Title: Glueing W-algebras

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Collection: Cohomological Hall Algebras in Mathematics and Physics

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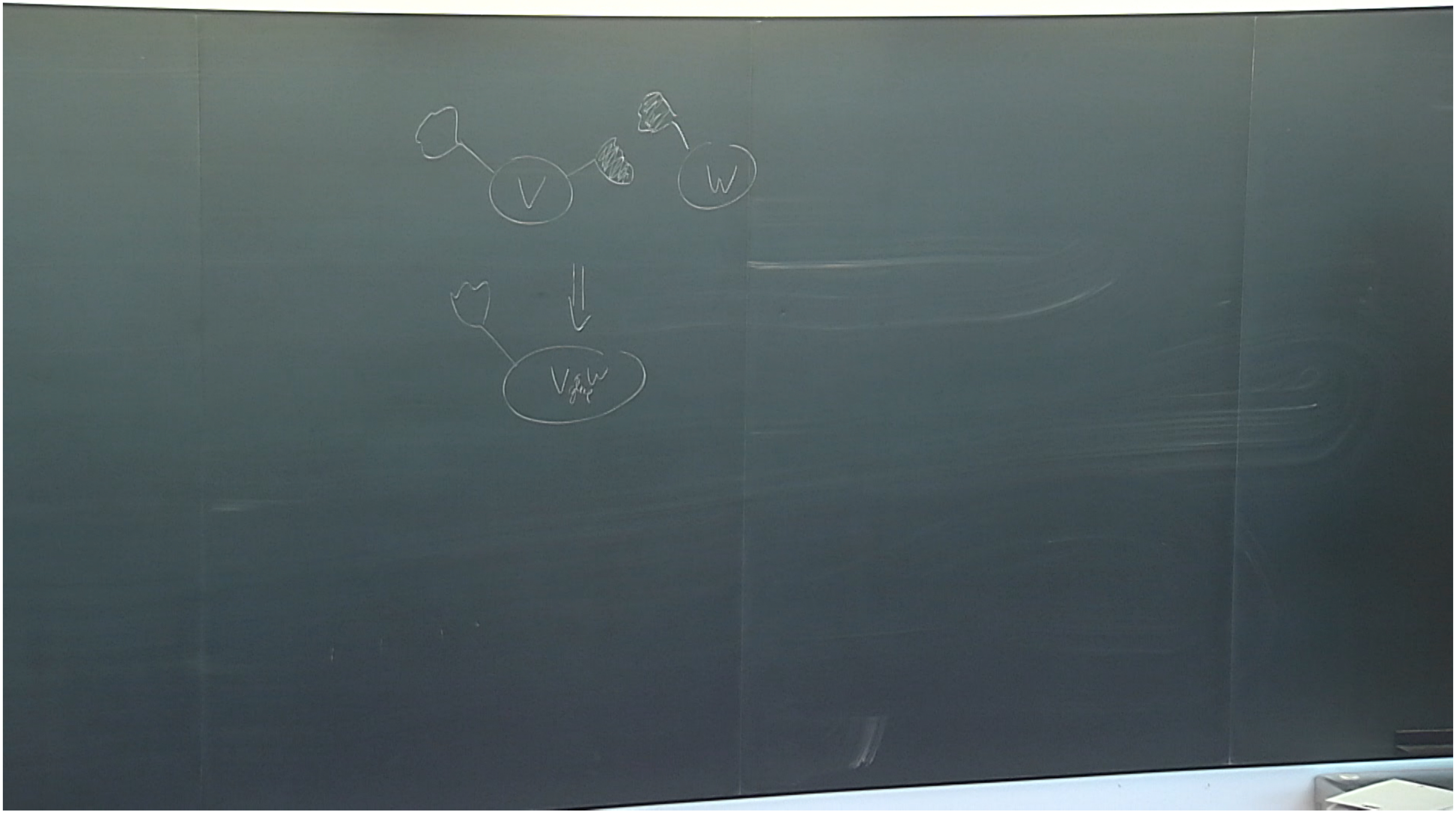
Abstract: S-duality predicts rather surprising isomorphisms of extensions of W-algebras. The aim of this talk is to present some explanation. Firstly, I will explain the concept of glueing W-algebras along certain categories of modules and then I will introduce what we call a W-algebra translation functor.


