Title: Angular momentum radiated by electromagnetic vs gravitational waves

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Collection: PI-CITA Day 2019

Date: April 02, 2019 - 11:30 AM

URL: http://pirsa.org/19020043

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Angular momentum radiated by electromagnetic versus gravitational waves

PERIMETER INSTITUTE

Béatrice Bonga – PI-CITA Day – Apr 2 2019

[Based on work with Abhay Ashtekar, Eric Poisson and Huan Yang]

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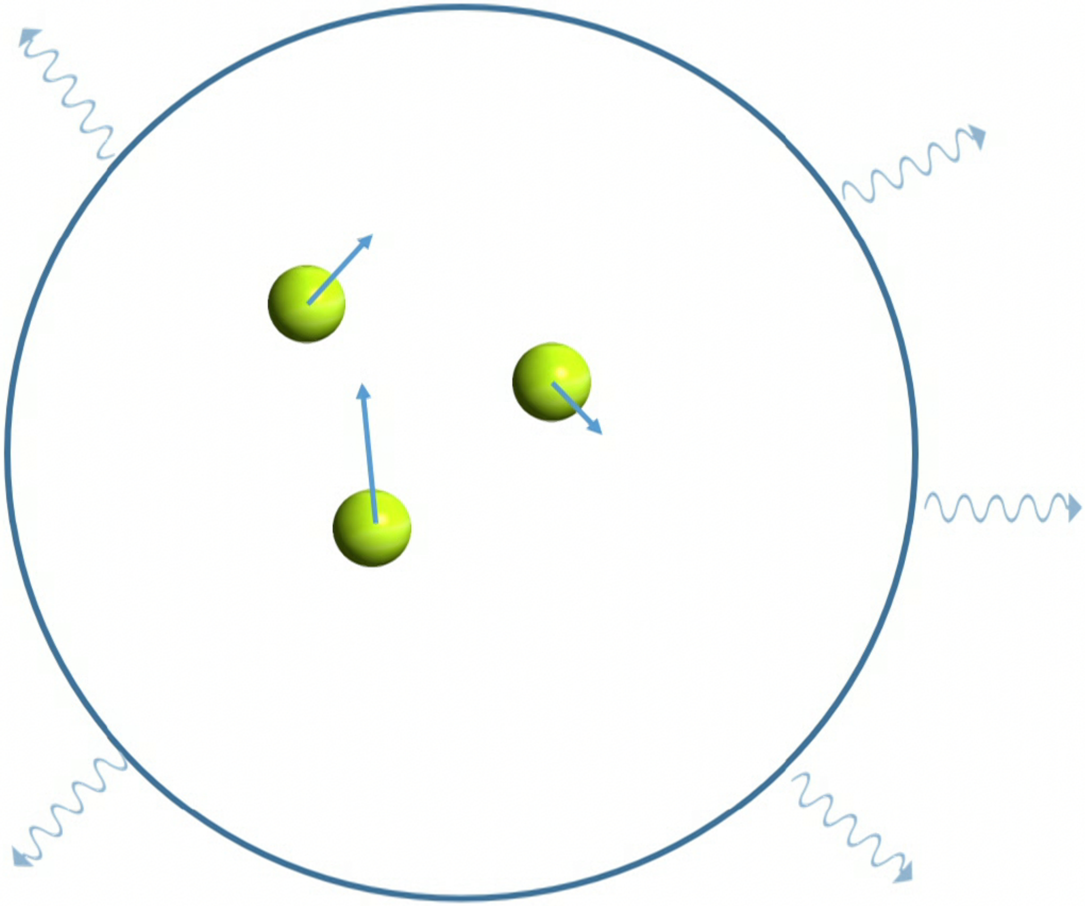
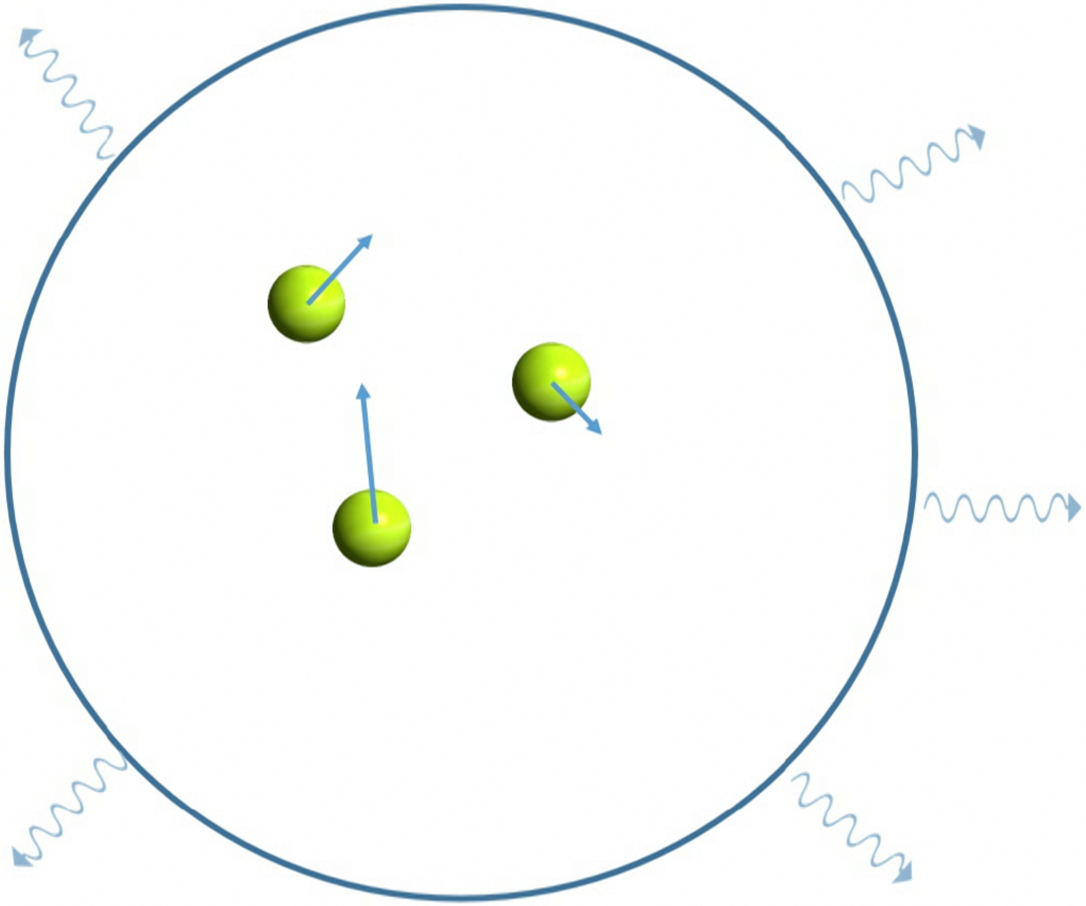
Fluxes in electromagnetism



