Title: A New Real-Time Picture of Vacuum Decay

Date: Feb 05, 2019 02:00 PM

URL: http://pirsa.org/19020041

Abstract: <p>Quantum decay of false vacuum states via the nucleation of bubbles may
have played an important role in the early history of our Universe. For
example, in multiverse models that utilize false vacuum eternal
inflation, the Big Bang of our observable Universe corresponds to one of
these bubble nucleation events. Further, our observable Universe may
have undergone a series of symmetry-breaking first-order phase
transitions as it cooled, which may have produced a remnant background
of gravitational waves.

I will present results from a new real-time picture of false vacuum
decay which, in contrast to existing semiclassical techniques, does not
rely on classically forbidden tunneling paths. Lattice simulations are
used to evolve initial realizations of fluctuations around the false
vacuum forward in time via the classical equations of motion. In these
simulations, we observe the false vacuum decay via the formation and
subsequent expansion and coalescence of true vacuum bubbles. By
sampling initial field realizations, we build up ensembles of these
decay histories and empirically determine the bubble nucleation rate.
The rates agree well with standard Euclidean techniques, which cannot
provide a time-dependent description of the decay. Some novel
applications of our new approach include investigation of bubble-bubble
correlation functions, decay of time-evolving metastable states, decay
of non-vacuum initial states, and the regime of rapid decays.</p>

A Real-Time Semiclassical Picture of Vacuum Decay

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w/ Matt Johnson, Hiranya Peiris, Andrew Pontzen, and Silke Weinfurtner
1712.02356, 1806.06069, and in progress

Perimeter Institute, February 5, 2019

How Quantum is QFT?

$$|\Psi\rangle$$

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \hat{H} |\Psi\rangle$$

$$P[\phi, \Pi]$$

$$\frac{\partial \phi}{\partial t} = \frac{\delta H}{\delta \Pi}$$

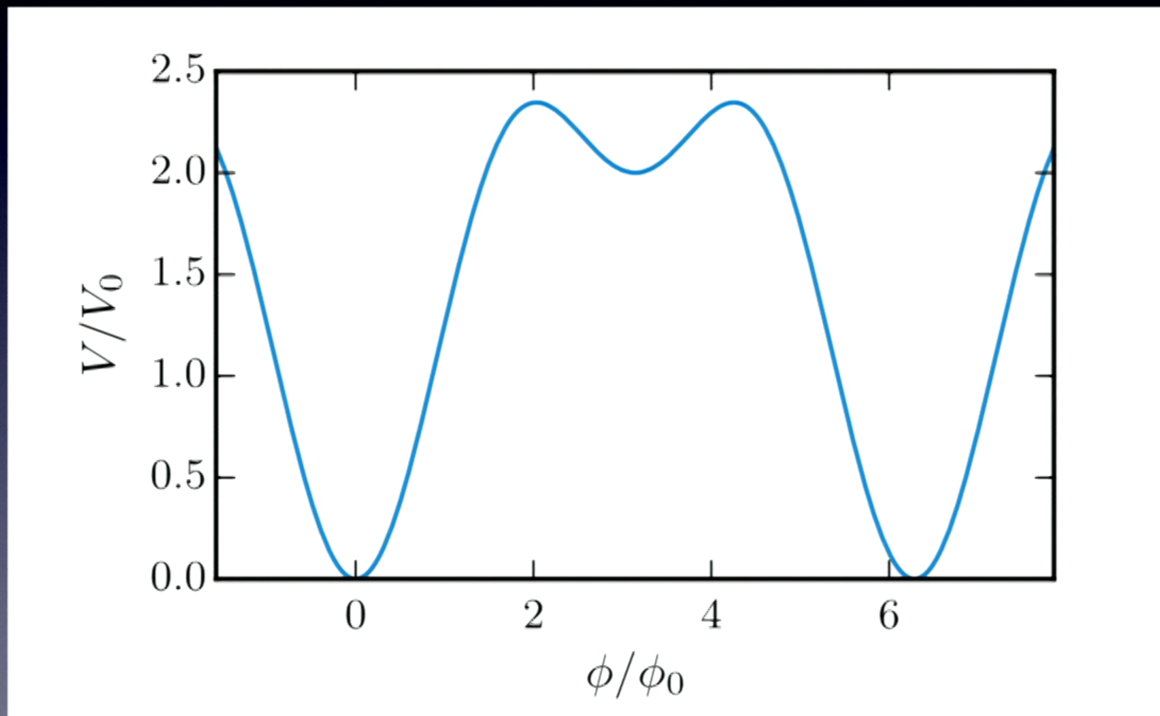
$$\frac{\partial \Pi}{\partial t} = -\frac{\delta H}{\delta \phi}$$

