Title: Decay of correlations in long-range interacting systems at non-zero temperature

Speakers: Senaida Hernandez Santana

Series: Perimeter Institute Quantum Discussions

Date: February 13, 2019 - 4:00 PM

URL: http://pirsa.org/19020071

Abstract: We study correlations in fermionic systems with long-range interactions in thermal equilibrium. We prove an upper-bound on the correlation decay between anti-commut-ing operators based on long-range Lieb-Robinson type bounds. Our result shows that correlations between such operators in fermionic long-range systems of spatial dimension D with at most two-site interactions decaying algebraically with the distance with an exponent $\alpha \geq 2\,D$, decay at least algebraically with an exponent arbitrarily close to α. Our bound is asymptotically tight, which we demonstrate by numerically analysing density-density correlations in a 1D quadratic (free, exactly solvable) model, the Kitaev chain with long-range interactions. Away from the quantum critical point correlations in this model are found to decay asymptotically as slowly as our bound permits.

Lieb-Robinson bounds

- Extension of Lieb-Robinson-like bound for long-range systems[13].

Lieb-Robinson-like bound

For any **two-site long-range** interacting and **fermionic** Hamiltonian and for any two anticommutative operators A, B separated by distance l:

$$\|\{A(t), B\}\| \leq c_0 \, \mathrm{e}^{v\,|t| - l/|t|^{\gamma}} + c_1 \, \frac{|t|^{\alpha\,(1+\gamma)}}{l^{\alpha}}$$

where c_0, c_1, v and α are positive constants depending on H and $\gamma = \frac{1+D}{\alpha - 2D}$.

(valid for $\alpha > 2D \Rightarrow$ energy is extensive)

[13] M. Foss-Feig, Z.-X. Gong et al., PRL 114 (2015).

20

Analytical approach: Results

Theorem

For **two-site** and **long-range** interacting **fermionic** systems with $\alpha > 2D$ at **finite temperature** and for any two **anticommutative operators** A, B separated by a distance l, we obtain that

$$|\mathrm{corr}(A, B)| = |\langle AB \rangle_\beta| \leq \mathcal{O}(l^{-\alpha}) \quad (l \to \infty).$$

Analytical approach: Results

- Corollary

For **quadratic models** this implies, by Wick's theorem[14], that **correlations between commuting operators** are upper-bounded by a power-law.

In particular, for **density-density correlations**

$$|\mathrm{corr}(n_i, n_j)| \leq \mathcal{O}(|i-j|^{-2\alpha}),$$

where $n_i = f_i^\dagger f_i \ \forall i$.

[14] M. Gluza, C. Krumnow et al., Phys. Rev. Lett. 117 (2016).

Analytical approach: Results

- Corollary

  For **quadratic models** this implies, by Wick's theorem[14], that **correlations between commuting operators** are upper-bounded by a power-law.

  In particular, for **density-density correlations**

$$|\mathrm{corr}(n_i, n_j)| \leq \mathcal{O}(|i-j|^{-2\alpha}),$$

  where $n_i = f_i^\dagger f_i \ \forall i$.

[14] M. Gluza, C. Krumnow et al., Phys. Rev. Lett. 117 (2016).

Numerical approach: Model

- Model: Kitaev chain with long-range p-wave pairing of length L

$$
\begin{aligned}
H &= -\frac{1}{2}\sum_{i=1}^{L}\left(f_i^\dagger f_{i+1} + f_{i+1}^\dagger f_i\right) - \mu \sum_{i=1}^{L}\left(f_i^\dagger f_i - \frac{1}{2}\right) \\
&+ \frac{1}{2}\sum_{i=1}^{L}\sum_{j=1}^{L-1} d_j^{-\alpha}\left(f_i f_{i+j} + f_i^\dagger f_{i+j}^\dagger\right) \text{ with } d_j = \min(j, L-j)
\end{aligned}
$$

  - This model has a **quantum phase transition at** $\mu = 1$.
  - It is quadratic and solvable by exact diagonalization ($L = 2000$ or further).

Quantum phase transition

- At $T = 0$.
- For short-range interactions: Correlations decay as a power-law, instead of exponentially.

