Title: Algebraic structures of T[M3] and T[M4]

Speakers: Sergei Gukov

Collection: Cohomological Hall Algebras in Mathematics and Physics

Date: February 26, 2019 - 11:00 AM

URL: http://pirsa.org/19020058

Abstract: The talk will focus on tensor categories associated with 3d N=2 theories and chiral algebras associated with 2d N=(0,2) theories, as well as their combinations that involve 3d N=2 theories "sandwiched" by half-BPS boundary conditions and interfaces. Such situations, originally studied in a joint work with A.Gadde and P.Putrov, have a variety of applications, including applications to topology of 3-manifolds and 4-manifolds where Kirby moves translate into novel dualities of 3d N=2 and 2d N=(0,2) theories and where the corresponding algebraic structures can be related to COHAs. After reviewing some elements of that story going back to 2013, I will focus on the latest developments in the area of "3d Modularity" where mock Jacobi forms, SL(2,Z) Weil representations, quantum modular forms, non-semisimple modular tensor categories, and chiral algebras of logarithmic CFTs make a surprising appearance (based on recent and ongoing work with M.Cheng, S.Chun, F.Ferrari, S.Harrison and B.Feigin).

1) Intro.

2) Conj

3) Characters

Topology | **Physics**

$M_3 \longrightarrow$ 3d $\mathcal{N}=2$ theory

$T[M_3]$

M_4

$G = U(N)$

$SU(N)$

Topology

M_3

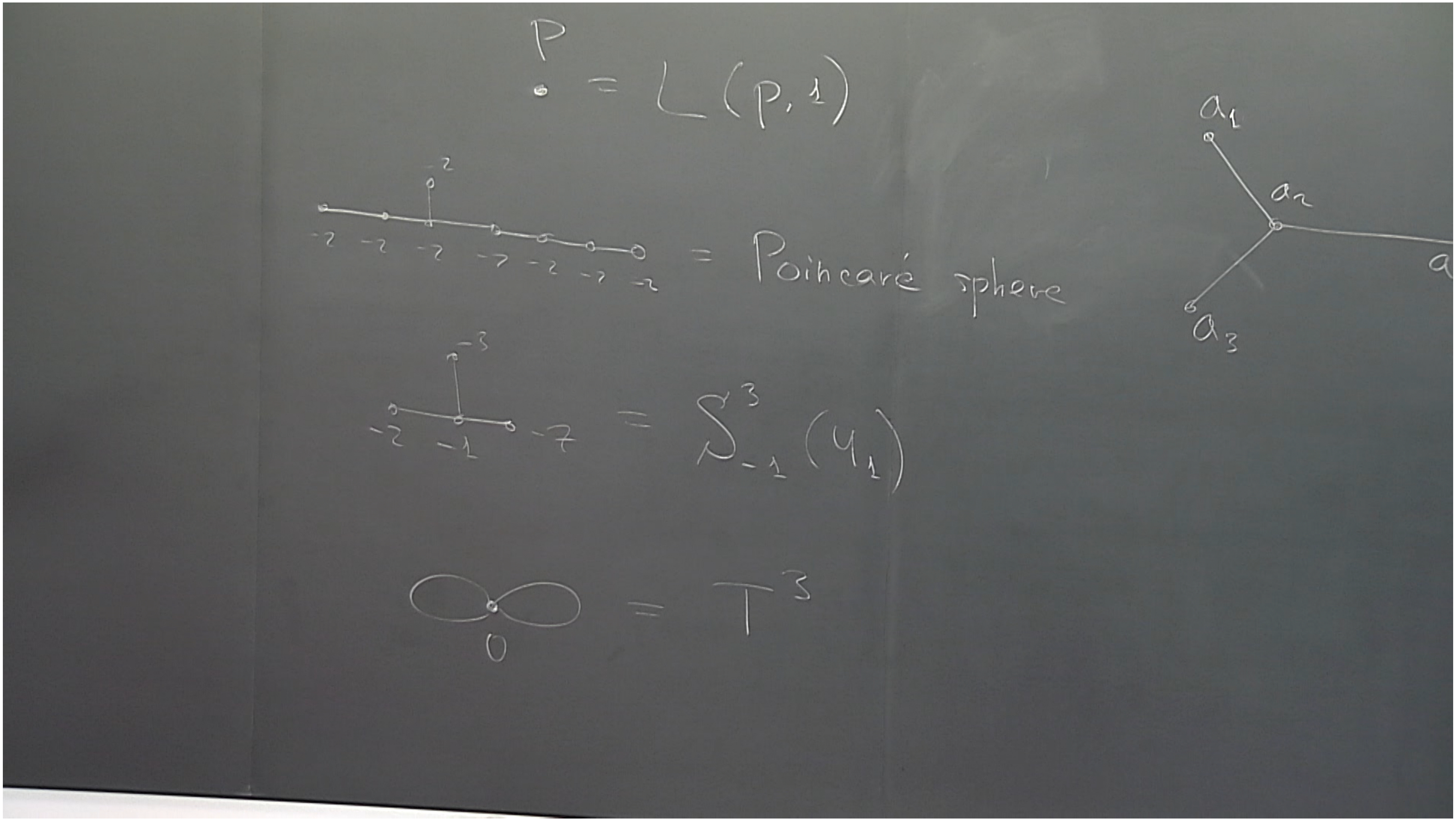
M_4

Topology | Physics

$M_3 \longrightarrow$ 3d $\mathcal{N}=2$ theory $T[M_3] \longrightarrow$;

$M_4 \longrightarrow$ 2d $\mathcal{N}=(0,2)$ theory $T[M_4] \longrightarrow$;

Topology	Physics	Algebra
M_3 $\longrightarrow$	3d $\mathcal{N}=2$ theory $T[M_3]$ $\longrightarrow$	$SW(M_3)$
M_4 $\longrightarrow$	2d $\mathcal{N}=(0,2)$ theory $T[M_4]$ $\longrightarrow$	$SW(M_4)$

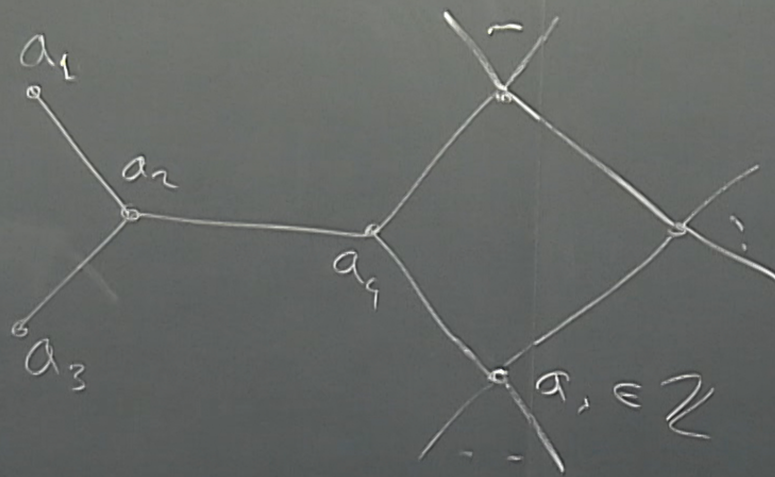

P
= L(p, 1)
-2
-2 -2 -2 -2 -2 -2 -2
= Poincaré sphere
-3
-2 -1 -7
= $S^3_{-1}(4_1)$
0
= T^3
a_1
a_2
a_3
a

$, 1)$

oincaré sphere

(Y_1)

