Title: N=4 SYM and IIB Supergravity at 1-Loop

Speakers: Francesco Aprile

Series: Quantum Fields and Strings

Date: February 26, 2019 - 2:30 PM

URL: http://pirsa.org/19020085

Abstract: Using knowledge about the spectrum of operators in N=4 SYM, consistency of OPE, and analytic bootstrap techniques, I will obtain 1-loop corrections for 4-pt functions of single particle half-BPS operators of IIB supergravity on AdS_5\timesS^5. Along the way, I will discuss a general formula for the leading anomalous dimension of all double-trace operators in the supergravity regime.

# $\mathcal{N}$=4 SYM & IIB Sugra @ 1-Loop

Francesco Aprile

[Milano Bicocca University]

based on
1706.02822, 1706.08456, 1711.03903, 1802.06889

with James Drummond, Paul Heslop and Hynek Paul


2019/02/26

## Outline:

How to Construct @1-loop the 4-pt Amplitude
of the superconformal primary in the
graviton multiplet of IIB Sugra

$$\langle \varphi_2 \varphi_2 \varphi_2 \varphi_2 \rangle$$

A direct computation remains a challenge in sugra!!

Instead we will bootstrap the answer from novel CFT data
in N=4 SYM extracted from tree-level correlators

## Reminder:

[Maldacena]

SuperStrings on $AdS_5 \times S^5$ $\longleftrightarrow$ $SU(N)$ $\mathcal{N}$=4 SYM in 4d

$g_s = g_{YM}$ and $L^4 = \lambda\alpha^2$ with $\lambda = g_s N$

Interesting regime: large 't Hooft coupling and large N

Supergravity: classical + 1/N expansion (dressed by a $\lambda^{-1/2}$ expansion)

## Basics of Holography

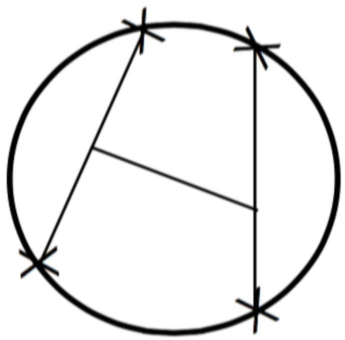
Single particles (KK modes) — Protected Half-BPS $\mathcal{O}_p \equiv \mathrm{Tr}\,\Phi^p$

$$\varphi_p \longleftrightarrow \mathcal{O}_p + \frac{1}{N}\sum \#_q \mathcal{O}_q \mathcal{O}_{p-q} + \dots$$

[Witten, van Nieuwenhuizen et al.]

## Gravitational Amplitudes dual to Boundary Correlators

[Witten, Klebanov-Gubser]



$$\langle \varphi_{p_1} \varphi_{p_2} \varphi_{p_3} \varphi_{p_4} \rangle$$

## *Witten diagram computations in IIB sugra*

[Arutyunov,Frolov, Uruchurtu, Freedman, Rastelli, Sokatchev, Dolan, Osborn]

- *3-pt functions,*  *i.e.* $1/N$ *bulk vertices of* $\langle \varphi_p \varphi_q \varphi_k \rangle$
- *the following list of 4pt functions at* $1/N^2$

$$\langle \varphi_{k+2} \varphi_{k+2} \varphi_{n-k} \varphi_{n+k} \rangle, \ \langle \varphi_3 \varphi_3 \varphi_3 \varphi_3 \rangle, \ \langle \varphi_4 \varphi_4 \varphi_4 \varphi_4 \rangle$$

*but: N=4 SYM is a very constrained SCFT*

*Partial Non-renormalization theorem:* [Eden,Petkou,Schubert,Sokatchev]

$$\langle \varphi_{p_1}\varphi_{p_2}\varphi_{p_3}\varphi_{p_4}\rangle \;=\; \text{Free Theory} \;+$$

$$(x-y)(x-\bar{y})(\bar{x}-y)(\bar{x}-\bar{y})\;\mathcal{H}_{p_1p_2p_3p_4}[u,v,\sigma,\tau]$$