23

Numerical approach: Figure of merit

- Density-density correlations

$$\mathrm{corr}(n_i, n_{i+d}) = \langle n_i n_{i+d} \rangle_\beta - \langle n_i \rangle_\beta \langle n_{i+d} \rangle_\beta$$

Numerical approach: Figure of merit

- Density-density correlations

$$\mathrm{corr}(n_i, n_{i+d}) = \langle n_i n_{i+d} \rangle_\beta - \langle n_i \rangle_\beta \langle n_{i+d} \rangle_\beta$$

$$\boxed{\mathrm{corr}(n_i, n_{i+d}) = f(d)\ \textbf{?}}$$

26

Numerical approach: Methods

- Bogoliubov transformation

- Wick's theorem

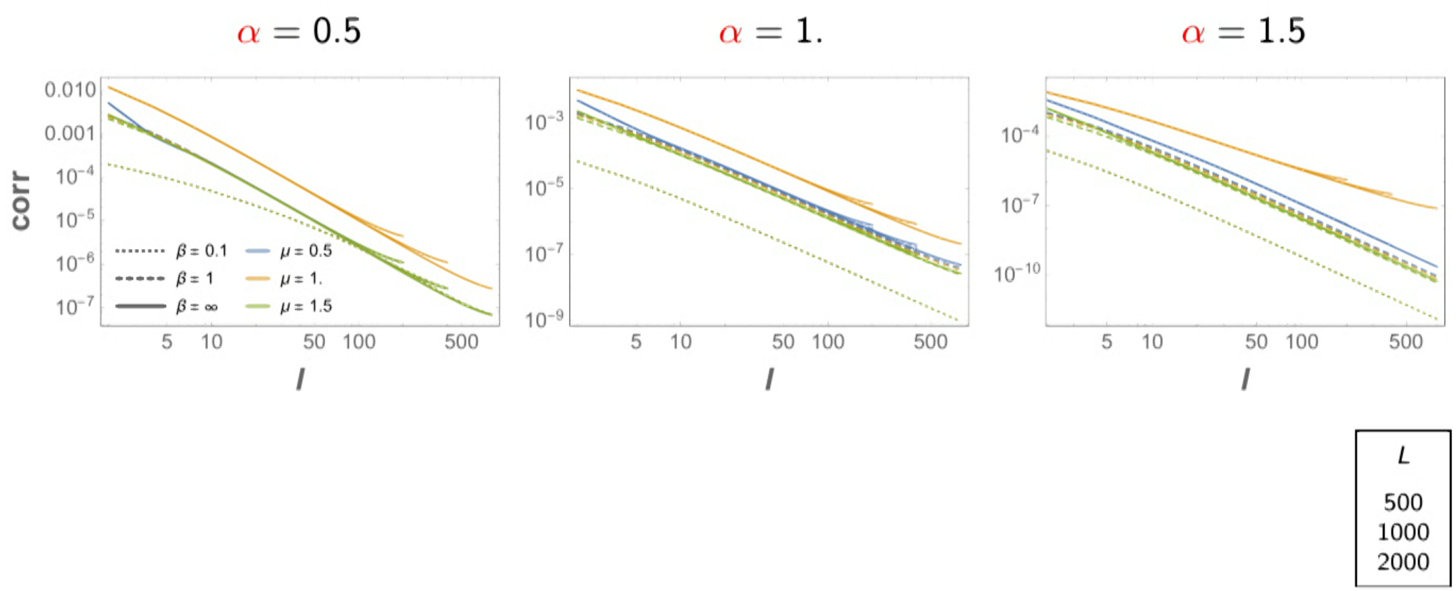
$$\mathrm{corr}_\beta(n_i, n_{i+d}) = \langle f_i^\dagger f_{i+d}\rangle_\beta \langle f_i f_{i+d}^\dagger\rangle_\beta - \langle f_i^\dagger f_{i+d}^\dagger\rangle_\beta \langle f_i f_{i+d}\rangle_\beta,$$

where $n_i = f_i^\dagger f_i$.