V
W


V
W

W-algebras

$\mathfrak{g}$ fin. dim Lie superalgebra

$B : \mathfrak{g} \times \mathfrak{g} \to \mathbb{C}$

$k \in \mathbb{C}$

$$\hat{\mathfrak{g}} = \mathfrak{g} \otimes_{\mathbb{C}} \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}K \oplus \mathbb{C}d$$

$$V_k(\mathfrak{g}) = \mathrm{Ind}_{\mathfrak{g}_+}^{\hat{\mathfrak{g}}} \mathbb{C}_k$$

$$V_k(\lambda) = \mathrm{Ind}_{\mathfrak{g}_+}^{\hat{\mathfrak{g}}} S_{\lambda, k}$$

$C^k(\mathfrak{g})$

simple quotient of $V_k(\mathfrak{g})$ is $L_k(\mathfrak{g})$

$L_k(\lambda)$

$C_k(\mathfrak{g})$

quantum Hamiltonian/

Drinfeld-Sokolov reduction

$f \in \mathfrak{g}$ nilpotent

$$H^{f}_{DS}\left(V_k(\mathfrak{g})\right)$$

Ex: $\mathfrak{g} = \mathfrak{sl}_2$

$$H^{f}_{DS}\left(V_k(\mathfrak{sl}_2)\right) = Vir_c$$

$$c = 1 - 6(\kappa + \kappa^{-1})$$

$$\kappa = k + 2$$

Thm (C – Kanade – McRae)

Let V, W be VOAs and $\mathcal{C}_V, \mathcal{C}_W$ full, braided, semi-simple rigid tensor subcategories of V resp. W-modules with at most countable inequivalent simples

Let $\tau : O$

Thm (C - Kanade - McRae)

Let V, W be VOAs and $\mathcal{C}_V, \mathcal{C}_W$ full, braided, semi-simple rigid tensor sub categories of V resp. W-modules with at most countable inequivalent simples

Let $\tau: \mathrm{SimObj}(\mathcal{C}_V) \xrightarrow{1-1} \mathrm{SimObj}(\mathcal{C}_W)$ $(\tau(V) = W)$

Then the following two are equiv.

1.) $\bigoplus_{X \in \mathrm{SimObj}(\mathcal{C}_V)} X \otimes \tau(X)^*$

can be given the structure of a simple VOA.

2.) There is a braid-reversed equivalence $\mathcal{C}_V \cong \mathcal{C}_W^{rev}$ mapping $X \mapsto \tau(X)$

, There is a braid-reversed

quivalence $\mathcal{C}_V \cong \mathcal{C}_W^{rev}$

pping $X \longmapsto \tau(X)$

$\mathcal{C}$ braided tensor cat.

$(A, \mu : A \otimes A, \iota : \mathbb{1} \to A)$

object

$$\begin{tikzcd}
(A\otimes A)\otimes A \arrow[rr] \arrow[d, "\mu\otimes\mathrm{id}"'] & & A\otimes(A\otimes A) \arrow[d, "\mathrm{id}\otimes\mu"] \\
A\otimes A \arrow[dr, "\mu"'] & \circlearrowleft & A\otimes A \arrow[dl] \\
& A &
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbb{1}\otimes A \arrow[r, "\iota"] \arrow[dr, "l_A"'] & A\otimes A \arrow[d] \arrow[dl, phantom, "\circlearrowleft"] \\
& A
\end{tikzcd}$$

infeld-So...