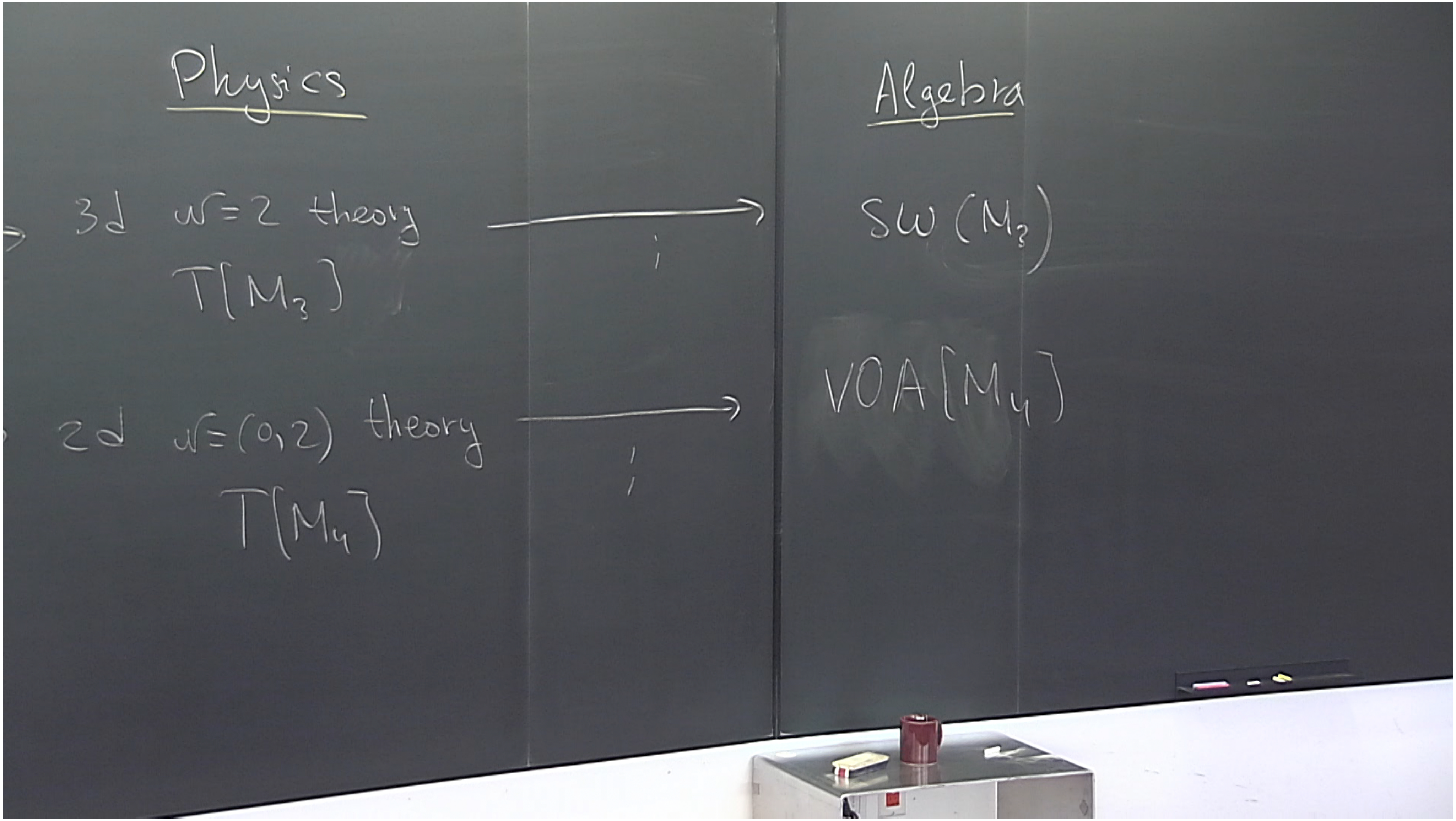
-3

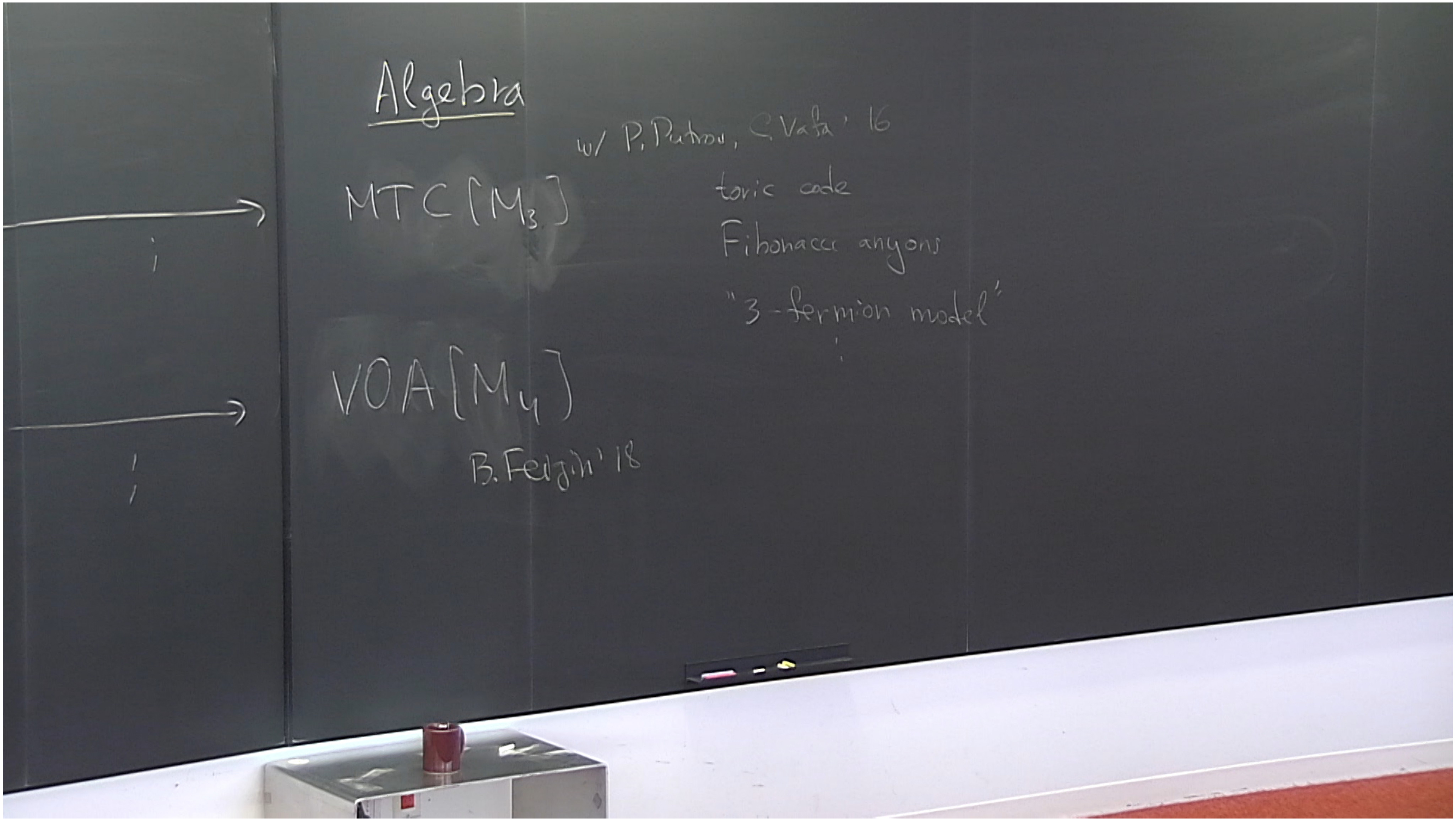


$G = U(N)$

$SU(N)$

Q = adjacency matrix


Physics
Algebra
3d $\mathcal{N}=2$ theory $T[M_3]$
$SW(M_3)$
2d $\mathcal{N}=(0,2)$ theory $T[M_4]$
$VOA[M_4]$


Algebra
w/ P. Putrov, C. Vafa '16
MTC[M3]
toric code
Fibonacci anyons
"3-fermion model"
VOA[M4]
B. Feigin '18

3d $\mathcal{N}=2$ on $S^1 \times \Sigma_g$

To

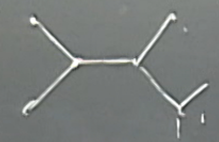
$\rightsquigarrow$ category

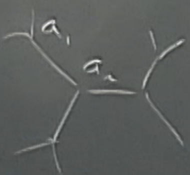
$$Z(S^1 \times \Sigma_g) =$$

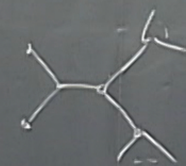
$$= \sum_{\lambda:\ \text{sol'ns to BAE}} \left(S_{0\lambda}\right)^{2-2g} \sim \sum_{\lambda} \left(\text{qdim}\, M_\lambda\right)^{2-2g}$$

λ: sol'ns to BAE

$$\exp\left(\frac{\partial \widetilde{W}(z)}{\partial \log z_i}\right) = 1$$

$SL(2,\mathbb{Z}) \subset \hat{V} = K^0(\mathrm{MTC}[$  $]) = \langle$ components of <u>all</u> $G_{\mathbb{C}}$ flat connections on M_3 $\rangle$

<u>Conj 1.</u> 

$\longrightarrow$ log-VOA[]

Conj ["Mirror Symmetry"]:

$a_i \mapsto -a_i$

"leaking" $\quad q \leftrightarrow \frac{1}{q}$

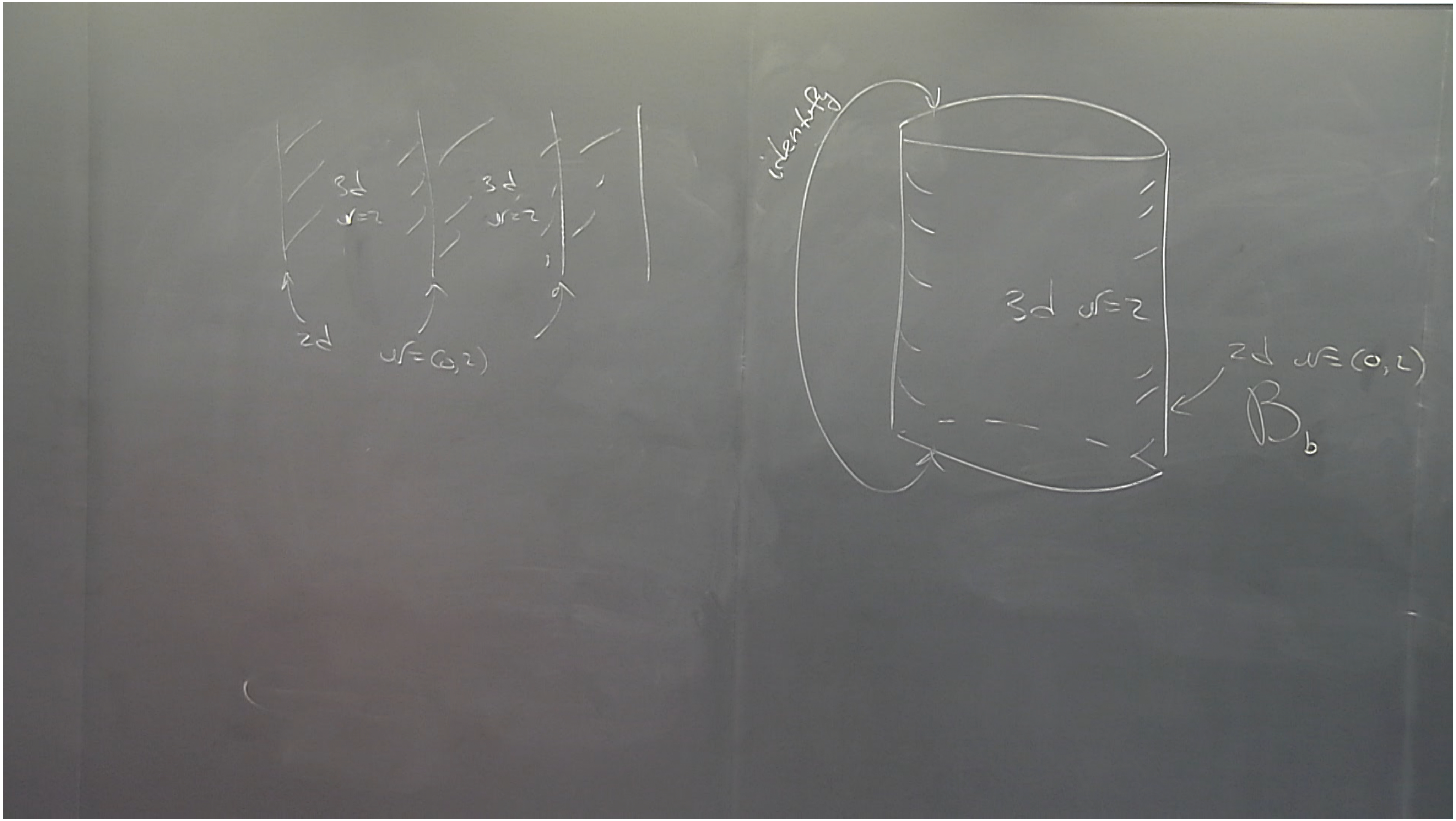
$\boxed{\tau}$

$$\chi_b(q) = \sum_n c_n \tau^n$$

$$\downarrow$$

$$\uparrow$$

$$\tilde{\chi}_b(q) = \sum_n c_n (-\tau)^n$$


3d
N=2
3d
N=2
2d
N=(0,2)
identify
3d N=2
2d N=(0,2)
B_b

$a \in (0,1)$

D_b

$$\hat{Z}_b(q) = Z(S^1 \times_q D^2 ; \mathcal{B}_a) = \mathrm{Tr}\,(-1)^F q^{\frac{R}{2}+J_3} =$$

$$= \int_{|x_v|=1} \frac{dx}{x}\, F_{3d}(x)\, \Theta^{(b)}_{2d}(x)$$

$$F_{3d}(x) = \prod_{v \in \mathrm{Vertices}} \left(x_v - \frac{1}{x_v}\right)^{2-\deg(v)}$$

