Title: PSI 2018/2019 - Quantum Field Theory III - Lecture 10

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Collection: PSI 2018/2019 - Quantum Field Theory III (Gomis, Wohns and Ali)

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URL: http://pirsa.org/19030002

QFT III

Instantons + Anomalies (≈ 3 + 3 lectures)

↑ non-trivial saddles of the Euclidean functional integral

↑ quantum effects that spoil classical symmetries

In QFT I + II

$$f(g) \overset{g \to 0}{\sim} \sum a_n g^n$$

- $f(g)$: exact quantity of interest, e.g. iM or $\langle \phi \cdots \phi \rangle$
- small parameter, e.g. coupling constant or $\hbar$
- $\sim$: asymptotic sum + integral

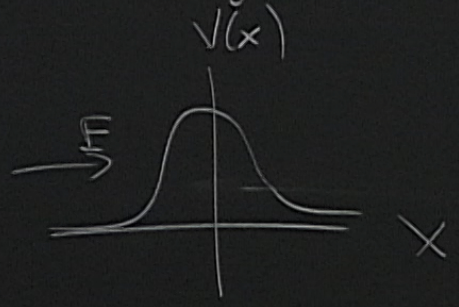
→ contributions like $e^{-A/g}$ (constant A) are invisible

↑ all terms in Taylor series are zero or $e^{-A/g} \overset{g \to 0}{<<} g^N$

In QM:

$\ln\left(\frac{1}{\text{Hole}}\right) \simeq 140$

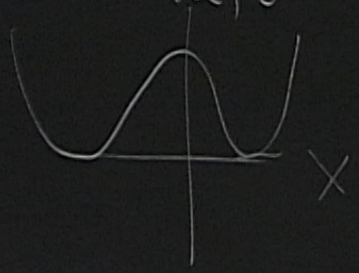
Tunneling through a barrier



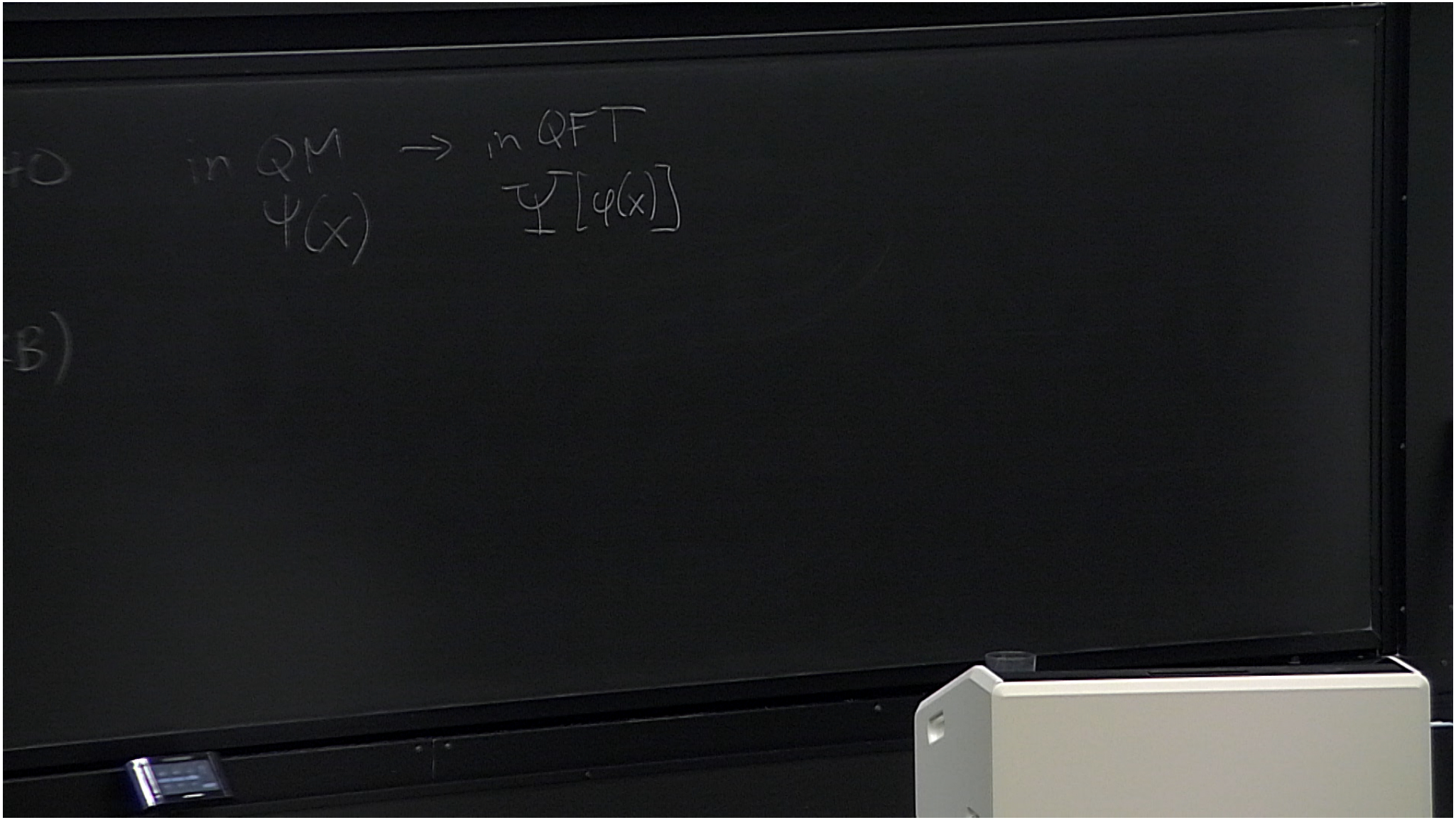
$$T \propto e^{-\frac{2}{\hbar}\int \sqrt{2m(V-E)}\,dx}$$