*and from* OPE $\quad \varphi_{p_i}\varphi_{p_j} \sim \sum_{\text{operators } O} \langle \varphi_{p_i}\varphi_{p_j}|O\rangle\, O$

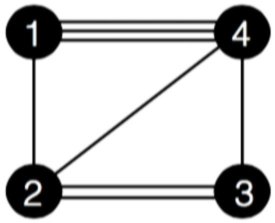
*Q: do we know these long operators ??* ✓

$$\mathcal{H}_{p_1p_2p_3p_4} = \sum_{K_{T,\ell,a,b}} C_{p_1p_2K_{T,\ell,a,b}} C_{p_3p_4K_{T,\ell,a,b}} \underbrace{\left[u^T(1-v)^\ell + \ldots\right]}_{\text{conformal block } \mathcal{B}^{[T,\ell]}_{p_{21},p_{43}}} \Upsilon_{[a,b,a]}(\sigma,\tau)$$

$$(T \equiv \Delta - \ell)$$

Heslop,Howe, Heslop-Doobary 1508.03611

# *Parenthesis: Our point of view on Super Conformal blocks*




$$= \mathcal{P}\,(u\sigma)^{\#_1}\left(\frac{u\tau}{v}\right)^{\#_2}$$

**Cross Ratios and Free Theory Diagrams**

$p_{43} \geq p_{21} \geq 0$

$$\mathcal{P} = g_{12}^{\frac{p_1+p_2-p_4+p_3}{2}}\, g_{14}^{\frac{-p_2+p_1+p_4-p_3}{2}}\, g_{24}^{\frac{p_2-p_1+p_4-p_3}{2}}\, g_{34}^{p_3}$$

$$\frac{g_{13}g_{24}}{g_{12}g_{34}} = u\sigma \qquad \frac{g_{14}g_{23}}{g_{12}g_{34}} = \frac{u\tau}{v}$$

## *1) refinement of the diagrammatic expansions*

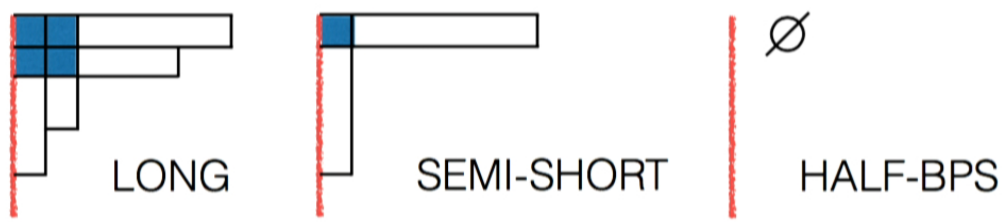
$$\langle \varphi_{p_1}\varphi_{p_2}\varphi_{p_3}\varphi_{p_4}\rangle = \mathcal{P}\sum_{\gamma}\left[(u\sigma)^{\frac{\gamma+p_{34}}{2}}\sum_{i=0}^{\frac{\gamma+p_{34}}{2}} a_\gamma[i]\left(\frac{\tau}{v\sigma}\right)^{i}\right]$$

$$\gamma = p_{43}, p_{43}+2, \ldots \min(p_1+p_2, p_3+p_4)$$

Heslop,Howe, Heslop-Doobary 1508.03611

## 2) *Superconformal Blocks and super YT of* GL(2|2)

$$p = \frac{\gamma + p_{34}}{2}$$




$$u = x_1 x_2 \quad v = (1-x_1)(1-x_2) \quad \sigma = \frac{1}{y_1}\frac{1}{y_2} \quad \tau = \frac{1-y_1}{y_1}\frac{1-y_2}{y_2}$$

