Title: Gauge theory, vertex algebras and COHA

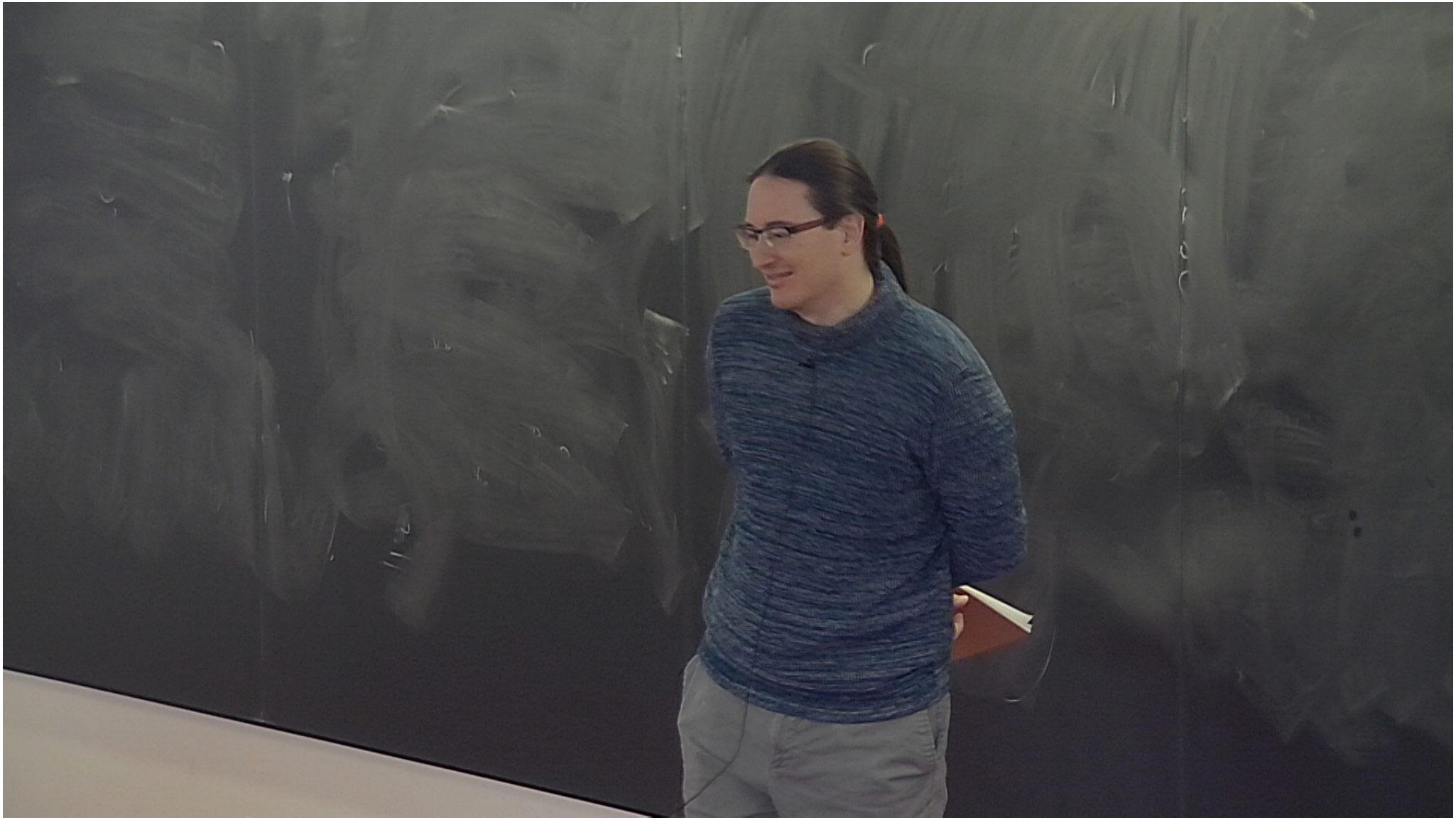
Speakers: Davide Gaiotto

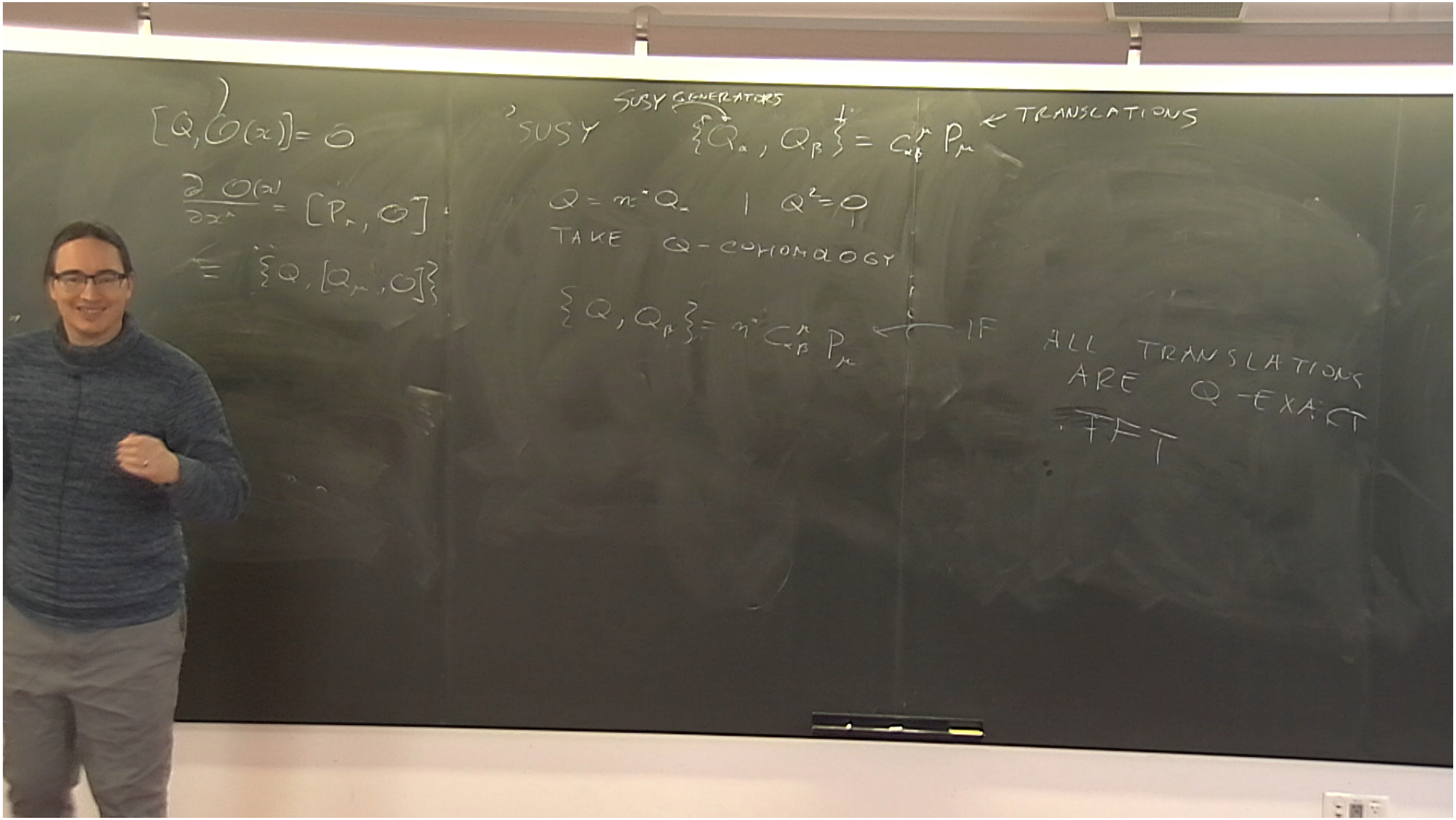
Collection: Cohomological Hall Algebras in Mathematics and Physics

Date: February 25, 2019 - 11:00 AM

URL: http://pirsa.org/19020054

Abstract: I will discuss some of the relations between supersymmetric gauge theories, vertex algebras and COHA.