Statement

Fluxes of E, P^i, J^{ij} are described by the radiative degrees of freedom of A_μ

Fluxes in electromagnetism



Statement

Fluxes of E, P^i, J^{ij} are described by the radiative degrees of freedom of A_μ

Practical!

radiative degrees of freedom described by A_i^t

(t = transverse to direction of wave propagation)

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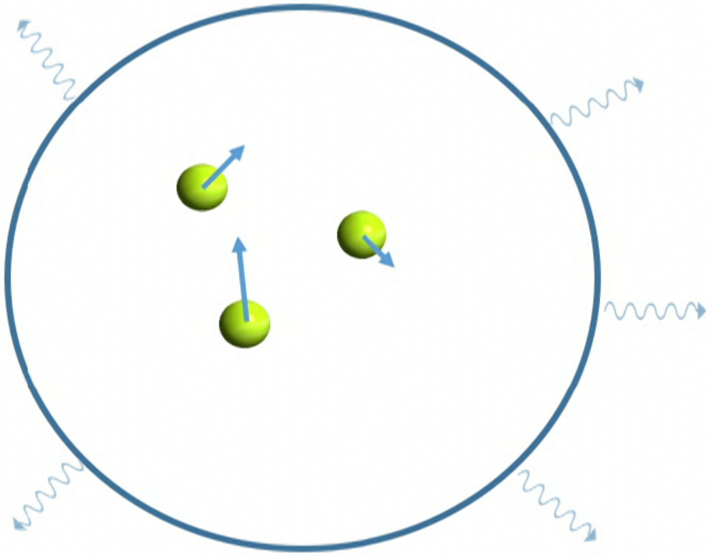
Fluxes in electromagnetism