Nonlinear, Nonperturbative, Nonequilibrium Phenomena

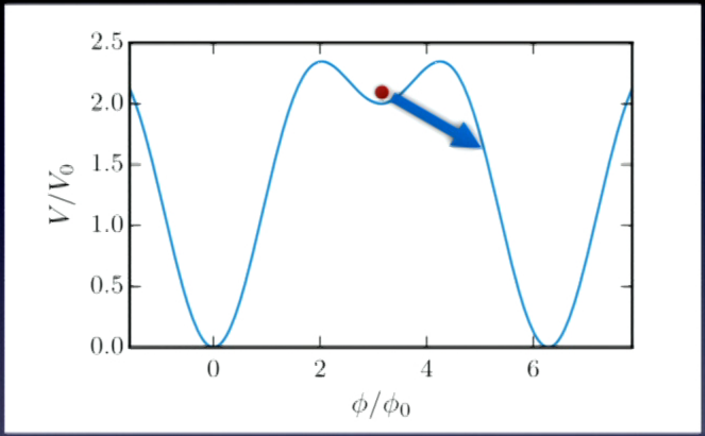
Outline

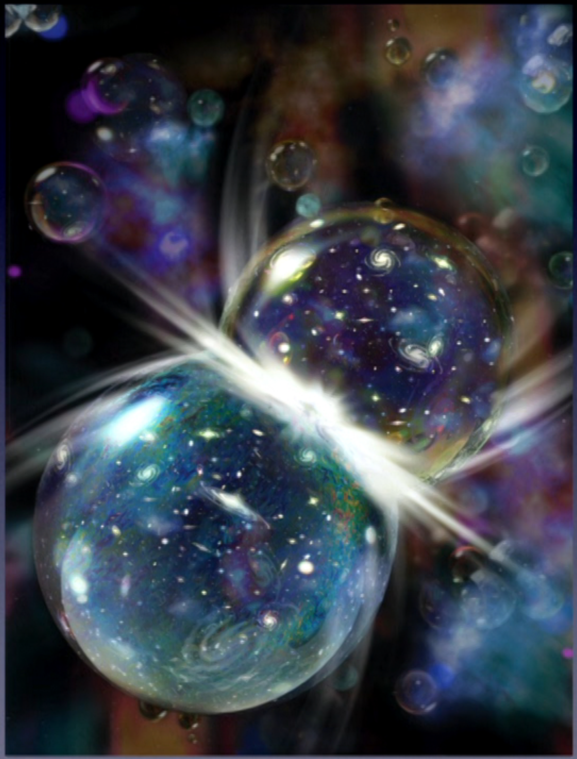
- Review of Vacuum Decay and 1st Order Phase Transitions
- Euclidean Description (including new computational method)
- Real-Time Description of Decay
- Novel Future Applications
- Connection to BECs (time permitting)