Numerical approach: Results

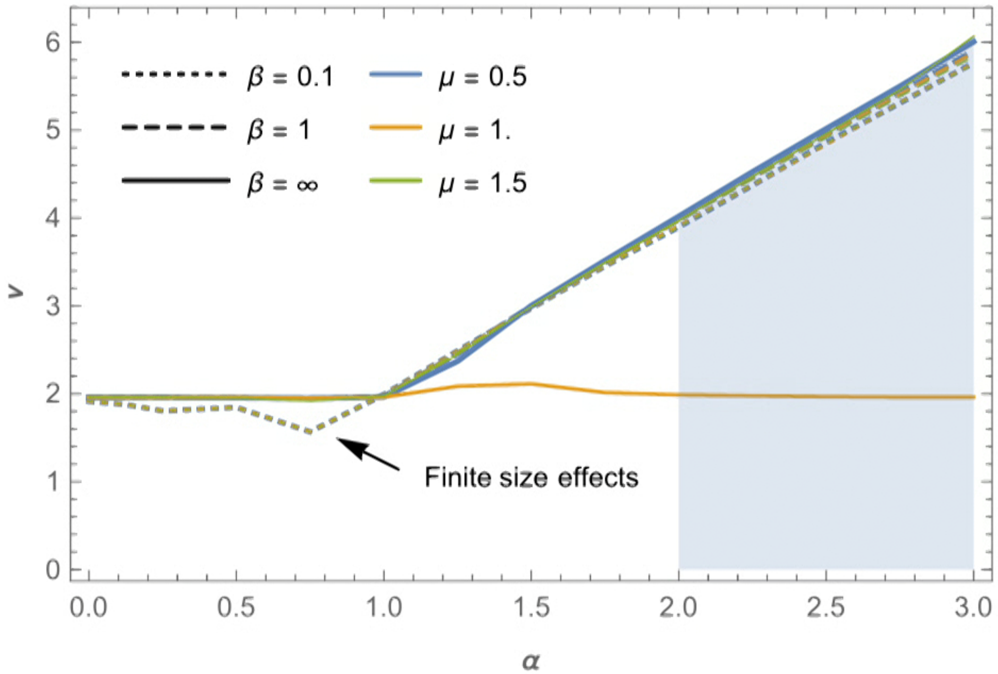
- Correlations asymptotically decay as a power law at any temperature and at both criticality and off-criticality regimes.

$$\mathrm{corr}(n_i, n_{i+d}) \propto \frac{1}{d^{\nu}}$$



Numerical approach: Results

- The asymptotic decay is characterized by α, μ and T. In particular:
  1. Critical regime ($T = 0$, $\mu = 1$) $\Rightarrow \nu \approx 2 \;\; \forall\alpha$
  2. Off-critical regime $\Rightarrow \nu \approx 2$ if $\alpha \leq 1$ or $\nu \approx 2\alpha$ if $\alpha > 1$



Summary

- For any **fermionic** system with **two-site long-range** interactions, **correlations between anticommutative operators** decay at least polinomially.

Summary

- For any **fermionic** system with **two-site long-range** interactions, **correlations between anticommutative operators** decay at least polinomially.
- For **quadratic Hamiltonians**, this can be extended to **commutative operators** such as density-density correlations.

Summary

- For any **fermionic** system with **two-site long-range** interactions, **correlations between anticommutative operators** decay at least polinomially.
- For **quadratic Hamiltonians**, this can be extended to **commutative operators** such as density-density correlations.
- For a **concrete quadratic model**, density-density **correlations decay as a power-law** asymptotically.

Summary

- For any **fermionic** system with **two-site long-range** interactions, **correlations between anticommutative operators** decay at least polinomially.
- For **quadratic Hamiltonians**, this can be extended to **commutative operators** such as density-density correlations.
- For a **concrete quadratic model**, density-density **correlations decay as a power-law** asymptotically.
- The **decay exponent depends** on the **interaction decay** and on the presence of a **phase transition**.

Summary

- For any **fermionic** system with **two-site long-range** interactions, **correlations between anticommutative operators** decay at least polinomially.
- For **quadratic Hamiltonians**, this can be extended to **commutative operators** such as density-density correlations.
- For a **concrete quadratic model**, density-density **correlations decay as a power-law** asymptotically.
- The **decay exponent depends** on the **interaction decay** and on the presence of a **phase transition**.
- Numerics show that the **upper-bound** is asymptotically **tight**.

35

Summary

- For any **fermionic** system with **two-site long-range** interactions, **correlations between anticommutative operators** decay at least polinomially.
- For **quadratic Hamiltonians**, this can be extended to **commutative operators** such as density-density correlations.
- For a **concrete quadratic model**, density-density **correlations decay as a power-law** asymptotically.
- The **decay exponent depends** on the **interaction decay** and on the presence of a **phase transition**.
- Numerics show that the **upper-bound** is asymptotically **tight**.

Physical Review Letters **119**, 110601 (2017)

Summary

- For any **fermionic** system with **two-site long-range** interactions, **correlations between anticommutative operators** decay at least polinomially.
- For **quadratic Hamiltonians**, this can be extended to **commutative operators** such as density-density correlations.
- For a **concrete quadratic model**, density-density **correlations decay as a power-law** asymptotically.
- The **decay exponent depends** on the **interaction decay** and on the presence of a **phase transition**.
- Numerics show that the **upper-bound** is asymptotically **tight**.

Physical Review Letters **119**, 110601 (2017)

Thank you!

36