### *Solution of the 4d SuperCasimir equation*

$$\mathbb{F}^{\alpha\beta\gamma\lambda} = \sum_{\mu\supset\lambda} R^{\alpha\beta\gamma\lambda}_{\mu} s_\mu(\vec{x}|\vec{y}) = (-)^{p+1} \frac{\prod_{i,j}(x_i - y_j)}{(x_1-x_2)(y_1-y_2)} \det \begin{pmatrix} F_{\lambda_j}[x_i] & \frac{1}{x_i - y_j} \\ -\delta_{i,j-\lambda_j} & G_i[y_j] \end{pmatrix}$$

$$F_{\lambda_j}[x_i] = \left( x_i^{\lambda_j - j}\ {}_2F_1[\lambda_j + 1 - j + \alpha, \lambda_j + 1 - j + \beta; 2\lambda_j + 2 - 2j + \gamma; x_i] \right)_{\substack{1\le i\le 2 \\ 1\le j\le p}}$$

$$G_i[y_j] = \left( y_j^{i-1}\ {}_2F_1[i-\alpha, i-\beta; 2i-\gamma; y_j] \right)_{\substack{1\le i\le p \\ 1\le j\le 2}} \quad \alpha = \frac{\gamma + p_{21}}{2} \quad \beta = \frac{\gamma - p_{43}}{2}$$

Heslop,Howe, Heslop-Doobary 1508.03611

## 3) *Unpacking the determinant* $\mathbb{F}^{\gamma\lambda} \to k^{\gamma\lambda} + \mathcal{S}^{\gamma\lambda} + (u\sigma)^p \mathcal{D}^{\gamma\lambda}$

$$\mathcal{S}^{\gamma\lambda} = \frac{\prod_{i,j}(x_i - y_j)}{(x_1 - x_2)(y_1 - y_2)} \left[ \left( \frac{f^{\gamma\lambda}(x_2, y_2)}{x_1 - y_1} - (y_1 \leftrightarrow y_2) \right) - (x_1 \leftrightarrow x_2) \right]$$

$$\mathcal{D}^{\gamma\lambda} = \prod_{i,j}(x_i - y_j)\, \mathcal{H}^{\gamma\lambda}(x_1, x_2, y_1, y_2)$$

Long : *4-variables function fully factorized*

$$\mathcal{H}^{\gamma[\lambda_1,\lambda_2]} = \underbrace{\frac{\det\left(F_{\lambda_j}[x_i]\right)_{1\le i,j\le 2}}{x_1 - x_2}}_{\mathcal{B}^{[T,\ell]}_{p_{21},p_{43}}} \; \underbrace{\frac{\det\left(G_{\lambda'_i - i+1}[y_j]\right)_{1\le i,j\le 2}}{y_1 - y_2}}_{\Upsilon_{[a,b,a]}(\sigma,\tau)}$$

$$T = \gamma + 2(\lambda_2 - 2) \qquad a = \lambda'_1 - \lambda'_2$$

$$\ell = \lambda_1 - \lambda_2 \qquad b = \gamma - 2\lambda'_1$$

## 4) SCPW decomposition

$$\text{Free Theory} = \mathcal{P} \sum_{\gamma} \left[ (u\sigma)^{\frac{\gamma+p_{34}}{2}} \sum_{i=0}^{\frac{\gamma+p_{34}}{2}} a_{\gamma}[i] \left(\frac{\tau}{v\sigma}\right)^{i} \right]$$

$$= \mathcal{P} \sum_{\gamma} \sum_{\mathrm{YT}\ \lambda} (u\sigma)^{\frac{\gamma+p_{34}}{2}} A^{\alpha\beta}_{\gamma\lambda}\ \mathbb{F}^{\alpha\beta\gamma\lambda}[\vec{x}, \vec{y}]$$

*CFT data*

$$A^{\alpha\beta}_{\gamma\lambda} = \sum_{K_{T,\ell,a,b}} C_{p_1 p_2 K_{T,\ell,a,b}} C_{p_3 p_4 K_{T,\ell,a,b}} \quad \textit{UNMIXING}$$

$$\mathcal{H}_{p_1p_2p_3p_4} = \sum_{K_{T,\ell,a,b}} C_{p_1p_2K_{T,\ell,a,b}} C_{p_3p_4K_{T,\ell,a,b}} \underbrace{\left[u^{T}(1-v)^{\ell} + \ldots\right]}_{\text{conformal block } \mathcal{B}^{[T,\ell]}_{p_{21},p_{43}}} \Upsilon_{[a,b,a]}(\sigma,\tau)$$

