Title: COHA and AGT for Spiked Instantons

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Collection: Cohomological Hall Algebras in Mathematics and Physics

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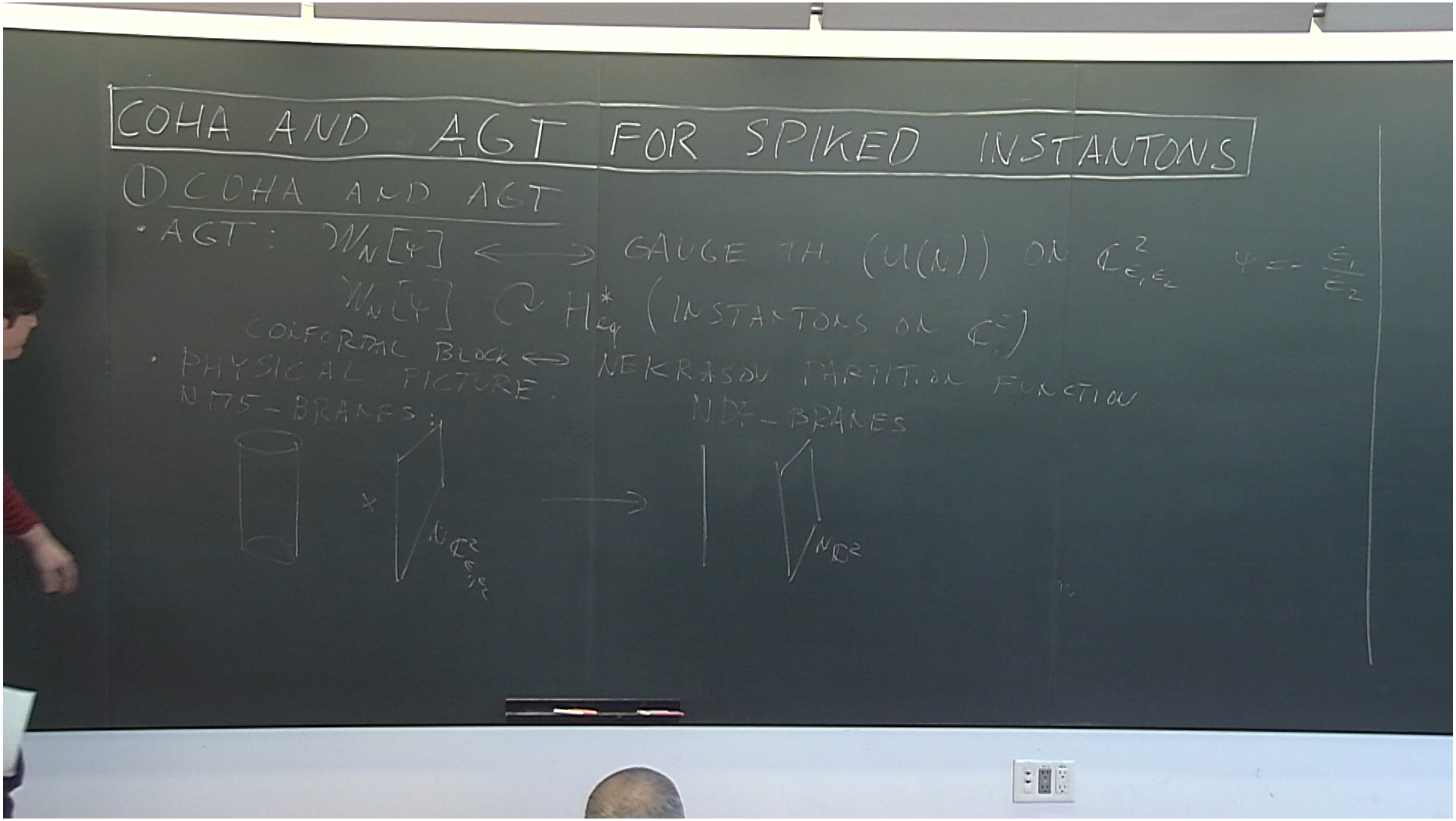
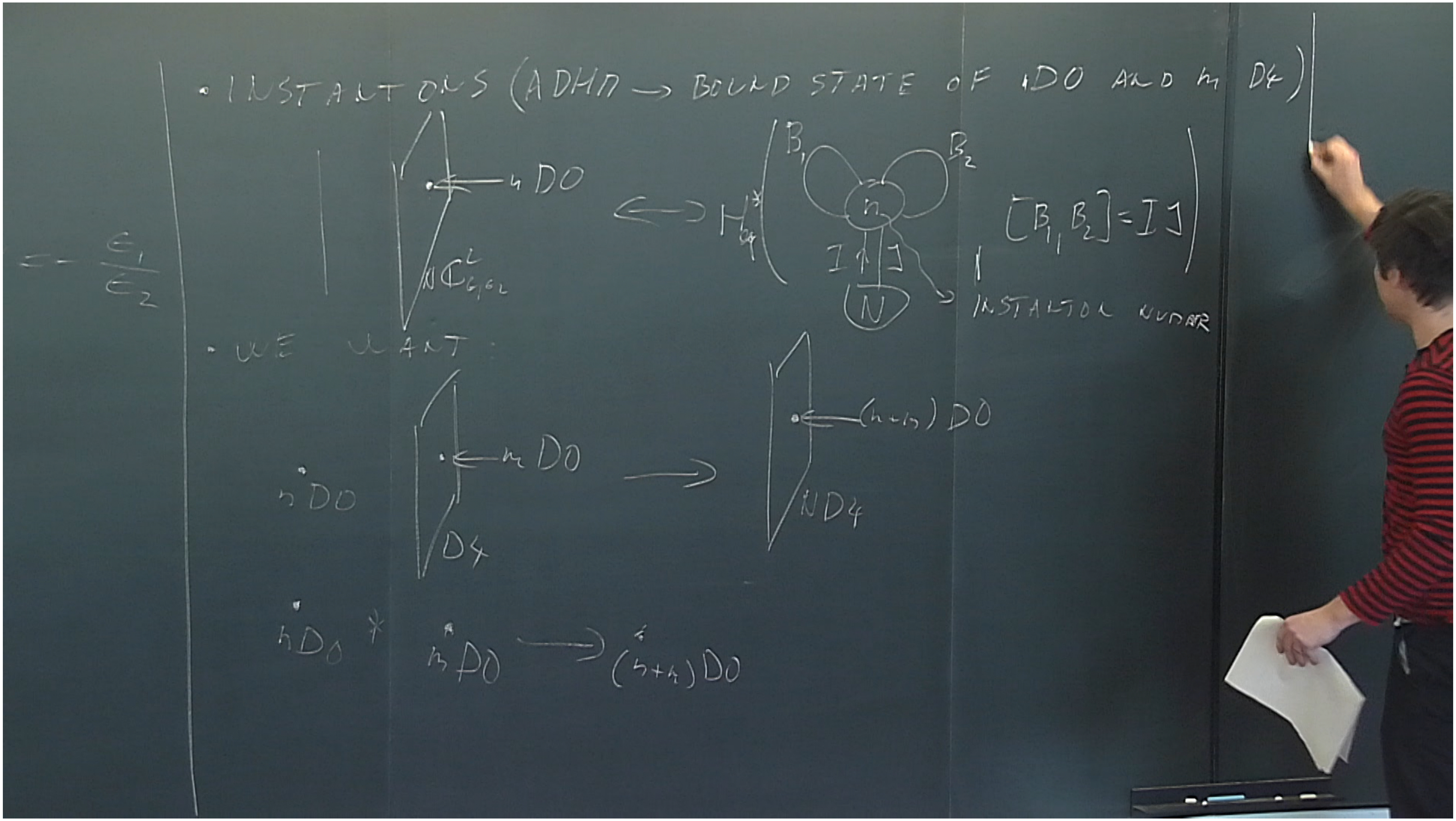
Abstract: The well-known AGT correspondence relates $\mathcal{W}_N$-algebras and supersymmetric gauge theories on $\mathbb{C}^2$. Embedding $\mathbb{C}^2$ as a coordinate plane inside $\mathbb{C}^3$, one can associate the COHA to $\mathbb{C}^3$ and derive the corresponding $\mathcal{W}_N$ as a truncation of its Drinfeld double. Building up on Zhao's talk, I will discuss a generalization of this story, where $\mathbb{C}^2$ is replaced by a more general divisor inside $\mathbb{C}^3$ with three smooth components supported on the three coordinate planes. Truncations of the Drinfeld double lead to a three-parameter family of algebras $\mathcal{W}_{L,M,N}$ determining the vertex algebras associated to Nekrasov's spiked instantons. Many interesting questions emerge when considering a general Calabi-Yau three-fold instead of $\mathbb{C}^3$. I will discuss a class of vertex algebras conjecturally arising from divisors inside more general toric Calabi-Yau three-folds.