T: transmission coefficient

energy splitting in symmetric double-well potential



$$\Delta E = E_+ - E_- \propto e^{-\frac{1}{\hbar}\int dx \sqrt{2V}}$$


in QM → in QFT
ψ(x) Ψ[φ(x)]

all terms in Tayl

In Math for QFT

$$I(z) = \int_C g(w)\, e^{z f(w)}\, dw \underset{z\to\infty}{\sim} \sum_{\substack{\text{relevant} \\ \text{saddles} \\ w_0}} \sqrt{\frac{2\pi}{z}}\, \frac{e^{z f(w_0)}}{\sqrt{-f''(w_0)}}\, g(w_0)$$

generalization to QFT?

- which saddles + contours?

Instanton method: studying ground state

Outline

Instantons in QM – E_0 from Euclidean functional integral

- energy splittings in double-well + periodic potentials
- Γ ← decay rates

Instantons in QFT – Decay of metastable vacuum in QFT

- θ vacua (ground state of QCD)

E_0 from Euclidean path integral

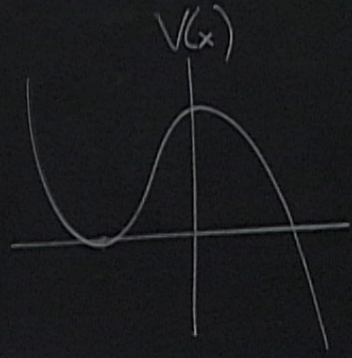
$$Z = \langle x_f | e^{-HT/\hbar} | x_i \rangle = \int_{x(-\frac{T}{2})=x_i}^{x(\frac{T}{2})=x_f} \mathcal{D}[x(t)]\, e^{-\frac{1}{\hbar} S_E[x(t)]}$$

$$H|n\rangle = E_n|n\rangle$$

↑ energy eigenstates

$$\langle x_f | e^{-HT/\hbar} | x_i \rangle = \sum_n e^{-E_n T/\hbar} \langle x_f | n \rangle \langle n | x_i \rangle$$

$$E_0 \propto -\lim_{T\to\infty} \frac{\hbar}{T} \ln Z$$



unstable potential

$$\frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$$

instanton $\bar{x}(\tau)$ = non-trivial saddle of Z

$$\frac{\delta S}{\delta \bar{x}(\tau)} = 0 = -\frac{d^2\bar{x}}{d\tau^2} + V'(x)$$

$$V' = \frac{\partial V}{\partial x}$$

can include fluctuations around saddles

$$S_E[x] = S_E[\bar{x}] + \frac{1}{2}\tilde{x}\cdot S''[\bar{x}]\cdot\tilde{x} + \cdots$$