$(T \equiv \Delta - \ell)$

*Q: do we know these long operators ??*

part I

## The spectrum of N=4 SYM at large λ

No heavy stringy states, only light multi-trace operators

- Simple consideration, say
- Expand in 1/N

$$K \sim \varphi_p \partial^l \varphi_q$$

$$C = C^0 + \frac{1}{N^2} C^1 + \dots$$

then the threshold "t" for exchange of a long DT operator in

$\langle \varphi_{p_1} \varphi_{p_2} \varphi_{p_3} \varphi_{p_4} \rangle$ is $\max(p_1 + p_2, p_3 + p_4)$

$$T = t + \frac{1}{N^2}\eta + \dots$$

K acquires anomalous dimension,

$$\mathcal{H}_{p_1 p_2 p_3 p_4} \supset \frac{1}{N^2} \log(u) \sum_{K_{t,\ell,a,b}} C^0_{p_1 p_2 K}\, \eta_K\, C^0_{p_3 p_4 K}\, \mathcal{B}^{[t,\ell]}_{p_{21},p_{43}} \Upsilon_{[a,b,a]}$$

[1608.06624 ] Leonardo and Xinan, Thank you!!

*Task: Construct a tree level function with the required properties*

*[Crossing, Ward Identity, Mellin Space, Flat space limit]*

[Dolan and Osborn] [Mack] [Penedones] [Penedones]

$$\frac{u^{\frac{p_3-p_4}{2}}}{v^{\frac{p_2+p_3}{2}}}\sum_{i,j}\frac{a_{ijk}}{i!j!k!}\frac{\sigma^i\tau^j}{(z-\tilde{\zeta}+2k)(w-\tilde{\omega}+2j)(\tilde{\mu}-z-w+2i)}$$

$$\mathcal{H}_{\rm RZ}=\oint dzdw\, u^{\frac{z}{2}}v^{\frac{w}{2}}\mathcal{R}[^{z}_{\sigma}\ ^{w}_{\tau}]\,\Gamma[\tfrac{p_1+p_2-z}{2}]\Gamma[\tfrac{p_3+p_4-z}{2}]\times \Gamma[\tfrac{p_1+p_4-w}{2}]\Gamma[\tfrac{p_2+p_3-w}{2}]\times \Gamma[\tfrac{z+w+4-p_1-p_3}{2}]\Gamma[\tfrac{z+w+4-p_2-p_4}{2}]$$

$$i+j+k=p_3-2+\min\left(0,\tfrac{p_1+p_2-p_3-p_4}{2}\right)$$

$$a_{ijk}=\frac{p_1p_2p_3p_4}{\left(\frac{|p_1+p_2-p_3-p_4|}{2}+k\right)!\left(\frac{p_{43}+p_{21}}{2}+i\right)!\left(\frac{p_{43}-p_{21}}{2}+j\right)!}$$

fixed in JD,FA,PH,HP 1802.06889
from the light-like limit

10d proof: Caron-Huot,Trinh 1809.09173

# [1608.06624 ] Leonardo and Xinan, Thank you!!

*Task: Construct a tree level function with the required properties*

*[Crossing, Ward Identity, Mellin Space, Flat space limit]*

[Dolan and Osborn] [Mack] [Penedones] [Penedones]

$$\frac{u^{\frac{p_3-p_4}{2}}}{v^{\frac{p_2+p_3}{2}}}\sum_{i,j}\frac{a_{ijk}}{i!j!k!}\frac{\sigma^i\tau^j}{(z-\tilde{\zeta}+2k)(w-\tilde{\omega}+2j)(\tilde{\mu}-z-w+2i)}$$

$$\mathcal{H}_{\rm RZ}=\oint dzdw\, u^{\frac{z}{2}}v^{\frac{w}{2}}\mathcal{R}\left[\begin{smallmatrix} z & w\\ \sigma & \tau\end{smallmatrix}\right]\Gamma\left[\tfrac{p_1+p_2-z}{2}\right]\Gamma\left[\tfrac{p_3+p_4-z}{2}\right]\times \Gamma\left[\tfrac{p_1+p_4-w}{2}\right]\Gamma\left[\tfrac{p_2+p_3-w}{2}\right]\times \Gamma\left[\tfrac{z+w+4-p_1-p_3}{2}\right]\Gamma\left[\tfrac{z+w+4-p_2-p_4}{2}\right]$$