$[Q, \mathcal{O}(x)] = 0$
$\frac{\partial \mathcal{O}(x)}{\partial x^\mu} = [P_\mu, \mathcal{O}]$
$= \{Q, [Q_\mu, \mathcal{O}]\}$
SUSY
SUSY GENERATORS
$\{Q_\alpha, Q_\beta\} = C^\mu_{\alpha\beta} P_\mu$
TRANSLATIONS
$Q = \pi^\alpha Q_\alpha \mid Q^2 = 0$
TAKE Q-COHOMOLOGY
$\{Q, Q_\beta\} = \pi^\alpha C^\mu_{\alpha\beta} P_\mu$
IF
ALL TRANSLATIONS ARE Q-EXACT
TFT

DEFECT IN QFT :

LOCAL MODIFICATION $\mathbb{R}^d \subset \mathbb{R}^D$

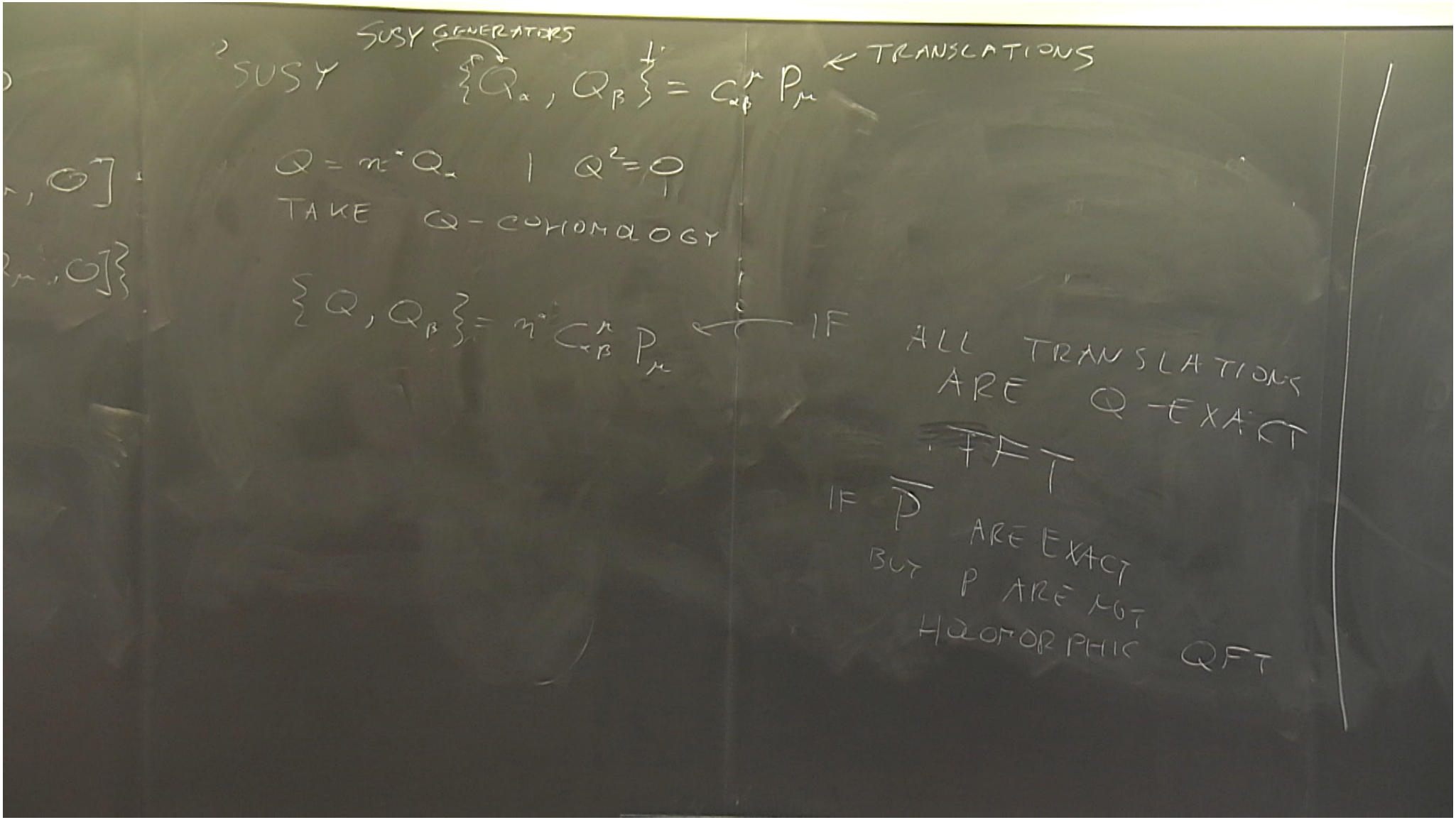
E.G BOUNDARY CONDITIONS

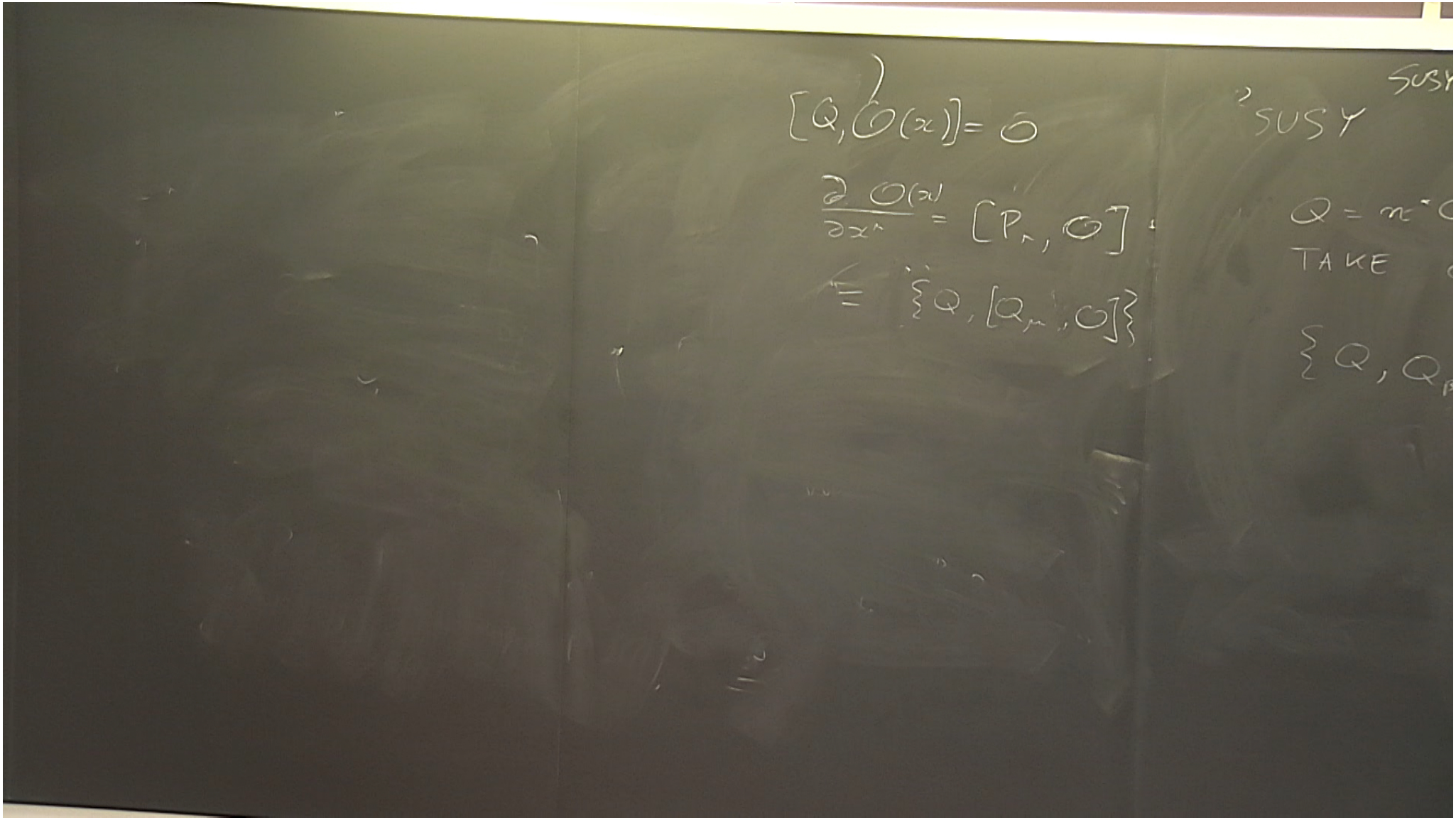
ALLOWS TO DEFINE

QFT ON $\mathbb{R}^+ \times \mathbb{R}^{D-1}$

E.G LINE DEFECT

MODIFY AT $\mathbb{R} \subset \mathbb{R}^D$


SUSY GENERATORS
SUSY
$\{Q_\alpha, Q_\beta\} = C^\mu_{\alpha\beta} P_\mu$
TRANSLATIONS
$Q = n^\alpha Q_\alpha \mid Q^2 = 0$
TAKE Q-COHOMOLOGY
$\{Q, Q_\beta\} = n^\alpha C^\mu_{\alpha\beta} P_\mu$
IF ALL TRANSLATIONS ARE Q-EXACT
TFT
IF $\overline{P}$ ARE EXACT
BUT P ARE NOT
HOLOMORPHIC QFT