$f_x = \cos\theta \sin\varphi$
$f_y = \cos\theta \cos\varphi$
$f_z = \sin\theta$



$$\dot{E} \sim \int d^3V\, \dot{A}_i^t \dot{A}_i^t$$

$$\dot{P}^j \sim \int d^3V\, \dot{A}_i^t \dot{A}_i^t\, f_j(\theta, \varphi)$$

$$j^{ij} \sim \int d^3V\, \dot{A}_i^t\, \eth\, A_i^t$$

Fluxes in electromagnetism

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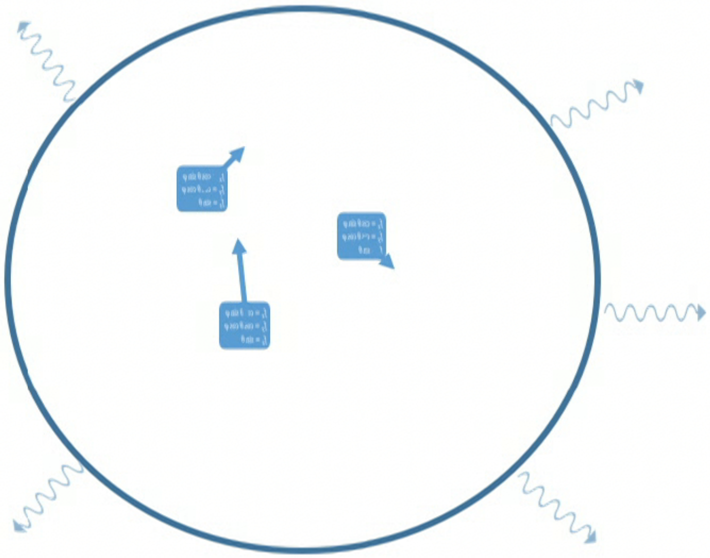

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$$j^{ij} \sim \int d^3V\, \dot{A}_i^t\, (\eth A_i^t + q(\theta,\varphi))$$

"Charge aspect"

$$Q \sim \int d^2\Omega\, q(\theta,\varphi)$$

Fluxes in electromagnetism

Statement

Fluxes of E, P^i, J^{ij} are described by the radiative degrees of freedom of A_μ

Correct statement I

Fluxes of E, P^i are described by the radiative degrees of freedom of A_μ

Fluxes in electromagnetism

Statement

Fluxes of E, P^i, J^{ij} are described by the radiative degrees of freedom of A_μ

Correct statement I

Fluxes of E, P^i are described by the radiative degrees of freedom of A_μ

Correct statement II

Flux of J^{ij} is described by the radiative degrees of freedom ***as well as Coulombic parts*** of A_μ

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Angular momentum radiated in EM

$$j^{ij} \sim \int d^3V\, \dot{A}_i^t\, (\eth\, A_i^t \;+ q(\theta, \varphi))$$

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Angular momentum radiated in EM

$$J^{ij} \sim \int d^3V \, \dot{A}_i^t \, (\eth \, A_i^t \; + q(\theta, \varphi))$$

Fine print

Charge aspect only important if $Q \neq 0$

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Angular momentum radiated in EM

$$j^{ij} \sim \int d^3V\, \dot{A}_i^t \,(\,\eth\, A_i^t \;+ q(\theta,\varphi))$$

Fine print

Charge aspect only important if $Q \neq 0$

Examples

1) Static/boosted charge → $j^{ij} = 0$
2) Oscillating dipole → $j^{ij} \neq 0$ but purely radiative
3) Example 1+2 → $j^{ij} \neq 0$ radiative + Coulombic interaction