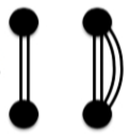
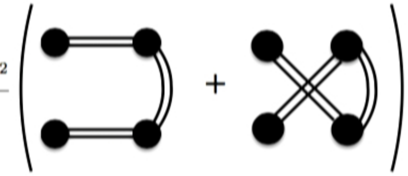
$$i+j+k=p_3-2+\min\left(0,\tfrac{p_1+p_2-p_3-p_4}{2}\right)$$

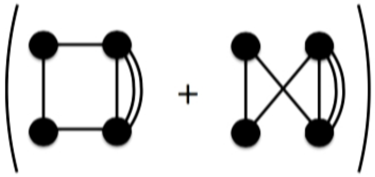
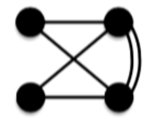
$$a_{ijk}=\frac{p_1p_2p_3p_4}{\left(\frac{|p_1+p_2-p_3-p_4|}{2}+k\right)!\left(\frac{p_{43}+p_{21}}{2}+i\right)!\left(\frac{p_{43}-p_{21}}{2}+j\right)!}$$

fixed in JD,FA,PH,HP 1802.06889
from the light-like limit

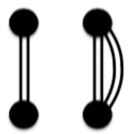
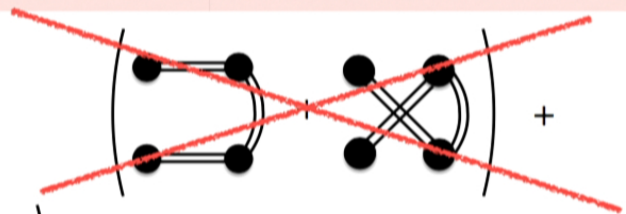
Page 12 of 23 Tod proof: Caron-Huot,Trinh 1809.09173

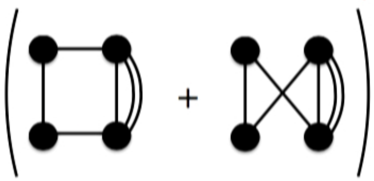
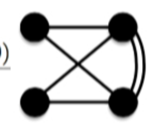
$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_4 \mathcal{O}_4 \rangle$

$\frac{8(N^2-1)^2(N^4-6N^2+18)}{N^2}$  $+$ $\frac{32(N^2-1)(2N^2-3)^2}{N^2}$  $+$

$\frac{64(N^2-1)(N^4-6N^2+18)}{N^2}$  $+$ $\frac{192(N^2-1)(N^4-6N^2+18)}{N^2}$ 

$\langle \varphi_2 \varphi_2 \varphi_4 \varphi_4 \rangle \qquad \varphi_2 = \mathcal{O}_2 \qquad \varphi_4 = \mathcal{O}_4 - \frac{2N^2-3}{N(N^2+1)} \mathcal{O}_2 \mathcal{O}_2$

$\frac{8(N^2-1)^2(N^2-4)(N^2-9)}{N^2+1}$  $+$  $+$

$\frac{64(N^2-1)(N^2-4)(N^2-9)}{N^2+1}$  $+$ $\frac{192(N^2-1)(N^2-4)(N^2-9)}{N^2+1}$ 

part III

## The spectrum of N=4 SYM at large λ

- More precise considerations: Long Double Trace

$$K_{pq} = \varphi_p\, \partial^l \Box^{\frac{1}{2}(t-p-q)}\, \varphi_q \qquad (p \leq q)$$

dimension of this space is $\mu(\frac{t-b}{2} - a - 1)$ with $\mu \equiv \lfloor \frac{b+1}{2} + \frac{1+(-1)^{a+\ell}}{4} \rfloor$

