Title: Quantum Fields and Strings Seminar

Speakers: Netta Engelhardt

Series: Quantum Fields and Strings

Date: February 19, 2019 - 2:30 PM

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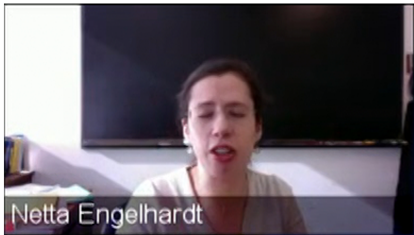
Abstract: Seminar given remotely.

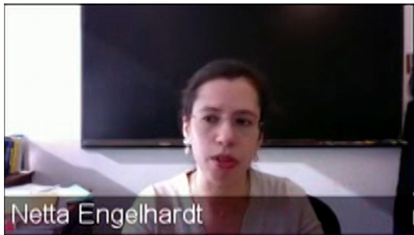


Bulk Constraints from Subregion Causality

Netta Engelhardt

Princeton

Based on work in progress w/ S. Fischetti



Motivation

General philosophy: we use fundamental principles in QFT to derive facts about QG with AdS boundary conditions (and hope that the boundary conditions don't really matter...)

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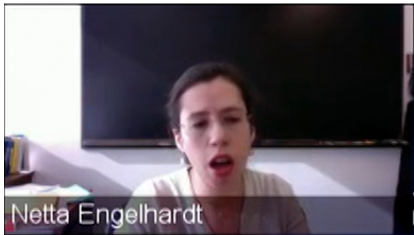

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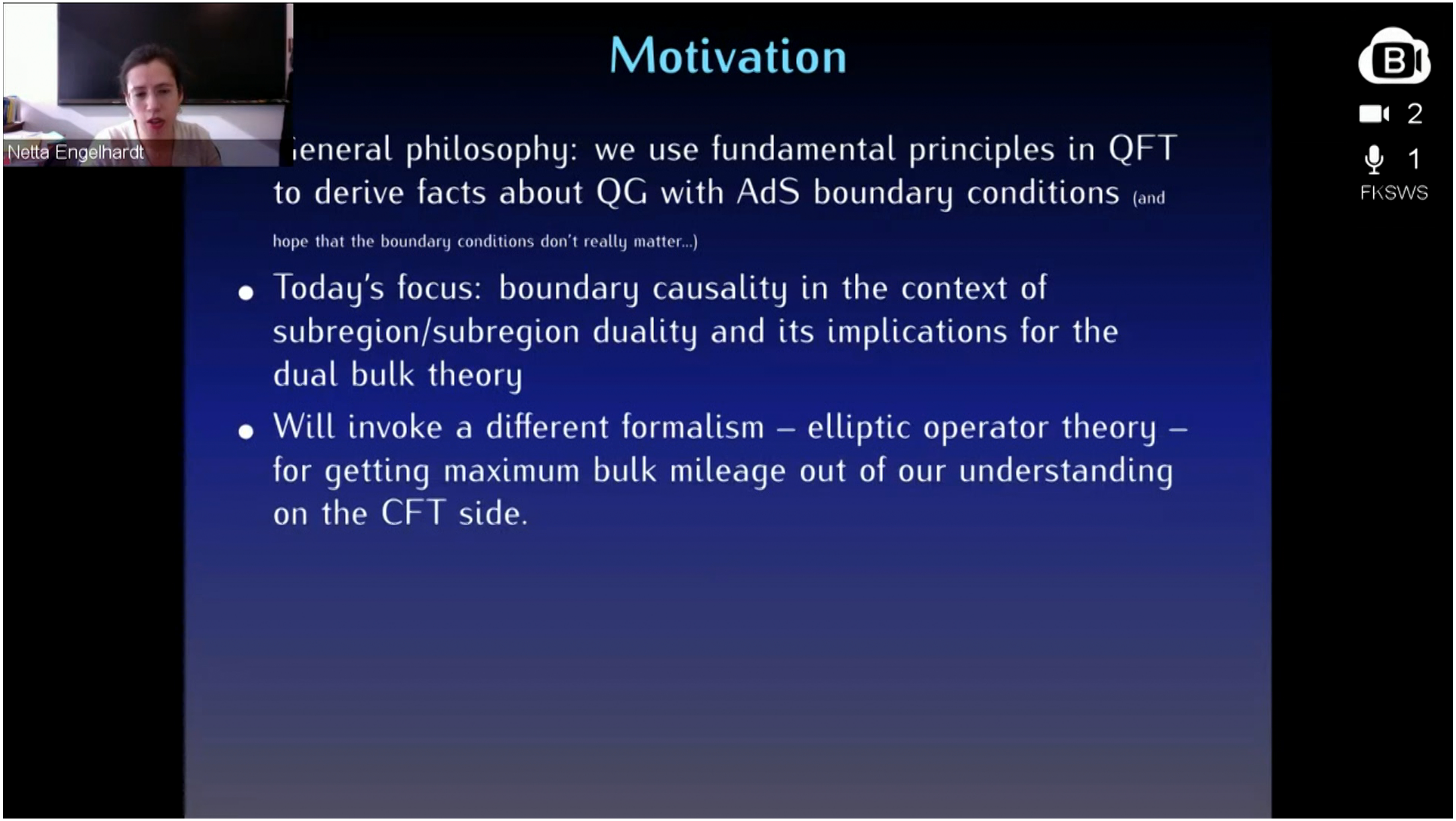
- Today's focus: boundary causality in the context of subregion/subregion duality and its implications for the dual bulk theory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Will invoke a different formalism – elliptic operator theory – for getting maximum bulk mileage out of our understanding on the CFT side.
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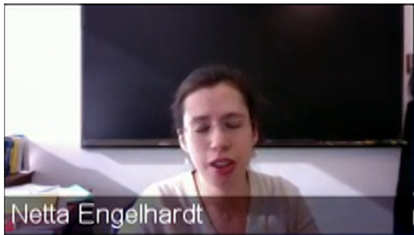

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- Today's focus: boundary causality in the context of subregion/subregion duality and its implications for the dual bulk theory
- Will invoke a different formalism – elliptic operator theory – for getting maximum bulk mileage out of our understanding on the CFT side.
- This formalism gives a powerful, tractable way of translating statements about subregion/subregion dualiity to the bulk; it also allows us to close some embarrassing gaps in our knowledge about extremal surfaces (e.g. whether they exist or not...)

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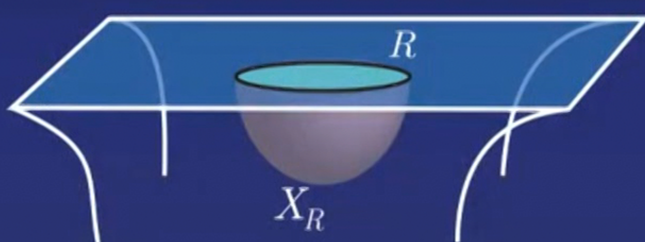
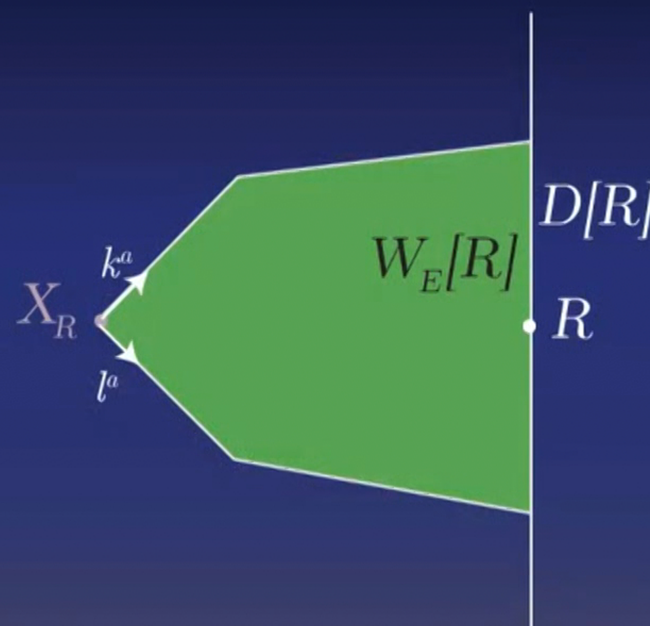
Reminder: Subregion/Subregion Duality

$$S_{vN}[\rho_R] = \frac{\text{Area}[X_R]}{4G\hbar} + \cdots$$ [Hubeny-Rangamani-Takayanagi, Ryu-Takayanagi]