nilpot...t

DS $(V_k(g))$

- reversed

$\in \rho_w^{rev}$

$x)$

Ex: Arakawa - C - Linshaw

Let $g \in ADE$. Let k, ℓ satisfy

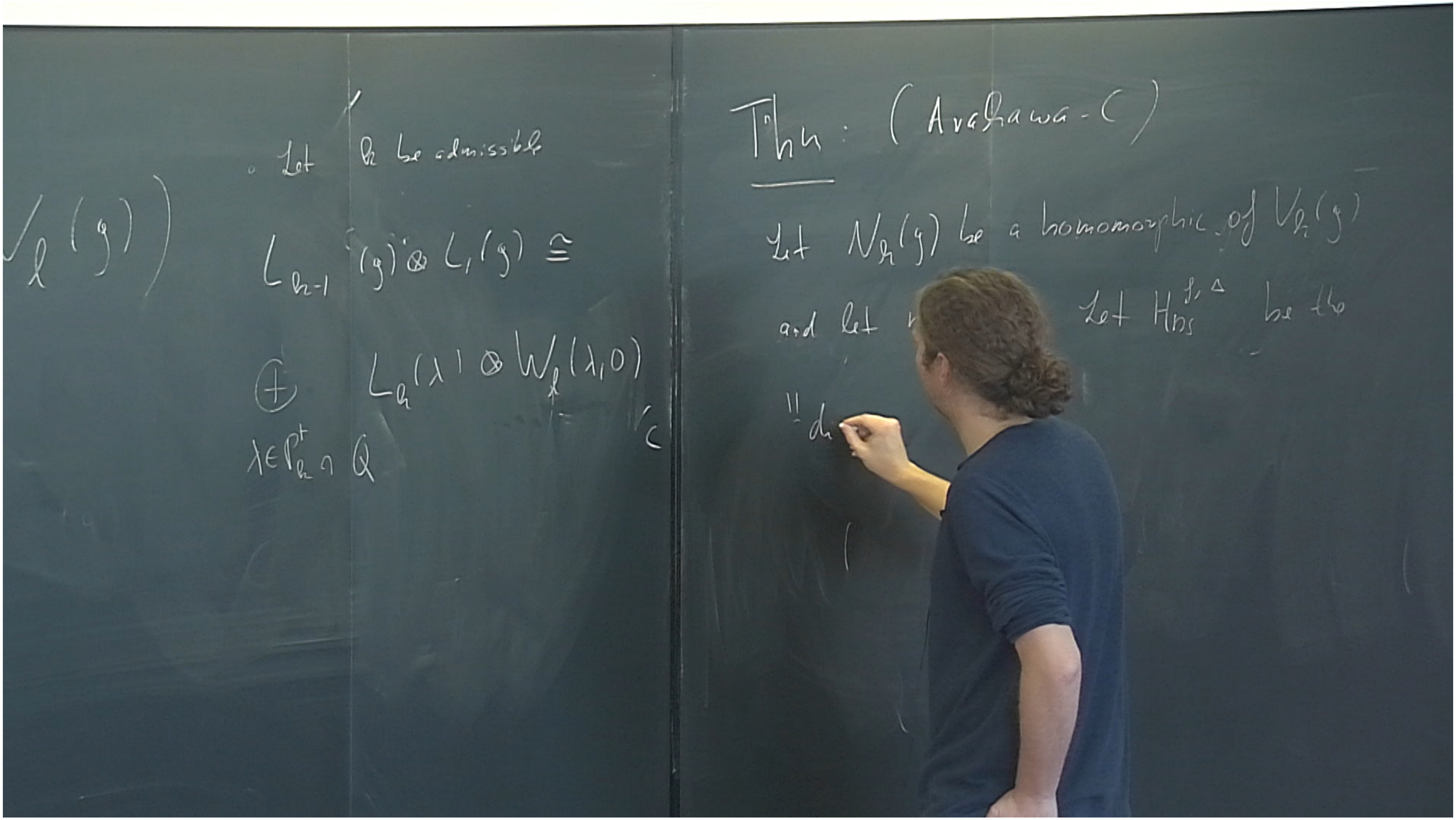
$$\frac{1}{k+h^\vee} + \frac{1}{\ell+h^\vee} = 1$$