$[Q, \mathcal{O}(x)] = 0$
$\frac{\partial \mathcal{O}(x)}{\partial x^\mu} = [P_\mu, \mathcal{O}]$
$= \{Q, [Q_\mu, \mathcal{O}]\}$
SUSY
SUSY
$Q = \pi^* Q$
TAKE
$\{Q, Q_\beta$

A DEFECT MAY PRESERVE Q

AND BREAK SOME Q_+

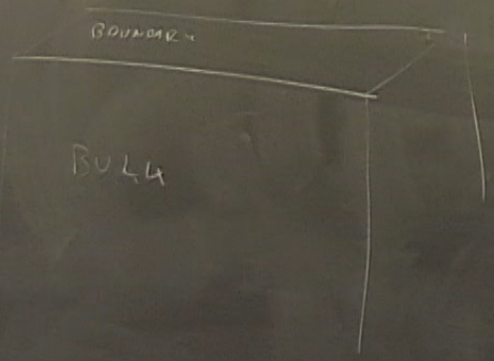
E.G. TFT_D

AND HOLOMORPHIC DEFECT $\mathbb{R}^2 \subset \mathbb{R}^D$

LOCAL OPERATORS AT D GIVE A VOA $A[T, D]$

3d TFT T

2d HOLOMORPHIC DEFECT VOA $\mathcal{A}$



$T[S^1]$ = CATEGORY OF (TOP) LINE DEFECTS IN T



DEFECT IN

LOCAL

E.G BOUN

ALLO

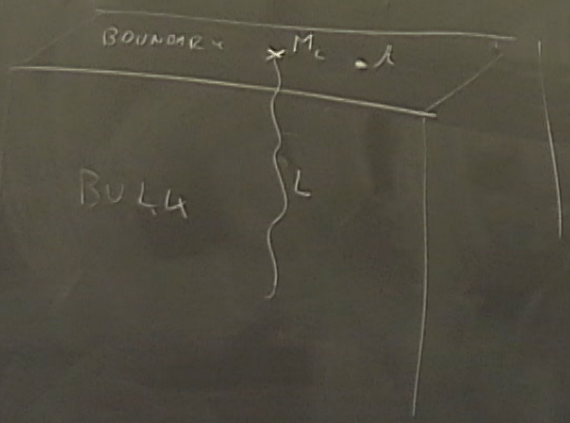
QFT

E.G LINE

3d TFT T

2d HOLOMORPHIC DEFECT T

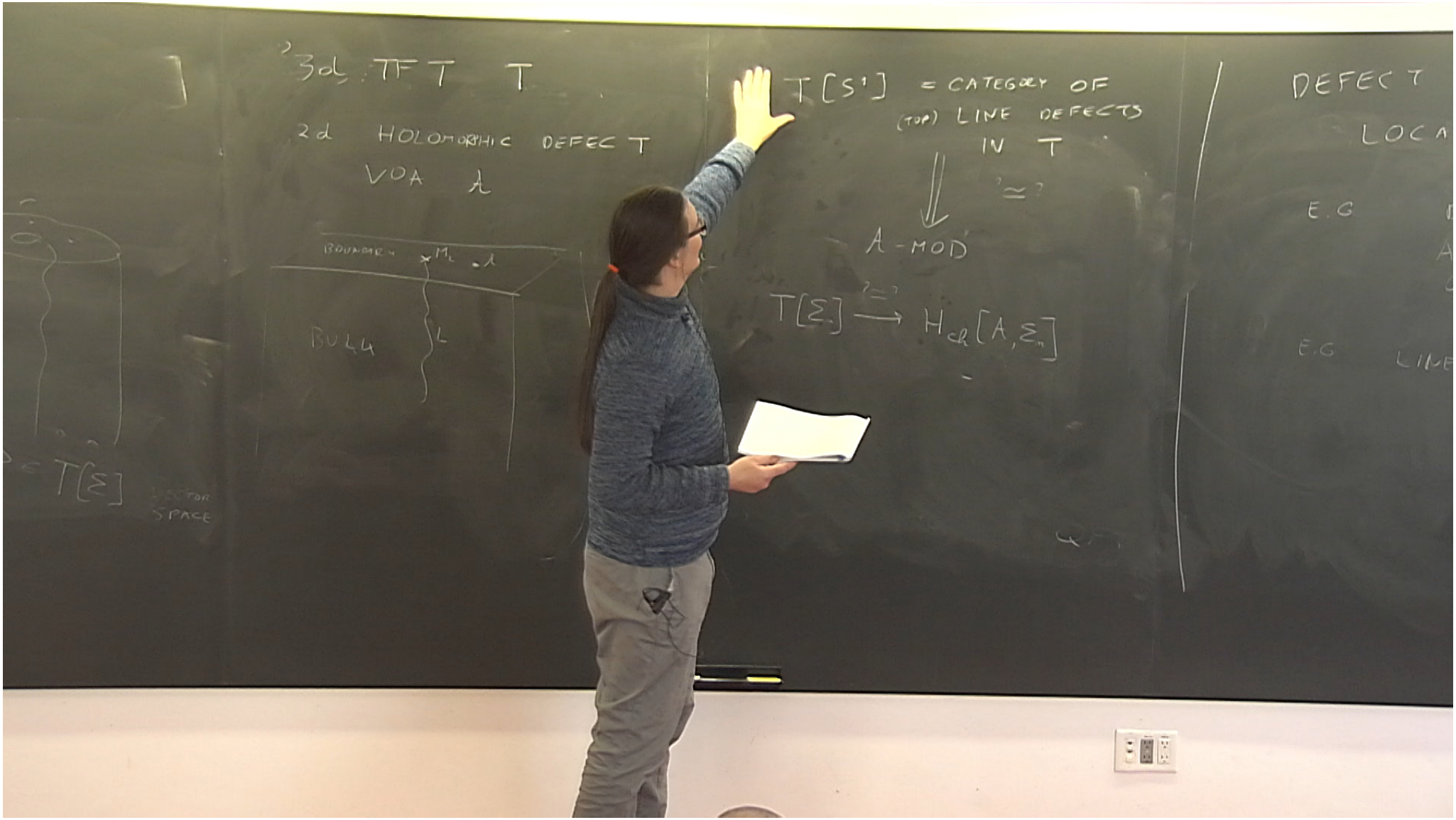
VOA $\mathcal{A}$



$T[S^1]$ = CATEGORY OF (TOP) LINE DEFECTS IN T.

$\Downarrow$

$\mathcal{A}$-MOD


3d TFT T
2d HOLOMORPHIC DEFECT T
VOA A
BOUNDARY
BULK
T[Σ]
VECTOR SPACE
T[S¹] = CATEGORY OF (TOP) LINE DEFECTS IN T
≃?
A-MOD
T[Σ] → H_ch[A, Σ_n]
DEFECT
LOCA
E.G
E.G
LINE

T

$T[S^1]$ = CATEGORY OF (TOP) LINE DEFECTS IN T

$\simeq$?