The information in $D[R]$ (ρ_R and $\mathcal{A}_R$) is dual to $W_E[R]$ proposed by [Van Raamsdonk; Czech et al.; Wall; Hubeny et al....; proved by Dong, Harlow, Wall; explicit construction by Faulkner, Lewkowycz] for quantum fields on a curved background

ubregion/Subregion Duality & Causality

1. Entanglement Wedge Nesting: if $D[R] \subset D[R']$, then $W_E[R] \subset W_E[R']$

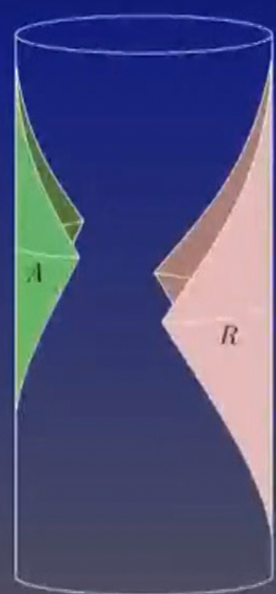
2. Causal wedge inclusion: $W_C[R] \subset W_E[R]$

The bulk implications of both will be presented in the upcoming paper.

Entanglement Wedge Nesting

FT Causality Implies EWN

Operators localized to $D[A]$ commute with operators localized to a subset of $D[A^c]$. So if $D[R] \subset D[A^c]$, then (1) $W_E[R] \subset W_E[A^c]$. and (2) $D[A] \subset D[R^c]$. Since $X_R = X_{R^c}$, this means that $W_E[A] \subset W_E[R^c]$ whenever $D[A] \subset D[R^c]$.



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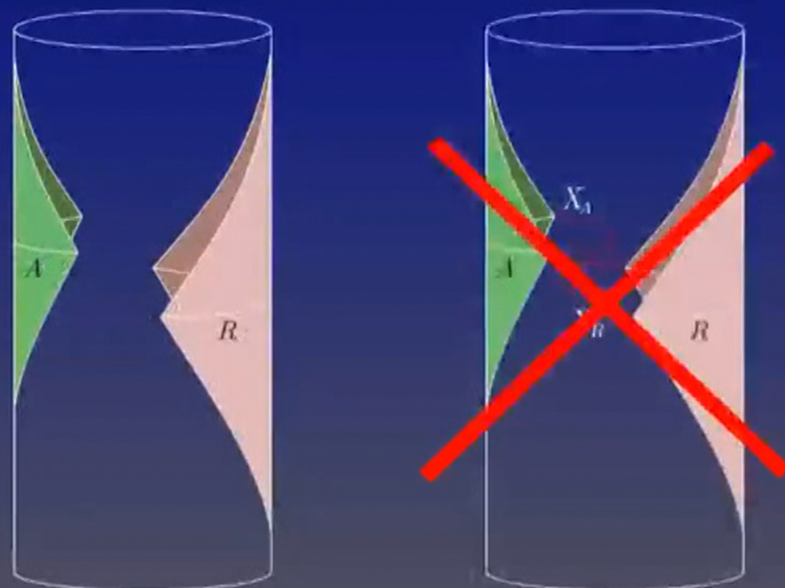
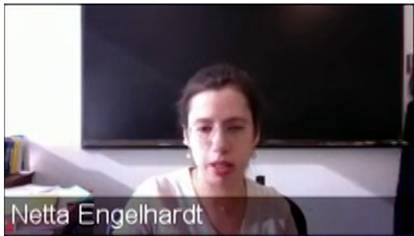
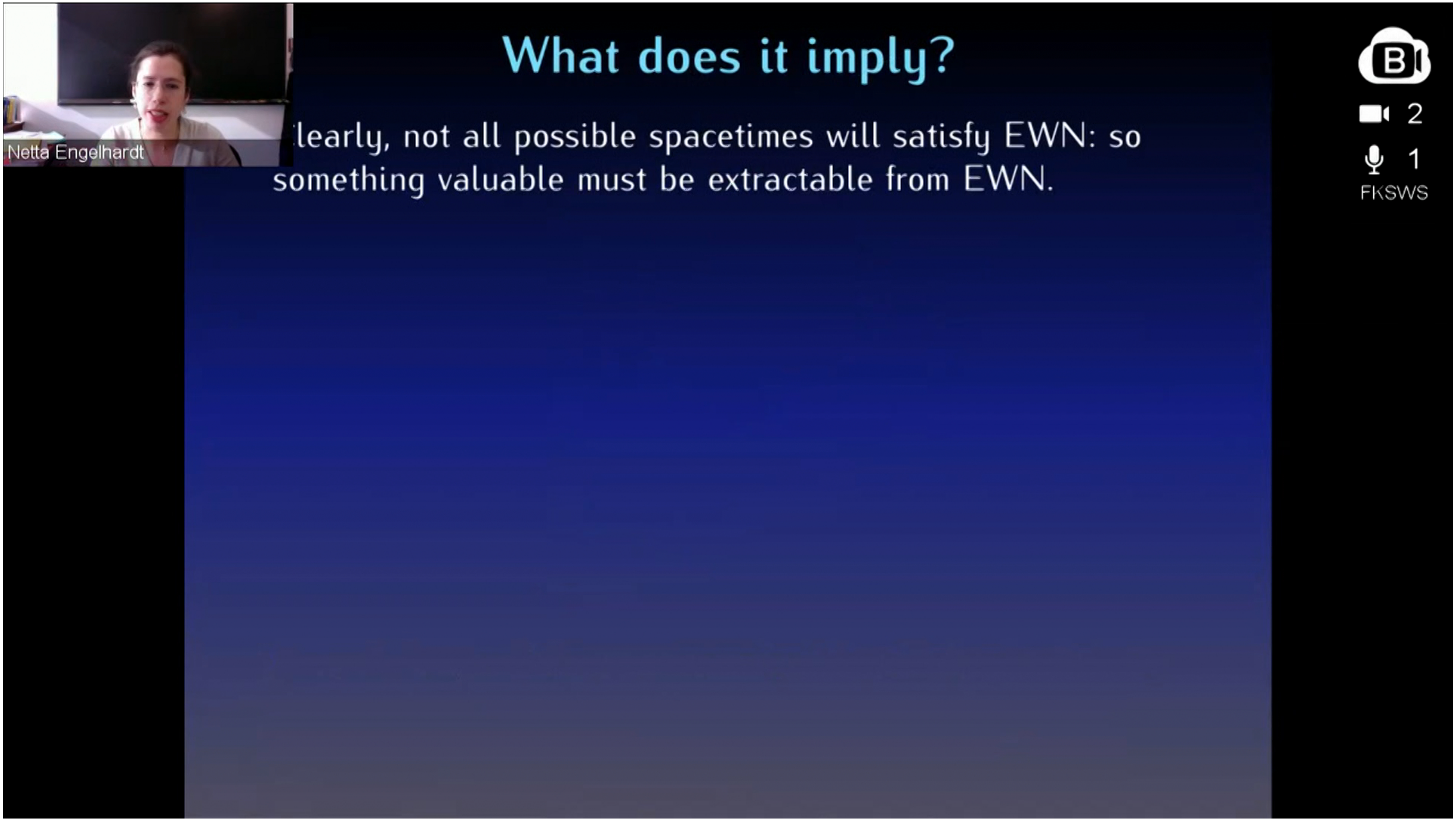




Proof for HRT surfaces Wall '12 assuming the null energy condition (NEC) and certain stability requirements. Later proved for non-minimal extremal surfaces assuming NEC and continuity conditions [NE, Wall '13].