COHA AND AGT FOR SPIKED INSTANTONS


COHA AND AGT FOR SPIKED INSTANTONS
① COHA AND AGT
• AGT: $W_N[\psi] \longleftrightarrow$ GAUGE TH. ($U(N)$) ON $\mathbb{C}^2_{\epsilon_1,\epsilon_2}$ $\psi = -\frac{\epsilon_1}{\epsilon_2}$
$W_N[\psi] \circlearrowright H^*_{eq}$(INSTANTONS ON $\mathbb{C}^2$)
CONFORMAL BLOCK ⟺ NEKRASOV PARTITION FUNCTION
• PHYSICAL PICTURE:
N M5-BRANES:
N D4-BRANES
N $\mathbb{C}^2_{\epsilon_1,\epsilon_2}$
N $\mathbb{C}^2$


• INSTANTONS (ADHM → BOUND STATE OF n D0 AND N D4)
n D0
B1
B2
[B1, B2] = IJ
I
J
n
N
INSTANTON NUMBER
• WE WANT:
m D0
D4
(n+m) D0
N D4
n D0
(n+m) D0

) AND in D4)

$B_2] = IJ$)

TOR NUMBER

· WE CAN USE COHA:

$$H^*_{eq}(\ldots) * H^*_{eq}(\ldots) \to H^*_{eq}(\ldots)$$

$$H^*_{eq}(\ldots) * H^*_{eq}(\ldots) \to H^*_{eq}(\ldots)$$

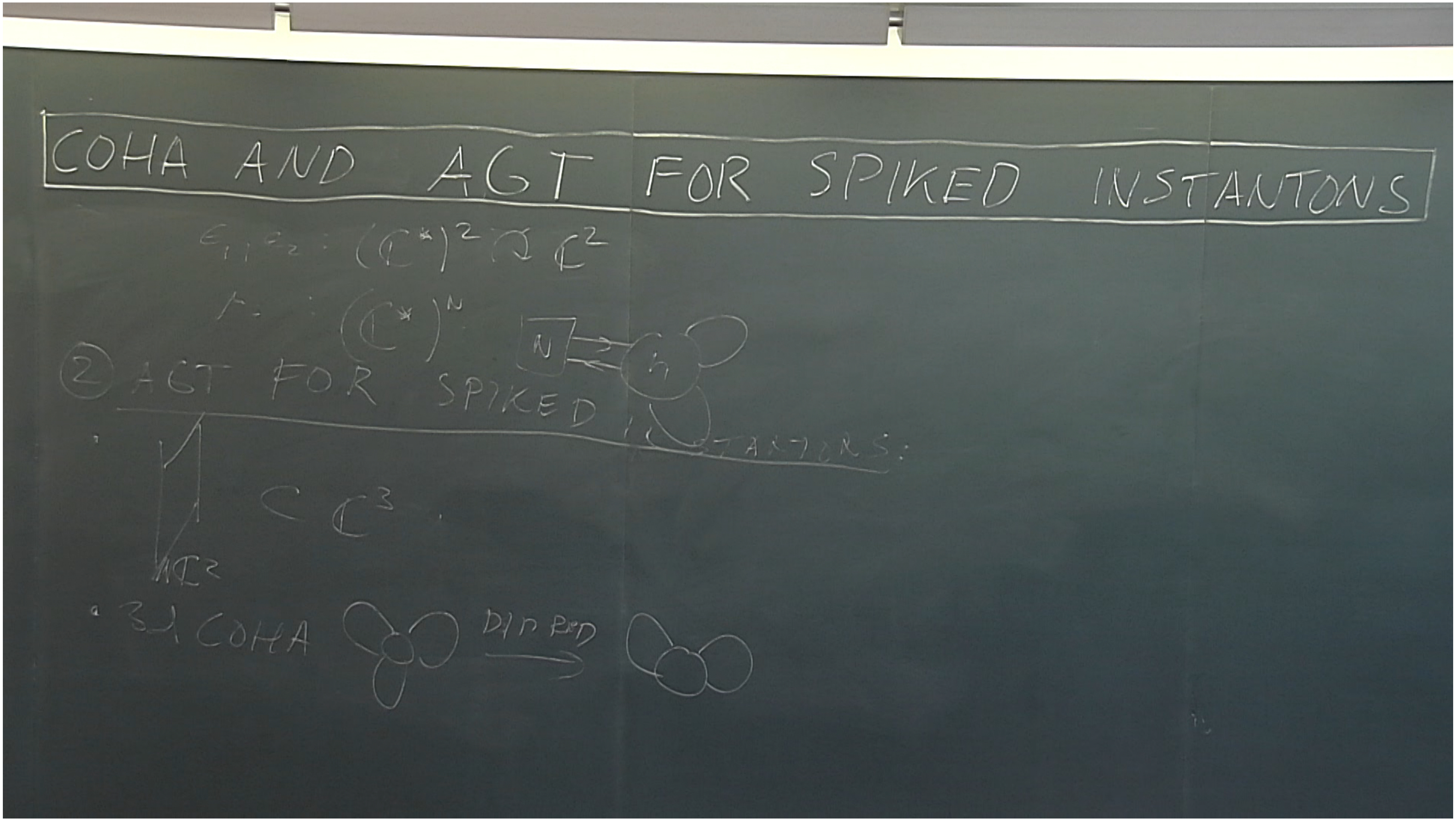
· $COHA^2 = Y(\widehat{gl}(1)) \otimes H^*_{eq}(\ldots)$