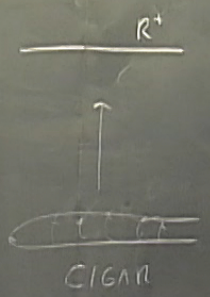
A-MOD

$T[\Sigma] \xrightarrow{?=} H_{cl}[A, \Sigma_A]$

3d TOP TWIST OF 3d N=4 SQFTs

$R^+ \times \mathbb{C}$

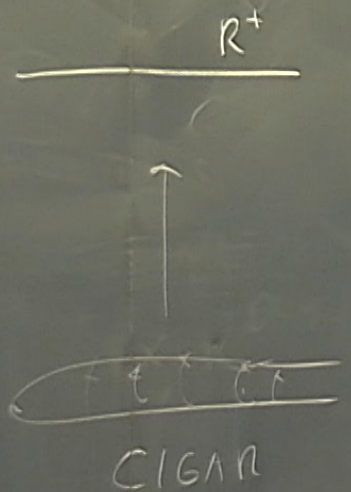
4d N=2 SCFTs ON CIGAR $\times \mathbb{C}$



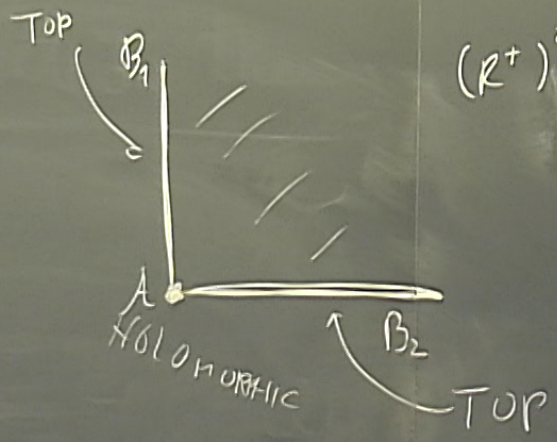
$\chi_A(q) = \mathrm{Tr}_{\mathcal{H}} S^+ S$

$S = \widehat{\Pi} K_\gamma^{\Omega_\gamma}$

F 3d N=4 SQFTs



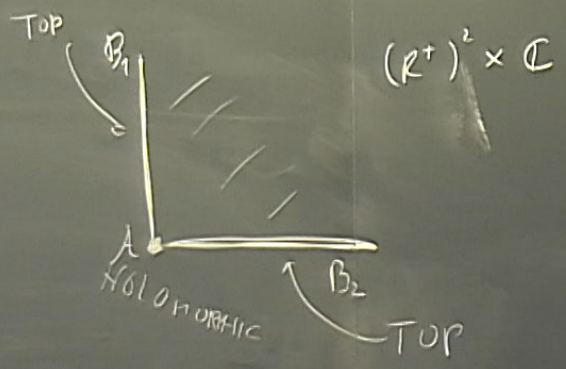
4d TFT



$(R^+)^2 \times \mathbb{C}$

TS

4d TFT



TFT_4 : GL-TWIST

N=4 SYM

G, κ $\kappa \in \mathbb{C}$

$TFT_4[\Sigma] \rightarrow$ CATEGORY

$\Downarrow$

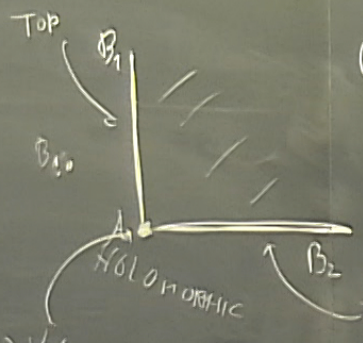
$TFT_4[\Sigma \times R^+_{B_1}] \rightarrow \mathcal{O}_1$ $\Downarrow$

$B_2 \longrightarrow \mathcal{O}_2$

$HOM(\mathcal{O}_1, \mathcal{O}_2) \longrightarrow H_{CH}(A, \Sigma)$

QFTs

4d TFT



$(\mathbb{R}^+)^2 \times \mathbb{C}$

$W_{G,u}$

TFT_4 : GL-TWIST

$\mathcal{N}=4$ SYM

G, u $\quad u \in \mathbb{C}$

$TFT_4[\Sigma] \longrightarrow$ CATEGORY

$\Vert$

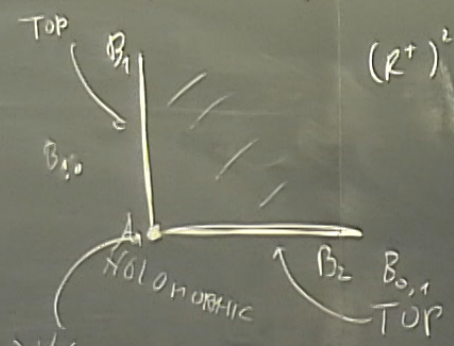
$TFT_1[\Sigma \times \mathbb{R}^+_{B_1}] \longrightarrow \mathcal{O}_1$ $\quad \Vert$