What does it imply?
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Netta Engelhardt
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- Clearly, not all possible spacetimes will satisfy EWN: so something valuable must be extractable from EWN.

- To derive geometric implications of EWN, need to control the way that extremal surfaces change under deformations of the boundary conditions.

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What does it imply?

- learly, not all possible spacetimes will satisfy EWN: so something valuable must be extractable from EWN.

- To derive geometric implications of EWN, need to control the way that extremal surfaces change under deformations of the boundary conditions.

- This is hard: fairly straightforward (if tedious) to use geometric arguments to constrain the behavior of extremal surfaces under assumptions about the spacetime, but very, very difficult to do the converse: use geometric arguments to constrain the spacetime.

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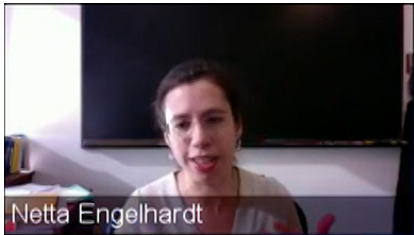

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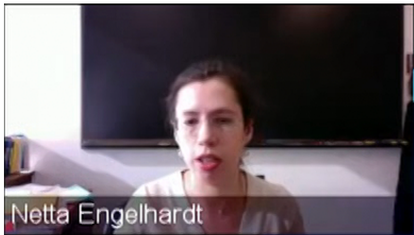

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$$T^c \nabla_c (T^b \nabla_b \eta^a) = -R^a_{\ cbd} T^c T^d \eta^b$$

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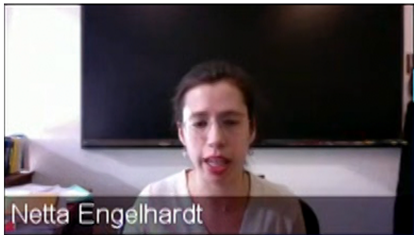

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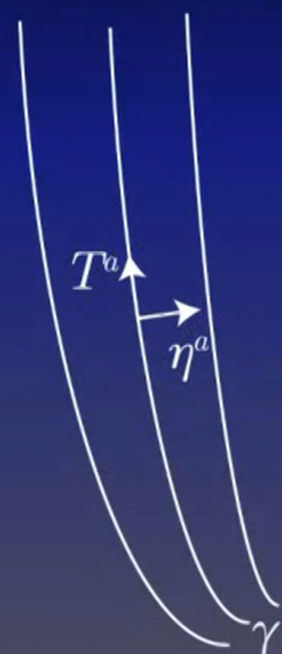
$$\Delta_\gamma\eta^a + R^a_{\ cbd}h^{cd}\eta^b = 0$$



Equation of Extremal Deviation

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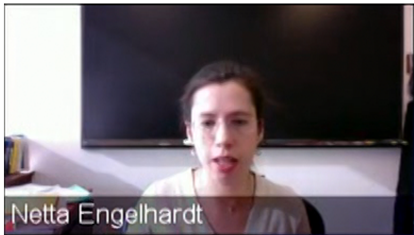
Right idea, but because higher-dim extremal surfaces have nonzero extrinsic curvature, you have to modify the equation a bit:



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Right idea, but because higher-dim extremal surfaces have nonzero extrinsic curvature, you have to modify the equation a bit:

$$P^a_b \left(\Delta_X \eta^b + 2 S^b_c \eta^c + R_{cbde} h^{cd} \eta^e \right) = 0$$

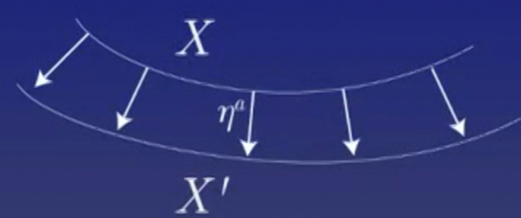
where P^a_b projects onto the directions normal to X (this is trivial for a geodesic) and S^a_b contains information about the extrinsic curvature of X.

Equation of Extremal Deviation

his can be rewritten as an operator equation:

$$(\mathbf{L}[\eta])^a = 0$$

- $\mathbf{L}$ is a *vector* operator, acting on vectors normal to X (unless X is codim 1, in which case it's a scalar operator)
- $\mathbf{L}[\eta] = 0$ is a vector equation for an extremal surface X' where η^a is normal to X and yields the deformation that gives X'.



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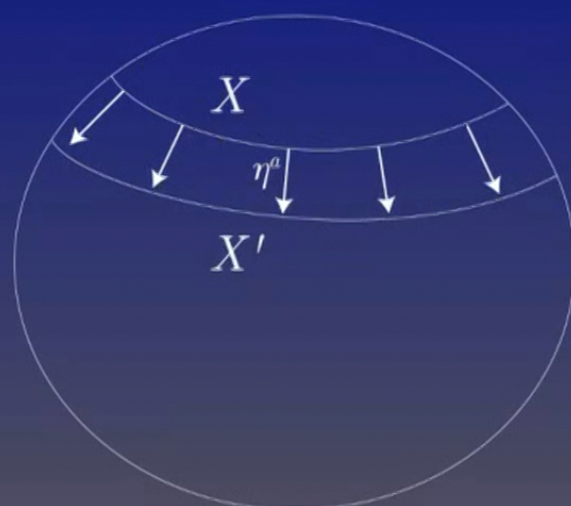

An elliptic operator problem

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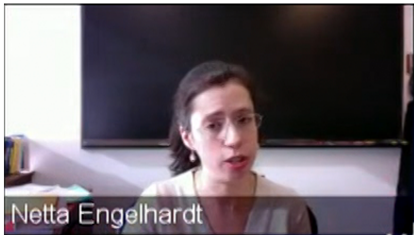
- A lot is known about elliptic operators and their various properties in mathematics, and we can use all of those results to study surfaces with vanishing mean curvature – of any codimension.



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- An aside: extremal deviation actually independently discovered in context of string worldsheets and branes, which are described by hyperbolic operators. Results here that are indep of elliptic operator theory are applicable to those cases as well.

Vector Operator Statement of EWN

$$\mathbf{L}[\eta] = 0$$

is the statement that there exists an extremal surface near X in the direction η^a.

EWN $\Leftrightarrow$ if $\eta^a \eta_a > 0$ asymptotically, then $\eta^a \eta_a > 0$ everywhere.

Another way of saying this is that if we expand $\eta^a = \alpha k^a + \beta \ell^a$, where k^a is future-directed, ℓ^a is past-directed, then $\alpha\beta > 0$ everywhere if $\alpha\beta > 0$ at ∂X_A.

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Minimum Principles

Recall: maximum principle from 1-variable calculus:

$$-u'' + u' + 5u \geq 0$$

u cannot attain a nonpositive local minimum. **If $u \geq 0$ at the boundary of the domain, then $u \geq 0$ everywhere.**

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This carries over to scalar elliptic operators, and also to certain types of vector elliptic operators ("cooperative" elliptic operators).

Max/Min Principle For Elliptic Systems

For codim≤ 2, $L[\eta] = 0$ is either a single (scalar) equation, or when the NEC is satisfied, it is a "cooperative" elliptic system.

