Title: Networks of intertwiners, 3d theories and superalgebras

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Collection: Cohomological Hall Algebras in Mathematics and Physics

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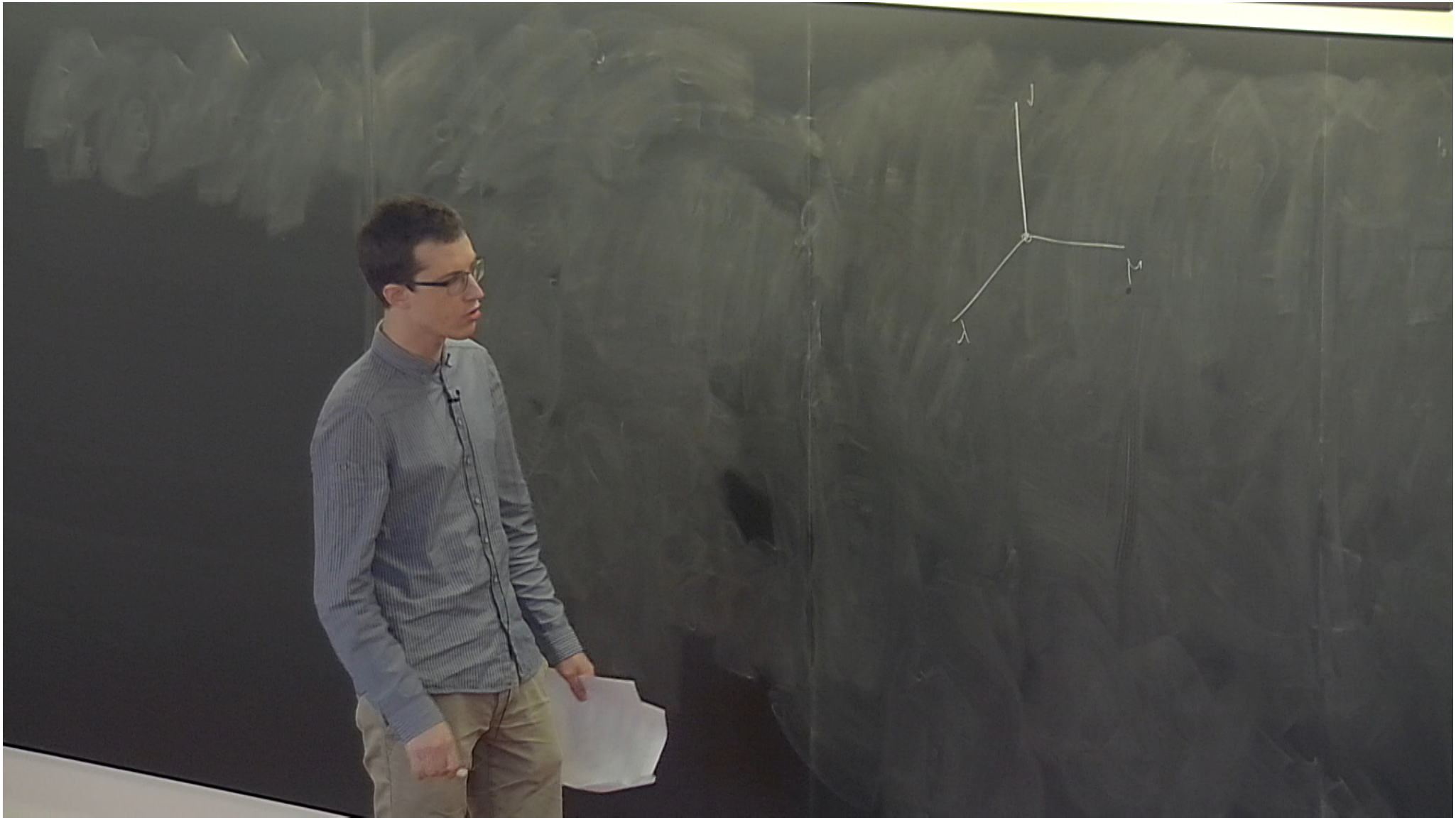
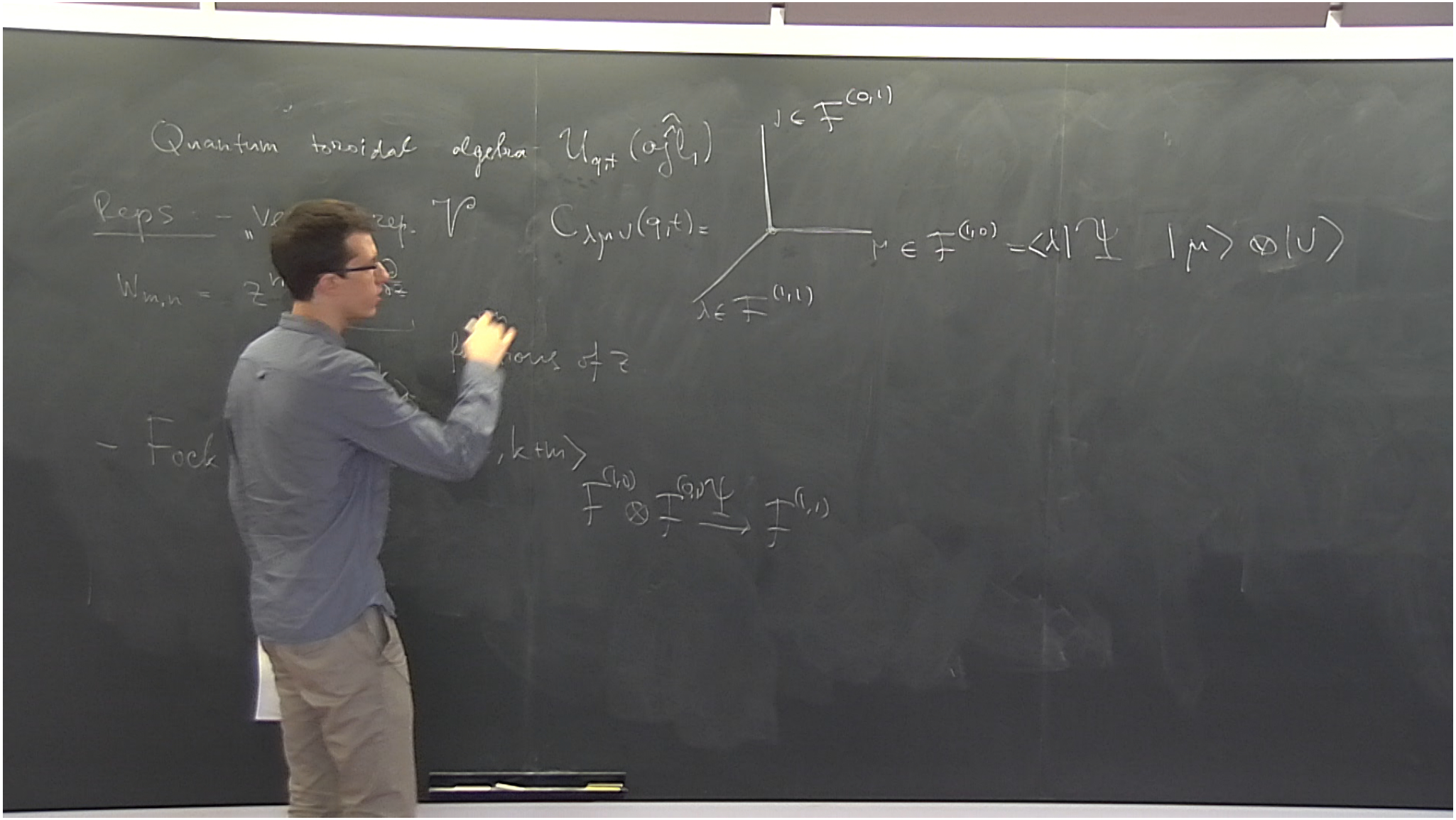
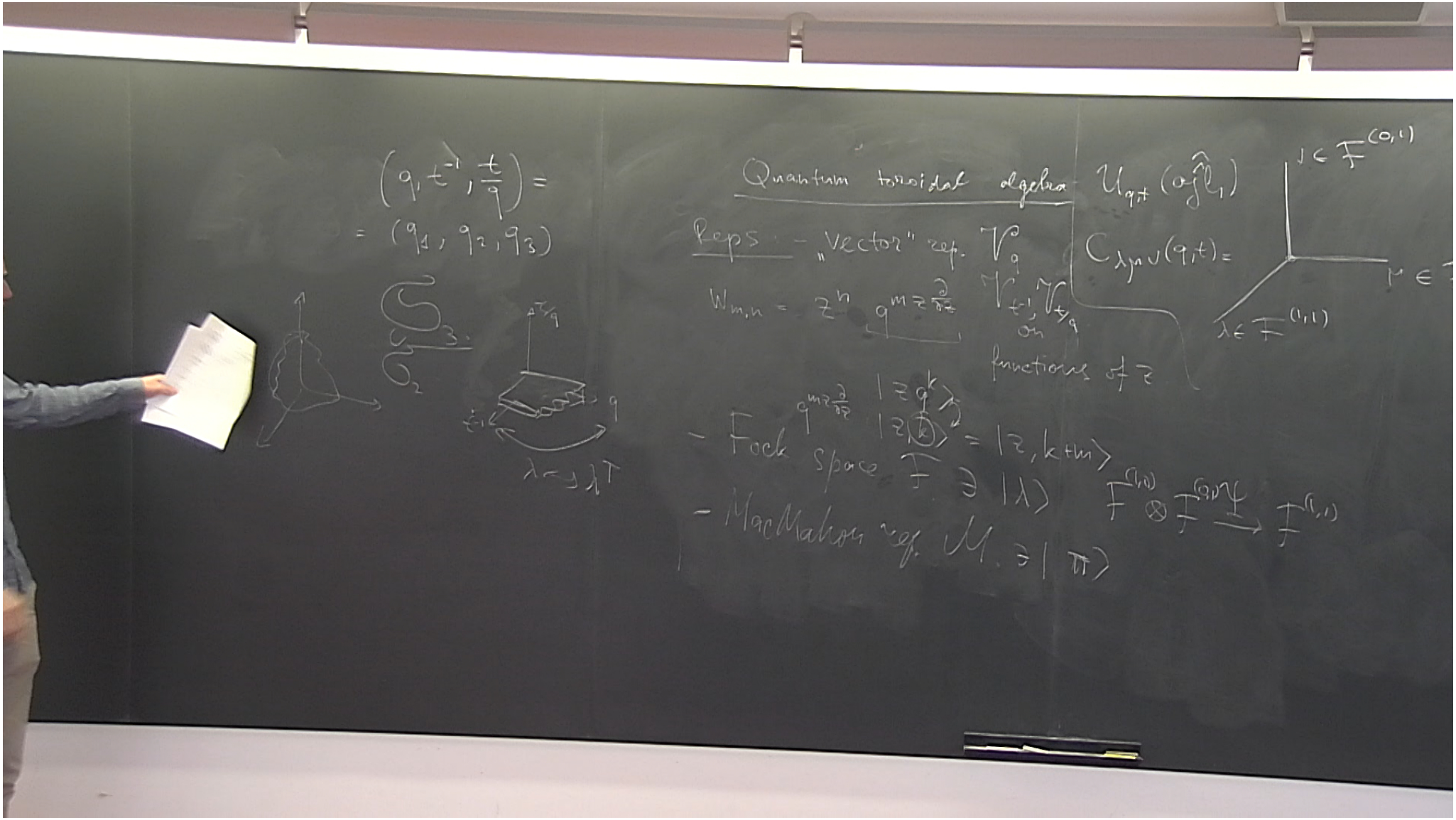
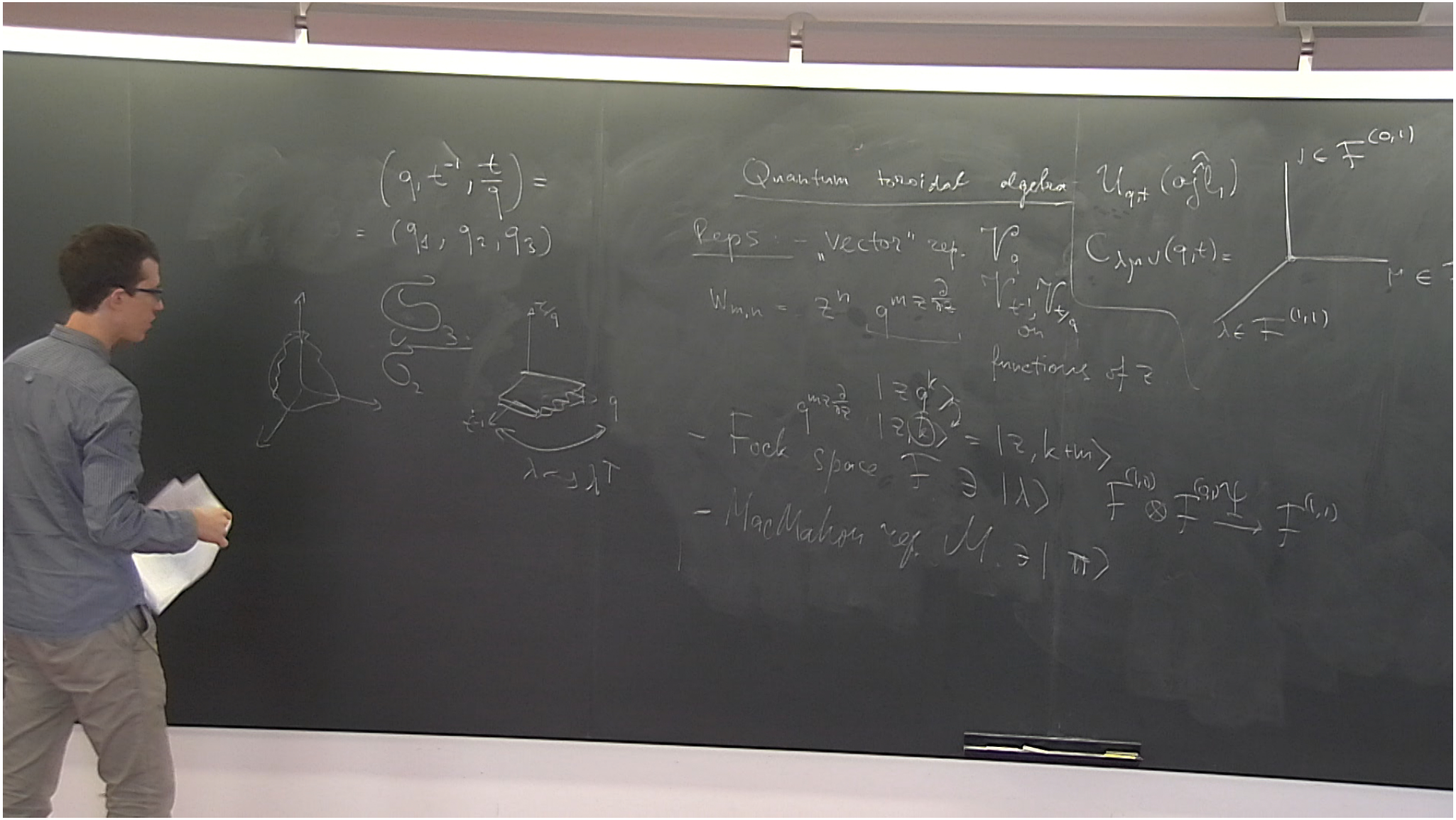
Abstract: Refined topological vertex formalism allows one to conveniently compute partition functions of topological strings on toric CY backgrounds. These partition functions reproduce instanton partition functions of 5d N=1 gauge theories, obtained from the CY by the geometric engineering procedure. In the algebraic language the vertices can be described as intertwiners of Fock representations of a quantum toroidal algebra. I will present a Â«HiggsedÂ» version of refined topological vertex formalism which computes vortex partition functions of certain N=2* 3d theories, and show how it naturally arises in the algebraic approach. The new formalism gives a streamlined way to write down the screening charges of a general class of q-deformed W-algebras, including those associated with superalgebras. The obtained partition functions are automatically eigenfunctions of Ruijsenaars-Schneider Hamiltonians or their supersymmetric generalizations.