quadratic form Q

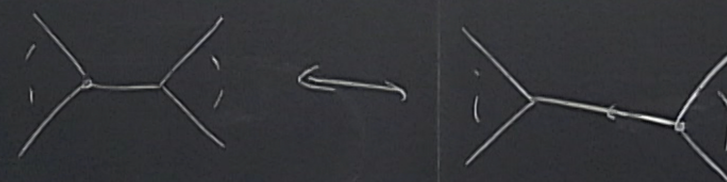
<u>Thm:</u> $\hat{Z}_b(q)$ converges in $|q|<1$

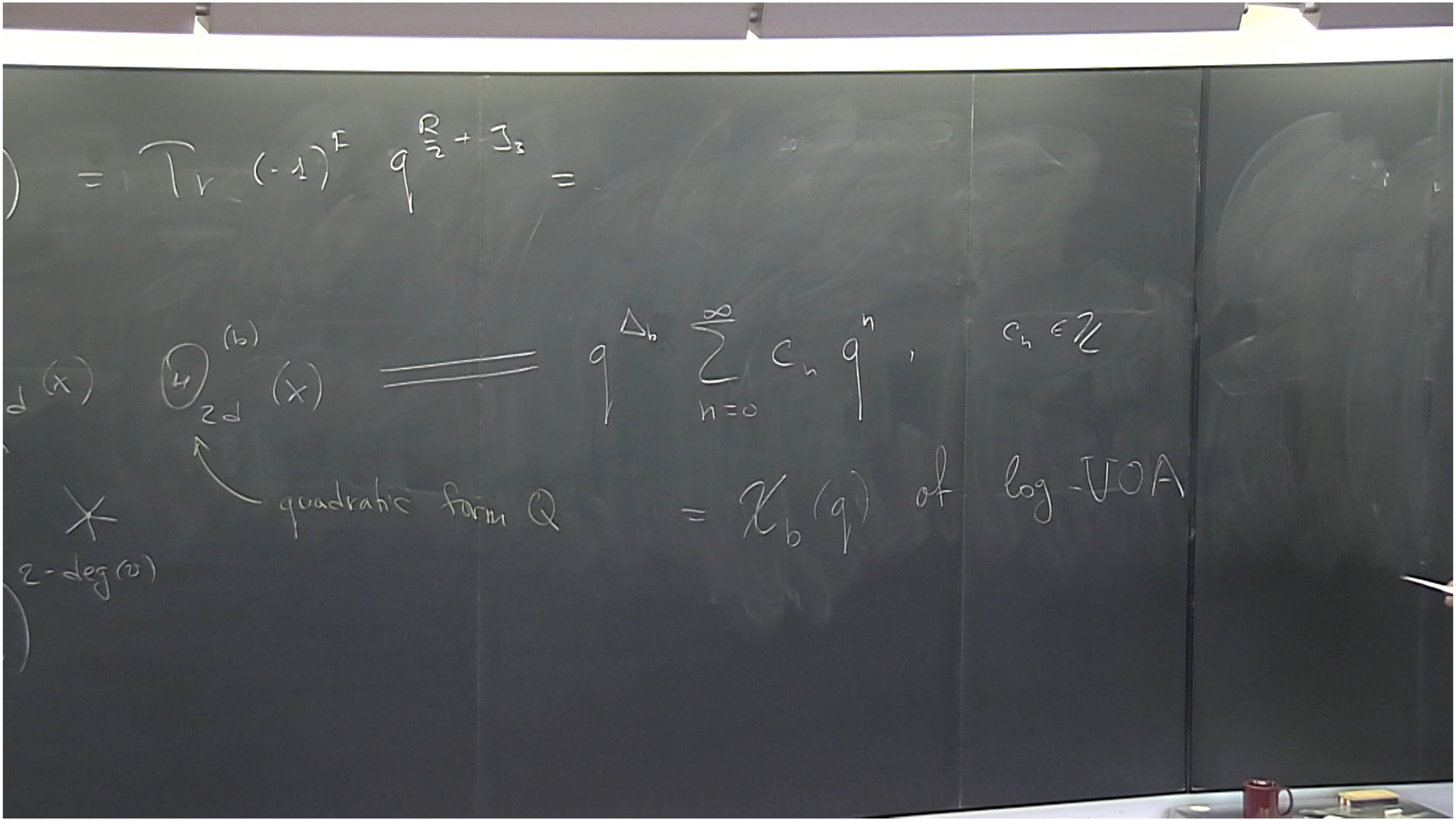
$$\hat{Z}_b(q) \in q^{\Delta_b}\mathbb{Z}[[q]]$$