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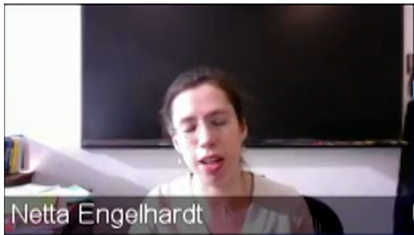

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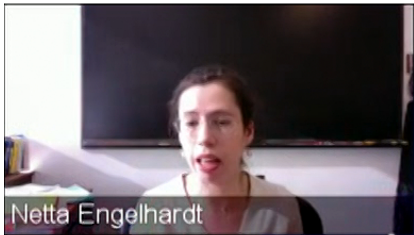
NEC implies (continuous) EWN

max principle $\Rightarrow$ if $\alpha > 0$ and $\beta > 0$ at ∂X_A, and the NEC is satisfied, then $\alpha > 0$, $\beta > 0$ everywhere on X_A. This is EWN.

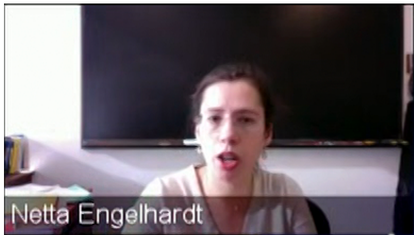
We already knew EWN followed from NEC, but this derivation takes exactly one line!

What about the other way? W/o NEC, system is not cooperative for codim 2.



Scalar Decomposition for EWN

- **L** has two components for the two normal directions to X.



Scalar Decomposition for EWN

- $\mathbf{L}$ has two components for the two normal directions to X.
- We can expand $\mathbf{L}$ in a basis $\{k^a, v^a\}$ where $k^a k_a = 0$ and v^a is any other (independent) vector normal to X.

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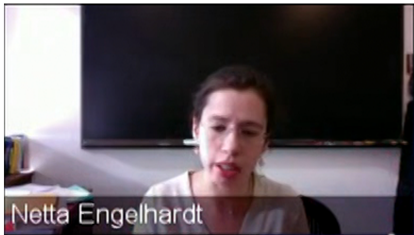
- After rearranging, we get two scalar equations for extremal deviation, one of which is:

$$\mathcal{L}_v[\psi] = \sigma_k^2 + R_{ab}k^a k^b \qquad (1)$$

where ψ is the component of η^a in the v^a direction.



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$$\mathcal{L}_v[\psi] = \sigma_k^2 + R_{ab}k^a k^b \tag{1}$$

where ψ is the component of η^a in the v^a direction.

- EWN is the statement that for any solution to (1) with $\psi \geq 0$ asymptotically, $\psi > 0$ everywhere.

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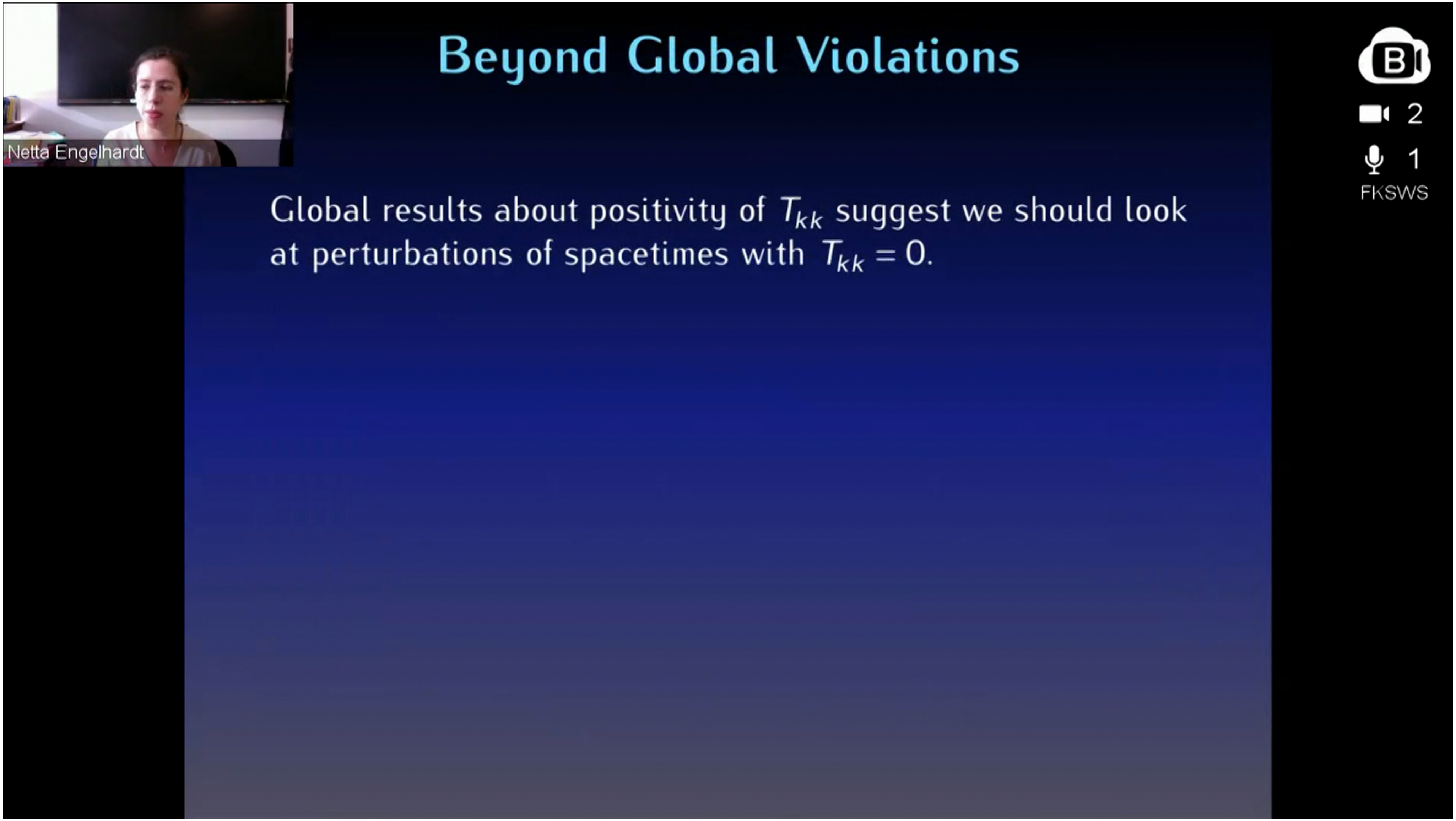
From EWN to NEC?

By writing the two-dimensional eqn $L[\eta]=0$ as two scalar equations, we can make use of Max Principle even without NEC. This allows us to derive a new result almost immediately:

New Result: QEI?

EWN implies that $T_{kk}+\sigma_k^2$ cannot be nonpositive *everywhere* on an HRT surface unless it is strictly zero everywhere. For example, if an arbitrarily small (classical) perturbation of pure AdS (which has $T_{kk}=0$) has $T_{kk}<0$ somewhere, it must have $T_{kk}>0$ elsewhere (for regions w/o shear).

Looks a lot like a quantum energy inequality.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Beyond Global Violations
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Beyond Global Violations

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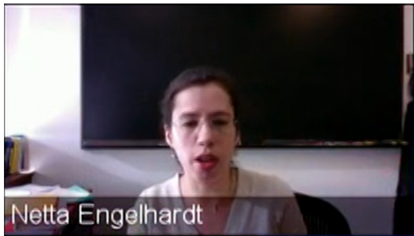
Global results about positivity of T_{kk} suggest we should look at perturbations of spacetimes with $T_{kk} = 0$. Like pure AdS.