Quantum toroidal algebra $U_{q,t}(\widehat{\mathfrak{gl}}_1)$
Reps
$W_{m,n} =$
$C_{\lambda\mu\nu}(q,t) =$
$\nu \in \mathcal{F}^{(0,1)}$
$\mu \in \mathcal{F}^{(1,0)}$
$\lambda \in \mathcal{F}^{(1,1)}$
$= \langle \lambda | \Psi | \mu \rangle \otimes (U)$
$\mathcal{F}^{(1,0)} \otimes \mathcal{F}^{(0,1)} \xrightarrow{\Psi} \mathcal{F}^{(1,1)}$


$(q, t^{-1}, \frac{t}{q}) =$
$= (q_1, q_2, q_3)$
Quantum toroidal algebra
$U_{q,t}(\widehat{\mathfrak{gl}}_1)$
Reps - „vector" rep. V_q
$W_{m,n} = z^n q^{m z \frac{\partial}{\partial z}}$
$V_{t^{-1}}, V_{t/q}$
on functions of z
$C_{\lambda\mu\nu}(q,t) =$
$\nu \in \mathbb{F}^{(0,1)}$
$\lambda \in \mathbb{F}^{(1,1)}$
$q^{m z \frac{\partial}{\partial z}} |z q^k\rangle = |z, k+m\rangle$
- Fock space $\mathbb{F} \ni |\lambda\rangle$
- MacMahon rep. $\mathcal{M} \ni |\Pi\rangle$
$\mathbb{F}^{(1,0)} \otimes \mathbb{F}^{(0,1)} \xrightarrow{\Psi} \mathbb{F}^{(1,1)}$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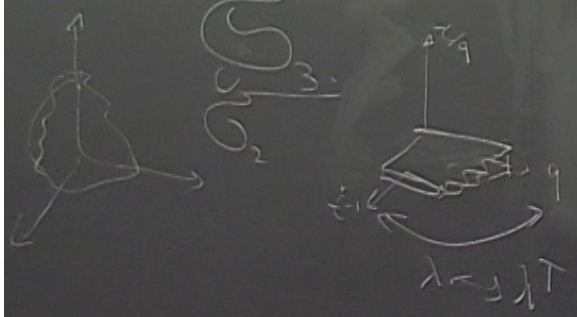


Quantum toroidal algebra $\mathcal{U}_{q,t}(\widehat{\mathfrak{gl}}_1)$

Reps - "vector" rep. V_q $(0,0)$

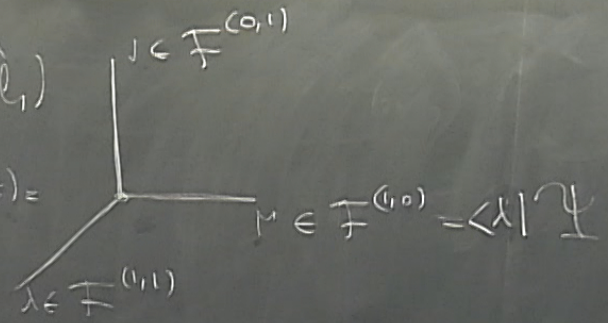
$W_{m,n} = z^n q^{m z \frac{\partial}{\partial z}}$ $V_{t^{-1}}, V_{t/q}$ on functions of z

$q^{m z \frac{\partial}{\partial z}} |z q^k\rangle = |z, k+m\rangle$

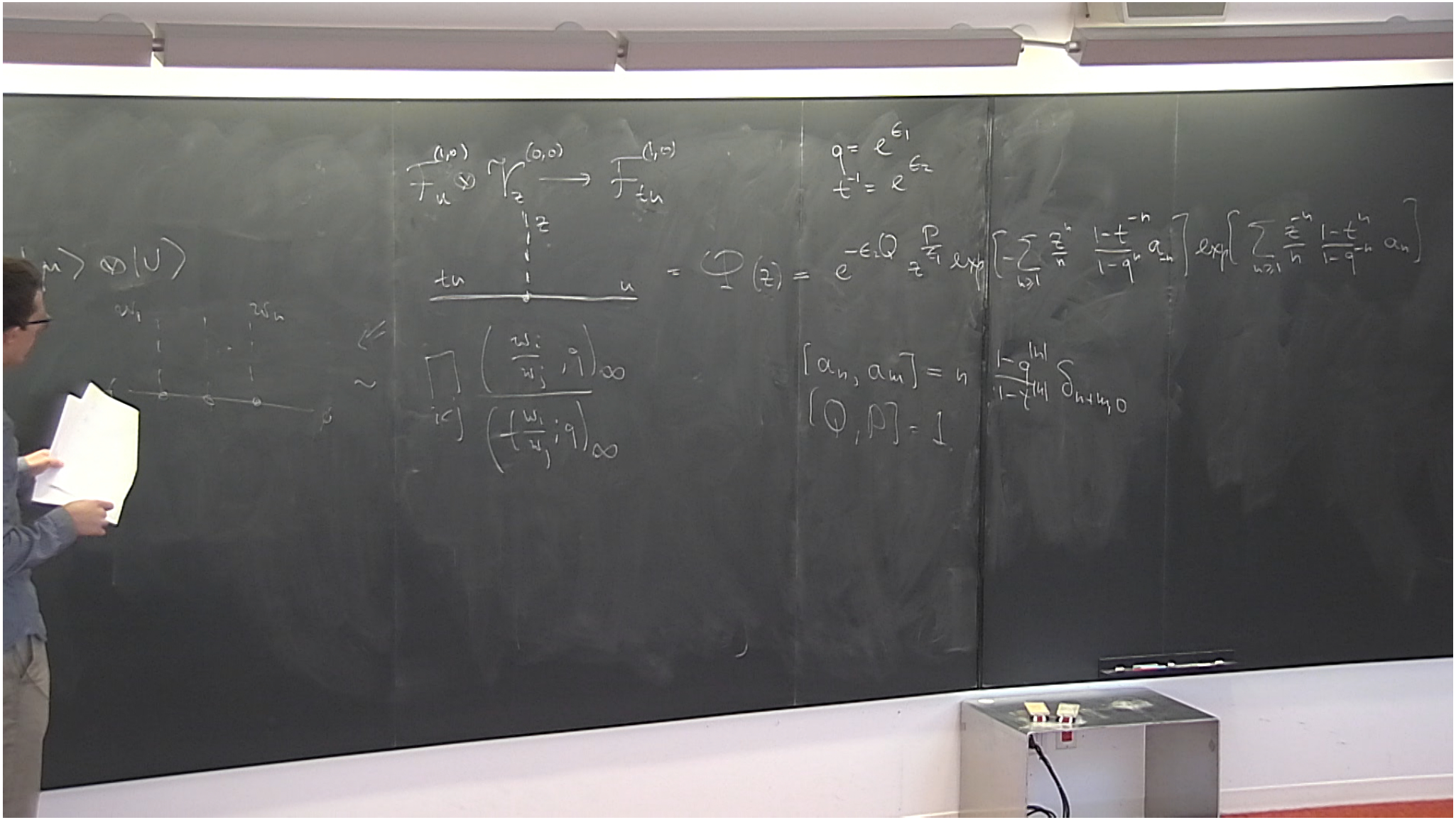
- Fock space $\mathcal{F} \ni |\lambda\rangle$

