Title: Coulomb Phases and Anomalies: Geometric Approach to 5d/6d Theories

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Series: Mathematical Physics

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Abstract: I will review the geometric approach to the description of Coulomb branches and Chern-Simons terms of gauge theories coming from compactifications of M-theory on elliptically fibered Calabi-Yau threefolds. Mathematically, this involves finding all the crepant resolutions of a given Weierstrass model and understanding the network of flops connecting them together with computing certain topological invariants. I will further check that the uplifted theory in 6d is anomaly-free using Green-Schwartz mechanism. I will give examples on the theory with semi-simple gauge groups such as SU(2)xG2, which plays a major role in the classification of 6d superconformal theories, and SU(2)xSU(3), which describes the non-abelian sector of the standard model.

Coulomb Phases and Anomalies :

Geometric Approach to 5d/6d Theories

Motivation

Understand physics <u>geometrically</u>

→ elliptic fibrations

Elliptic Fibration

→ "Weierstrass Model"

heories

s

$\mathcal{L}$: a line bundle over a quasi-var. B

Def the proj. bundle

$$\pi: X_0 = \mathbb{P}[\mathcal{O}_B \oplus \mathcal{L}^{\otimes 2} \oplus \mathcal{L}^{\otimes 3}] \longrightarrow B$$

= an elliptic fibration $\varphi: Y \to B$ cut out by the zero locus of a section of the line bundle $\mathcal{O}(3) \otimes \pi^* \mathcal{L}^{\otimes 6}$ in X_0.

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$\Delta = 4f^3 + 27 g^2$ (locus of a sing. fiber)

$j = 1728 \dfrac{4f^3}{\Delta}$.

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$\to B$

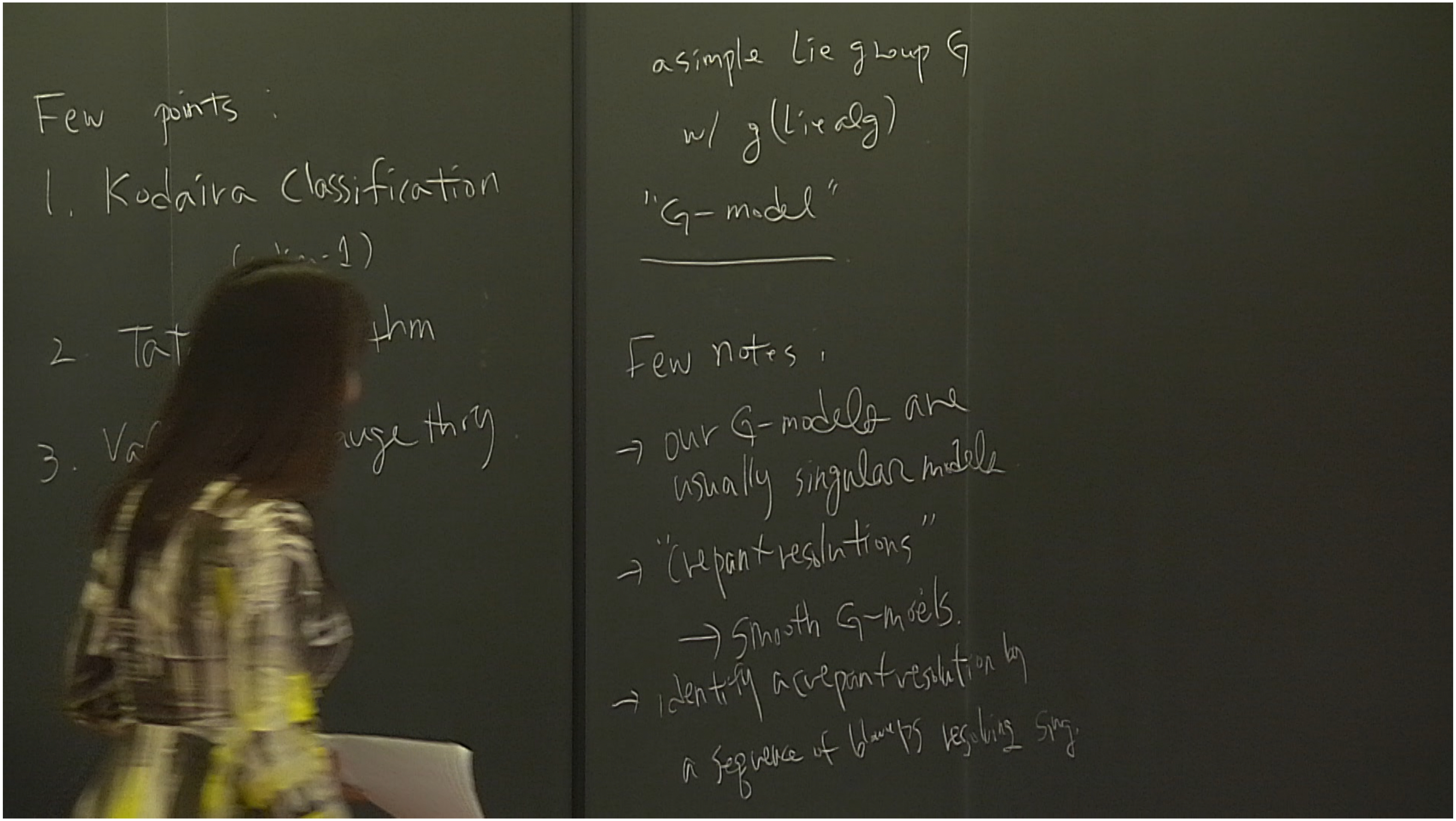
$x \quad \mathcal{O}_{X_0}(1) \otimes \pi^* \mathcal{L}^{\otimes 2}$
$y \quad \mathcal{O}_{X_0}(1) \otimes \pi^* \mathcal{L}^{\otimes 3}$
$z \quad \mathcal{O}_{X_0}(1)$