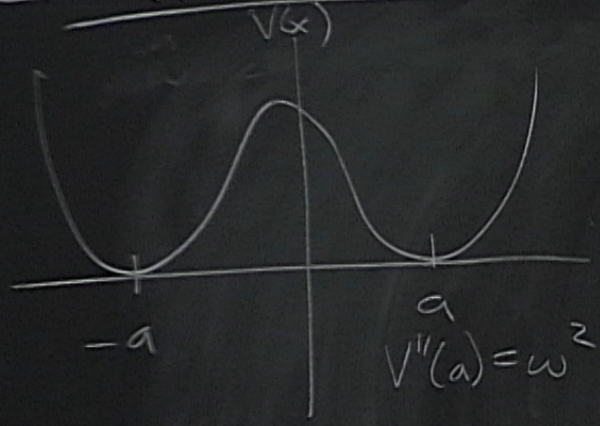
$$x(\tau) = \bar{x}(\tau) + \hbar^{1/2}\tilde{x}(\tau)$$

$$\tilde{x} \cdot S''[\bar{x}] \cdot \tilde{x} = \int dx_1 dx_2 \, \tilde{x}(x_1) \frac{\delta^2 S[\bar{x}]}{\delta x(x_1) \delta x(x_2)} \tilde{x}(x_2)$$