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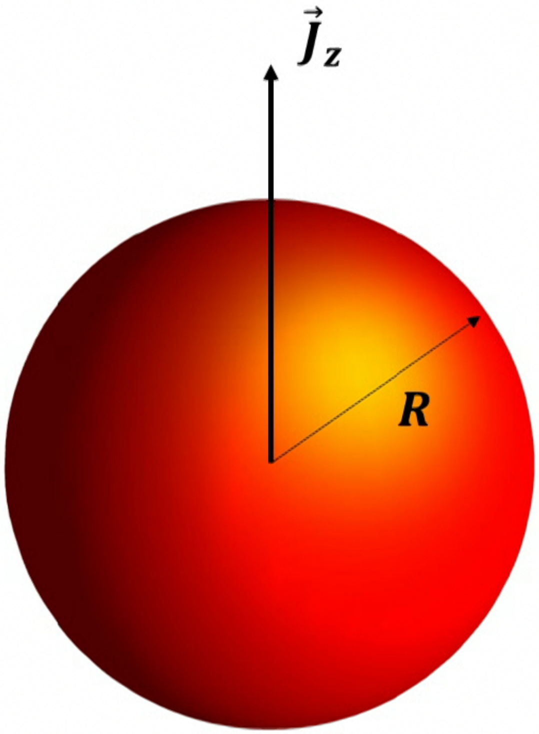
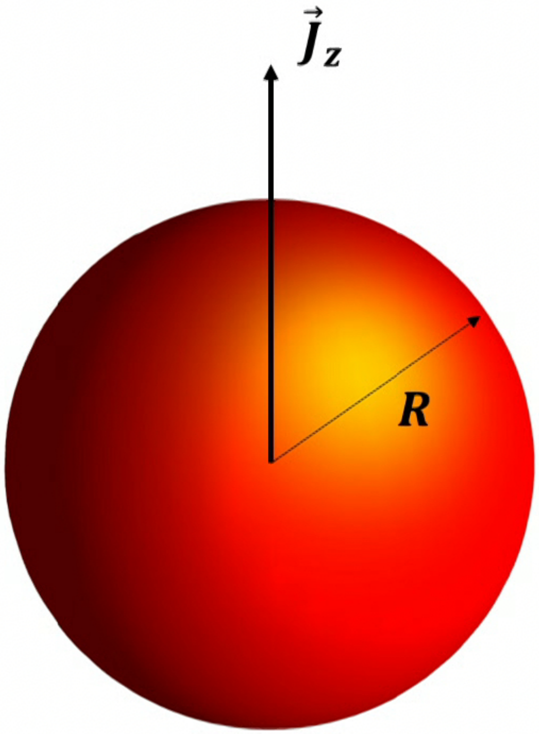
Spinning charged sphere



Variable angular velocity $\Omega = \Omega(t)$

Spinning charged sphere



$$m(t) = \frac{1}{3}\, q\, R^2\, \Omega(t)$$

$$= \frac{\mu_0 q}{6\pi c}\left[\ddot{m} + \frac{1}{10}\tau^2 m^{(4)} + \frac{1}{280}\tau^4 m^{(6)} + \frac{1}{15120}\tau^6 m^{(8)} + \cdots\right]_{t-r_0/c}$$

Variable angular velocity $\Omega = \Omega(t)$

From EM to GR

$$\dot{E} \sim \int d^3V\, \dot{A}_i^t \dot{A}_i^t$$

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$$j^{ij} \sim \int d^3V\, \dot{A}_i^t\, (\eth\, A_i^t + q(\theta,\varphi))$$

Replace

$$A_i^t \longrightarrow h_{ij}^{tt}$$

$$q(\theta,\varphi) \longrightarrow M(\theta,\varphi)$$

?

From EM to GR

$$\dot{E} \sim \int d^3V\, \dot{A}_i^t \dot{A}_i^t$$

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Replace

$$A_i^t \longrightarrow h_{ij}^{tt}$$

$$q(\theta,\varphi) \longrightarrow M(\theta,\varphi)$$

?

But no such “mass aspect” contribution in the literature…

Angular momentum in literature

Approaches to angular momentum radiated

Hamiltonian/Lagrangian methods

Restrict to the radiative phase space

[Ashtekar & Steubel, Penrose, Dray & Streubel, …]

Landau-Lifschitz approach

Restrict to *periodic* sources at *rest*

[DeWitt, Thorne, …]

Why Landau-Lifschitz?

$$\frac{d}{du} J^{ab} = -\mathcal{T}^{ab}$$

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Landau-Lifschitz & Bondi formalism

$$ds^2 = -UV\,du^2 - 2U\,dudr + \gamma_{AB}(r\,d\theta^A + W^A\,du)(r\,d\theta^B + W^B\,du)$$