Few points :

1. Kodaira Classification
(codim-1)

2. Tate's algorithm

3. Vafa $\to$ gauge thry.


Few points:
1. Kodaira Classification
2. Tat
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3. Va
uge thry
a simple Lie group G
w/ g (Lie alg)
"G-model"
Few notes:
→ our G-models are usually singular models.
→ "crepant resolutions"
→ smooth G-models.
→ identify a crepant resolution by a sequence of blowups resolving sing.

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M-thry ←

g Loup G
eg)

le are
gular models
ations"
G-models.
point resolution by

$$
\begin{tikzcd}
\text{M-thry} \arrow[rr, leftrightarrow, "\text{dual}"] \arrow[d, "CY_3"] & & \text{F-theory} \arrow[d, "CY_3"] \\
\text{5d } \mathcal{N}=1 \text{ thry} \arrow[rr, bend right, "\text{uplift}^*"'] & & \text{6d } \mathcal{N}=(1,0) \text{ thry} \arrow[ll, "S^1"']
\end{tikzcd}
$$

Goal: geometric & top. data

→ Describe the Coulomb branch

→ 5d/6d spectra

5d field contents

gravity mult ($g_{\mu\nu}, A_\mu, \psi_{\mu I}$).

vector mult ($A_\mu^A, \lambda_I^A, \phi^A$)

hypermult (q^m, A_I^m)

"quaternion"