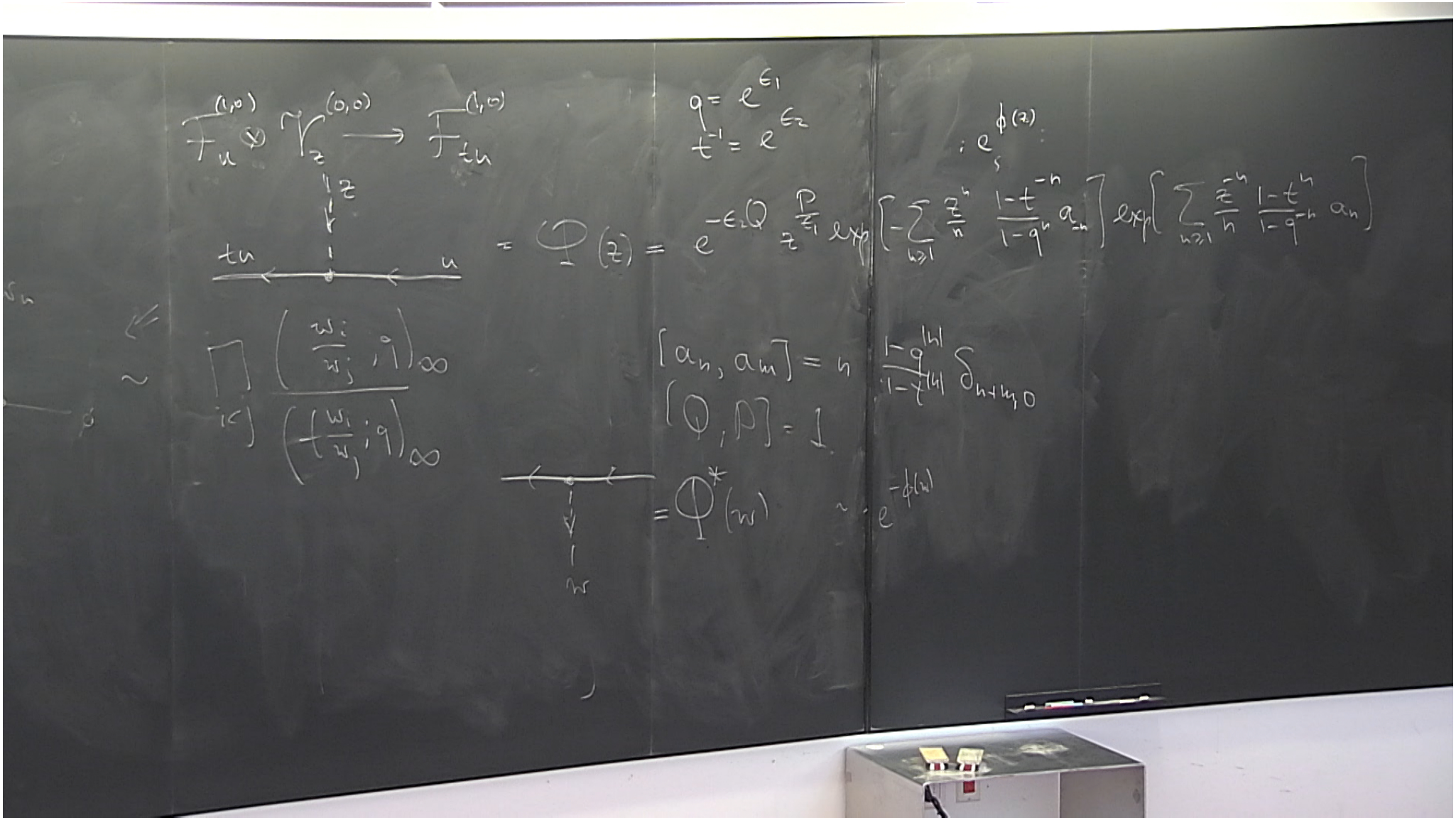
- MacMahon rep. $\mathcal{M} \ni |\Pi\rangle$