Landau-Lifschitz & Bondi formalism

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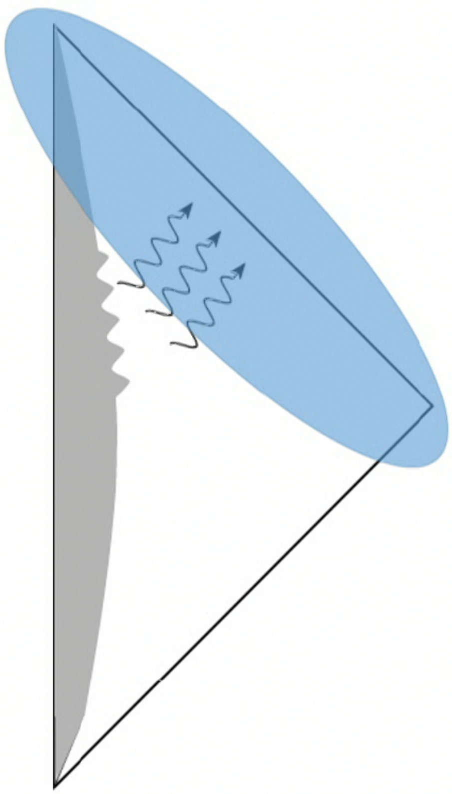

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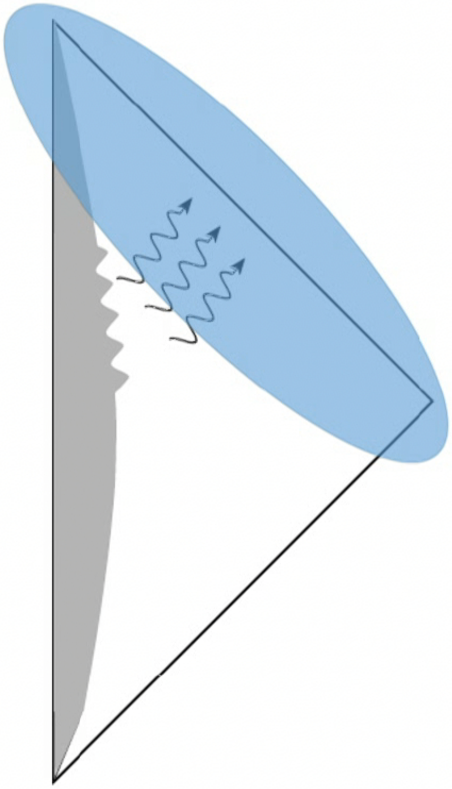

$$U = 1 + B/r^2 + O(r^{-3}),$$
$$V = 1 - 2M/r + N/r^2 + O(r^{-3}),$$
$$W^A = A^A/r + B^A/r^2 + O(r^{-3}),$$
$$\gamma_{AB} = \Omega_{AB} + f_{AB}/r + \tfrac{1}{4} f^2 \Omega_{AB}/r^2 + O(r^{-3})$$

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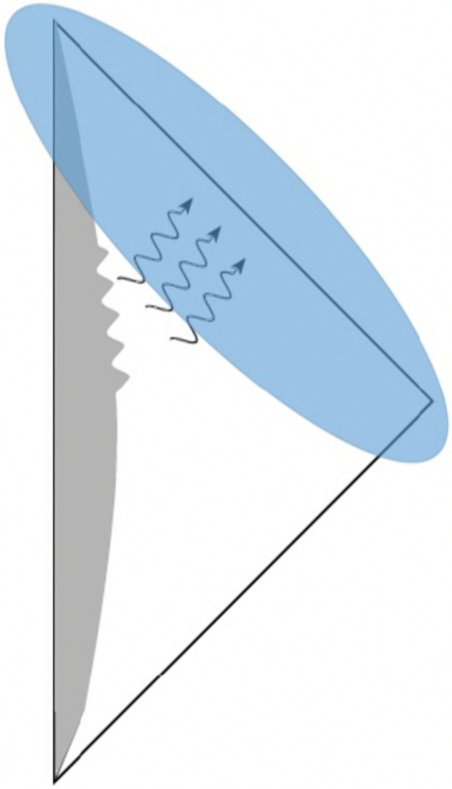