First Order Phase Transitions

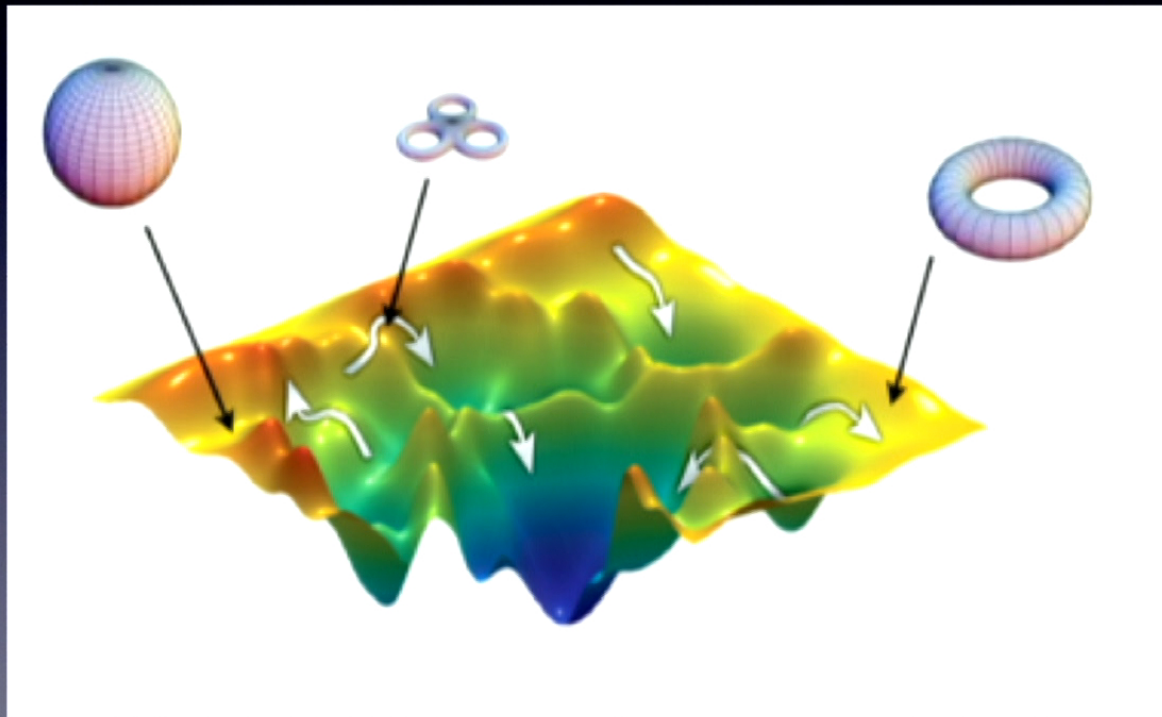


First Order Phase Transitions

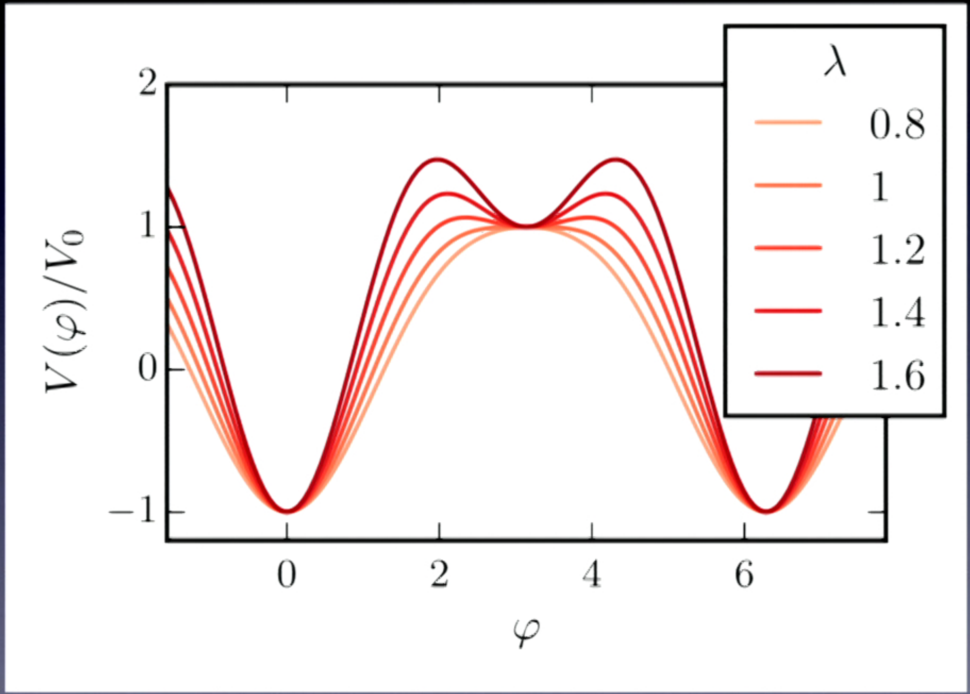




First Order Phase Transitions



Model



$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

0th Order Questions

- How fast does the vacuum decay?
- Do bubbles form?
- What do the bubbles look like?