A careful analysis of the equation of extremal deviation around AdS results in a derivation of a positive energy theorem, first established by [Czech et al.] in a completely different way:

$$\int_X \delta T_{kk} > 0$$

where δT_{kk} is the perturbation in the stress-energy tensor due to the spacetime perturbation.



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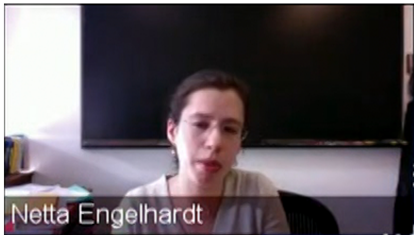
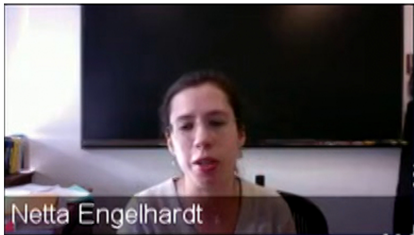


An Excellent Question

Last summer, I gave a talk at a math conference, and the first question I was asked was:

How do you know extremal surfaces exist in general?



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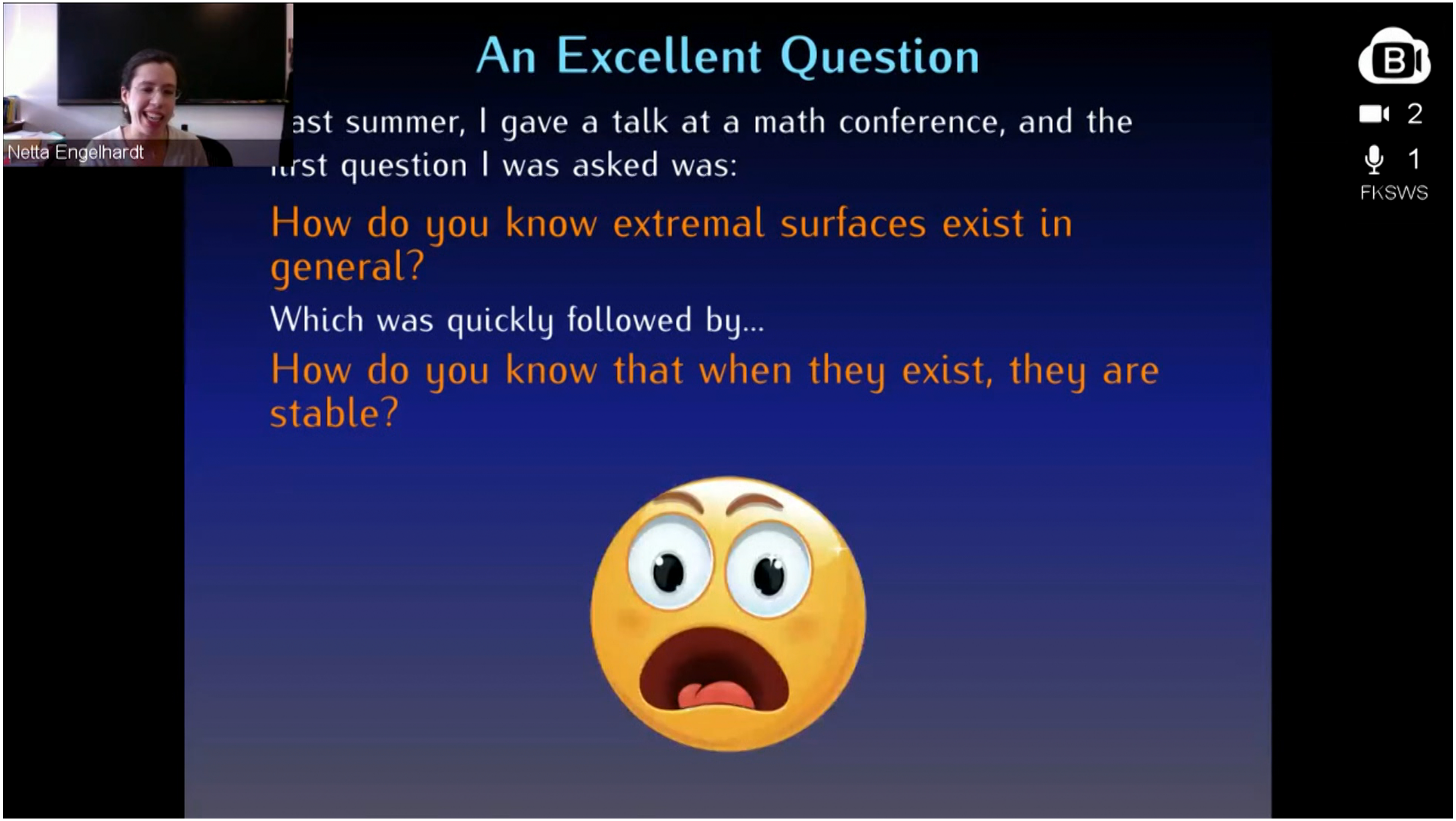
How do you know that when they exist, they are stable?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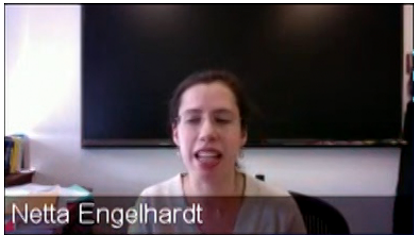

Sourced Equation of Extremal Deviation

If an extremal surface X_R exists in a given spacetime (M, g), how can we be sure it doesn't disappear under perturbations $(M, g + \delta g)$?

Sourced Extremal Deviation

$$\mathbf{L}[\eta] = f(\delta g) \tag{2}$$

where $f(\delta g)$ involves $K\delta g$ and $\delta\Gamma$. *Stability* of an extremal surface under small perturbations of $g \Leftrightarrow$ (2) has a solution for any δg.