$\Vert$ $\quad B_2 \longrightarrow \mathcal{O}_2$

$HOM(\mathcal{O}_1, \mathcal{O}_2) \longrightarrow H_{CH}(A, \Sigma)$

FTs

4d TFT



$(\mathbb{R}^+)^2 \times \mathbb{C}$

TFT_4 : GL-TWIST

$N=4$ SYM

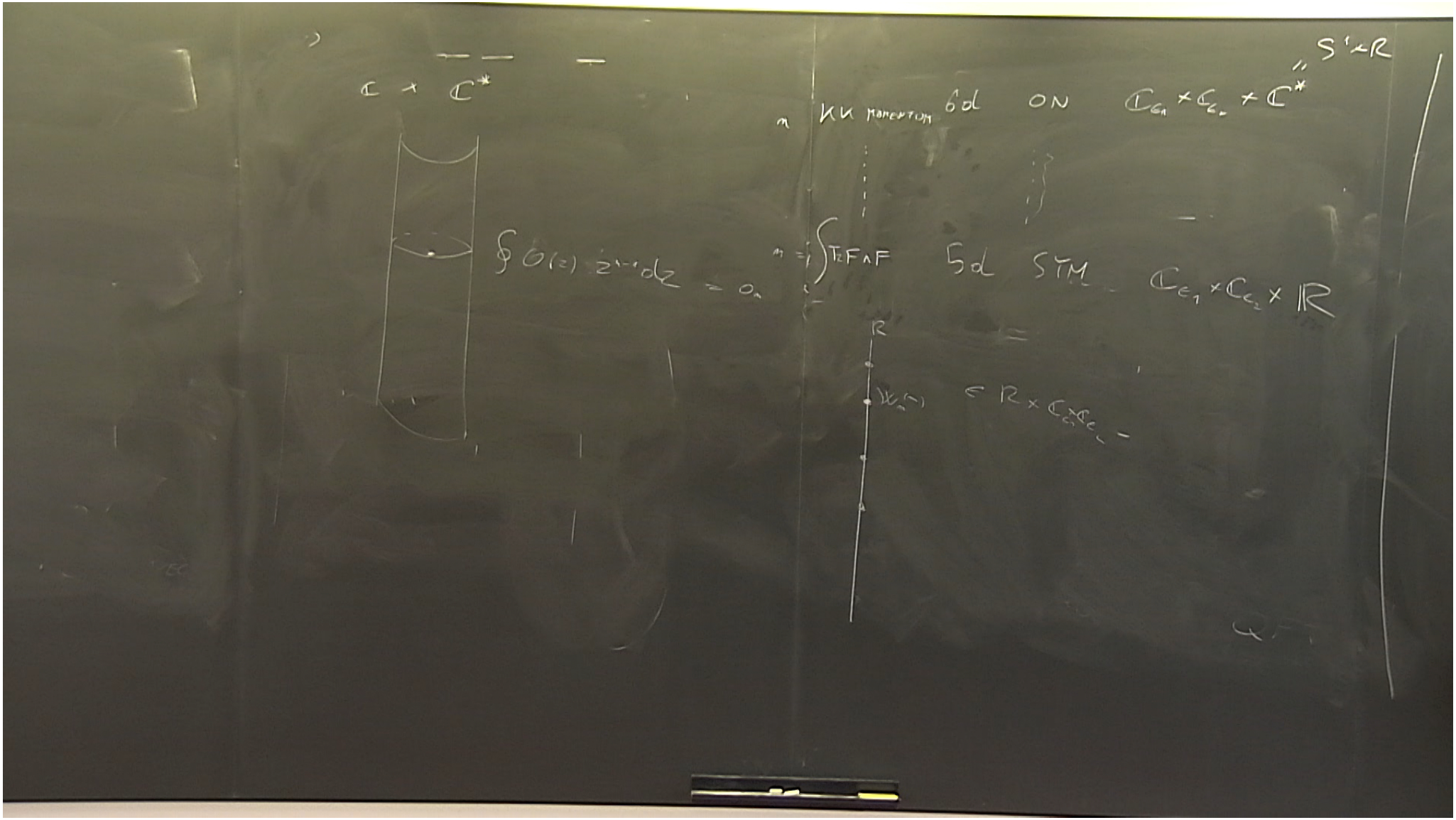
G, τ $\quad \tau \in \mathbb{C}$