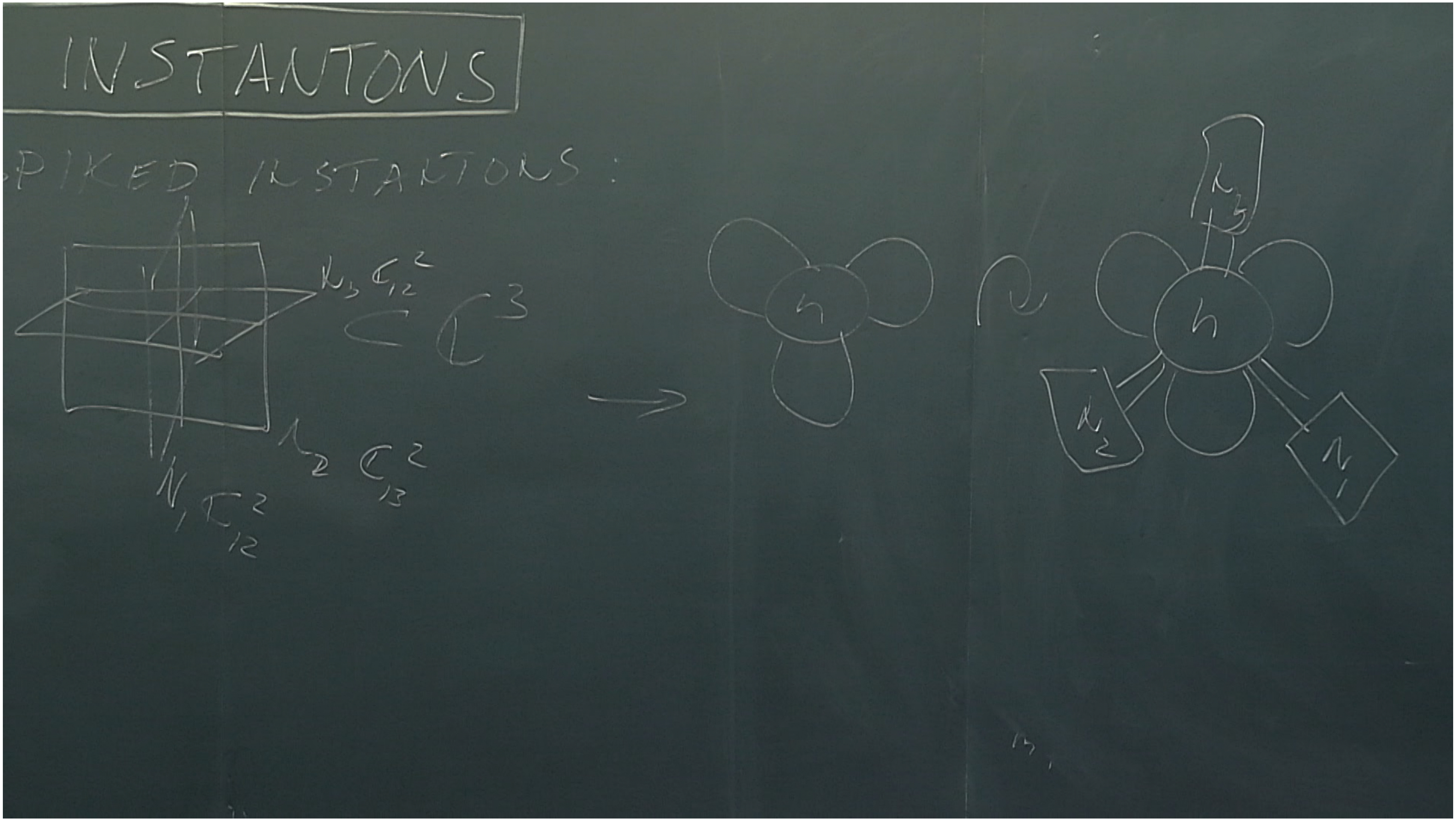
$\Rightarrow \mathcal{W}_N[\Psi]$ ALGEBRA $(W_1(z), W_2(z), \ldots, W_N(z))$