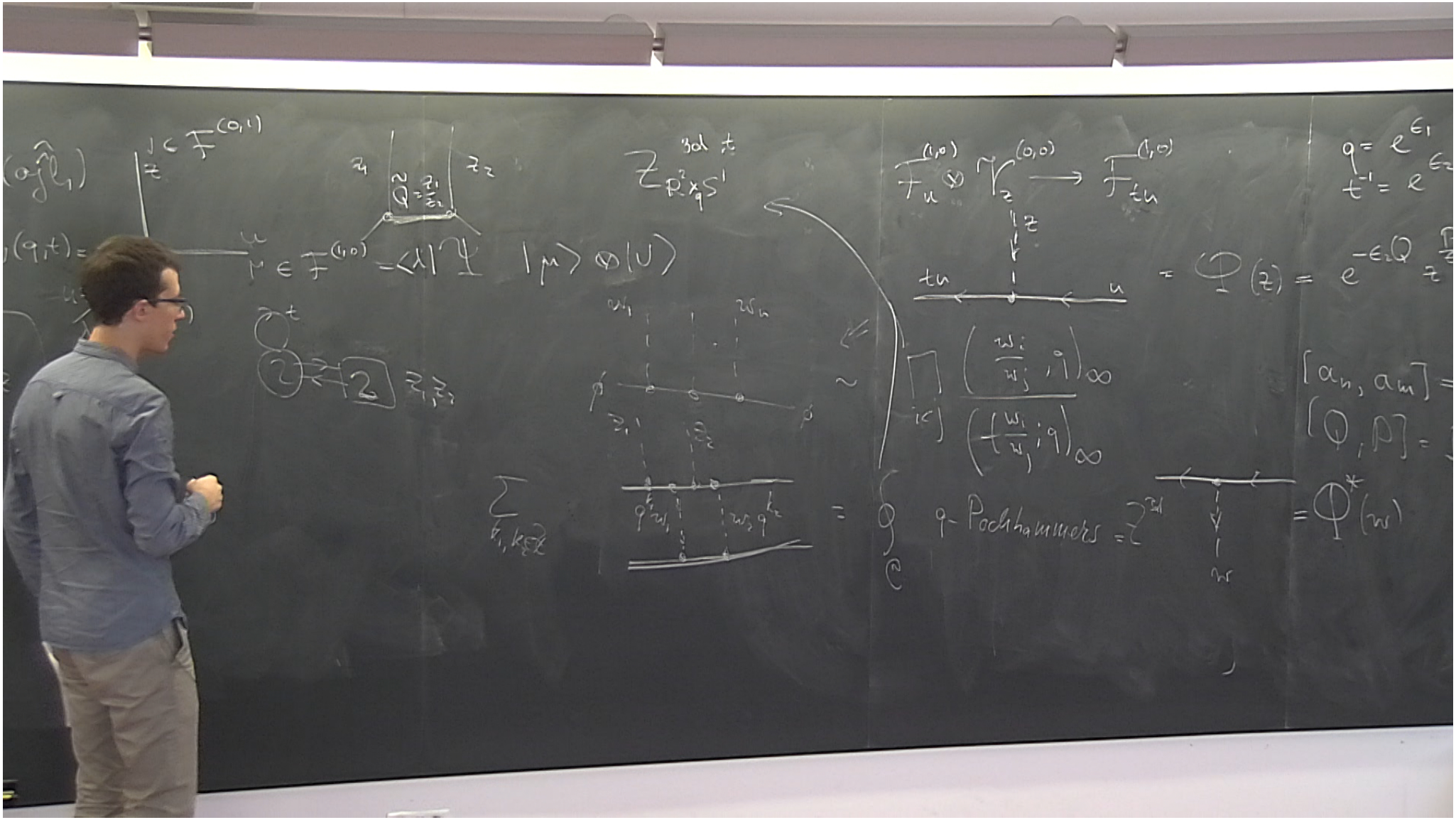
$C_{\lambda\mu\nu}(q,t) = \langle \lambda | \Psi$