$$Z = N \left(\det S''[\bar{x}] \right)^{-1/2} e^{-S_E[\bar{x}]/\hbar}$$

focus of this course

In QM Double Well Potential

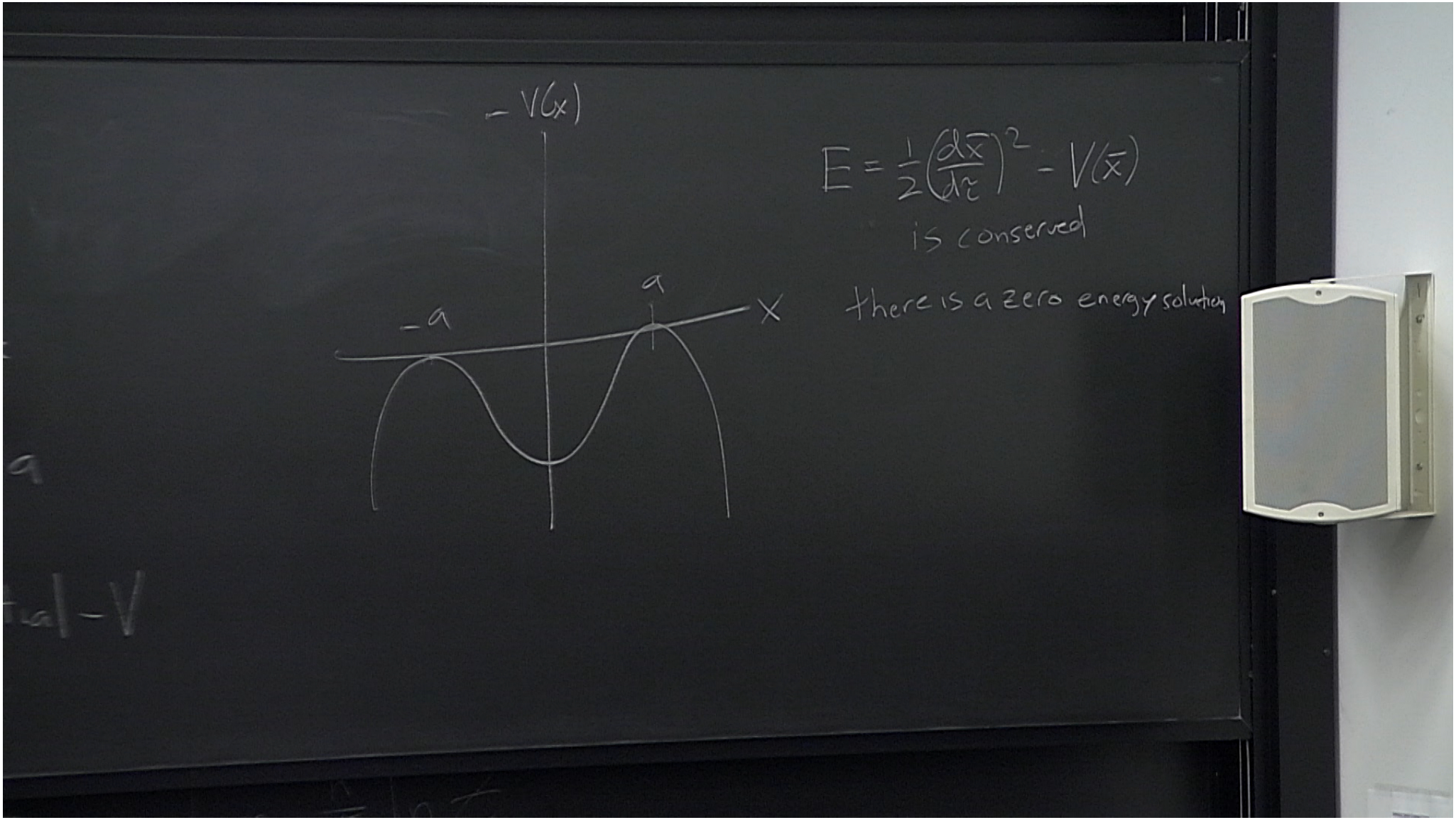


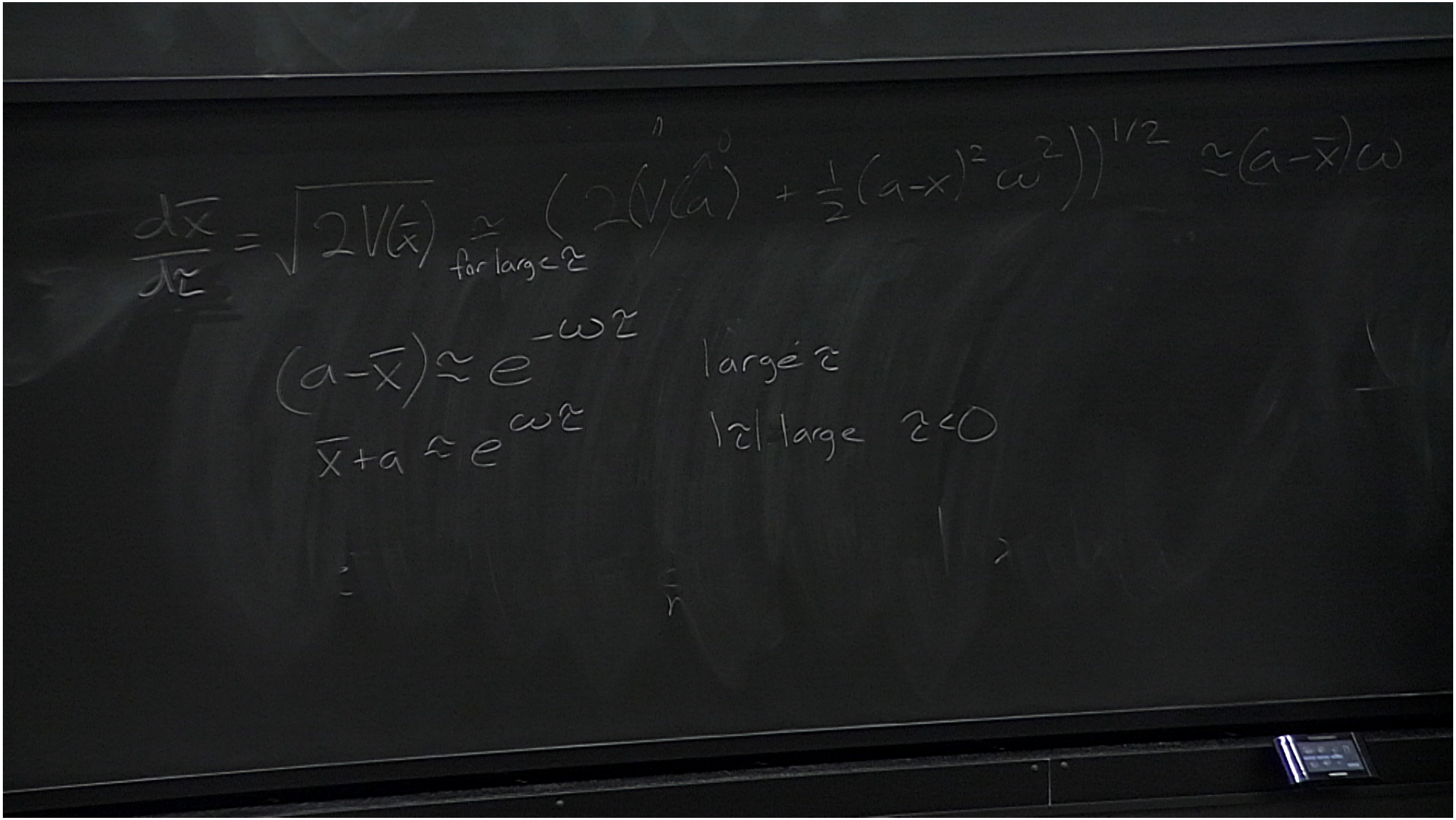
symmetric $V(x) = V(-x)$

want $\langle a | e^{-HTA} | -a \rangle$ using saddle-point approx

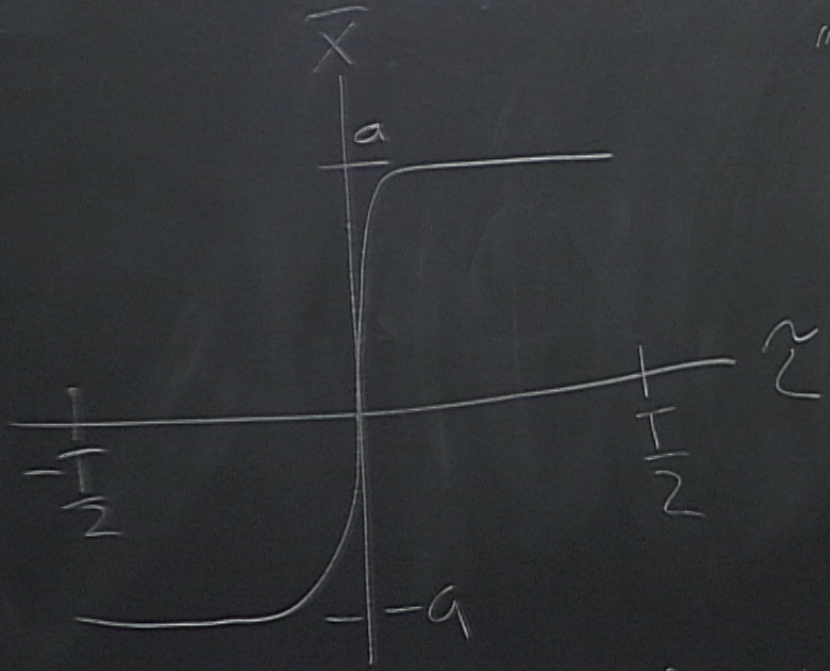
$-\frac{d\bar{x}}{d\tau^2} + V'(\bar{x}) = 0$ with $\bar{x}(\pm\frac{T}{2}) = \pm a$

EOM is for a particle moving in a potential $-V$


-V(x)
-a
a
X
$E = \frac{1}{2}\left(\frac{d\bar{x}}{d\bar{\tau}}\right)^2 - V(\bar{x})$
is conserved
there is a zero energy solution


$\frac{d\bar{x}}{d\tau} = \sqrt{2V(\bar{x})} \underset{\text{for large } \tau}{\simeq} \left(2\left(V(a) + \frac{1}{2}(a-x)^2\omega^2\right)\right)^{1/2} \simeq (a-\bar{x})\omega$
$(a-\bar{x}) \simeq e^{-\omega\tau}$ large τ
$\bar{x}+a \simeq e^{\omega\tau}$ $|\tau|$ large $\tau<0$

$\tilde{x} = (a - \bar{x})/\omega$



"instanton" is localized in Euclidean time

$\tilde{\tau} = \frac{\tau}{\omega}$ center of instanton can be anywhere in $T \to \infty$ limit

anti-instanton is $\bar{x}(-\tau)$

$\langle -a | e^{-HT/\hbar} | a \rangle$