Thm: - $\hat{Z}_b(q)$ converges in $|q|<1$

- $\hat{Z}_b(q) \in q^{\Delta}\mathbb{Z}[[q]]$

- invariant under Kirby moves




$= \mathrm{Tr}\, (-1)^F q^{\frac{R}{2} + J_3} =$
$\hat{Z}_b$
$d\,(x)$
$(u)^{(b)}_{2d}\,(x) \equiv q^{\Delta_b} \sum_{n=0}^{\infty} c_n q^n, \quad c_n \in \mathbb{Z}$
quadratic form Q
$= \chi_b(q)$ of log-VOA
2 - deg (v)

$=$

$= q^{\Delta_b} \sum_{n=0}^{\infty} c_n q^n , \quad c_n \in \mathbb{Z}$

form Q

$= \chi_b(q)$ of log-VOA

Thm: $- \hat{Z}_b(q$

$- \hat{Z}_b($

$b \in \begin{pmatrix} \text{Abelian} \\ \text{flat conn.} \\ \text{on } M_3 \end{pmatrix}$

$-$ invari

Q: Why? "3d Modularity"

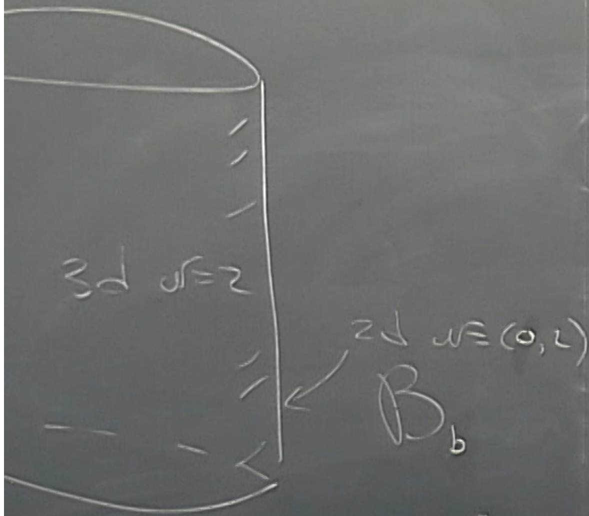
$$\Theta^{(b)}_{2d}(x) = q^{\Delta_b} \sum_{n=0}^{\infty} c_n q^n , \quad c_n \in \mathbb{Z}$$

quadratic form Q

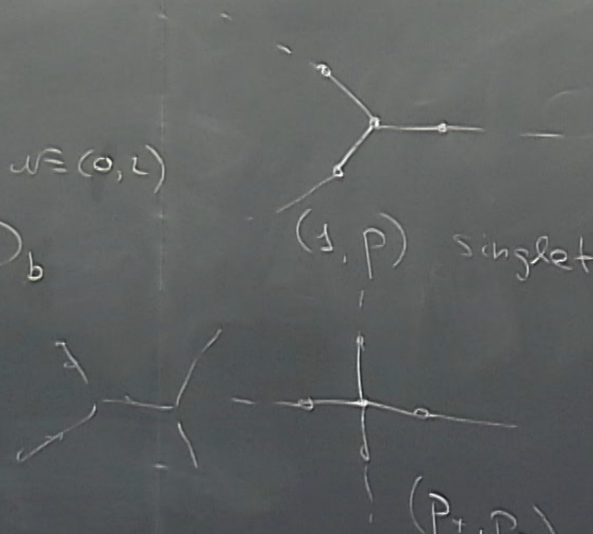
deg(v)