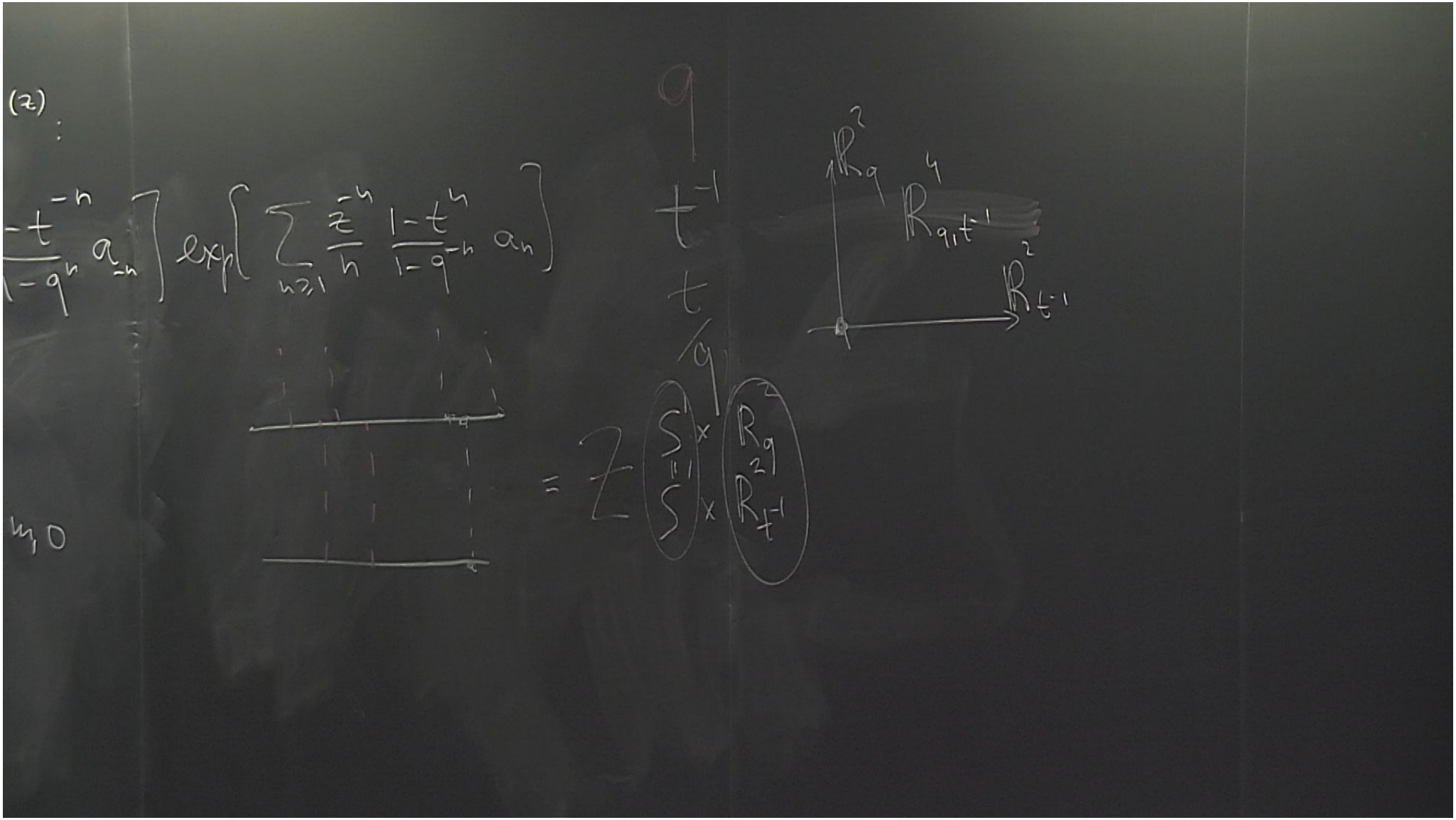


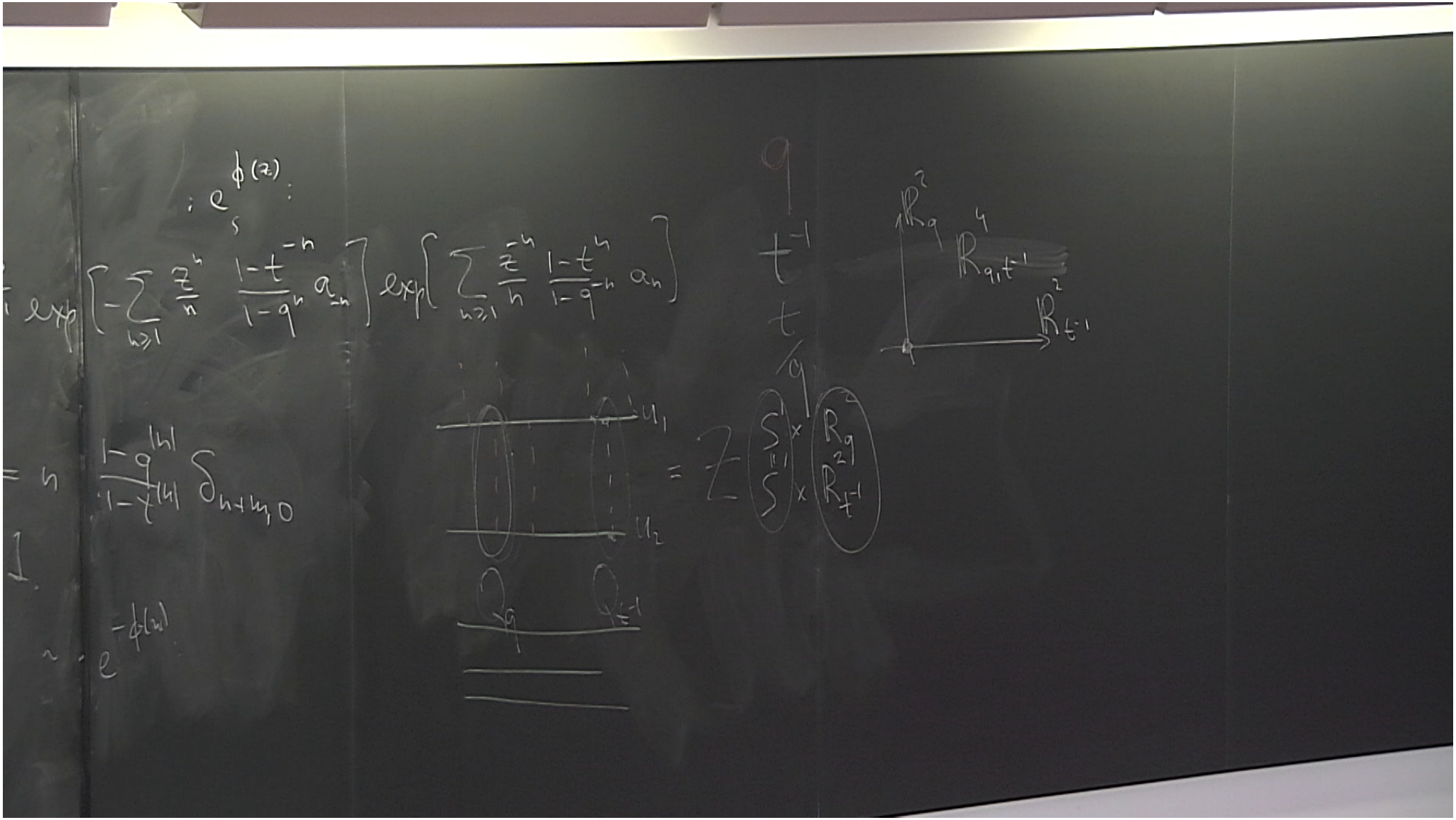
$\mathcal{F}^{(1,0)} \otimes \mathcal{F}^{(0,1)} \xrightarrow{\Psi} \mathcal{F}^{(1,1)}$