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Flat Space

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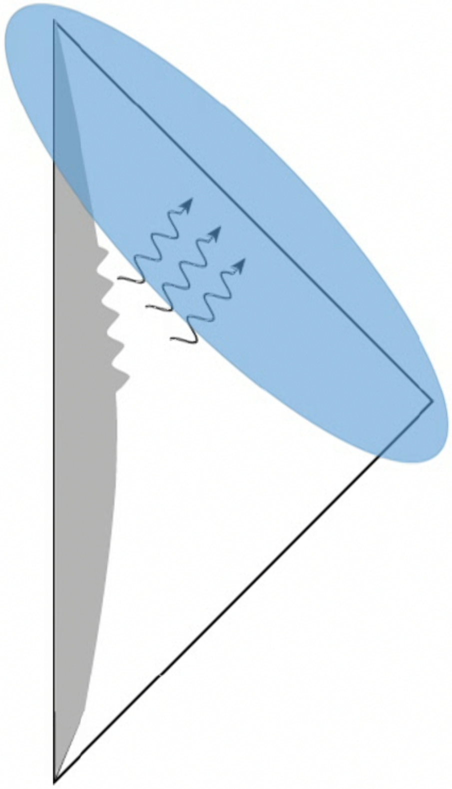

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Flat Space

Mass aspect

Landau-Lifschitz & Bondi formalism

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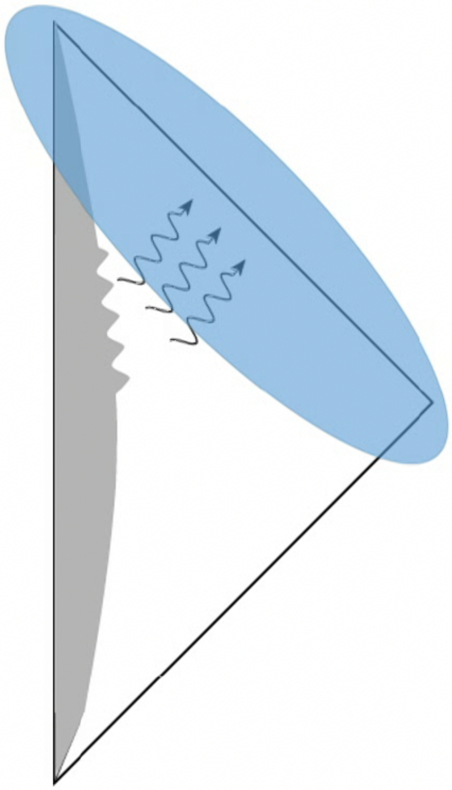

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Flat Space | Mass aspect | Radiative modes

Landau-Lifschitz & Bondi formalism

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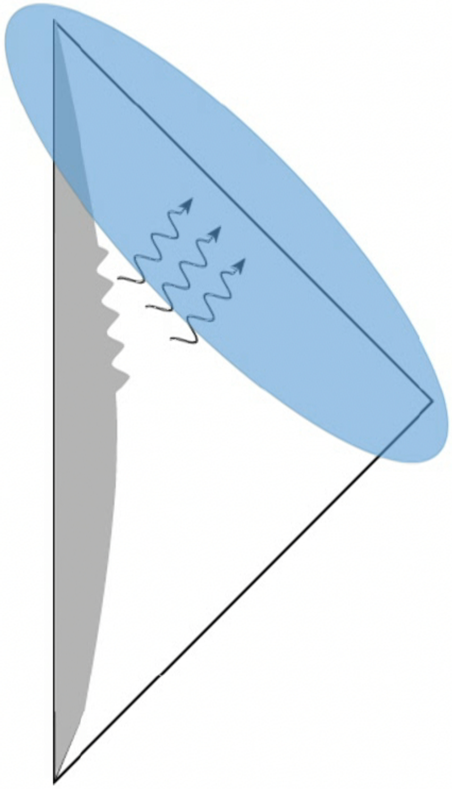

$$\begin{aligned} U &= 1 + B/r^2 + O(r^{-3}), \\ V &= 1 - 2M/r + N/r^2 + O(r^{-3}), \\ W^A &= A^A/r + B^A/r^2 + O(r^{-3}), \\ \gamma_{AB} &= \Omega_{AB} + f_{AB}/r + \tfrac{1}{4}f^2\Omega_{AB}/r^2 + O(r^{-3}) \end{aligned}$$