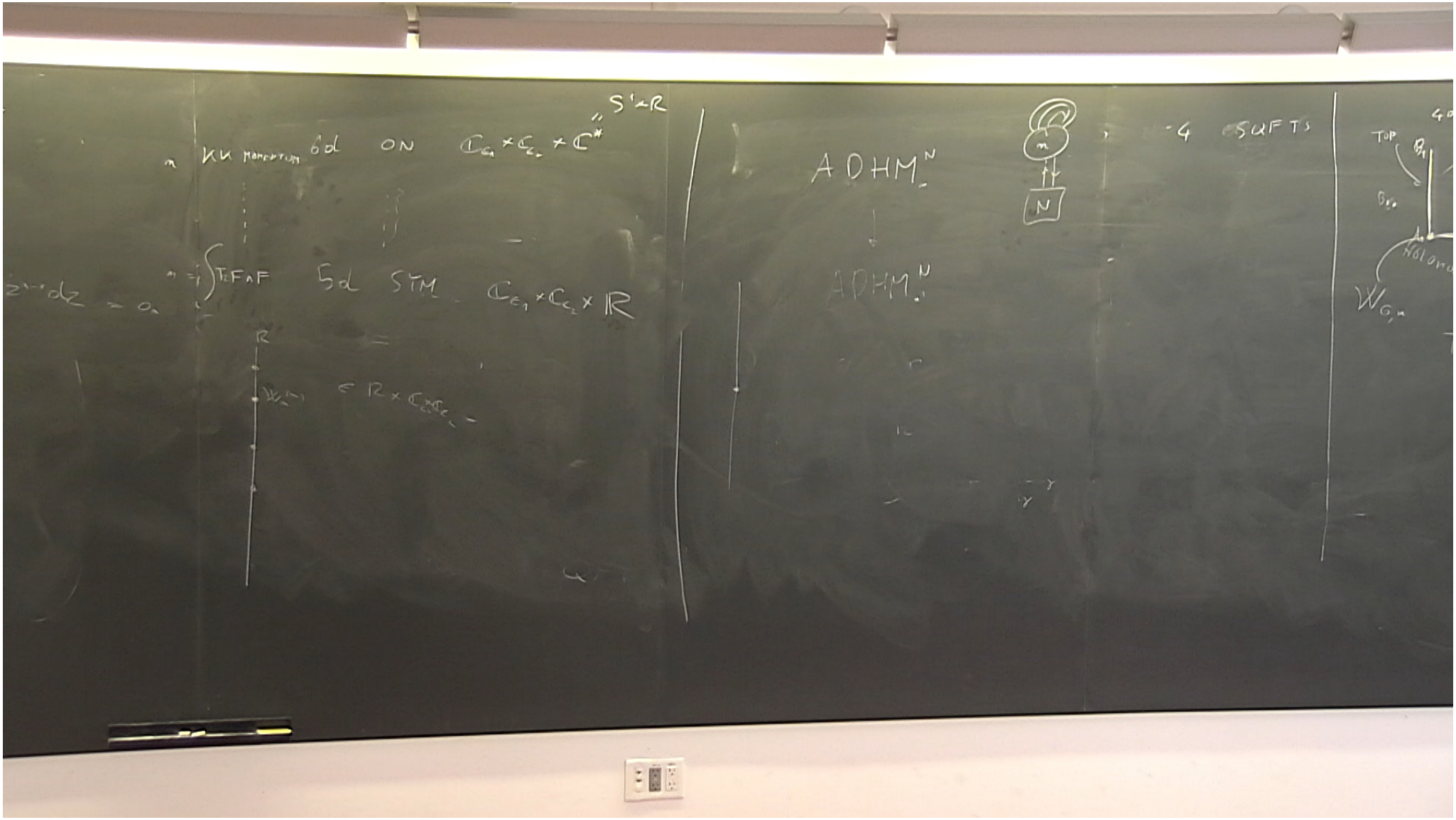
6d SCFT $\mathcal{J}$

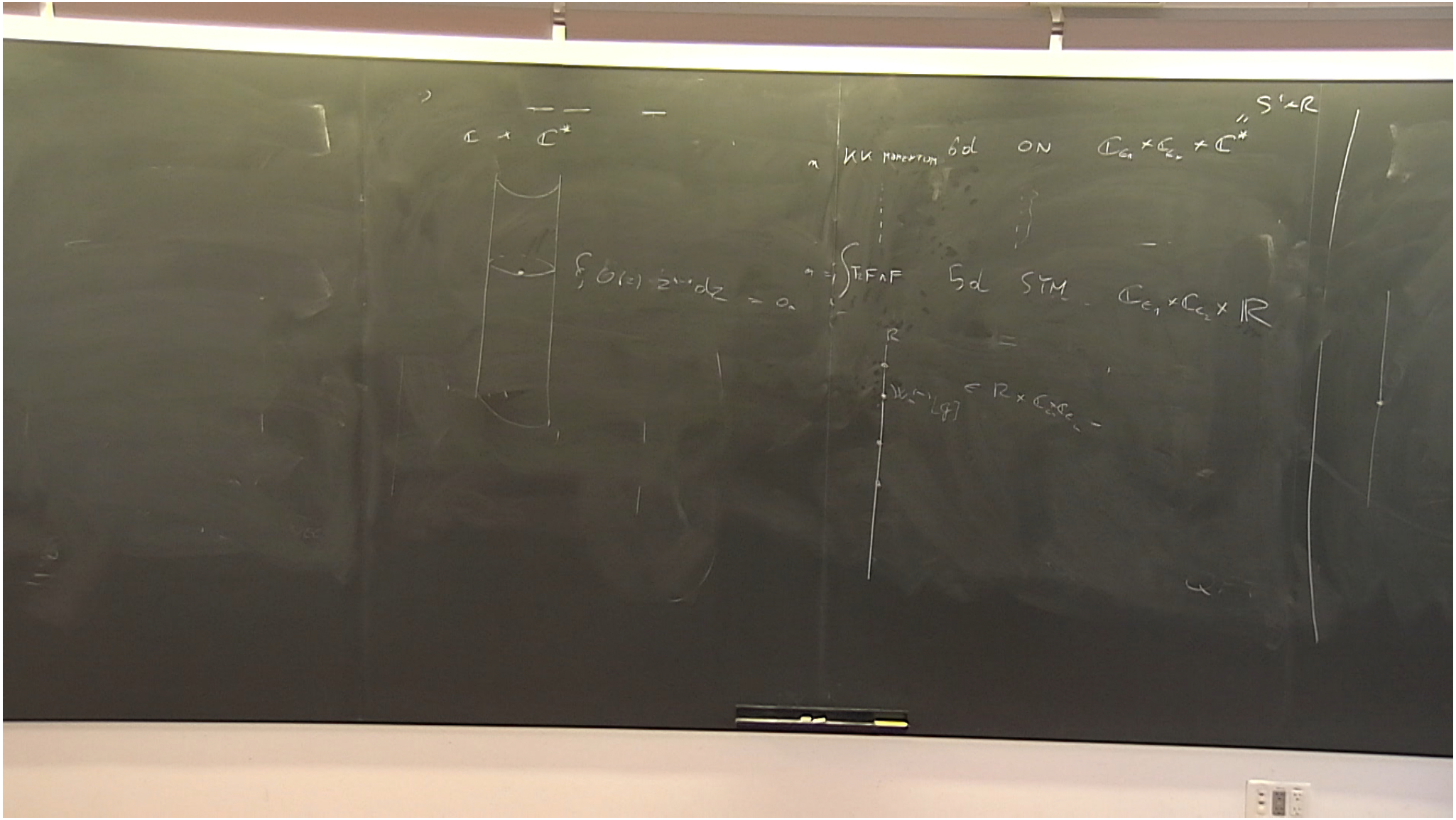
ON

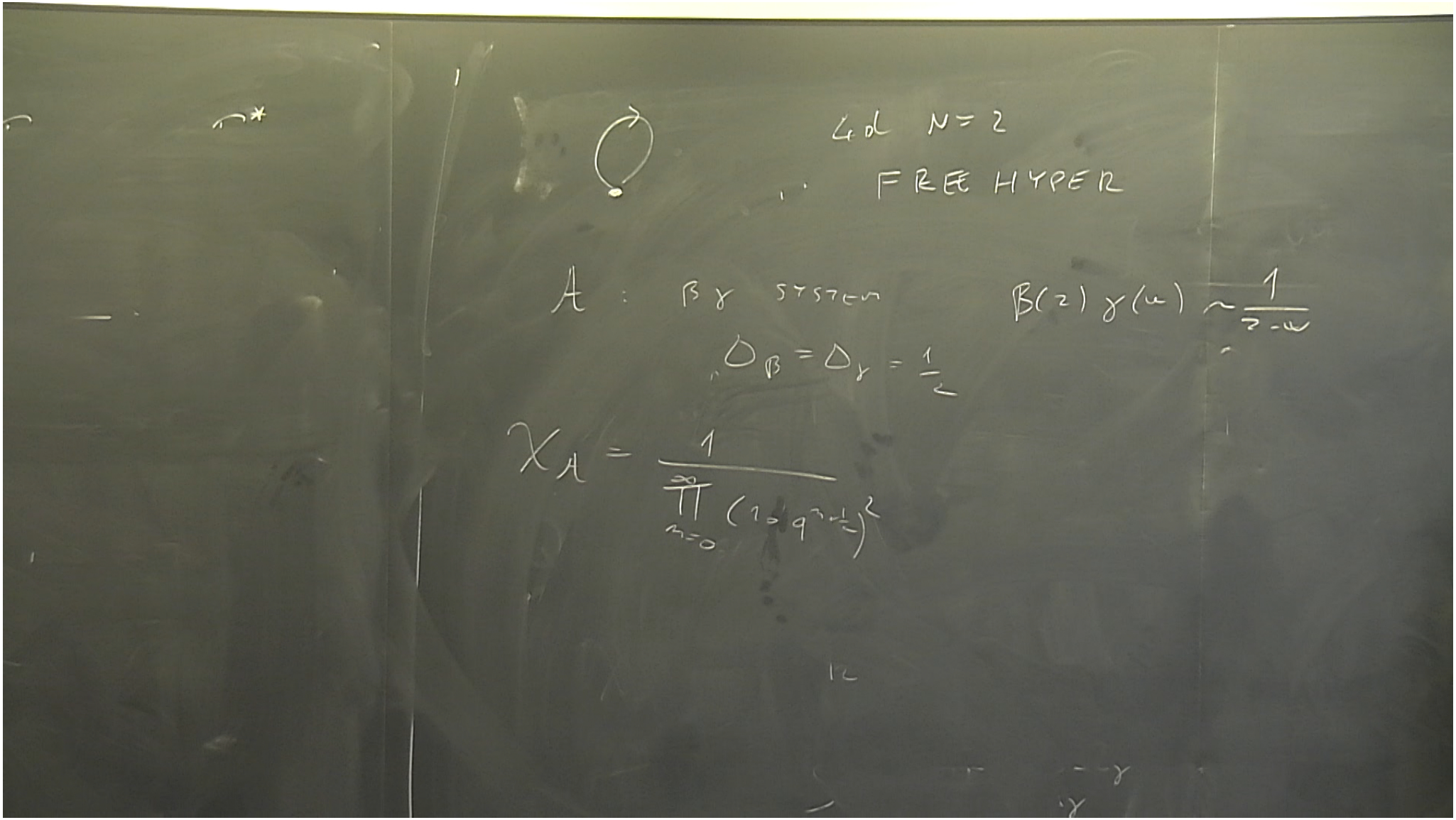
$\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{C}$ $\quad \tau = \frac{\epsilon_1}{\epsilon_2}$