Associate to it a matrix of correlators

$$\begin{pmatrix} \begin{bmatrix} \langle 2626\rangle & \langle 3726\rangle & \langle 4826\rangle \\ & \langle 3737\rangle & \langle 4837\rangle \\ & & \langle 4848\rangle \end{bmatrix} & \begin{bmatrix} \langle 3526\rangle & \dots & \dots \\ \dots & \langle 4637\rangle & \dots \\ \dots & \dots & \langle 5748\rangle \end{bmatrix} & \begin{bmatrix} \langle 4426\rangle & \dots & \dots \\ \dots & \langle 5537\rangle & \dots \\ \dots & \dots & \langle 6648\rangle \end{bmatrix} \\ & \begin{bmatrix} \langle 3535\rangle & \langle 4635\rangle & \langle 5735\rangle \\ & \langle 4646\rangle & \langle 5746\rangle \\ & & \langle 5757\rangle \end{bmatrix} & \begin{bmatrix} \langle 4435\rangle & \dots & \dots \\ \dots & \langle 5546\rangle & \dots \\ \dots & \dots & \langle 6657\rangle \end{bmatrix} \\ & & \begin{bmatrix} \langle 4444\rangle & \langle 5544\rangle & \langle 6644\rangle \\ & \langle 5555\rangle & \langle 5566\rangle \\ & & \langle 6666\rangle \end{bmatrix} \end{pmatrix}$$

[0,4,0] "even spin" t=12

part III

# The spectrum of N=4 SYM at large λ

such a *symm* matrix carries two pieces of information

1) disconnected free theory $\mathcal{A}^{t,\ell}_{a,b} = \mathbb{C}_{t,\ell,a,b} \cdot \mathbb{C}^T_{t,\ell,a,b}$
2) log(u) data $\mathcal{M}^{t,\ell}_{a,b} = \mathbb{C}_{t,\ell,a,b} \cdot \widehat{\eta} \cdot \mathbb{C}^T_{t,\ell,a,b}$

Observation: the number of $\langle \varphi_p \varphi_q K^t_{pq} \rangle$ + the anomalous dim. equals the number of independent entries of $\mathcal{A}^{t,\ell}_{a,b}$ and $\mathcal{M}^{t,\ell}_{a,b}$

General solution of the eigenvalue problem

$$\delta^{(4)}_t = (t-1)(t+a)(t+a+b+1)(t+2a+b+2)$$

$$\eta_{pq} = -\frac{2}{N^2} \frac{\delta^{(4)}_t \, \delta^{(4)}_{t+\ell+1}}{(\ell+2p-2-a+\frac{1+(-1)^{a+\ell}}{2})_6}$$