Decay Rate

$$P_{\text{undecayed}} = |\langle \Omega_{\text{FV}}(t)|\Omega_{\text{FV}}(t=0)\rangle|^2 \sim e^{-\Gamma t}$$

Schematically

$$\langle \Omega_{\text{FV}}|\Omega_{\text{FV}}(t)\rangle = \langle \Omega_{\text{FV}}|e^{-iHt}|\Omega_{\text{FV}}\rangle$$

Work in Euclidean Time

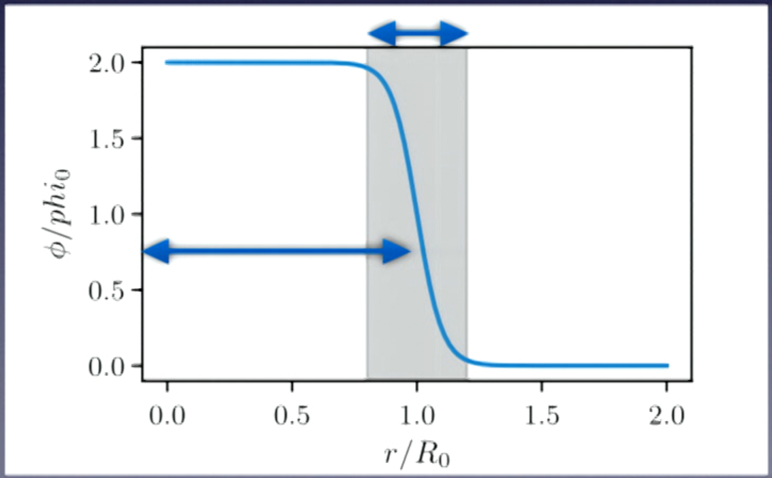
$$\langle \Omega_{\text{FV}}|e^{-HT}|\Omega_{\text{FV}}\rangle \sim e^{-E_0 T}$$

Imaginary Part of Energy Gives Decay in Real Time

Standard Description

$$r_{\mathrm{E}}^2 = \tau^2 + \mathbf{r}^2 \qquad \tau = it$$

$$\frac{\partial^2 \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}^2} + \frac{d}{r_{\mathrm{E}}}\frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}} - \frac{\partial V}{\partial \phi} = 0 \qquad \frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}}(0) = 0 \qquad \phi(\infty) = \phi_{\mathrm{fv}}$$

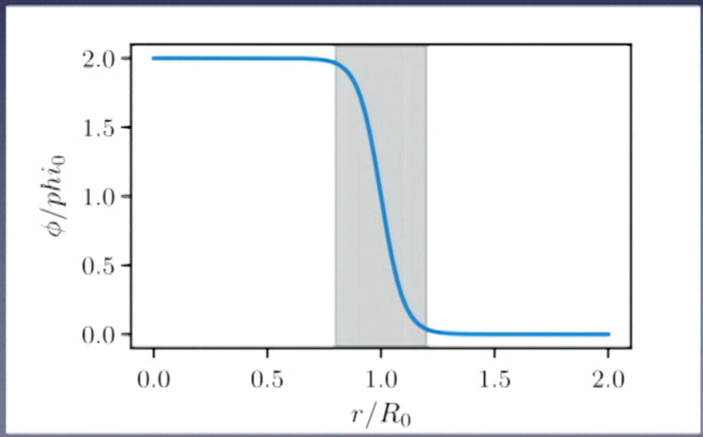


Typical Solution

Pseudospec Solution

$$\phi_{\rm I}(r) = \sum_n c_n B_{2n}\left(y\left(\frac{r}{\sqrt{r^2+L^2}}\right)\right)$$

$$y(x) = \frac{1}{\pi}\tan^{-1}\left(d^{-1}\tan\left(\pi\left[x-\frac{1}{2}\right]\right)\right)+\frac{1}{2}$$