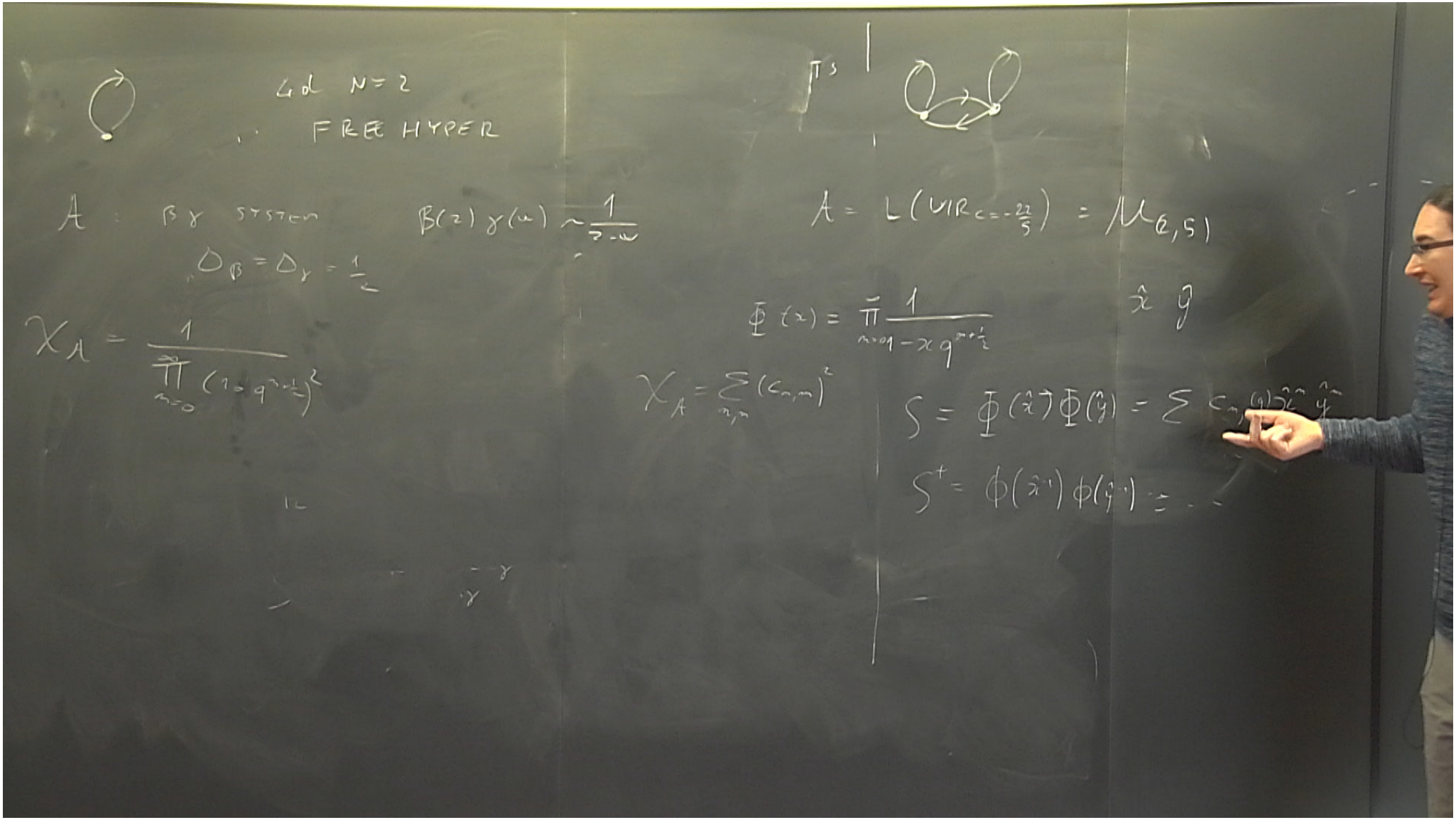
$W_{G,u}$

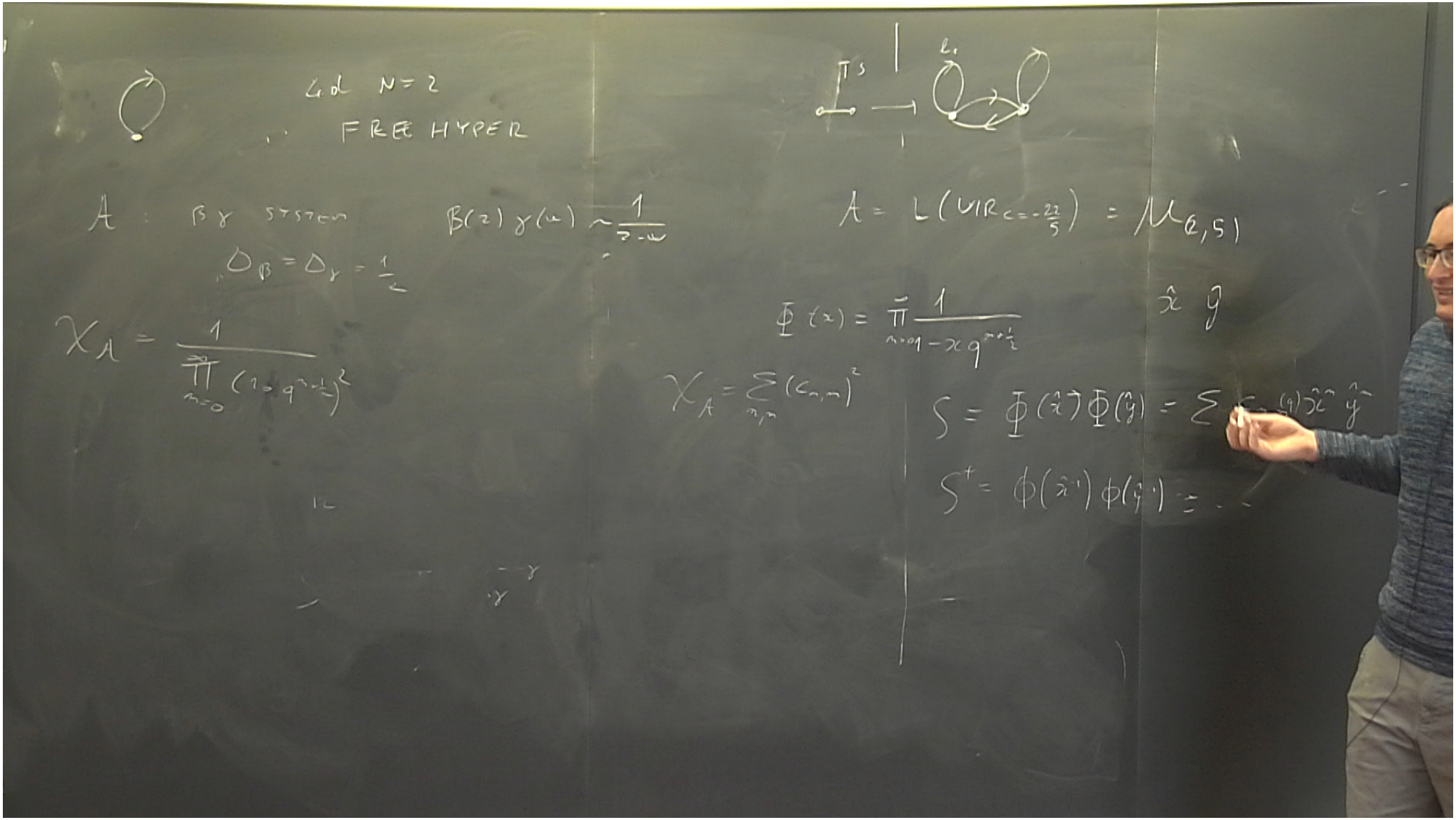

$\mathbb{C} \times \mathbb{C}^*$
$\oint G(z)\, z^{n-1} dz = 0_n$
KK momentum
6d ON $\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{C}^*$
$S^1 \times R$
$m = \int_{\Sigma} F \wedge F$
5d SYM $\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{R}$
R
$\in R \times \mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2}$

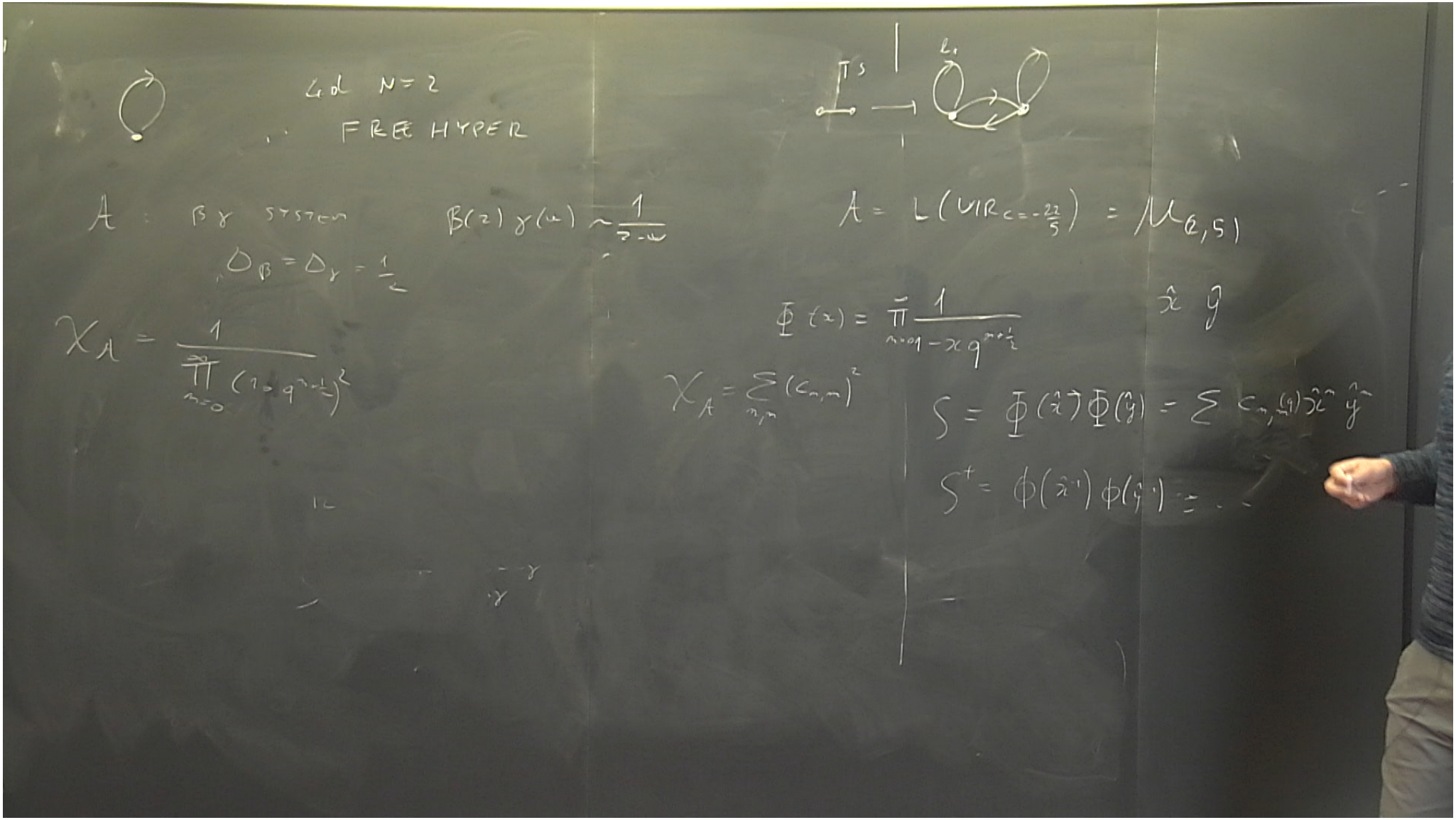

KK momentum
6d
ON
$\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{C}^*$
$S^1 \times R$
$\int T \wedge F \wedge F$
5d
SYM
$\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{R}$
R
$\in R \times \mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2}$
ADHM$^N_-$
ADHM$^N_+$
m
N
4 SUSY
TOP
Hol