FA,JD,PH,HP 1802.06889

# *The spectrum of N=4 SYM at large λ*

*part* III

*Example*: [0,2,0] "even spin" t=8

rixForm=

$$\begin{pmatrix}
\frac{504\left(50+11l+l^2\right)}{(1+l)(5+l)(6+l)(10+l)} & -\frac{3024\sqrt{\frac{1}{(2+l)(9+l)}}\left(50+11l+l^2\right)}{(1+l)(5+l)(6+l)(10+l)} & \frac{1008\sqrt{15}\sqrt{\frac{1}{(5+l)(6+l)}}}{(1+l)(10+l)} & -\frac{16128\sqrt{2}\sqrt{\frac{1}{(2+l)(5+l)(6+l)(9+l)}}}{(1+l)(10+l)} \\
-\frac{3024\sqrt{\frac{1}{(2+l)(9+l)}}\left(50+11l+l^2\right)}{(1+l)(5+l)(6+l)(10+l)} & \frac{504\left(2160+902l+203l^2+22l^3+l^4\right)}{(1+l)(2+l)(5+l)(6+l)(9+l)(10+l)} & -\frac{6048\sqrt{15}\sqrt{\frac{1}{(2+l)(5+l)(6+l)(9+l)}}}{(1+l)(10+l)} & \frac{1512\sqrt{2}\sqrt{\frac{1}{(5+l)(6+l)}}\left(74+11l+l^2\right)}{(1+l)(2+l)(9+l)(10+l)} \\
\frac{1008\sqrt{15}\sqrt{\frac{1}{(5+l)(6+l)}}}{(1+l)(10+l)} & -\frac{6048\sqrt{15}\sqrt{\frac{1}{(2+l)(5+l)(6+l)(9+l)}}}{(1+l)(10+l)} & \frac{504\left(40+11l+l^2\right)}{(1+l)(5+l)(6+l)(10+l)} & -\frac{504\sqrt{30}\sqrt{\frac{1}{(2+l)(9+l)}}\left(42+11l+l^2\right)}{(1+l)(5+l)(6+l)(10+l)} \\
-\frac{16128\sqrt{2}\sqrt{\frac{1}{(2+l)(5+l)(6+l)(9+l)}}}{(1+l)(10+l)} & \frac{1512\sqrt{2}\sqrt{\frac{1}{(5+l)(6+l)}}\left(74+11l+l^2\right)}{(1+l)(2+l)(9+l)(10+l)} & -\frac{504\sqrt{30}\sqrt{\frac{1}{(2+l)(9+l)}}\left(42+11l+l^2\right)}{(1+l)(5+l)(6+l)(10+l)} & \frac{504\left(1644+792l+193l^2+22l^3+l^4\right)}{(1+l)(2+l)(5+l)(6+l)(9+l)(10+l)}
\end{pmatrix}$$

$$\left\{-\frac{504\,(7+l)\,(8+l)}{(1+l)\,(2+l)\,(5+l)\,(6+l)},\ -\frac{504}{(5+l)\,(6+l)},\ -\frac{504}{(5+l)\,(6+l)},\ -\frac{504\,(3+l)\,(4+l)}{(5+l)\,(6+l)\,(9+l)\,(10+l)}\right\}$$

- ***Results***

*Complete Unmixing in* [000] *and* [010]

FA,JD,PH,HP 1706.08456
FA,JD,PH,HP 1711.03903

*Surprising pattern of degeneracies!*

FA,JD,PH,HP 1802.06889

*Accidental* 10d *symmetry*

Caron-Huot,Trinh 1809.09173

## *Bootstrap of* $\langle \varphi_2\varphi_2\varphi_2\varphi_2 \rangle$ *at one-loop*

FA,JD,PH,HP 1706.02822

- *Prediction from the* OPE

$$\mathcal{H}_{2222} \supset \frac{1}{(N^2-1)^2}\log^2(u) \sum_{K_{t,\ell}} C^0_{22K}\, \tfrac{1}{2}\eta^2_K\, C^0_{22K}\, \mathcal{B}^{[t,\ell]}$$

- *double disc. resums into* $\quad u = x\overline{x}, \quad v = (1-x)(1-\overline{x})$

$$\frac{1}{(x-\overline{x})^{14}}\Big[\mathcal{P}_1 + \mathcal{P}_2\frac{\mathrm{Li}[1,x]-\mathrm{Li}[1,\overline{x}]}{x-\overline{x}} + \mathcal{P}_3\frac{\mathrm{Li}[2,x]-\mathrm{Li}[2,\overline{x}]}{x-\overline{x}} + \log(\mathrm{v})\Big(\tilde{\mathcal{P}}_1 + \tilde{\mathcal{P}}_2\frac{\mathrm{Li}[1,x]-\mathrm{Li}[1,\overline{x}]}{x-\overline{x}}\Big)\Big]$$

- *consider the space of transcendental functions up to weight-4*

*make an ansatz* $\quad \dfrac{\big[\mathcal{P}\text{olynomials} \times \text{svHPL}\big]}{(x-\overline{x})^{15}}$

HPL by Remiddi,Vermaseren, & Maitre, SVHPL by Brown
Dixon,Duhr,Pennington,1207.1086