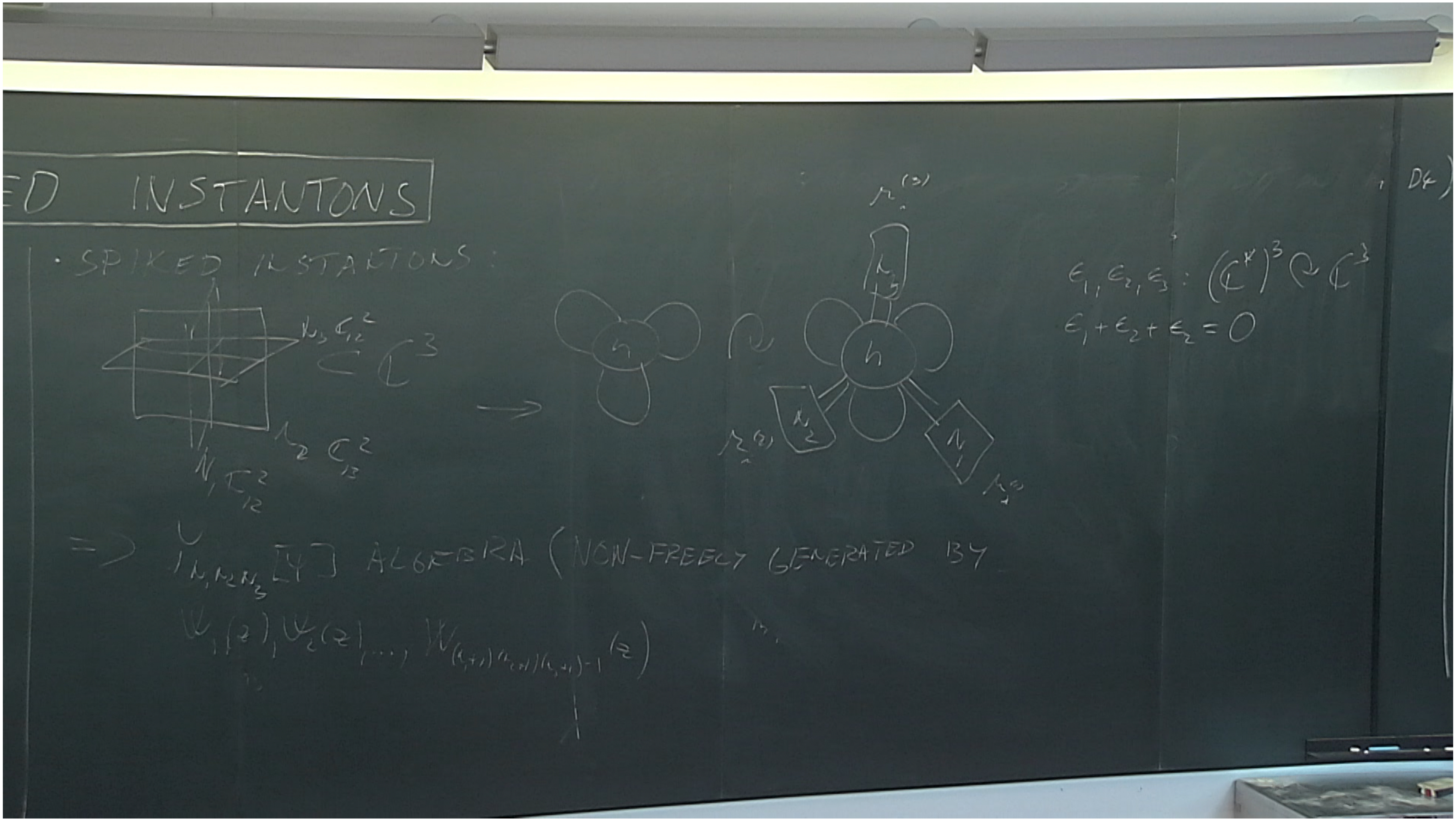
ACTING ON A GENERIC HW MODULE:

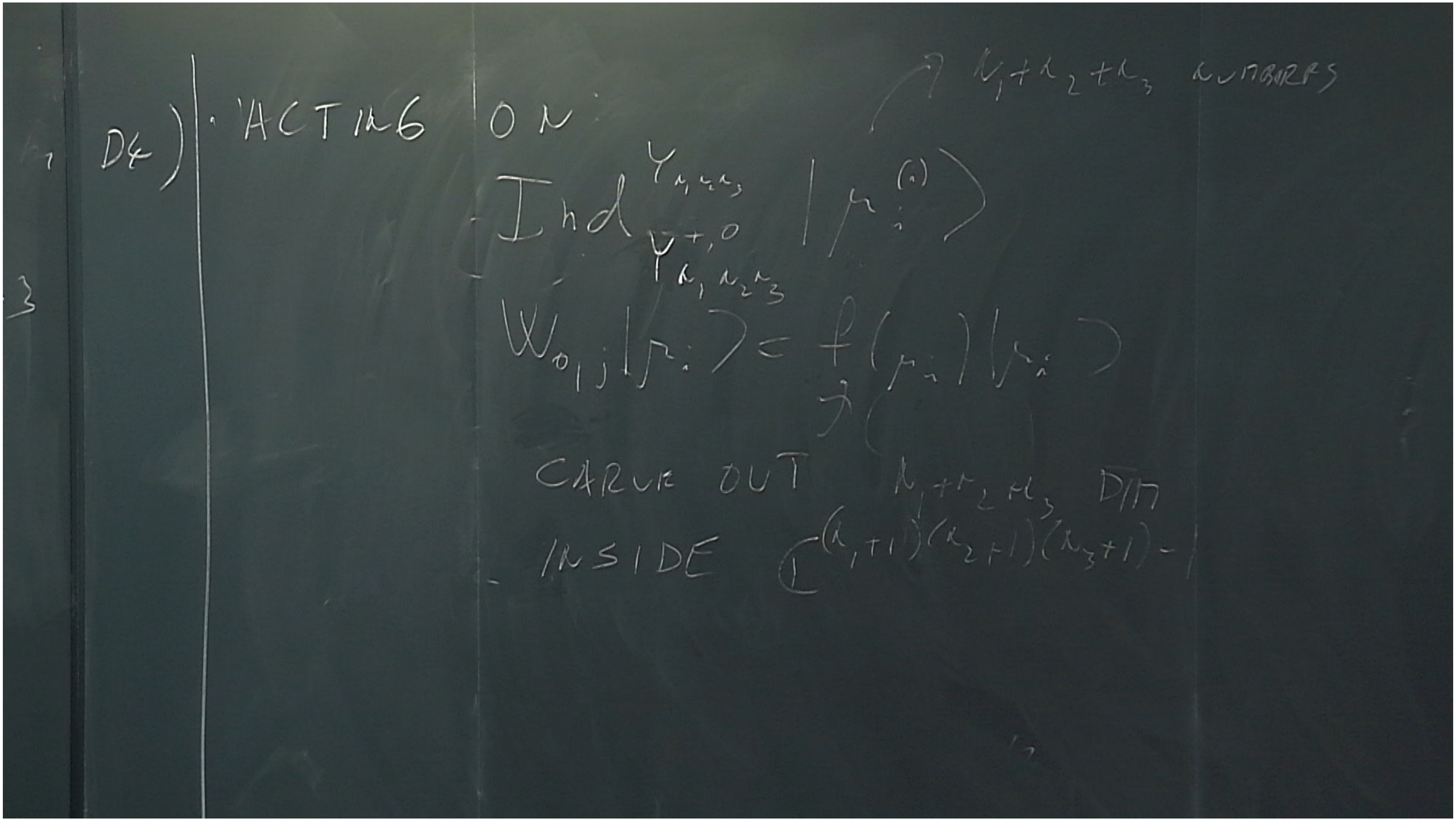
$$W_{n,i}|\mu\rangle = 0 \quad n>0 \quad \Rightarrow \quad M = \mathrm{Ind}^{\mathcal{W}_N}_{\mathcal{W}_N^{\geq 0}}|\mu\rangle$$