$$= \chi_b(q) \text{ of log-VOA}$$

$b \in$ (Abelia[n]
flat
on M



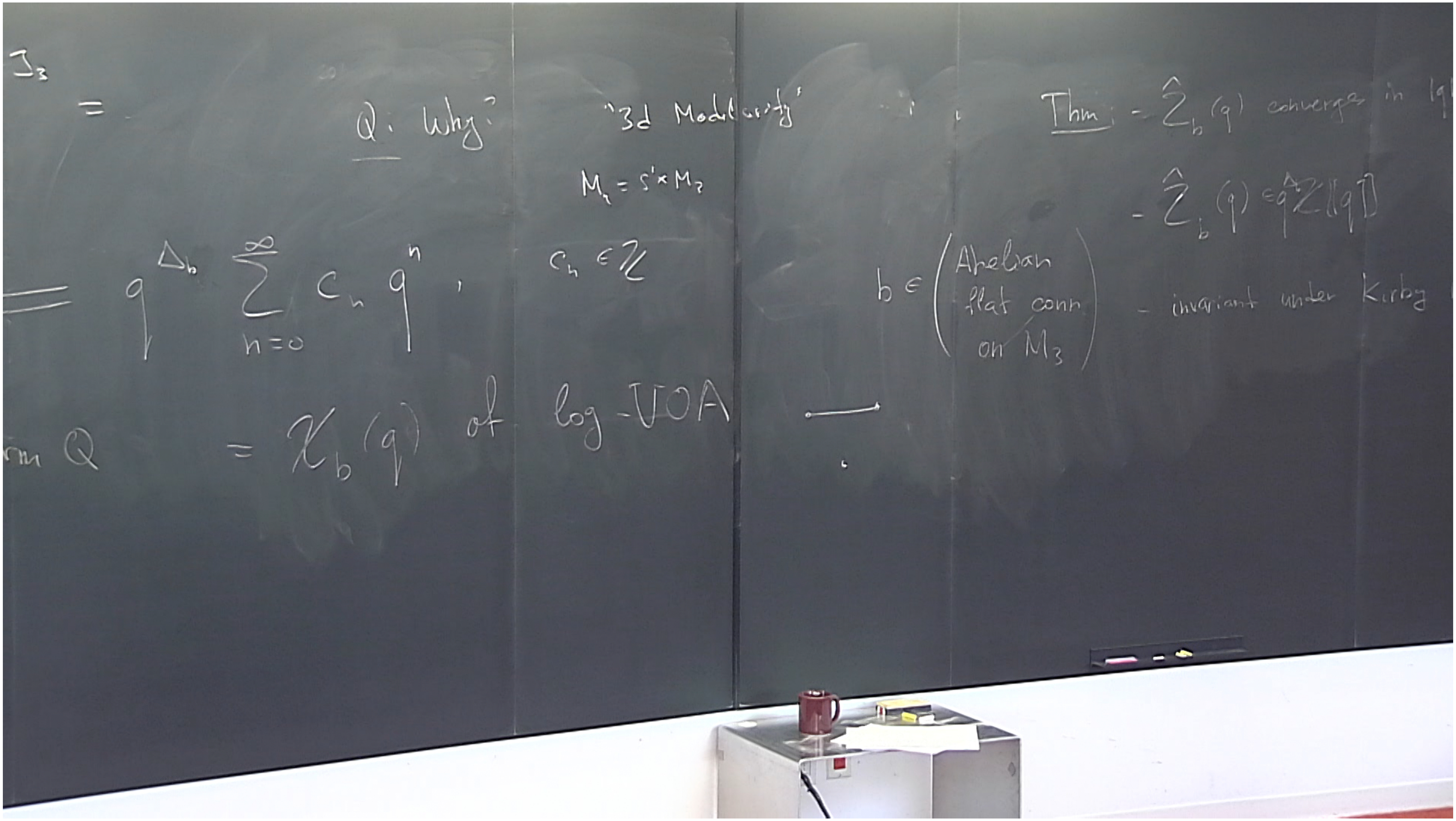
$$\hat{Z}_b(q) = Z(S^1 \times_q D^2 ; \mathcal{B}_a) = \mathrm{Tr}\,(-1)^F$$



$$= \int_{|x_v|=1} \frac{dx}{x} \, F_{3d}(x) \; \Theta^{(b)}_{2d}(x)$$

$$\prod_{v \in \text{Vertices}} \left(x_v - \frac{1}{x_v} \right)^{2-\deg(v)}$$

quadrat


Q: Why?
"3d Modularity"
$M_4 = S^1 \times M_3$
Thm: - $\hat{Z}_b(q)$ converges in |q|
- $\hat{Z}_b(q) \in q^{\Delta}\mathbb{Z}[[q]]$
- invariant under Kirby
$q^{\Delta_b} \sum_{n=0}^{\infty} c_n q^n$, $c_n \in \mathbb{Z}$
$b \in$ (Abelian flat conn on M_3)
$= \chi_b(q)$ of log-VOA