Majorana

CS lvls

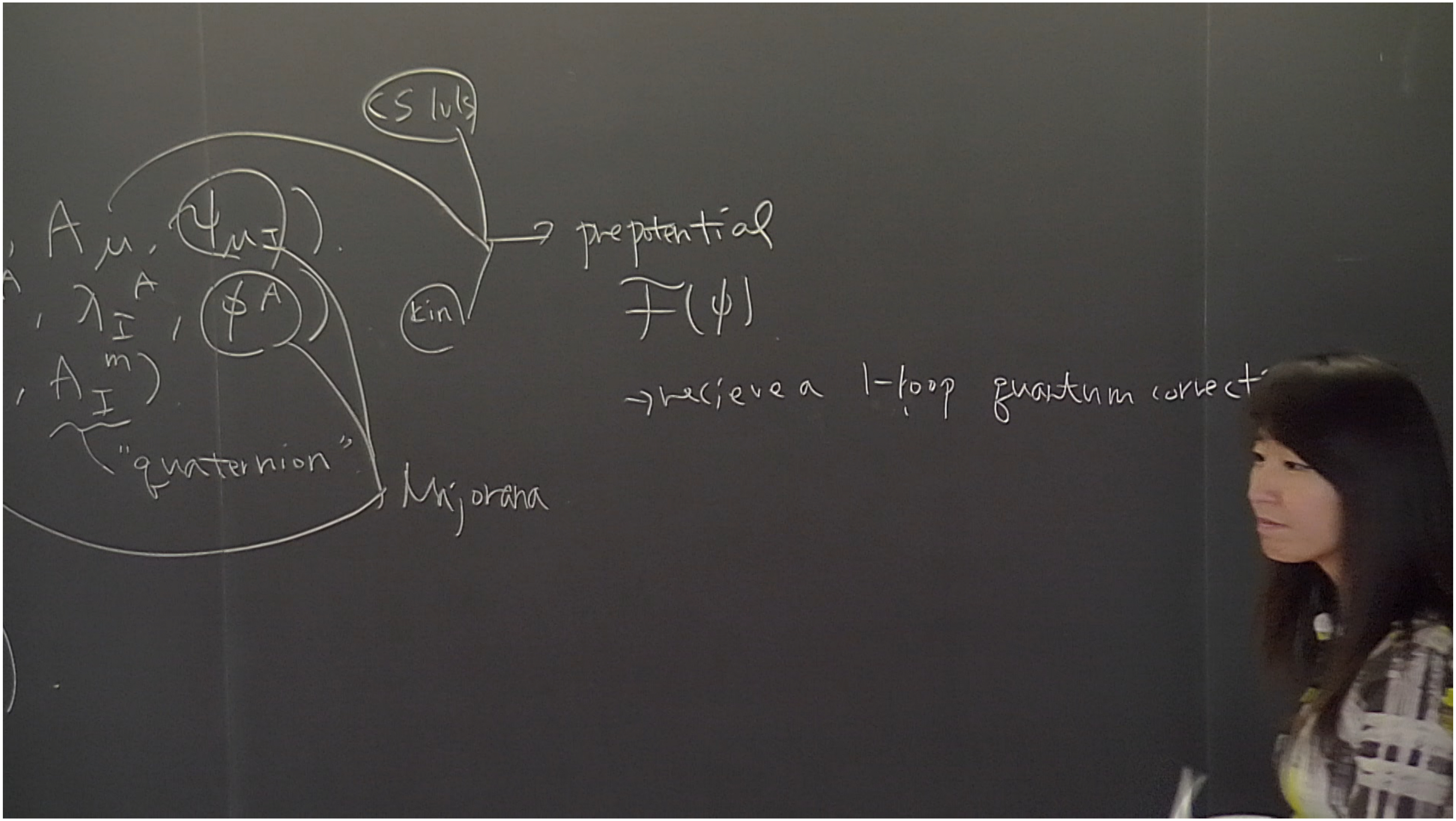
kin

$\rightarrow$ prepotential $\mathcal{F}(\phi)$.

$I : SU(2)_R$

$A = 1, \cdots, (h^{1,1} - 1)$

$m = 1, \cdots, 2(h^{2,1} + 1)$.


CS lvls
, A_μ, $\psi_{\mu I}$).
, λ_I^A, ϕ^A))
, A_I^m)
"quaternion"
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→ prepotential
$\mathcal{F}(\phi)$
→ recieve a 1-loop quantum correct
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, λ_I^A, ϕ^A kin → prepotential

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, A_I^m)

"quaternion" → Majorana

→ recieve a 1-loop quantum correction

Dictionary

elliptic fibration	gauge thry
codim 1 sing	gauge alg ($\mathfrak{g}$)
codim 2 "	representation (R)
crepant resolution	Coulomb phase
triple intersection poly	prepotential
Mordell-Weil group	the fund. group of the gauge group

6d f

grav

6d field contents

gravity mult ($g_{\mu\nu}$, $B^+_{\mu\nu}$, χ^{A-}_{μ})

tensor mult ($B^-_{\mu\nu}$, χ^{A+} , ϕ)

hypermult (q , η^+)

quaternion

vector mult (A_μ , λ^{A-})

symplectic Majorana

tensor mult $(B_{\mu\nu}, \chi^{A+}, \phi)$

hypermult : (q, η^{+})

quaternion

vector mult (A_μ, λ^{A-})

symplectic Majorana

A : $SU(2)_R$

ϕ scalar $SO(1,n_T)/SO(n_T)$ # tensor

$B_{\mu\nu}$ $SO(1,n_T)$

q dim n_H # hyper.

