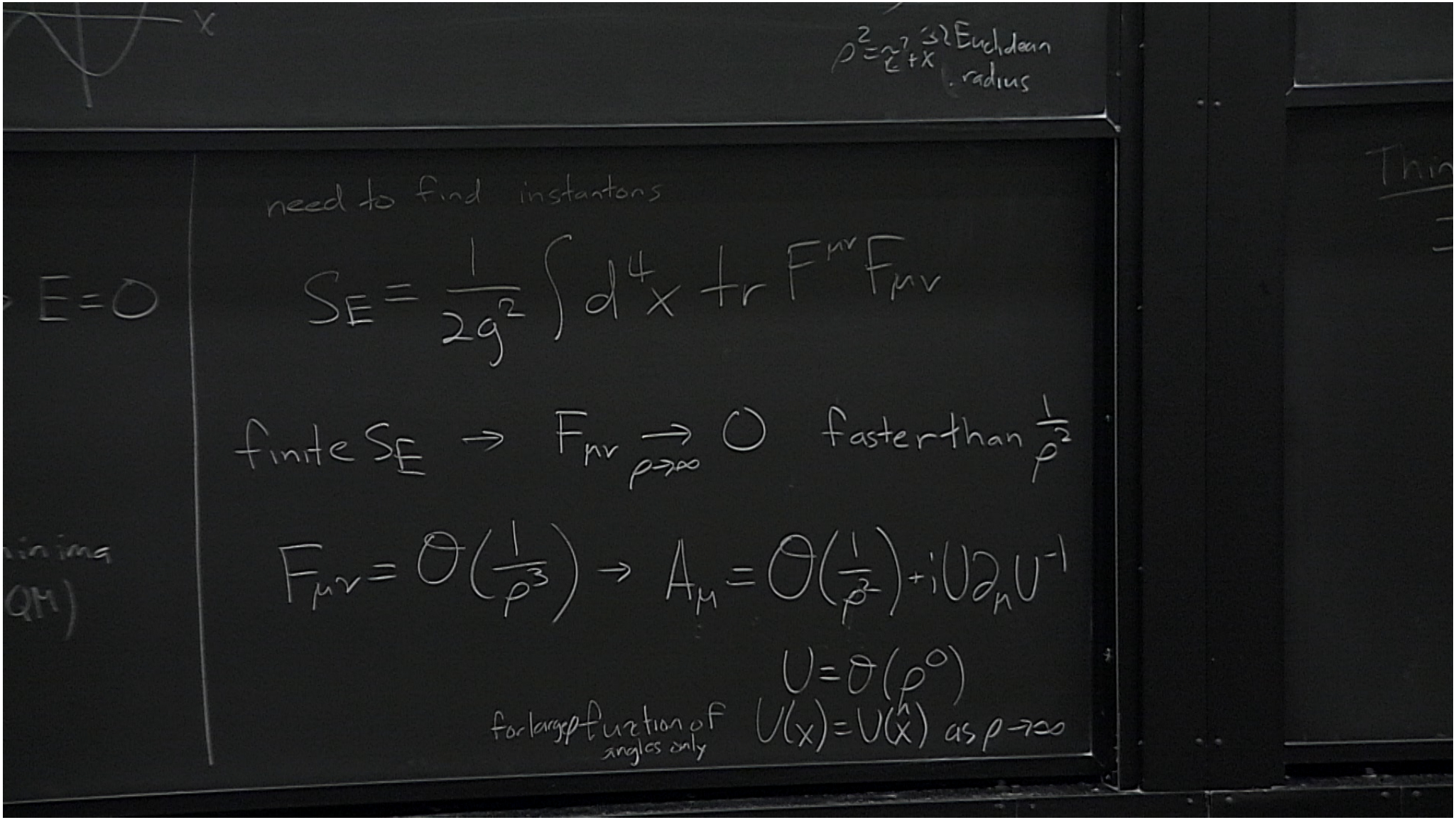
θ Vacua

Euclidean non-abelian gauge theory

what is the vacuum state?

$$g^{\mu\nu} = \delta^{\mu\nu}$$


classical vacua
$A_\mu = iU\partial_\mu U^{-1} \rightarrow F_{\mu\nu} = 0 \rightarrow E = 0$
set of pure gauge field configurations
is not connected
barriers between degenerate minima
(like periodic potential in QM)


$\rho^2 = x_4^2 + \vec{x}^2$ Euclidean radius
need to find instantons
$S_E = \frac{1}{2g^2}\int d^4x \, \mathrm{tr} F^{\mu\nu} F_{\mu\nu}$
finite $S_E \rightarrow F_{\mu\nu} \xrightarrow[\rho\to\infty]{} 0$ faster than $\frac{1}{\rho^2}$
$F_{\mu\nu} = O(\frac{1}{\rho^3}) \rightarrow A_\mu = O(\frac{1}{\rho^2}) + iU\partial_\mu U^{-1}$
$U = O(\rho^0)$
for large ρ function of angles only $U(x) = U(\hat{x})$ as $\rho \to \infty$

Simpler case: $U(1)$ in $D=2$

$$F_{\mu\nu} = \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$A_\mu = \mathcal{O}\left(\frac{1}{r^2}\right) + iU\partial_\mu U^{-1}$$

$$U \xrightarrow[\rho\to\infty]{} U(\theta) = e^{i\alpha(\theta)}$$

maps from S^1 (at $r=\infty$) $\to$ S^1 ($U(1)$)

(like periodic potential in QM)

$F_{\mu\nu} = O(\frac{1}{\rho^3}) \to A_\mu = O(\frac{1}{\rho^2}) + iU\partial_\mu U^\dagger$

$U = \theta(\rho^0)$

for large ρ function of angles only $U(x) = U(\hat{x})$ as $\rho \to \infty$

standard maps

$U^{(0)}(\theta) = 1$ trivial

$U^{(1)}(\theta) = e^{i\theta}$ identity

$U^{(\nu)}(\theta) = e^{i\nu\theta}$ wind ν times $\nu \in \mathbb{Z}$ around S^1

any map from $S^1 \to S^1$ continuously deformable to 1 and only 1 of these maps (homotopic)