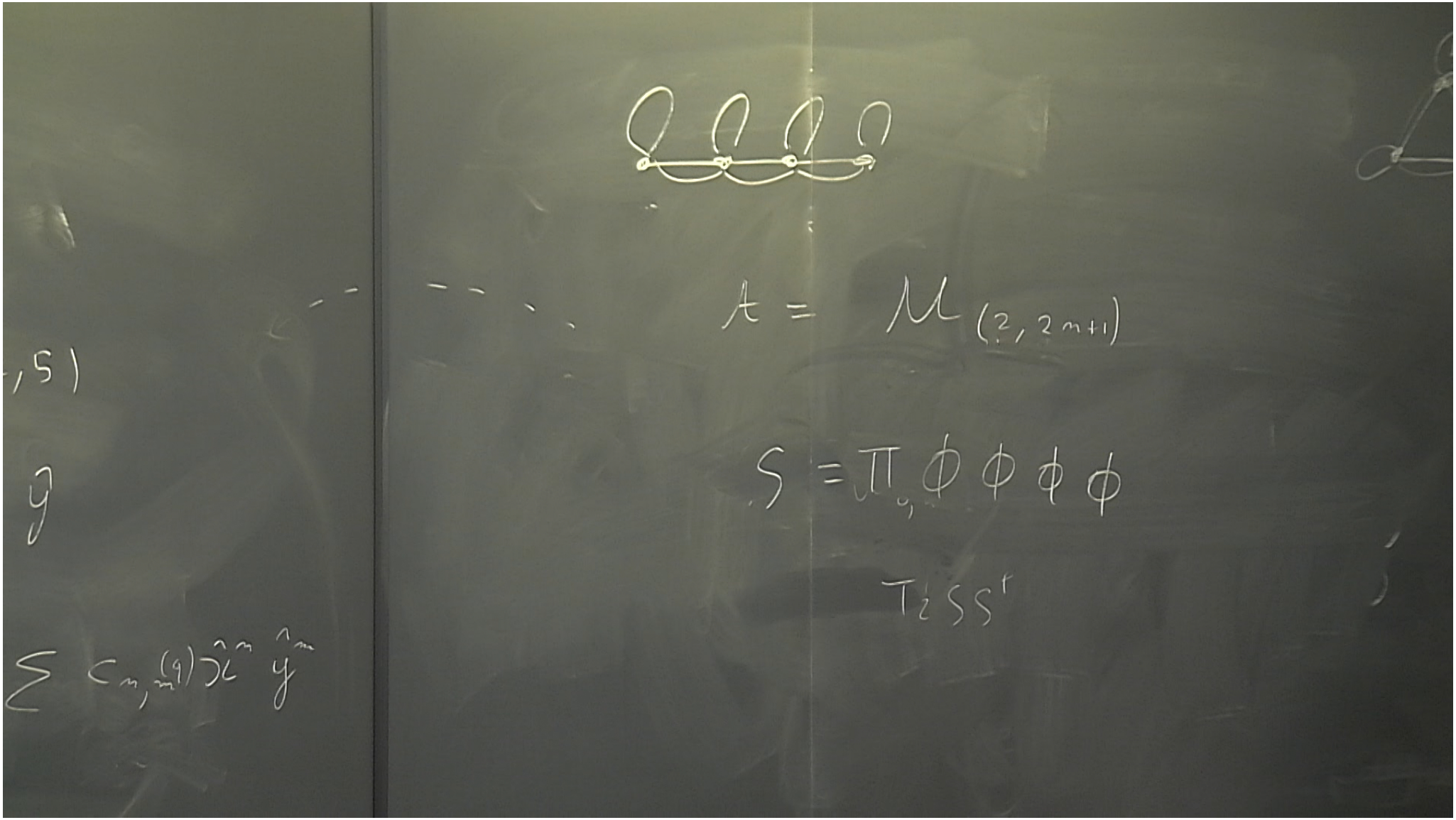

$\mathbb{C} \times \mathbb{C}^*$
$\oint O(z)\, z^{n-1} dz = 0$
KK momentum
6d ON $\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{C}^*$
$S^1 \times R$
$= \int \mathrm{Tr} F \wedge F$ 5d SYM $\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{R}$
R
$\in R \times \mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2}$

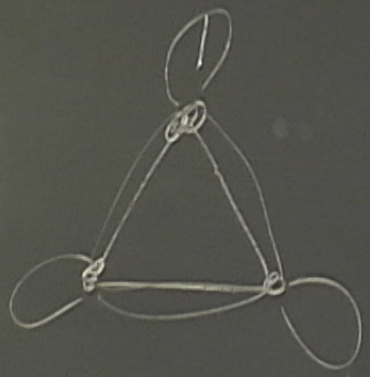

4d N = 2
∴ FREE HYPER
A : βγ system
$B(z)\gamma(w) \sim \frac{1}{z-w}$
$\Delta_\beta = \Delta_\gamma = \frac{1}{2}$
$\chi_A = \frac{1}{\prod_{n=0}^{\infty}(1-q^{n+\frac{1}{2}})^2}$


GL N=2
FREE HYPER
A : βγ system
$\beta(z)\gamma(w) \sim \frac{1}{z-w}$
$\Delta_\beta = \Delta_\gamma = \frac{1}{2}$
$\chi_A = \frac{1}{\prod_{n=0} (1-q^{n+\frac{1}{2}})^2}$
$\chi_A = \sum_{n,m} (c_{n,m})^2$
$A = L(VIR_{c=-\frac{22}{5}}) = M_{(2,5)}$
$\Phi(z) = \prod_{n=0} \frac{1}{1-xq^{n+\frac{1}{2}}}$
$\hat{x}$ $\hat{y}$
$S = \Phi(\hat{x})\Phi(\hat{y}) = \sum c_{n,m} \hat{x}^n \hat{y}^m$
$S^+ = \phi(\hat{x}^{-1})\phi(\hat{y}^{-1}) =$


GL N=2
FREE HYPER
A : βγ system
$\beta(z)\gamma(w) \sim \frac{1}{z-w}$
$\Delta_\beta = \Delta_\gamma = \frac{1}{2}$
$\chi_A = \frac{1}{\prod_{n=0} (1-q^{n+\frac{1}{2}})^2}$
$\chi_A = \sum_{n,m} (c_{n,m})^2$
$A = L(VIR_{c=-\frac{22}{5}}) = M_{(2,5)}$
$\Phi(x) = \prod_{n=0} \frac{1}{1-xq^{n+\frac{1}{2}}}$
$\hat{x}\ \hat{y}$
$S = \Phi(\hat{x})\Phi(\hat{y}) = \sum$
$S^+ = \phi(\hat{x}^{-1})\phi(\hat{y}^{-1}) =$


Gol N=2
FREE HYPER
A : βγ system
$\beta(z)\gamma(w) \sim \frac{1}{z-w}$
$\Delta_\beta = \Delta_\gamma = \frac{1}{2}$
$\chi_A = \frac{1}{\prod_{n=0} (1-q^{n+\frac{1}{2}})^2}$
$\chi_A = \sum_{n,m} (c_{n,m})^2$
$A = L(VIR_{c=-\frac{22}{5}}) = M_{(2,5)}$
$\Phi(x) = \prod_{n=0} \frac{1}{1-xq^{n+\frac{1}{2}}}$
$\hat{x}$ $\hat{y}$
$S = \Phi(\hat{x})\Phi(\hat{y}) = \sum c_{n,m} \hat{x}^n \hat{y}^m$
$S^+ = \phi(\hat{x}^{-1})\phi(\hat{y}^{-1}) = \ldots$


$A = M_{(2,2m+1)}$
$S = \Pi_y \phi\,\phi\,\phi\,\phi$
$T_i S S^\dagger$
$\sum c_{m,m'}(q)\,\hat{z}^m\,\hat{y}^{m'}$



$N=2^*$ $SU(2)$

$$\mathcal{A} = {}_5SVIR^{N=4}_{c=-9}$$