Motivat

Unders

Elliptic F

→ "Weiers

M-thry

$\downarrow CY_3$

6d $\mathcal{N}=1$ thry $\xrightarrow[\text{uplift}^*]{S^1}$ 6d $\mathcal{N}=(1,0)$ thry

CY_3

gravity mult

vector mult

hypermult

$$\begin{cases} n_V^{(5)} = n_V^{(6)} + n_T + 1 = h^{1,1}(Y) - 1 \\ n_H^0 = h^{2,1}(Y) + 1 \end{cases}$$

$$\begin{cases} n_V^{(6)} = h^{1,1}(Y) - h^{1,1}(B) - 1 \\ n_H^0 = h^{2,1}(Y) + 1 \\ n_T = h^{1,1}(B) - 1 \end{cases}$$

$I : SU(2)_R$

$A = 1, \cdots, (h^{1,1}$

$m = 1, \cdots, 2($

Sing.

6d $\mathcal{N}=1$ thry

uplift

$$\begin{cases} n_V^{(5)} = n_V^{(6)} + n_T + 1 = h^{1,1}(Y) - 1 \\ n_H^{0} = h^{2,1}(Y) + 1 \end{cases}$$

uncharged.

$$\begin{cases} n_V^{(\,)} \\ n_H^{0} = \\ n_T = \end{cases}$$

...dels.

...tion by

resolving sing.

))
— scalar
)

— symplectic
Majorana

\# tensor

Coulomb Phases and Anomalies:

Geometric Approach to 5d/6d Theories

Top & Geometric Data to 5d/6d Spectra

1. a sing. Weierstrass model w/ Kodaira fibers.
2. a crepant resolution

   Batyrev & Kontsevich: χ, $h^{1,1}$ & $h^{2,1}$ inv.
3. Pushforward formulas $\longrightarrow$ the total Chern class $\longrightarrow \chi$

4. Hodge #s

$$h^{0,1}(B) = h^{0,2}(B) = 0$$

Shioda-Tate-Wazir Thm

$$h^{1,1}(Y) = h^{1,1}(B) + \underbrace{f}_{= \operatorname{rank}(G)} + 1$$

fibers.

$$h^{2,1}(Y) = h^{1,1}(Y) - \frac{1}{2}\chi(Y)$$

inv.

5. The fiber structure.

lass

6. geometric weights, representation

$^2(B) = 0$

eir Thm

$h^{1,1}(B) + f + 1$ = rank(G)

$h^{1,1}(Y) - \frac{1}{2}\chi(Y)$

ture

nts, representation

7. triple intersection poly

$$\mathcal{F}_{trip} = \varphi_*\left(\left(\Sigma D_a \phi_a\right)^3 \underbrace{\varphi^* M}_{= c_1(Y_3)}\right)$$

$\|$ for $\varphi : Y \to B$

$$6\,\mathcal{F}_{pre} = \frac{1}{2}\left(\sum_a |\langle \alpha, \phi\rangle|^3 - \sum_R \sum_{w \in N} n_R |\langle w, \phi\rangle|^3\right) + \cdots$$

Few points:

1. Kodaira Classifica–
(codim-1)
2. Tate's algorithm
3. Vafa → gauge th

ion

hry.

pushforward thms

ex) $SU(2): X_0 \xleftarrow{(x,y,s\,|\,e_1)} X_1$

$SU(3): X_0 \xleftarrow{(x,y,s|e_1)} X_1 \xleftarrow{(y,e_1|e_2)} X_2$

1. total Chern class after a blowup along a local complete intersection

Thm (Aluffi)

$Z \subset X$ the complete intersection of d nonsing. hypersurfaces.

$Z_1, \cdots, Z_d$ meeting transversally in X.

$f: \tilde{X} \to X$ th

cen

E the except

The Chern cla

$c(T\tilde{X}) = (1+$

$f : \widetilde{X} \to X$ the blowup of X centered at Z

2 (of th

E the exceptional divisor of f

Thm (

The chern class of $\widetilde{X}$

ong

$$c(T\widetilde{X}) = (1+E)\left(\prod_{i=1}^{d} \frac{1+f^*z_i - E}{1+f^*z_i}\right) f^* c(TX)$$

d nonsing hypersurfaces .

ally in X .