$$W_{0,i}|\mu\rangle = f_i(\mu)|\mu\rangle$$


COHA AND AGT FOR SPIKED INSTANTONS
② AGT FOR SPIKED INSTANTONS:
3d COHA
DIM RED


INSTANTONS
PIKED INSTANTONS:


ED INSTANTONS
· SPIKED INSTANTONS:
ε1, ε2, ε3 : (ℂ*)^3 ↻ ℂ^3
ε1 + ε2 + ε3 = 0
⇒ Y_{n1,n2,n3}[Ψ] ALGEBRA (NON-FREELY GENERATED BY
Ψ1(z), Ψ2(z), ..., Ψ_{(n1+1)(n2+1)(n3+1)-1}(z)


ACTING ON:
Ind
CARVE OUT
INSIDE

COHA AND AGT FOR SPIKED

③ FREE FIELDS

• GENERALIZED MIURA:

$$Y_{N_1,N_2,N_3} \subset \left(\mathcal{H}^{(1)}\right)^{\otimes N_1} \otimes \left(\mathcal{H}^{(2)}\right)^{\otimes N_2} \otimes \left(\mathcal{H}^{(3)}\right)^{\otimes N_3}$$

$$[J_{i,n}, J_{j,m}] = -\frac{h_{\mathcal{H}_i}}{h_1 h_2 h_3}\,\delta_{ij}\,\delta_{n,-m}$$

$h_{\mathcal{H}_i}$ → COLOR

$$J(z) = \sum_{n=-\infty}^{\infty} \frac{J_n}{z^{n+1}} \Rightarrow J_i(z) J_j(w) \sim -\frac{h_{\mathcal{H}_i}}{h_1 h_2 h_3} \frac{1}{(z-w)^2}$$

• FR

ED INSTANTONS

• FREE BOSONS :

$$\phi_i(z)\,\phi_j(w) \sim -\frac{h_{x_i}\,\delta_{ij}}{h_1 h_2 h_3}\log(z-w)$$

$$J_i(z) = \partial\phi_i(z)$$

$$\Lambda_j = \exp\left[-\frac{i}{h_{x_j}}\phi_j(z)\right](h_3\partial)^{\frac{h_{x_j}}{h_3}}\exp\left[\frac{i}{h_{x_j}}\phi_j(z)\right]$$

$$j = 1, \dots, N_1+N_2+N_3$$

$\mathcal{H}_n = 3$:

EG: i) $\mathcal{L}_n = h_3 \partial + J_n(z)$, D4)

ii) $\mathcal{H}_n = 1$

$\mathcal{L} = (h_3 \partial)^{\frac{h_1}{h_3}} + \frac{h_1}{h_3} J_n(z) h_3 \partial^{\frac{h_1}{h_3} - 1} + \dots$

$\log(z - w)$

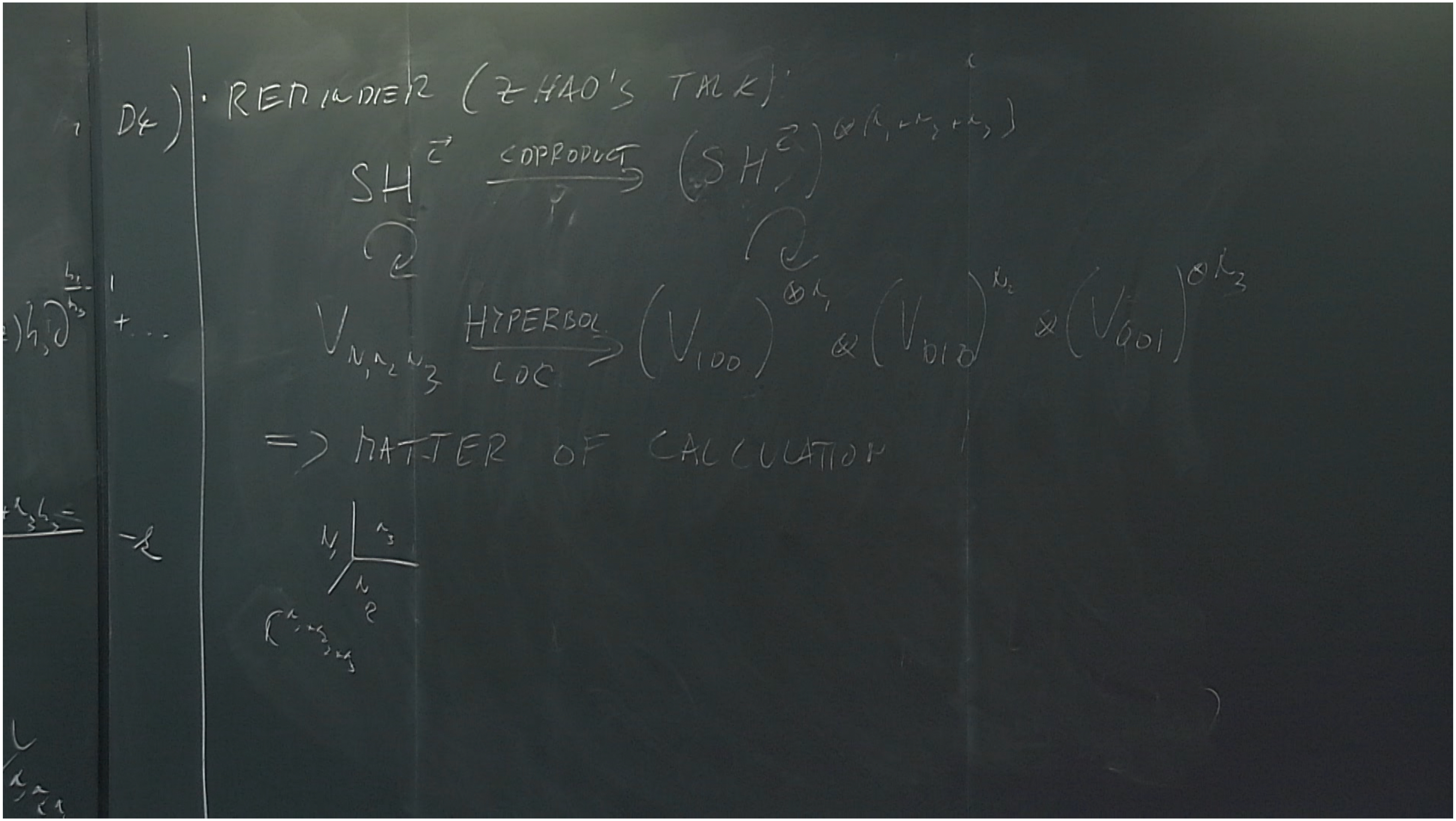
$)^{\frac{h_{x_j}}{h_3}} \exp\left[\frac{n}{h_{x_i}} \phi_j(z)\right]$