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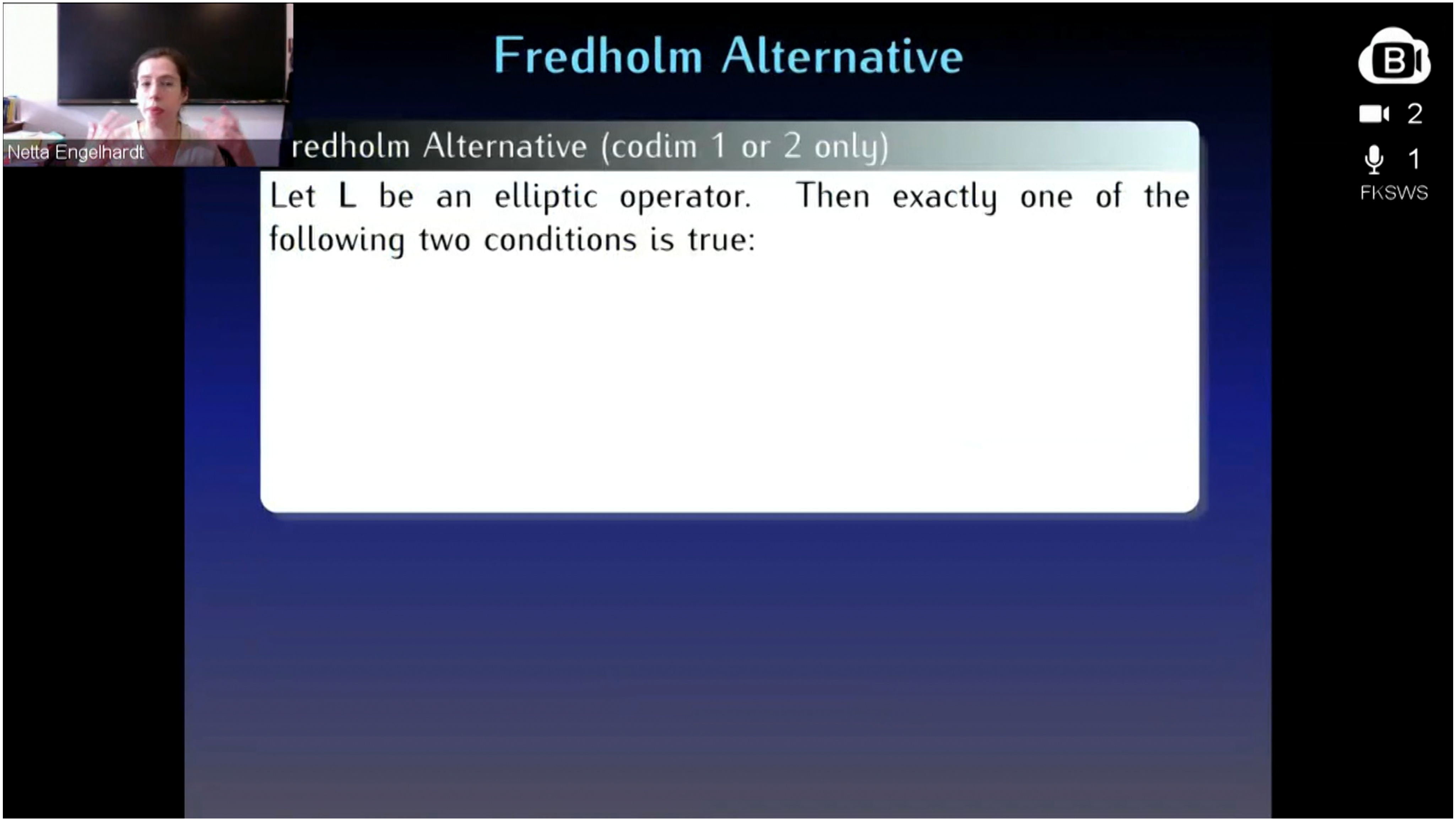
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Fredholm Alternative
redholm Alternative (codim 1 or 2 only)
Let L be an elliptic operator. Then exactly one of the following two conditions is true:
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1. The equation $\mathbf{L}[\eta] = \mathbf{0}$ with $\eta_{\partial M} = 0$ has nontrivial solutions, or

2.The equation $\mathbf{L}[\eta] = \mathbf{f}$ has a unique nontrivial solution for any f, $\eta_{\partial M} \neq 0$.

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An extremal surface X_R is stable under perturbations iff X_R is locally unique.

Alternatively, X_R is stable iff the spectrum of L contains no zero eigenvalue.

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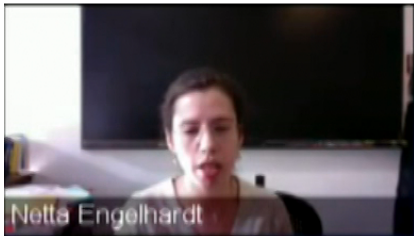
A Different notion of Stability

Time for a small complaint.

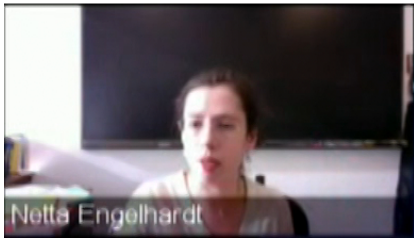


A Different notion of Stability

Time for a small complaint.

Extremal surfaces in the HRT prescription aren't actually extrema of the area functional. They are stationary points, but not extrema! The distinction is important.

This is because there is a different, intuitive notion of stability for surfaces that are actually extremal, like maximal volume slices. This notion doesn't make sense for HRT surfaces.



tability for (Actually) Extremal Surfaces

Example: consider a great circle on a sphere. It's a minimal surface between the north and south poles. But if you extend it beyond the south pole, there are small variations of it that cause the length to decrease.
(This is simply due to the phenomenon of conjugate points at the poles).
We can extend this to higher-dim'l surfaces which are actual extrema by studying the second variations of the area functional. The details are fairly technical, but it turns out that there is a unifying principle.

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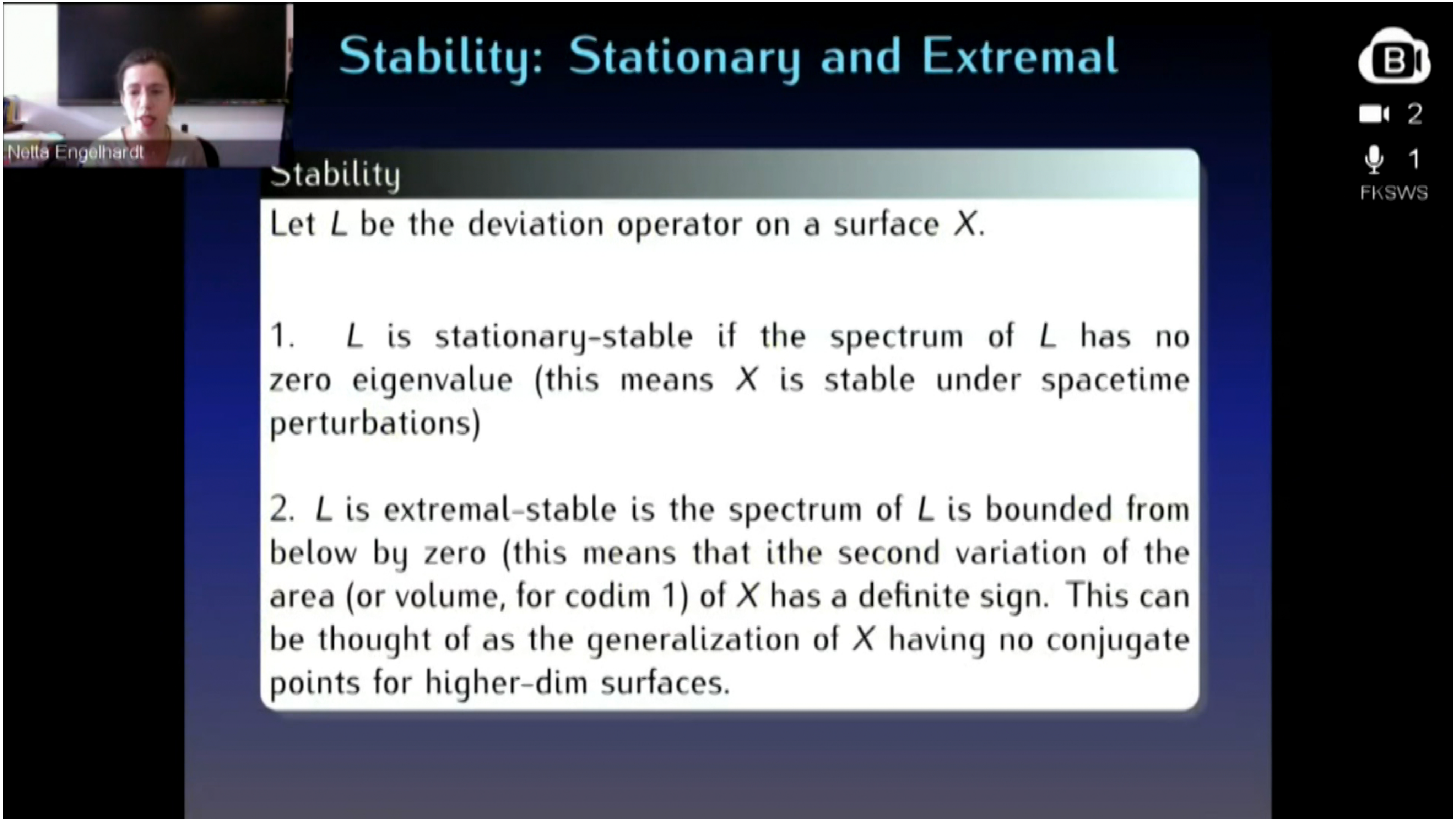
Netta Engelhardt
Stability: Stationary and Extremal
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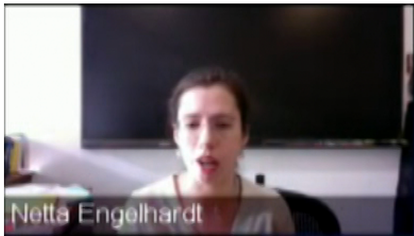
Stability
Let L be the deviation operator on a surface X.
1. L is stationary-stable if the spectrum of L has no zero eigenvalue (this means X is stable under spacetime perturbations)
2. L is extremal-stable is the spectrum of L is bounded from below by zero (this means that ithe second variation of the area (or volume, for codim 1) of X has a definite sign. This can be thought of as the generalization of X having no conjugate points for higher-dim surfaces.