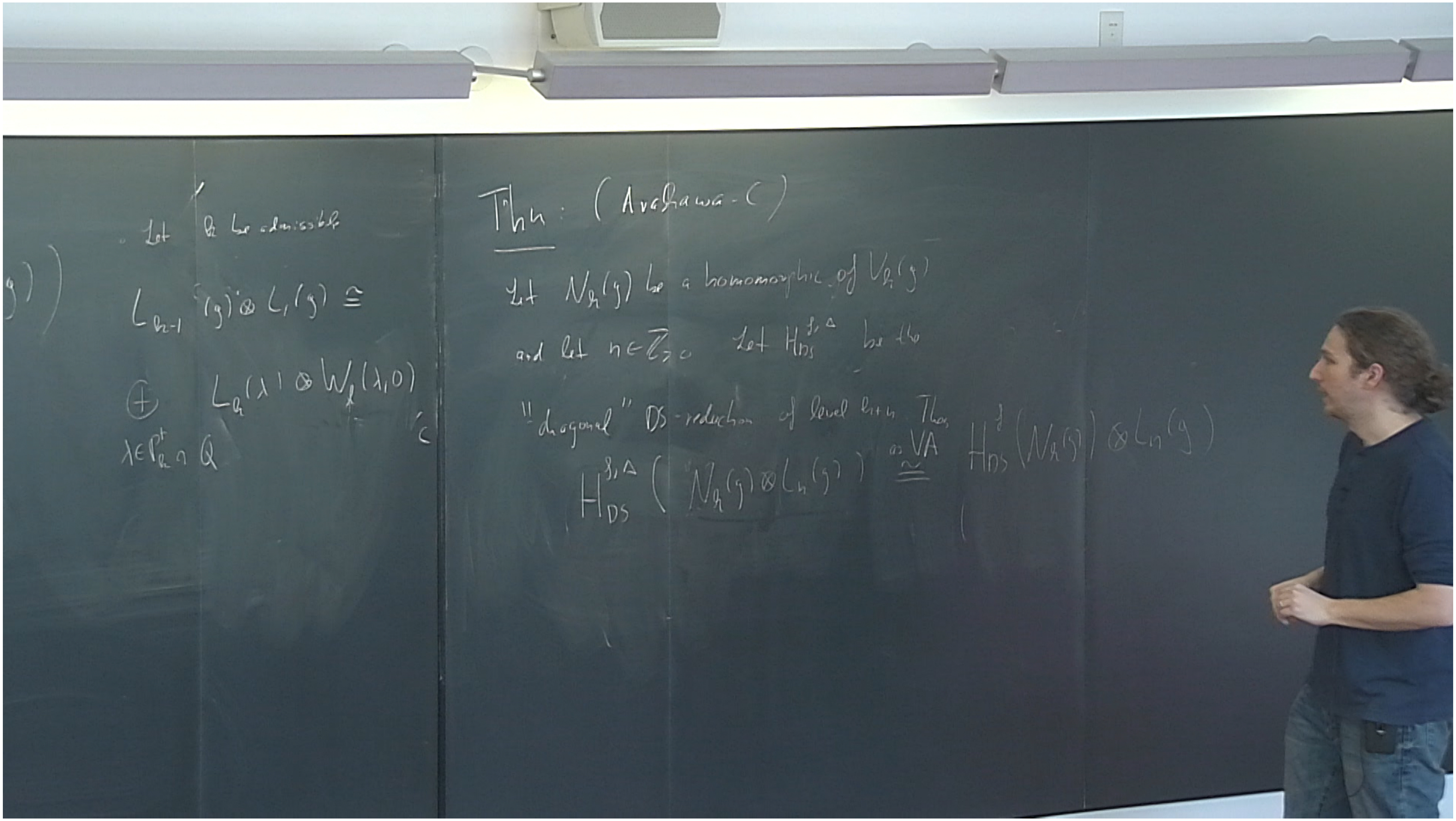
• Let k be generic

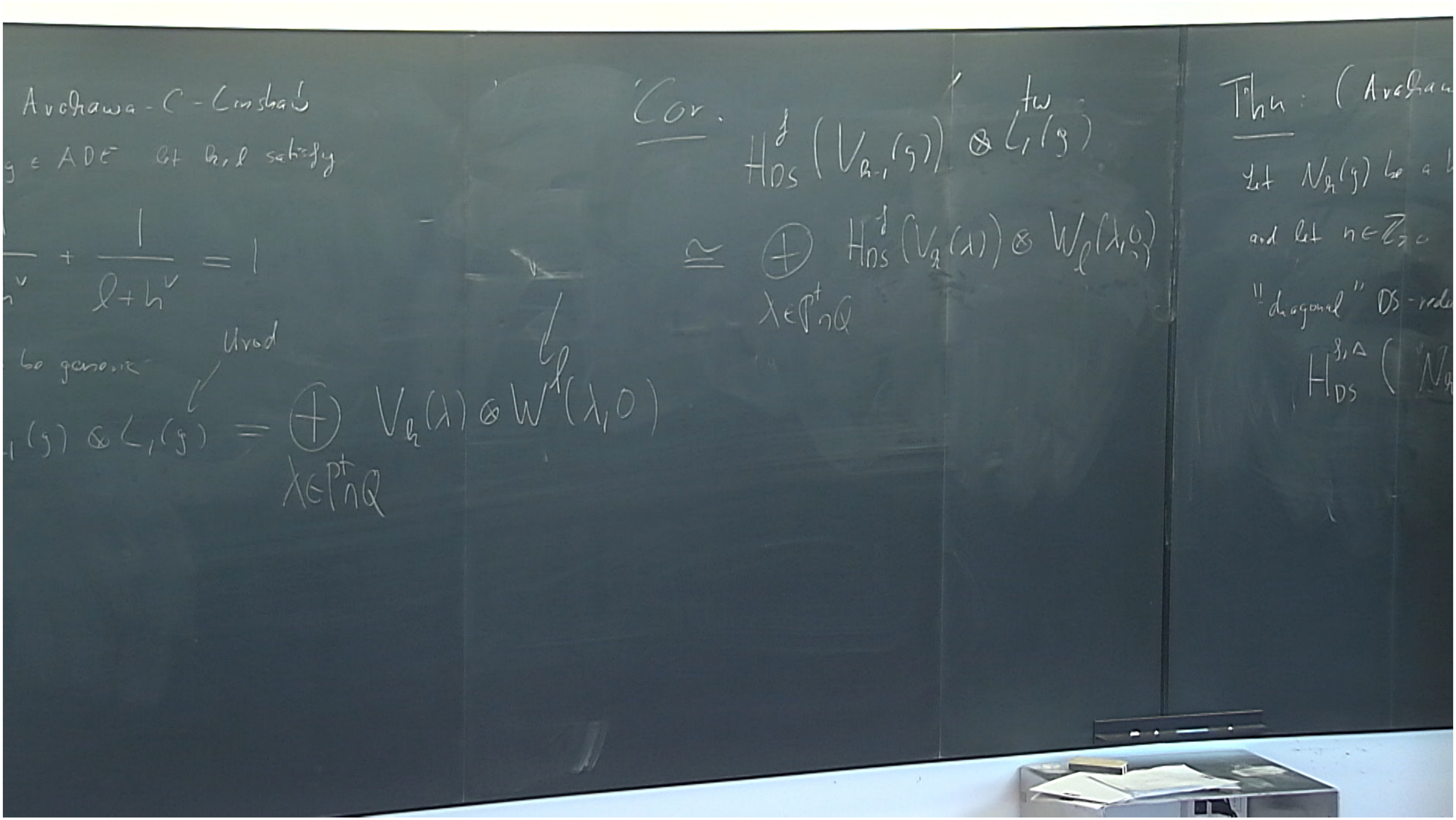
Urod

princ W-algebra

$$V_{k-1}(g) \otimes L_1(g) = \bigoplus_{\lambda \in P^+ \cap Q} V_k(\lambda) \otimes W_\ell(\lambda, 0)$$


Let k be admissible
L_{k-1}(g) ⊗ L_1(g) ≅ ⊕_{λ∈P_k^+ ∩ Q} L_k(λ) ⊗ W_ℓ(λ,0)
Thm: (Arakawa-C)
Let N_k(g) be a homomorphic of V_k(g)
and let
Let H_DS^{f,Δ} be the


Let k be admissible
$L_{k-1}(g) \otimes L_1(g) \cong \bigoplus_{\lambda \in P_k^+ \cap Q} L_k(\lambda) \otimes W_\ell(\lambda, 0)$
Thm: (Arakawa-C)
Let $N_k(g)$ be a homomorphic of $V_k(g)$
and let $n \in \mathbb{Z}_{\geq 0}$. Let $H_{DS}^{f,\Delta}$ be the
"diagonal" DS-reduction of level $k+n$. Then
$H_{DS}^{f,\Delta}(N_k(g) \otimes L_n(g)) \overset{\text{as VA}}{\cong} H_{DS}^{f}(N_k(g)) \otimes L_n(g)$


Arakawa - C - Linshaw
g ∈ ADE let k, l satisfy
$\frac{1}{k+h^\vee} + \frac{1}{l+h^\vee} = 1$
be generic
Urod
$L_1(g) \otimes L_1(g) = \bigoplus_{\lambda \in P^+ \cap Q} V_k(\lambda) \otimes W^l(\lambda, 0)$
Cor.
$H_{DS}^{g}(V_{k-1}(g)) \otimes L_1(g)$
tw
$\cong \bigoplus_{\lambda \in P^+ \cap Q} H_{DS}^{g}(V_k(\lambda)) \otimes W_l(\lambda, 0)$
Thm: (Arakawa
Let $N_k(g)$ be a
and let $n \in \mathbb{Z}_{\geq 0}$
"diagonal" DS-red
$H_{DS}^{g,\Delta}(N_k$