• DEF: $Y_{h_1 h_2 h_3}$:

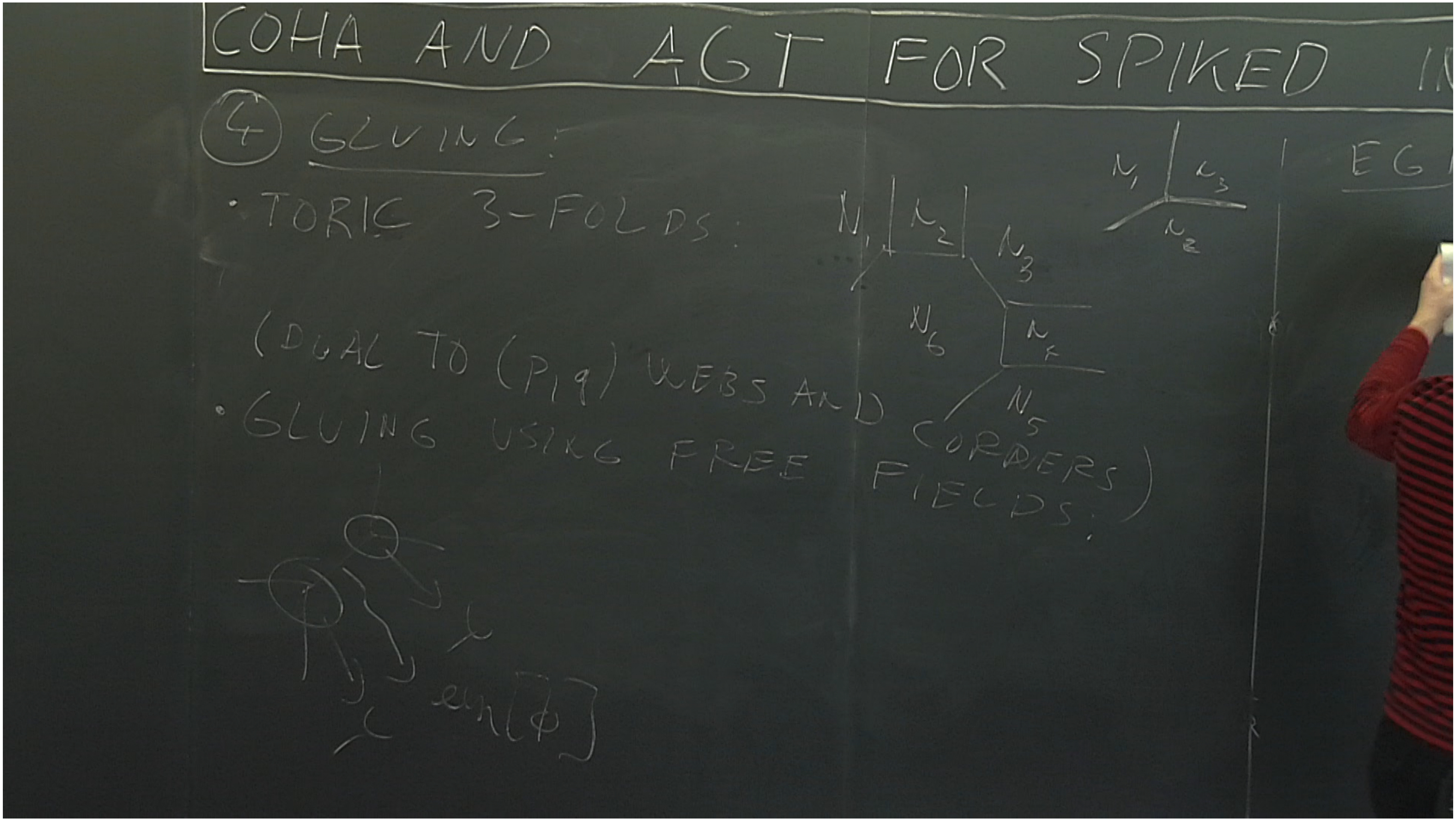
$$\prod_{n=1}^{h_1+h_2+h_3} \mathcal{L}_n = \sum_{\ell=0}^{\infty} U_\ell(z)\, \partial^{\frac{h_1 h_1 + h_2 h_2 + h_3 h_3 -}{h_3} - \ell}$$