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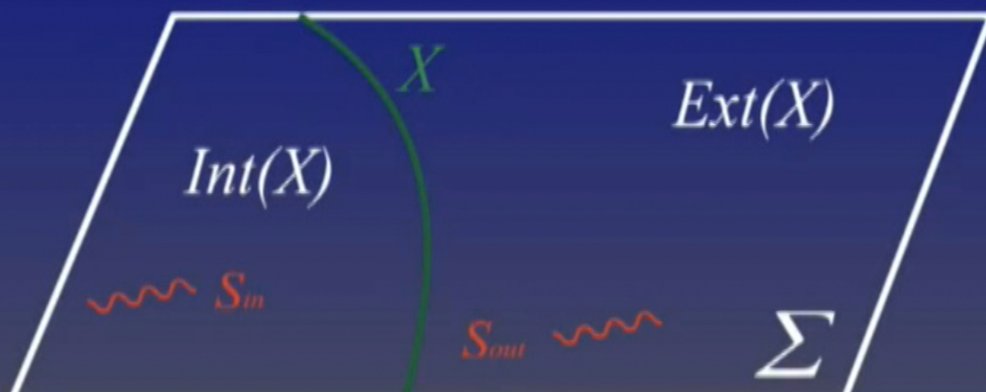
Quantum Extremal Surfaces

o leading order in N and λ, $S_{vN}[R]$ is given by the HRT surface X_R, which is a stionary pt of the area, and $S_{vN}[R]$ is the area of the HRT surface.

- To first subleading order in N: [Faulkner, Lewkowycz, Maldacena]

$$S_{vN}[R] = \frac{\text{Area}[X_R]}{4G_N\hbar} + S_{\text{out}}[X_R]$$



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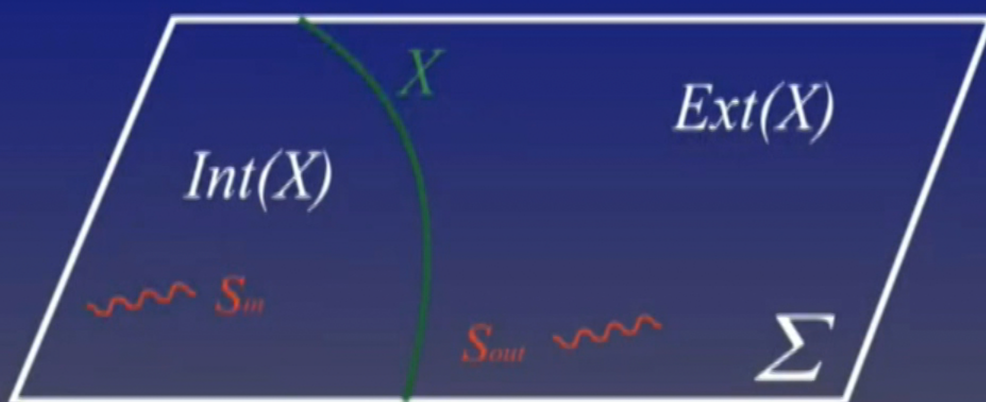
Quantum Extremal Surfaces

…o higher subleading orders in N, the surface itself changes: …he surface computing $S_{vN}[R]$ is a stationary pt of $A + S$ [NE, Wall '14] proved by [Dong, Lewkowycz '17]:

$$S_{vN}[R] = \frac{\text{Area}[\chi_R]}{4G_N\hbar} + S_{\text{out}}[\chi_R]$$

where χ_R is a stationary pt of $A + S$.



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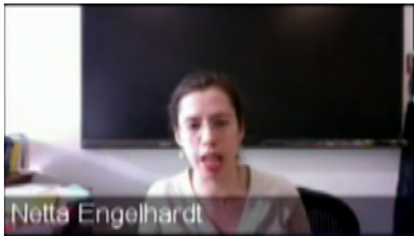

:quation of Quantum Extremal Deviation

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$$L[\eta]_a + 4G\hbar \int_{X_R} \frac{\mathcal{D}^2 S_{\mathrm{out}}}{\mathcal{D}X_R^b(p')\mathcal{D}_R^a(p)} (\eta')^b = 0.$$

where $\mathcal{D}/\mathcal{D}X_R^a$ is a functional derivative wrt to variations of X_R.

In the classical regime, we got an integrated null energy from EWN. What will we get in the semiclassical regime? (hint: quantum focusing)

otivation: Quantum Focusing Conjecture

/hat is quantum focusing and why should we care?

- Pillars of GR – singularity thms, good causal structure, grav. thermo. – all rely on focusing of light rays.
- Focusing requires the NEC: easily violated in QFT.
- But GR is a good effective description of the universe! Expect that most of the pillars of GR should be robust against quantum corrections.
- Quantum focusing [Bousso, Fisher, Leichenauer, Wall] "corrects" focusing so quantum corrected analogies of those results are still valid.
- It also implies the QNEC in the nongrav. limit.
- It's also unproven.

EWN in the classical regime gives a smeared NEC. Can it give a smeared QFC in the quantum regime?

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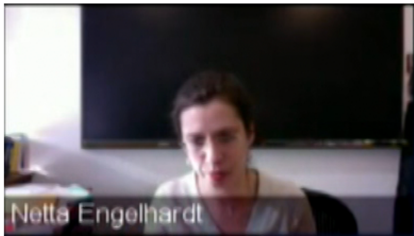
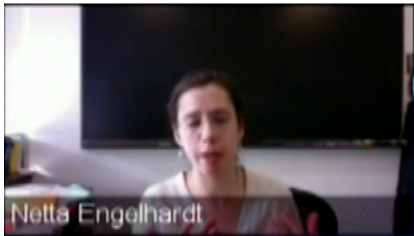


Quantum Focusing Conjecture

Classical	Quantum
$\frac{\text{Area}}{4G\hbar}$	$S_{gen} = \frac{\text{Area}}{4G\hbar} + S_{\text{out}}$
$\theta \propto \frac{d\text{Area}}{d\lambda}$	$\Theta \propto \frac{\mathcal{D}}{\mathcal{D}V(y)} S_{gen}$
Classical focusing: $\frac{d\theta}{d\lambda} < 0$	Quantum focusing: $\frac{\mathcal{D}\Theta}{\mathcal{D}V(y')} < 0$

where $\delta V(y)$ is a null variation of the null congruence fired from the surface localized to the null generator at y.



om Quantum EWN to Quantum Focusing

χ_R extremizes $S_{gen} \Leftrightarrow$ small variations of the location of χ_R do not change S_{gen}.

$\Rightarrow \Theta[\chi_R] = 0$. This is how we make contact with the QFC via EWN.