Flat Space | Mass aspect | Radiative modes | Angular momentum aspect

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Energy and linear momentum are as expected

$$\frac{dE}{du} = -\int \mathfrak{p}\, d\Omega$$

$$\mathfrak{e} = \frac{1}{4\pi} M - \frac{1}{32\pi} D_A D_B f^{AB}$$

$$\mathfrak{p} = \frac{1}{32\pi} \dot{f}_{AB} \dot{f}^{AB}$$

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LHS = total energy/momentum = "Coulombic" pieces (+ radiative modes)
RHS = fluxes = ***purely*** radiative

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$$\frac{dP^a}{du} = -\int \mathfrak{f}^a\, d\Omega$$

$$\mathfrak{p}^a = \frac{1}{4\pi} M \Omega^a - \frac{1}{32\pi} D_A \left(\Omega^a D_B f^{AB}\right)$$

$$\mathfrak{f}^a = \frac{1}{32\pi} \dot{f}_{AB} \dot{f}^{AB}\, \Omega^a$$

LHS = total energy/momentum = “Coulombic” pieces (+ radiative modes)
RHS = fluxes = ***purely*** radiative

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Angular momentum balance law

$$\frac{d}{du} J^{ab} = -\mathcal{T}^{ab}$$

$$J^{ab} = \int \mathrm{j}^{ab}\, d\Omega$$

$$\mathrm{j}^{ab} = -\frac{1}{4\pi} r \Omega^{[a} A^{b]} - \frac{1}{8\pi} \Omega^{[a} \left(3B^{b]} + f^{b]}{}_c A^c \right) + O(r^{-1})$$

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=0 after angular integration

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$$= -\frac{3}{8\pi}\Omega^{[a}B^{b]} - \mathfrak{r}^{ab}$$

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Flux of angular momentum

$$\mathcal{T}^{ab} = \int \mathfrak{t}^{ab}\, d\Omega$$

$$\mathfrak{t}^{ab} = -\frac{1}{16\pi}\Omega^{[a}\Omega^{b]}_{B}\left(3\dot{f}^{B}{}_{C}\, D_{D} f^{CD} - \dot{f}^{CD}\, D_{C} f^{B}{}_{D}\right) + \frac{\partial}{\partial u}\frac{1}{16\pi}\Omega^{[a}\Omega^{b]}_{B} f^{B}{}_{C}\, D_{D} f^{CD}$$

Flux of angular momentum

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Only radiative terms!

Flux of angular momentum

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Only **radiative terms!**

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Nevertheless, ***difference*** with standard expression

$$\mathfrak{t}^{ab} - \mathfrak{t}^{ab}_{\text{standard}} = \frac{\partial \mathfrak{p}^{ab}}{\partial u} - D_D\left(\frac{1}{8\pi}\Omega^{[a}\Omega^{b]}_{B}\dot{f}^{B}{}_{C}\, f^{CD}\right)$$

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Nevertheless, ***difference*** with standard expression

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=0 after orbit averaging

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=0 after orbit averaging

=0 after angular integration

Balance law redefined

$$\frac{d}{du} J^{ab} = -\mathcal{T}^{ab}$$

$$\frac{\partial}{\partial u}\int d\Omega\left(-\frac{3}{8\pi}\Omega^{[a}B^{b]} - \mathfrak{r}^{ab}\right) =$$

$$\int d\Omega\left(-\frac{1}{16\pi}\Omega^{[a}\Omega^{b]}_{B}\left(3\dot{f}^{B}{}_{C}\, D_D f^{CD} - \dot{f}^{CD}\, D_C f^{B}{}_{D}\right) + \frac{\partial \mathfrak{r}^{ab}}{\partial u}\right)$$

Balance law redefined

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$$\int d\Omega\left(-\frac{1}{16\pi}\Omega^{[a}\Omega^{b]}_{B}\left(3\dot{f}^{B}_{C}\,D_{D}f^{CD} - \dot{f}^{CD}\,D_{C}f^{B}_{D}\right) + \frac{\partial\mathfrak{r}^{ab}}{\partial u}\right)$$

$$d\mathsf{J}^{ab}/du = -\mathsf{T}^{ab}$$

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Conclusion

Radiative processes in electromagnetism and general relativity are similar, *except for* angular momentum radiated:

* EM: radiative modes + Coulombic parts
* GR: *only* radiative modes

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