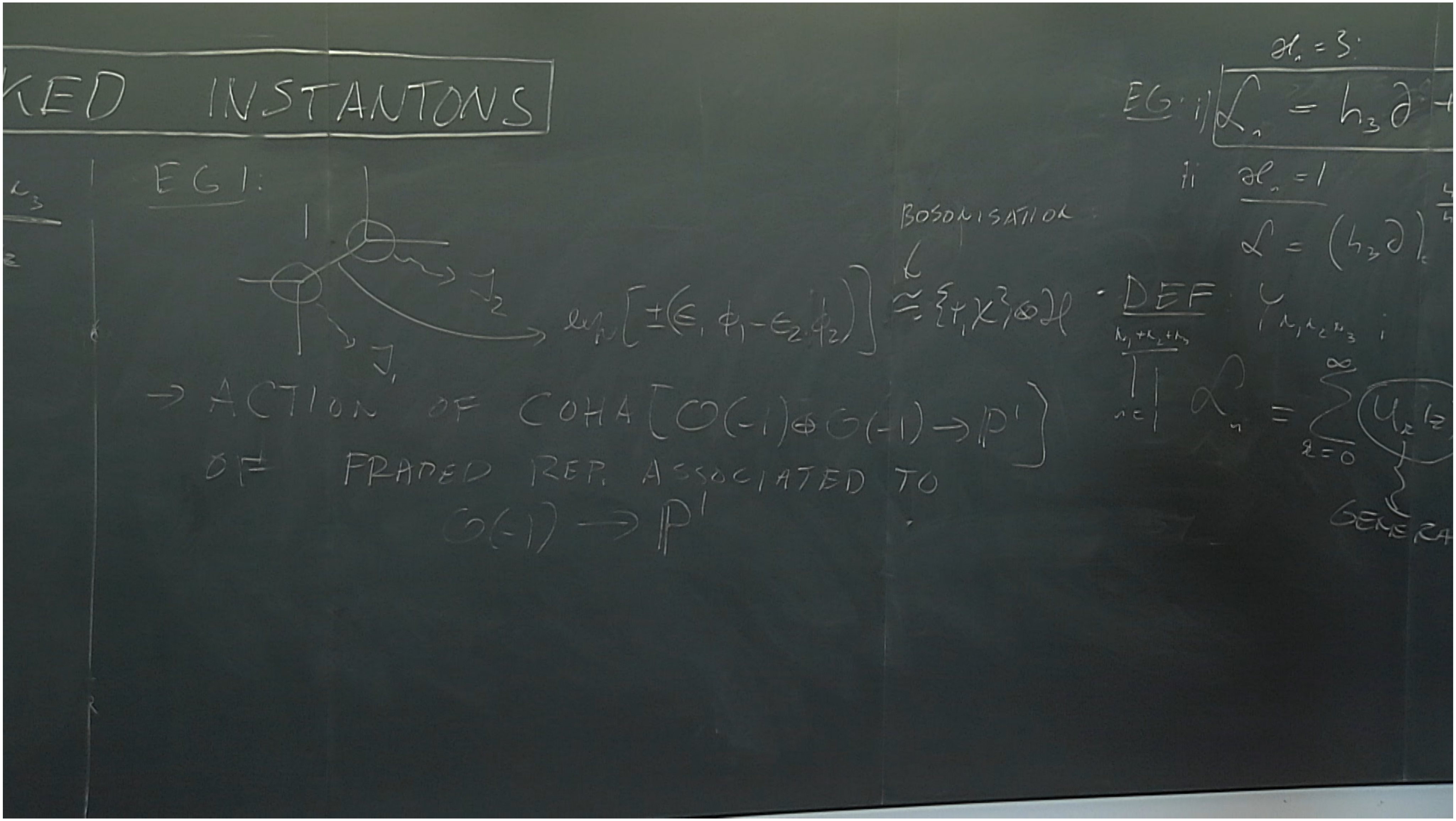
GENERATORS of $Y_{h_1 h_2 h_3}$

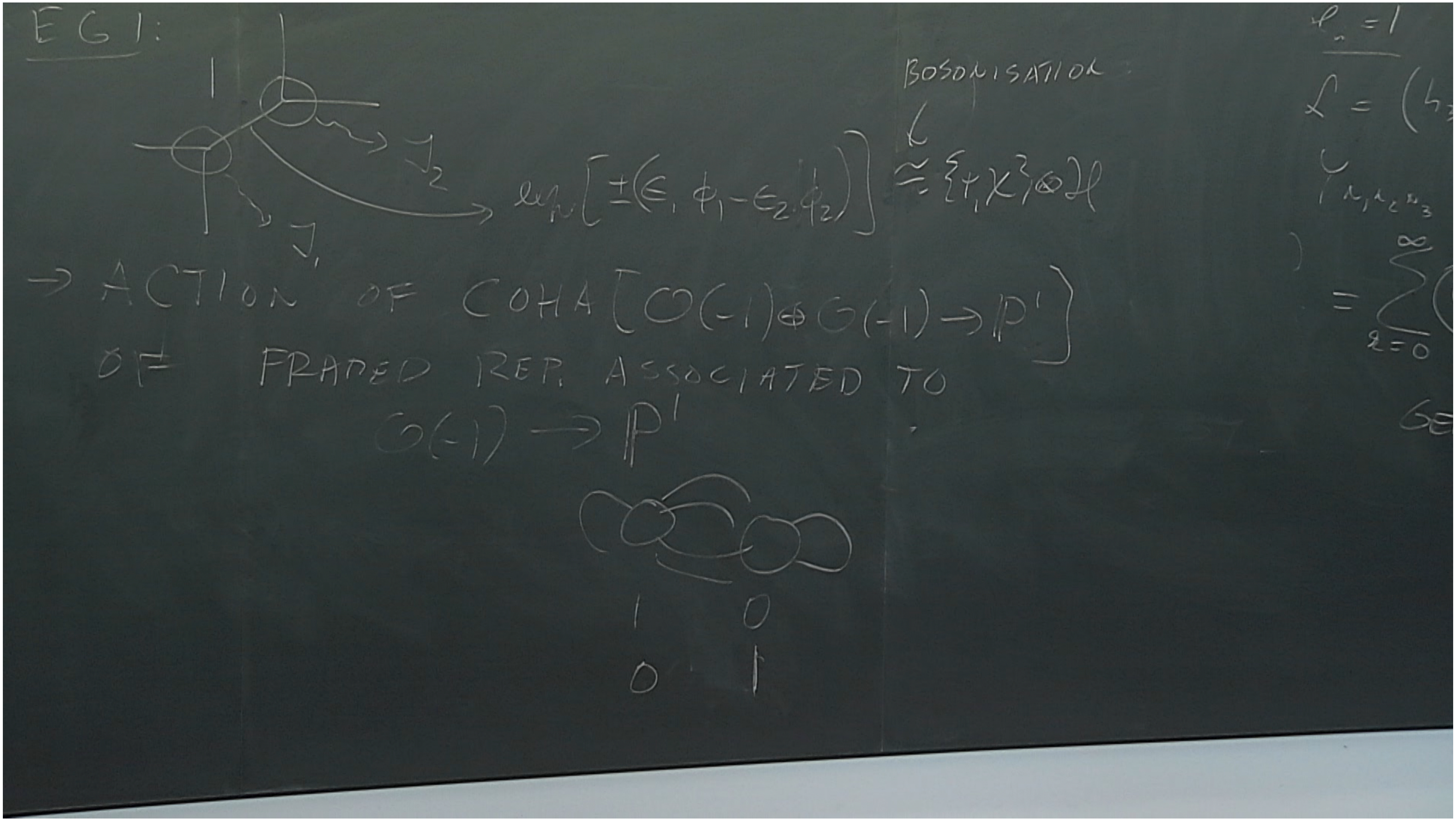
• ACTING

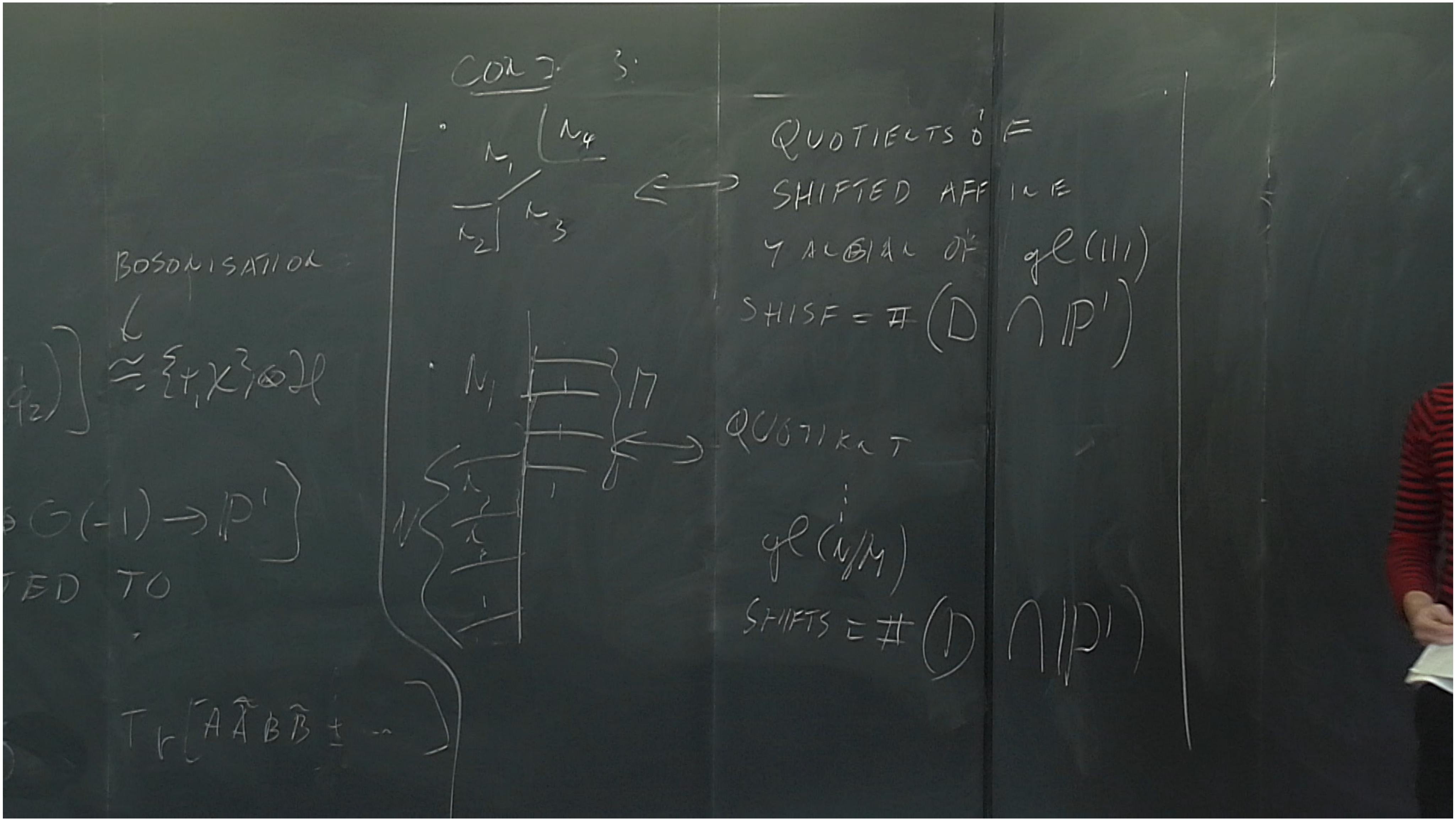


· REMINDER (ZHAO'S TALK):
SH COPRODUCT (SH)
HYPERBOL LOC
=> MATTER OF CALCULATION


COHA AND AGT FOR SPIKED IN
④ GLUING:
· TORIC 3-FOLDS:
(DUAL TO (p,q) WEBS AND CORNERS)
· GLUING USING FREE FIELDS:
N_1 N_2 N_3 N_4 N_5 N_6
EG


KED INSTANTONS
EG I:
BOSONISATION
DEF
→ ACTION OF COHA
OF FRAMED REP. ASSOCIATED TO
GENERA


EG 1:
BOSONISATION
J_2
J_1
$\exp\left[\pm(\epsilon_1 \phi_1 - \epsilon_2 \phi_2)\right] \simeq \{\psi, \chi\} \otimes \mathcal{H}$
→ ACTION OF COHA$[\mathcal{O}(-1) \oplus \mathcal{O}(-1) \to \mathbb{P}^1]$
OF FRAMED REP. ASSOCIATED TO
$\mathcal{O}(-1) \to \mathbb{P}^1$
1 0
0 1


BOSONISATION
QUOTIENTS OF
SHIFTED AFFINE
YANGIAN OF gl(1|1)
QUOTIENT
gl(N/M)
TO