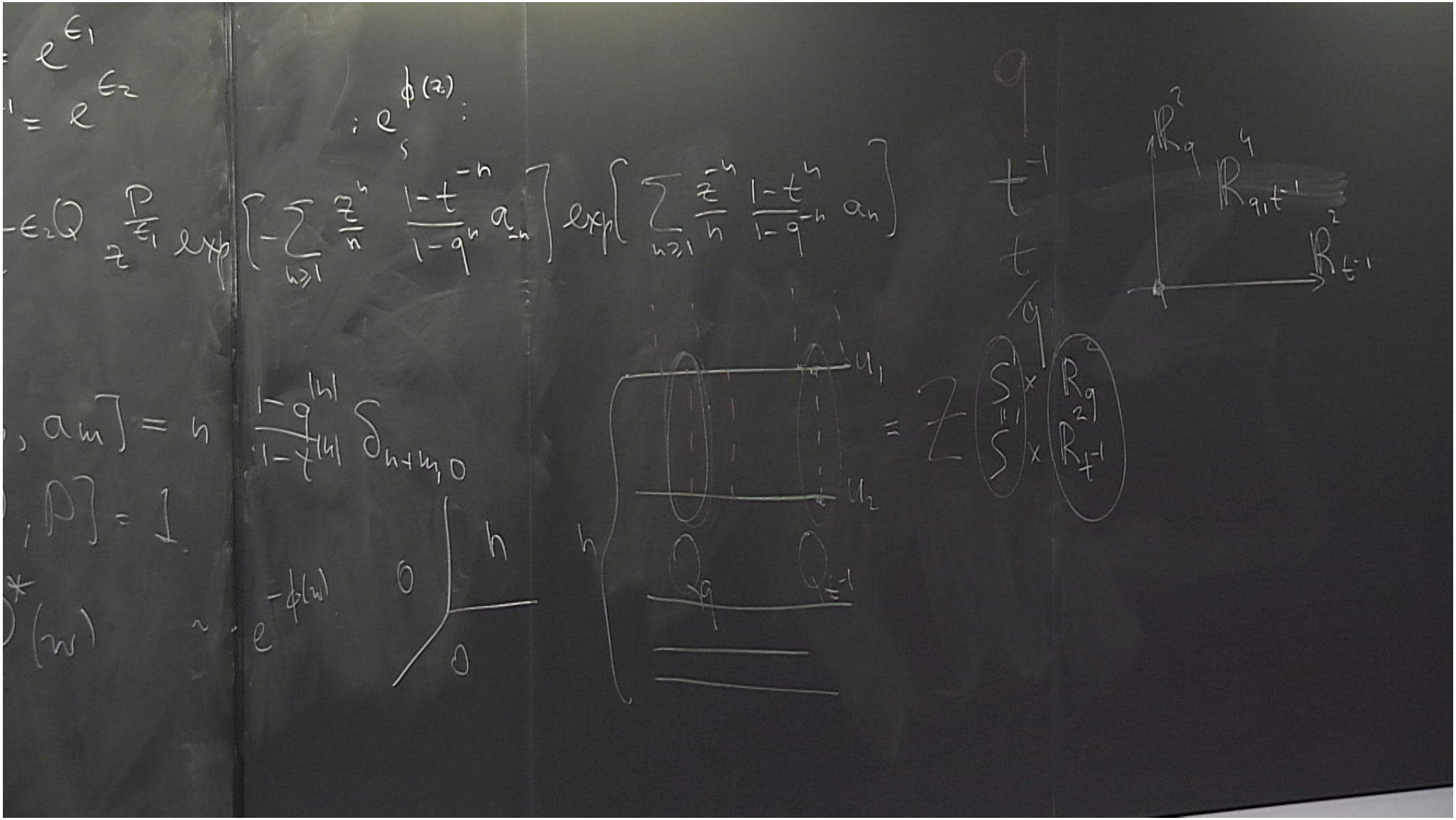

$q = e^{\epsilon_1}$
$t^{-1} = e^{\epsilon_2}$
$[a_n, a_m] = n$
$[Q, P] = 1$

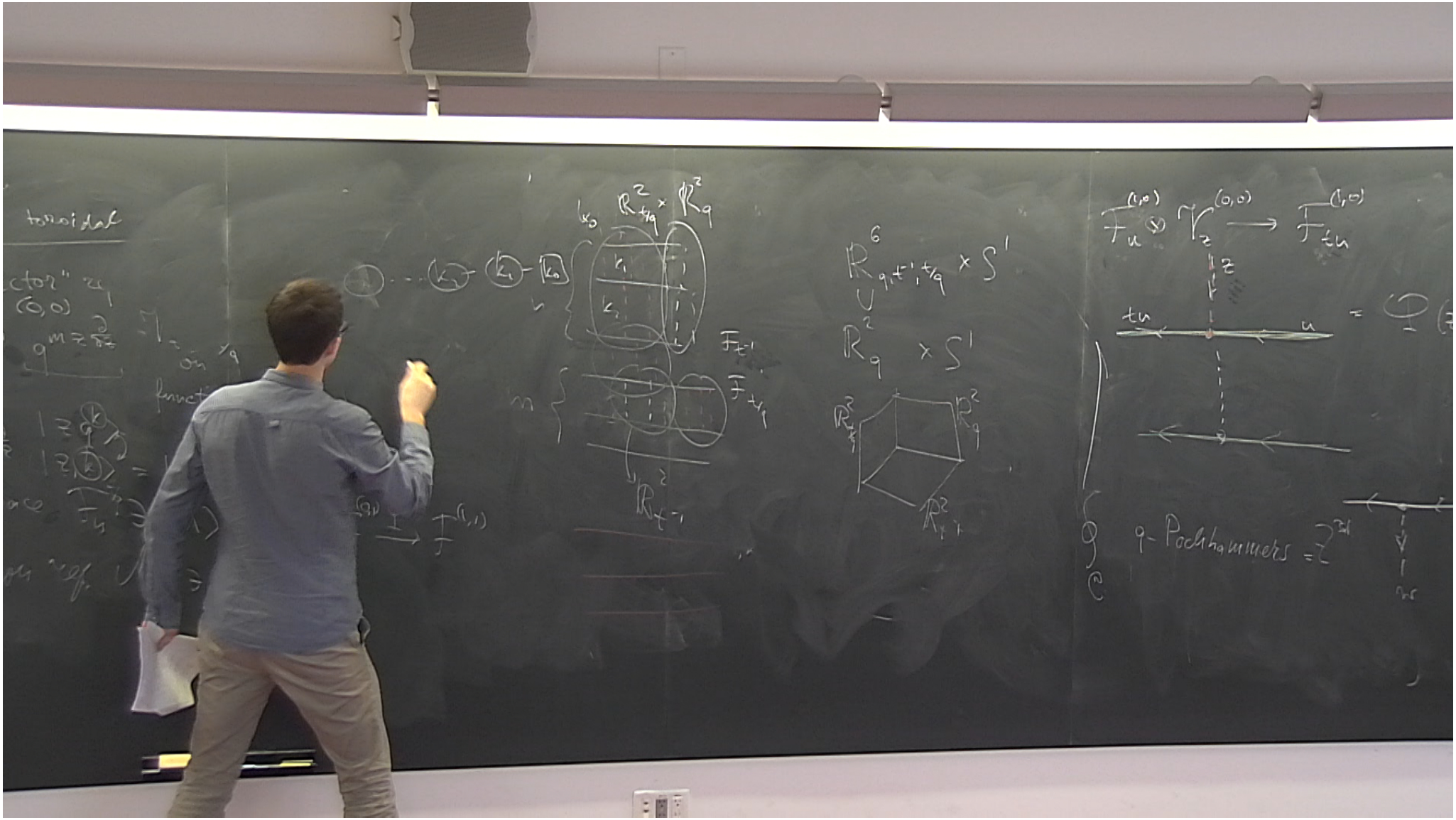

$q = e^{\epsilon_1}$
$t^{-1} = e^{\epsilon_2}$
$[a_n, a_m] = n \frac{1-q^{|n|}}{1-t^{|n|}} \delta_{n+m,0}$
$[Q, P] = 1$
$: e^{\phi(z)} :$
$\sim e^{-\phi(w)}$