2 (of the original space)

Thm (Esole - Jefferson - Kang)

$\widetilde{Q}(t) = \sum_a f^* Q_a t^a$ a formal power series of $Q_a \in A_*(X)$

$E(TX)$ $\quad Q(t) = \sum_a \underline{Q_a} \widetilde{t^a}$ the associated power series

whose coeff. pullback to the coeff. of $\widetilde{Q}(t)$

$$\rightarrow f_* \widetilde{Q}(E) = \sum_{\ell=1}^{d} Q(z_\ell) M_\ell \quad w/\ M_\ell = \prod_{\substack{m=1 \\ m\neq \ell}}^{d} \frac{z_m}{z_m - z_\ell}.$$

a formal power series

of $Q_a \in A_*(X)$

the associated power series

pullback to the coeff. of $\tilde{Q}(t)$

$)M_\ell$ w/ $M_\ell = \prod_{\substack{m=1 \\ m\neq \ell}}^{d} \frac{z_m}{z_m - z_\ell}$.

3. Pushforward Analytic expressions in the Chow ring of the proj. bundle X_0 to the Chow ring of its base

Thm (Esole-Jefferson-Kang)

$\pi : X_0 = \mathbb{P}[\mathcal{O}_B \oplus \mathcal{L}^{\otimes 2} \oplus \mathcal{L}^{\otimes 3}] \to B$.

$L = c_1(\mathcal{L})$

$H = c_1(\mathcal{O}_{X_0}(1))$

$$\to \pi_* \tilde{Q}(H) = -2 \left.\frac{Q(H)}{H^2}\right|_{H=-2L} + 3 \left.\frac{Q(H)}{H^2}\right|_{H=-3L} + \frac{Q(0)}{6L^2}.$$

geometric & top. data

1) # crepant resolutions

& graph of flips

2) the fiber structure of each crepant resolution

3) weights & representation

4) top. inv: χ, $h^{1,1}$, $h^{2,1}$

5) triple intersection poly in each crepant resol.

Coulomb P

Geometri

Coulomb Phases and Anomalies :

Geometric Approach to 5d/6d Theories

esolution

crepant resol

5d/6d theories

1. The structure Coulomb branch of 5d $\mathcal{N}=1$ thry w/ g, R

2. Number of hypers charged under each irred. rep. n_{R_i}

3. Anomaly Cancellation cond. @ 6d $\rightarrow n_{R_i}$

4.

5. Th

6.

Semi-simple groups

$SU(2) \times G \qquad rk(G) \leq 3$

1. $D_2 = A_1 + A_1$

$$\begin{cases} SO(4) \simeq (SU(2)\times SU(2))/\mathbb{Z}_2 \\ Spin(4) = SU(2)\times SU(2) \end{cases}$$

2. $SU(2)\times SU(3)$

$SU(2)\times Sp(4)$

$SU(2)\times G_2 \to$ NHC

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$SU(2)\times G_2 \to$ NHC

3. $SU(2)\times SU(4)$

SO(4) & Spin(4)