Chebyshev Polynomials

$$B_n(x) = \cos(n\cos^{-1}(x))$$

Pseudospec Solution

Zero deriv.
at origin

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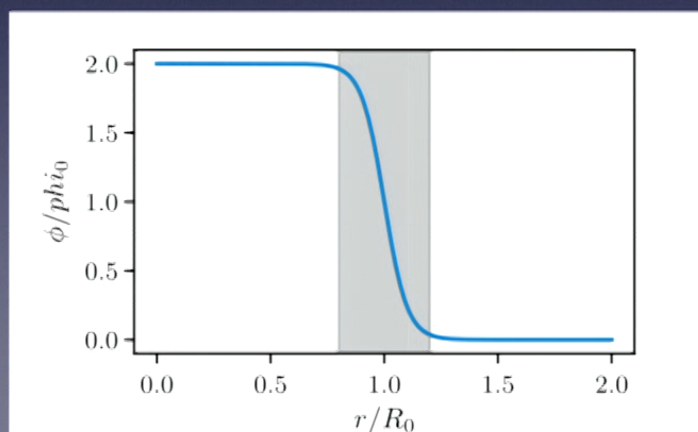

Chebyshev Polynomials

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Pseudospec Solution

Zero deriv. at origin

Infinite Domain, feature at L

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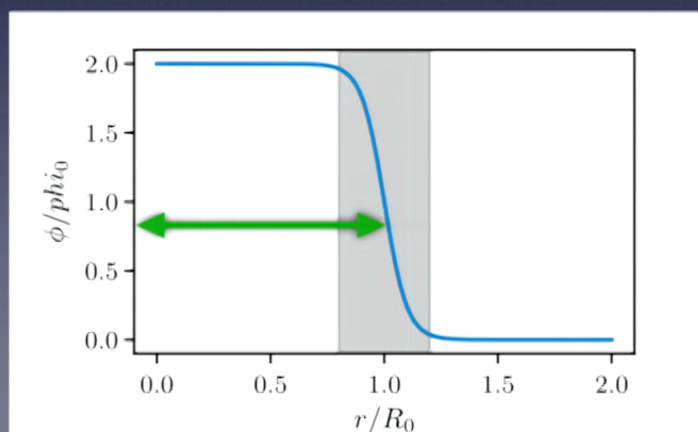


Chebyshev Polynomials

$$B_n(x) = \cos(n\cos^{-1}(x))$$

Pseudospec Solution

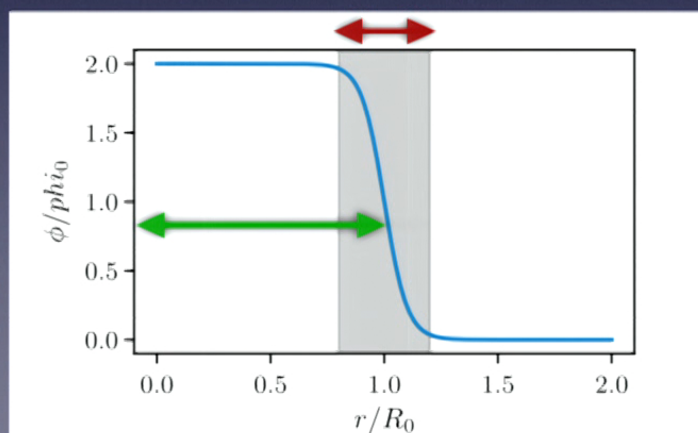
Zero deriv. at origin

Width of Feature

Infinite Domain, feature at L

$$\phi_{\mathrm{I}}(r) = \sum_n c_n B_{2n}\left(y\left(\frac{r}{\sqrt{r^2+L^2}}\right)\right)$$