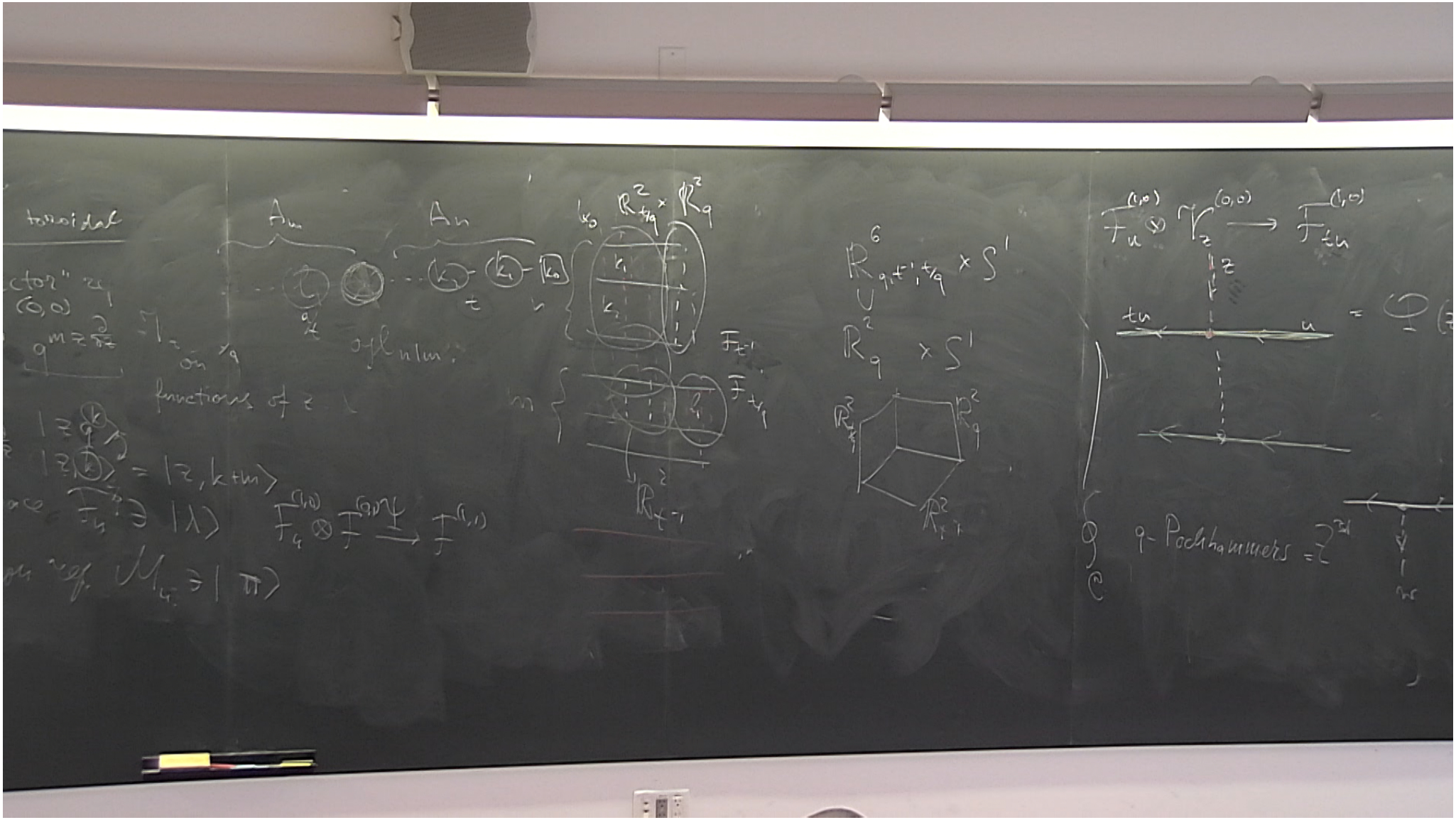

3d t
q-Pochhammers

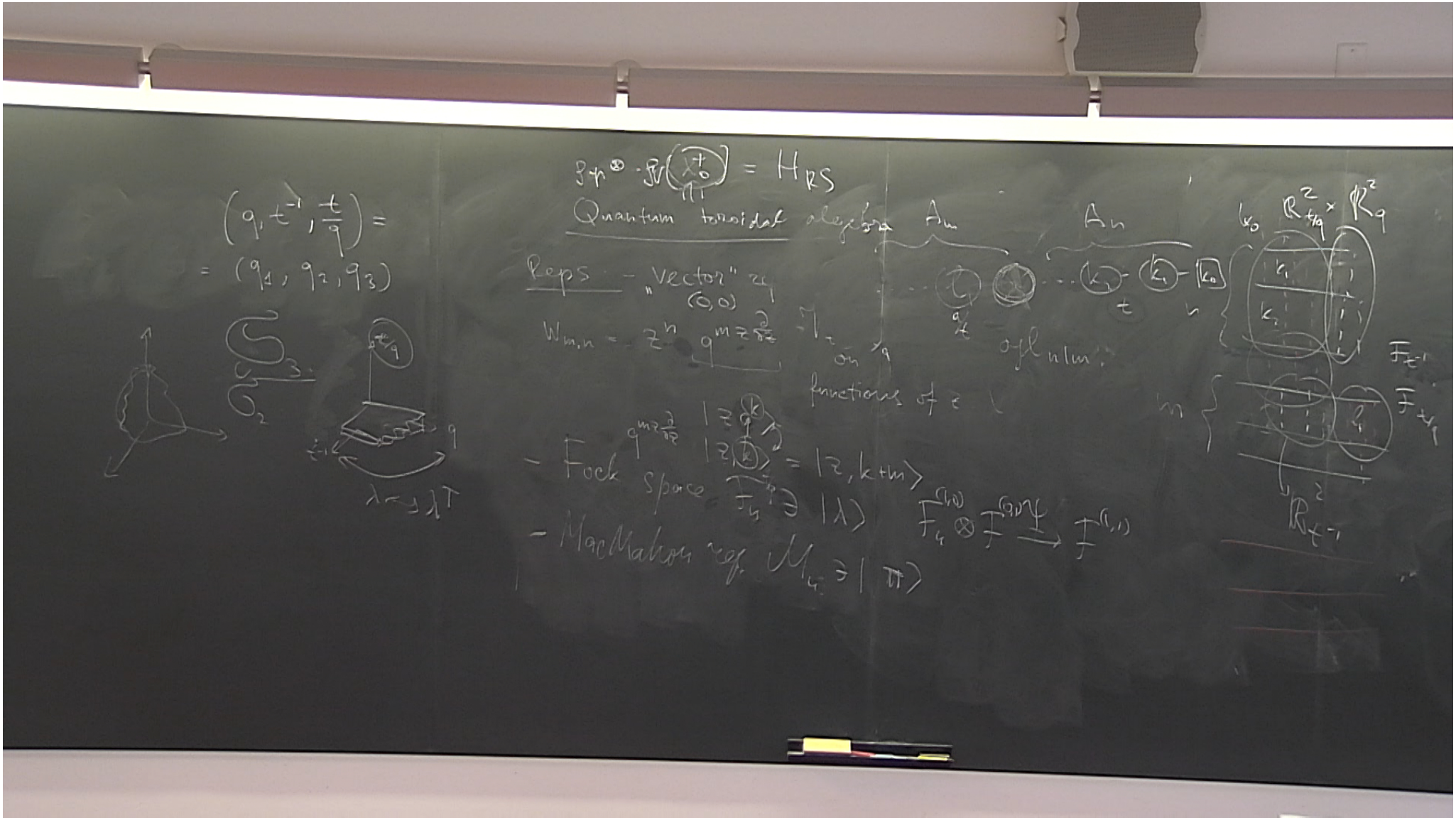

(z)
$\frac{-t^{-n}}{1-q^n} a_{-n}$
$\exp\left[\sum_{n\geq 1} \frac{z^{-n}}{n} \frac{1-t^n}{1-q^{-n}} a_n\right]$
$n, 0$
$= Z$
S_{11}^{1}
S
R_q^2
$R_{t^{-1}}^2$
q
t^{-1}
t
$1/q$
R_q^2
$R_{q,t^{-1}}^4$
$R_{t^{-1}}^2$


$: e^{\phi(z)} :$
$\exp\left[-\sum_{n\geq 1} \frac{z^n}{n} \frac{1-t^{-n}}{1-q^n} a_{-n}\right] \exp\left[\sum_{n\geq 1} \frac{z^{-n}}{n} \frac{1-t^n}{1-q^{-n}} a_n\right]$
$= n \frac{1-q^{|n|}}{1-t^{|n|}} \delta_{n+m,0}$
$\sim e^{-\phi(w)}$
u_1
u_2
Q_q
$Q_{t^{-1}}$
$= Z$
S
S^{-1}
R_{q^2}
$R_{t^{-1}}$
q
t^{-1}
t
$1/q$
R^2_q
$R^4_{q,t^{-1}}$
$R^2_{t^{-1}}$


$= e^{\epsilon_1}$
$= e^{\epsilon_2}$
$: e^{\phi(z)} :$
$-\epsilon_2 Q \; z^{\frac{P}{\epsilon_1}} \exp\left[-\sum_{n\geq 1} \frac{z^n}{n} \frac{1-t^{-n}}{1-q^n} a_{-n}\right] \exp\left[\sum_{n\geq 1} \frac{z^{-n}}{n} \frac{1-t^n}{1-q^{-n}} a_n\right]$
$, a_m] = n \frac{1-q^{|n|}}{1-t^{|n|}} \delta_{n+m,0}$
$, P] = 1$
$\phi^*(w) \sim e^{-\phi(w)}$
$= Z$
u_1
u_2
Q_q
$Q_{t^{-1}}$
R^2_q
$R^4_{q,t^{-1}}$
$R^2_{t^{-1}}$
t^{-1}
q
t
$1/q$
S
S^{-1}
R_q^2
$R^2_{t^{-1}}$
h
0
0
h


toroidal
q-Pochhammers


toroidal
Am
An
q-Pochhammers


$(q, t^{-1}, \frac{t}{q}) =$
$= (q_1, q_2, q_3)$
$\mathcal{H}_{RS} = H_{RS}$
Quantum toroidal algebra
Reps
"Vector"
$W_{m,n} = z^n q^{m z \frac{\partial}{\partial z}}$
A_m
A_n
$\mathbb{R}^2_{t/q} \times \mathbb{R}^2_q$
$\mathbb{R}^2_{t^{-1}}$
$F_{t^{-1}}$
$F_{t/q}$
$\lambda = (\lambda_1, \lambda_2, \lambda_3)$
- Fock space
- MacMahon rep.