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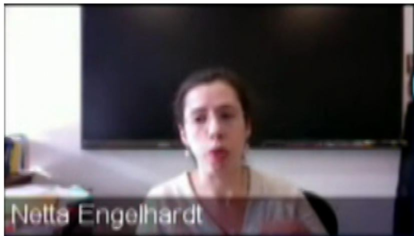
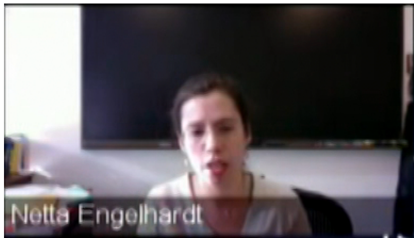


om Quantum EWN to Quantum Focusing

χ_R extremizes $S_{gen} \Leftrightarrow$ small variations of the location of χ_R do not change S_{gen}.

$\Rightarrow \Theta[\chi_R] = 0$. This is how we make contact with the QFC via EWN.

Because whenever classical focusing is satisfied strictly, quantum corrections can't result in violations, to test the QFC we need a situation where classical focusing is satisfied *marginally*: vacuum in pure AdS, where the χ_R all lie on a null horizon.



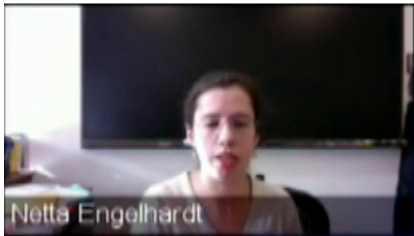
Smeared QFC

WN (ask me later to see details if interested!) implies (schematic notation only):

$$\int_{\chi_R} f \frac{\mathcal{D}\Theta[y]}{\mathcal{D}X_R(y')} d^{D-2}y \, d^{D-2}y' \leq 0$$

for perturbations of vacuum AdS due to quantum backreaction: a proof of the QFC to *second* subleading order. Makes no assumptions about the EOM. (if you assume backreaction of quantum fields is described by a semiclassical Einstein equation, then you can derive this from the proof of the QNEC on Killing horizons [Bousso et al.]

Here f is a known smearing function which is finite at ∂M, so integral is not dominated by asymptotics: reflects genuine physics of the deep bulk.



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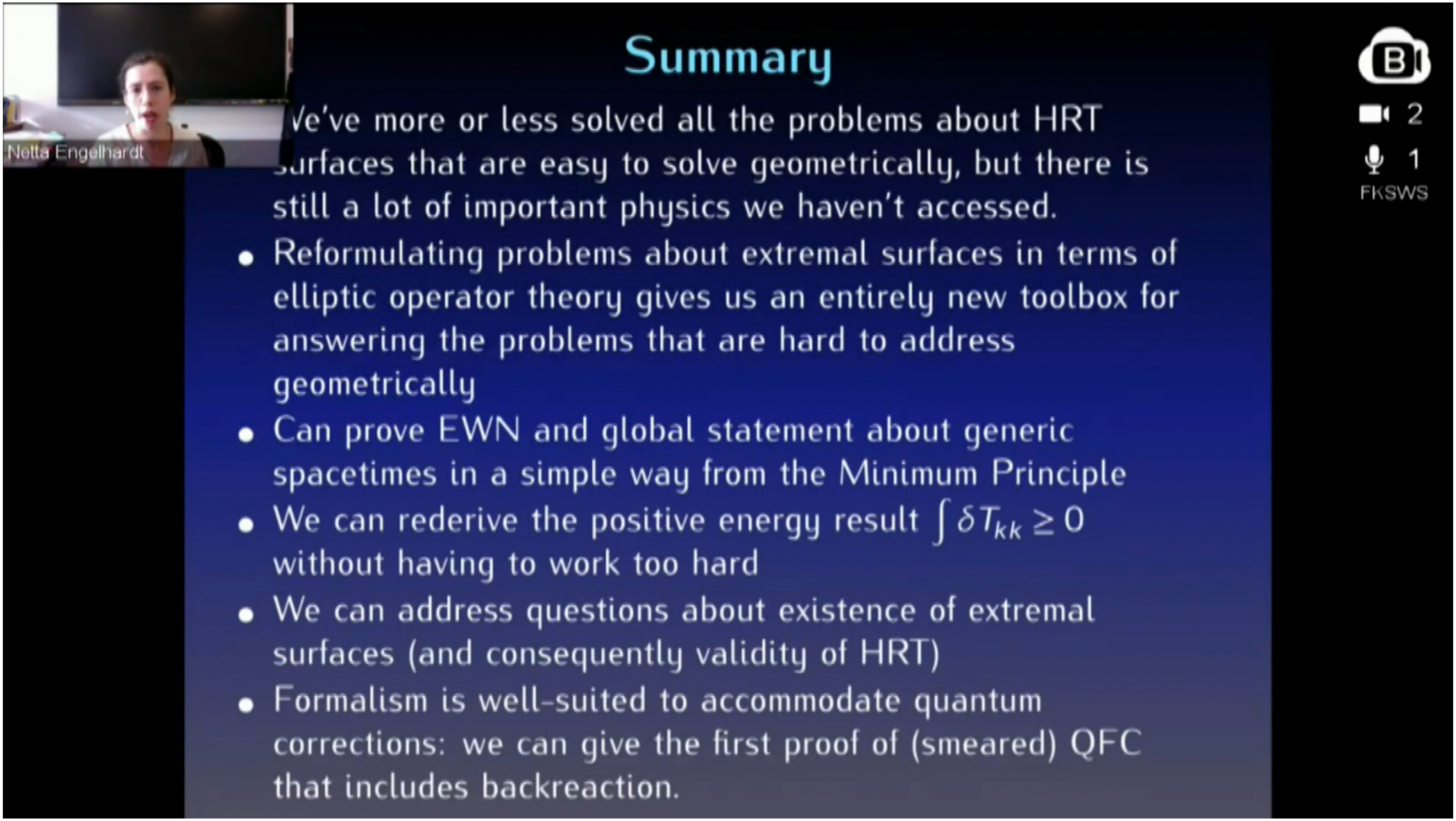
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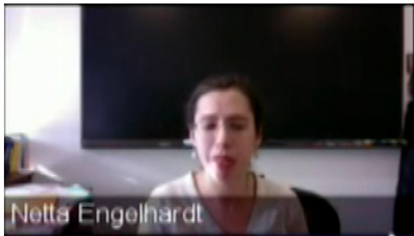
Summary
e've more or less solved all the problems about HRT
urfaces that are easy to solve geometrically, but there is still a lot of important physics we haven't accessed.
• Reformulating problems about extremal surfaces in terms of elliptic operator theory gives us an entirely new toolbox for answering the problems that are hard to address geometrically
• Can prove EWN and global statement about generic spacetimes in a simple way from the Minimum Principle
• We can rederive the positive energy result $\int \delta T_{kk} \geq 0$ without having to work too hard
• We can address questions about existence of extremal surfaces (and consequently validity of HRT)
• Formalism is well-suited to accommodate quantum corrections: we can give the first proof of (smeared) QFC that includes backreaction.



Upcoming and To do

- Math literature is hard to decipher...
- Upcoming paper with S. Fischetti will contain pedagogical introduction to all of this.
- Many theorems/results we haven't used yet, but can probably be used to prove analogues of $\int \delta T_{kk} \geq 0$ for spacetimes that are not perturbations of AdS
- Could potentially be used to generalize the results of [Faulkner et al.] on the Einstein equation to include quantum corrections at higher orders and derive semiclassical EOM.
- Scope is only limited by our creativity in applying known results about EOT (e.g.: EOT was used in the derivation of the CFT dual of the apparent horizon [NE, Wall '17, '18]).

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