$$Y^- \xleftrightarrow{\text{flop}} Y^+$$

$$Y^- \xrightarrow{f^-} Y_0 \xleftarrow{f^+} Y^+$$

$$SU(2) \times SU(2)$$
$$\uparrow \qquad\quad \uparrow$$
$$S \qquad\quad T$$

$SO(4)$: $R = (3,1) \oplus (1,3) \oplus (2,2)$

ℓ_2 $(S+T=2k)$

$$\chi = -4(9k^2 + 4kT + T^2)$$

$$h^{1,1} = 13 - k^2$$

$$h^{2,1} = 13 + 17k^2 + 8kT + 2T^2$$

$$\mathcal{F}^+ = -2(2k\,k^2 + 10kT + T^2)\psi_1^3$$

$$+ (T(T+4k))\psi_1\phi_1^2 - 4T(k+T)\phi_1^3,$$

push

ex) S

S

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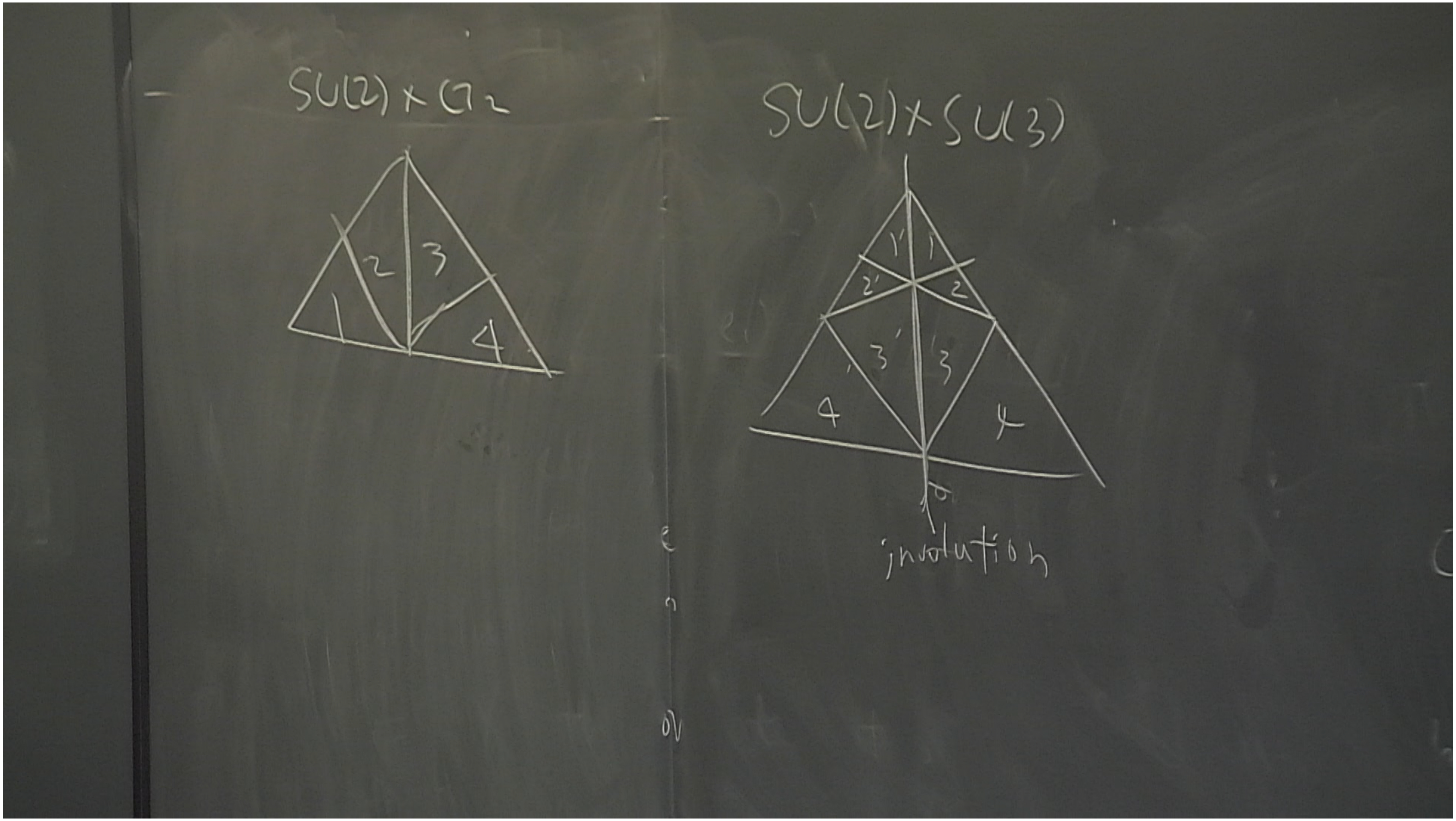
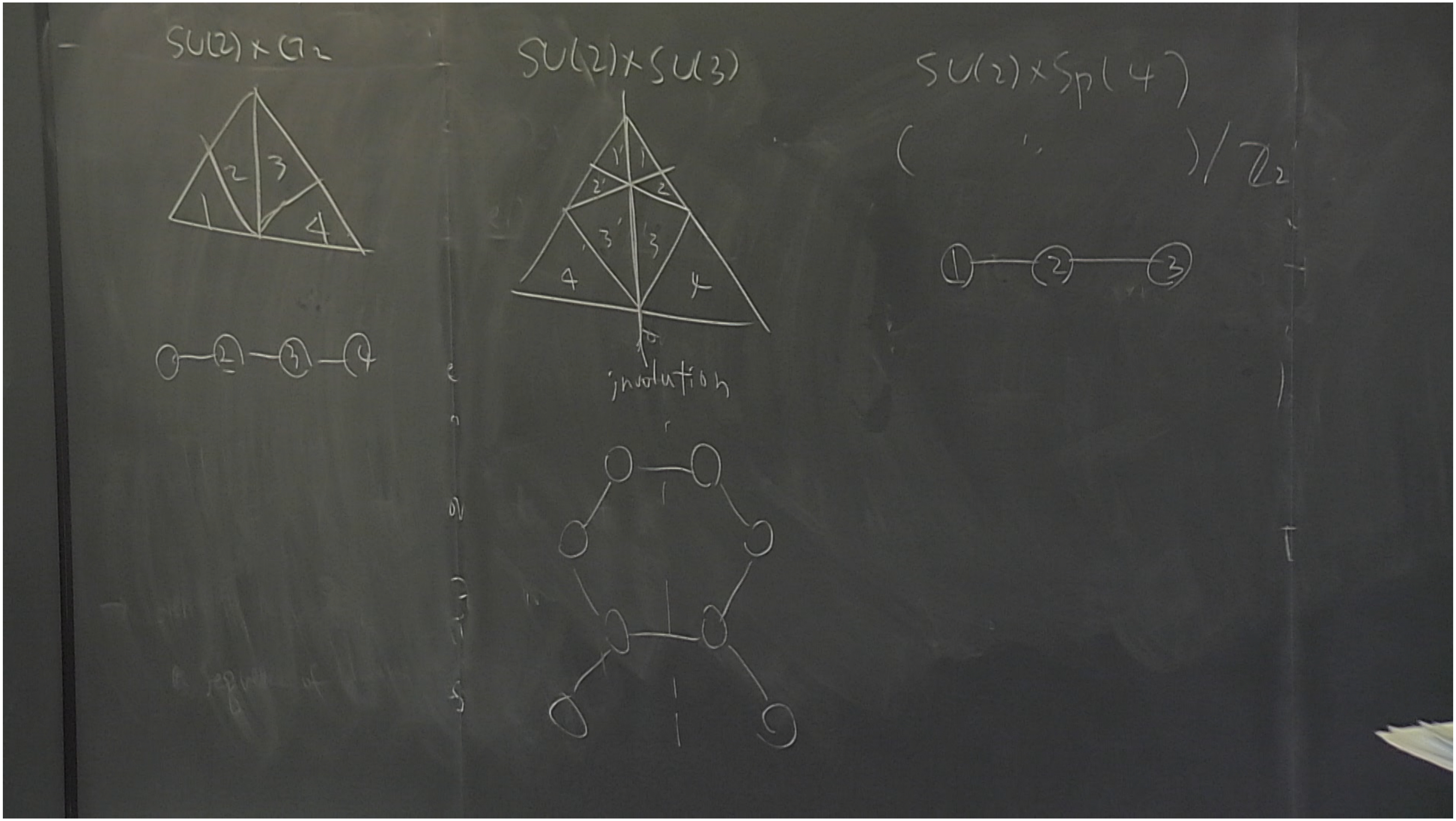
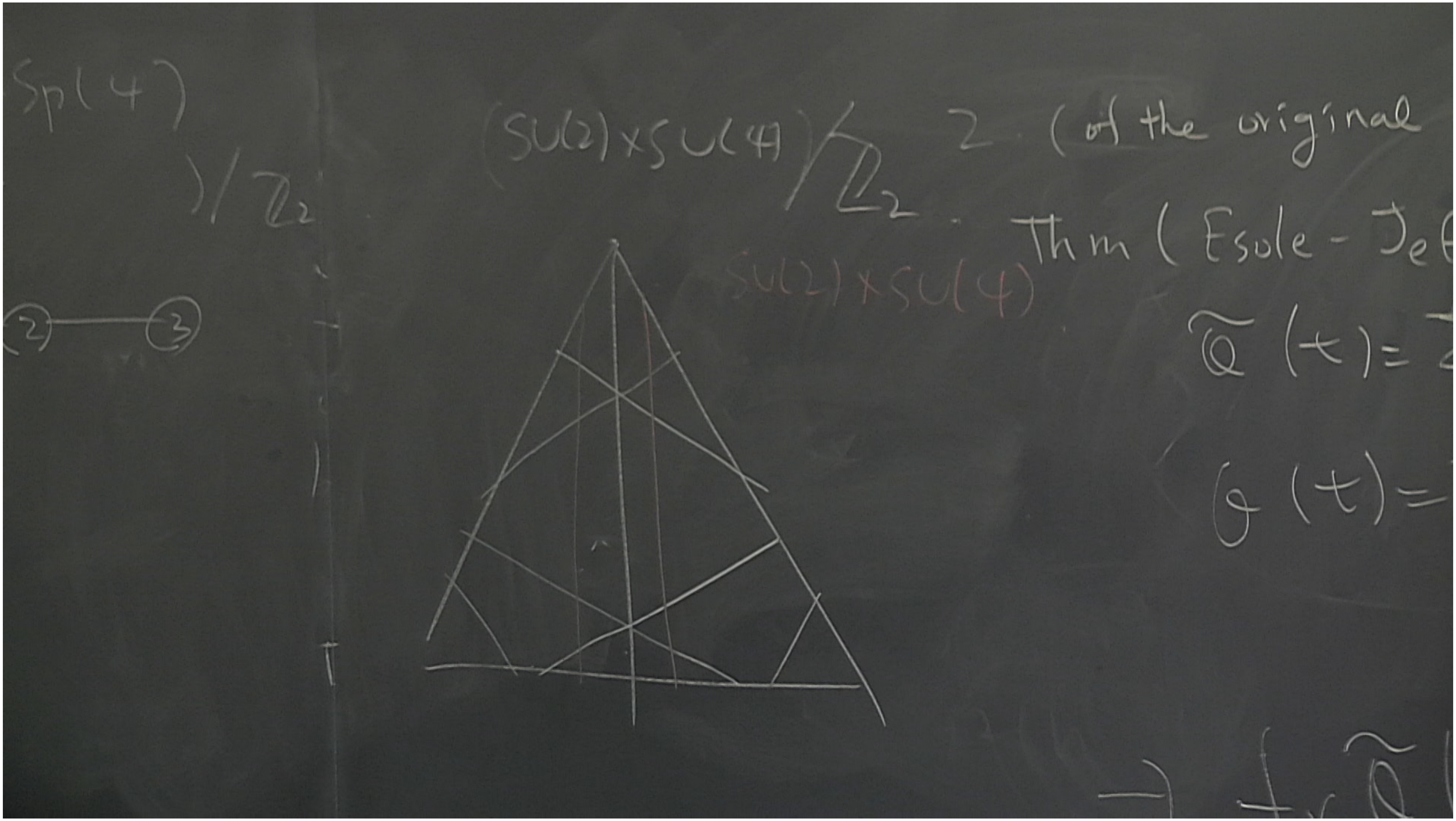

SU(2) × G₂
2
3
1
4
SU(2) × SU(3)
1'
1
2'
2
3'
3
4
4
involution


SU(2) × G₂
SU(2)×SU(3)
SU(2)×Sp(4)
( )/Z₂
involution


Sp(4)
)/ℤ2
(SU(2)×SU(4))/ℤ2
2 (of the original
Thm (Esole - Je
SU(2)×SU(4)
Q̃ (t) =
Θ (t) =

Anomaly Cancellation

$G = G_1 \times G_2$

$R = \bigoplus_i R_i$

CY_3 : $\begin{cases} n_V^{(6)} = \dim G \\ n_T = h^{1,1}(B) - 1 = 9 - K^2 \\ n_H = n_H^{\circ} + n_H^{ch} = h^{2,1}(Y) + 1 + \sum_i n_{R_i} (\dim R_i - \dim R_i^{\circ}) \end{cases}$

1. $n_H - n_V^{(6)} + 29 n_T - 273 = 0$

2. $B_{a,adj} - \sum n_{R_i a} B_{R_i a} = 0$

3. I_8 has to factor

$\longrightarrow \frac{1}{2} \Omega_{ij} Y_i^{(4)} X_j^{(4)}$ Thm