## ... More on the ansatz

$$\mathcal{L}^{(g)} = \frac{1}{x-\overline{x}} \sum_{r=0}^{g} (-)^r \frac{(2g-r)!}{r!(g-r)!g!} \log^r(u/v) \left[ \mathrm{Li}_{2g-r}\left(\tfrac{x}{x-1}\right) - \mathrm{Li}_{2g-r}\left(\tfrac{\overline{x}}{\overline{x}-1}\right) \right]$$

$$\begin{aligned} \mathcal{H}_{ADHP} = & \left[ A_1(u,v)\mathcal{L}^{(2)} + \mathrm{cross} \right] + \\ & \left[ A_4(u,v) x(1-x)\partial_x \mathcal{L}^{(2)} \pm (x \leftrightarrow \overline{x}) + \mathrm{cross} \right] \\ & \left[ A_9(u,v) \log^2(u) + \mathrm{cross} \right] \\ & \left[ A_{12}(u,v)\mathcal{L}^{(1)} + A_{13}(u,v)\log(u) + A_{14}(u,v)\log(v) + A_{15}(u,v) \right] \end{aligned}$$

*Task:* 
1) *impose crossing*
2) *match the double disc.*
3) *cancel spurios poles*
4) *odd other predictions!*

*Bake 30 minuts*

FA,JD,PH,HP 1706.02822
FA,JD,PH,HP 1711.03903

*also* $\langle \varphi_2 \varphi_2 \varphi_3 \varphi_3 \rangle$

The function is fixed up to a free coefficient,
same ambiguity that string theory would induce from R^4

Goncalves 1411.1675

New Data:

$$\eta^{1-\text{loop}}_{K_{4,l,0,0}} = \begin{cases} \alpha + \frac{1148}{3} & l = 0 \\ \frac{1344(l-7)(l+14)}{(l-1)(l+1)^2(l+6)^2(l+8)} - \frac{2304(2l+7)}{(l+1)^3(l+6)^3} & l = 2, 4, \ldots \end{cases}$$

$$\eta^{1-\text{loop}}_{K_{4,l,0,1}} = \begin{cases} \frac{320(9l^4+68l^3-1151l^2-5738l-3688)}{(l-1)(l+1)^3(l+4)^3(l+8)} & l = 2, 4, \ldots \\ \frac{320(9l^4+140l^3-487l^2-11262l-29400)}{l(l+4)^3(l+7)^3(l+9)} & l = 3, 5, \ldots \end{cases}$$

Summary

Obtained 1-Loop sugra correction from OPE consistency
and Analytic Bootstrap
after solving the mixing problem and resumming the ddisc

HPL by Remiddi,Vermaseren, & Maitre, SVHPL by Brown

Dixon,Duhr,Pennington,1207.1086

## ... More on the ansatz

$$\mathcal{L}^{(g)} = \frac{1}{x-\bar{x}} \sum_{r=0}^{g} (-)^r \frac{(2g-r)!}{r!(g-r)!g!} \log^r(u/v) \Big[ \mathrm{Li}_{2g-r}(\tfrac{x}{x-1}) - \mathrm{Li}_{2g-r}(\tfrac{\bar{x}}{\bar{x}-1}) \Big]$$

$$\begin{aligned} \mathcal{H}_{ADHP} = & \Big[A_1(u,v)\mathcal{L}^{(2)} + \text{cross}\Big] + \\ & \Big[A_4(u,v)x(1-x)\partial_x\mathcal{L}^{(2)} \pm (x \leftrightarrow \bar{x}) + \text{cross}\Big] \\ & \Big[A_9(u,v)\log^2(u) + \text{cross}\Big] \\ & \Big[A_{12}(u,v)\mathcal{L}^{(1)} + A_{13}(u,v)\log(u) + A_{14}(u,v)\log(v) + A_{15}(u,v)\Big] \end{aligned}$$

Task:
1) impose crossing
2) match the double disc.
3) cancel spurios poles
4) odd other predictions!

Bake 30 minuts

FA,JD,PH,HP 1706.02822

FA,JD,PH,HP 1711.03903

also $\langle \varphi_2\varphi_2\varphi_3\varphi_3 \rangle$