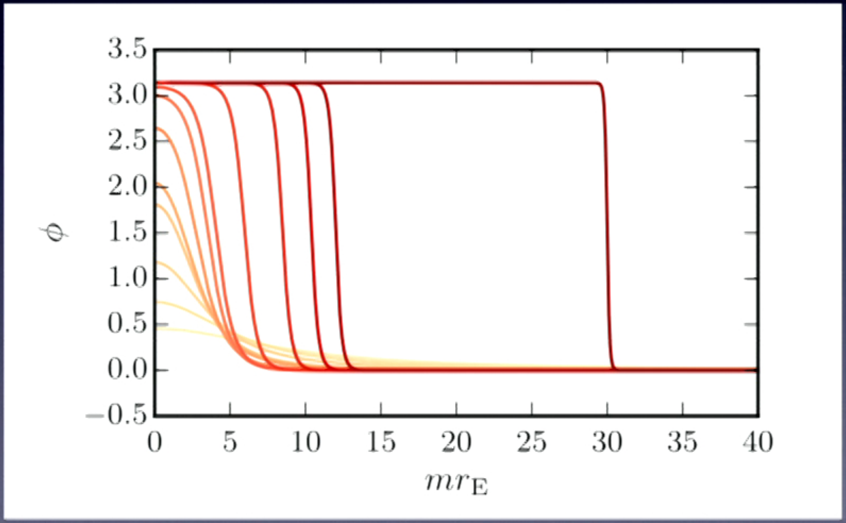
$$y(x) = \frac{1}{\pi}\tan^{-1}\left(d^{-1}\tan\left(\pi\left[x-\frac{1}{2}\right]\right)\right)+\frac{1}{2}$$



Chebyshev Polynomials

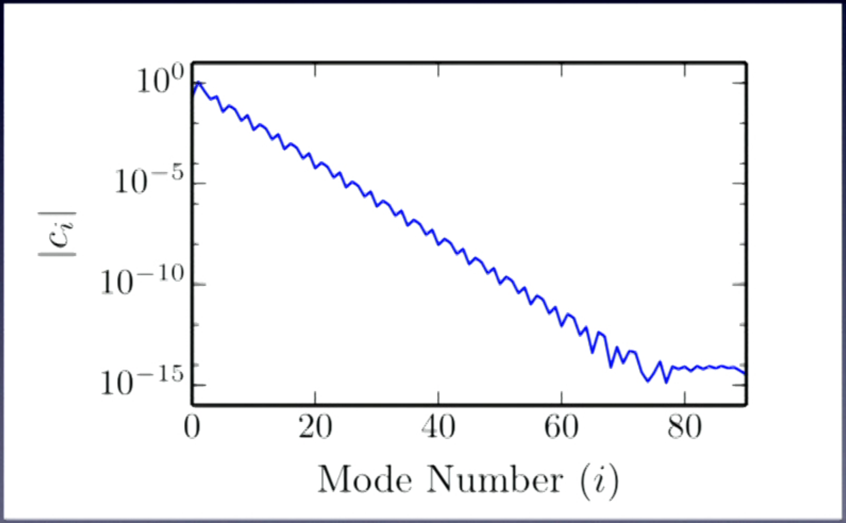
$$B_n(x) = \cos(n\cos^{-1}(x))$$

Bounce Profiles



- Outer boundary at ∞
- $\mathcal{O}(10^{-15})$: ~100 modes
- $N^3_{\mathrm{fields}}\mathcal{O}(10^{-3})$s
- ~~Arbitrary precision arithmetic~~

Bounce Profiles



- Outer boundary at ∞
- $\mathcal{O}(10^{-15})$: ~100 modes
- $N^3_{\text{fields}}\mathcal{O}(10^{-3})$s
- ~~Arbitrary precision arithmetic~~

Decay Rates

$$S_{\rm E} = A_{\rm d+1} \int dr_{\rm E} r_{\rm E}^d \left(\frac{\phi'^2}{2} + V(\phi) \right)$$

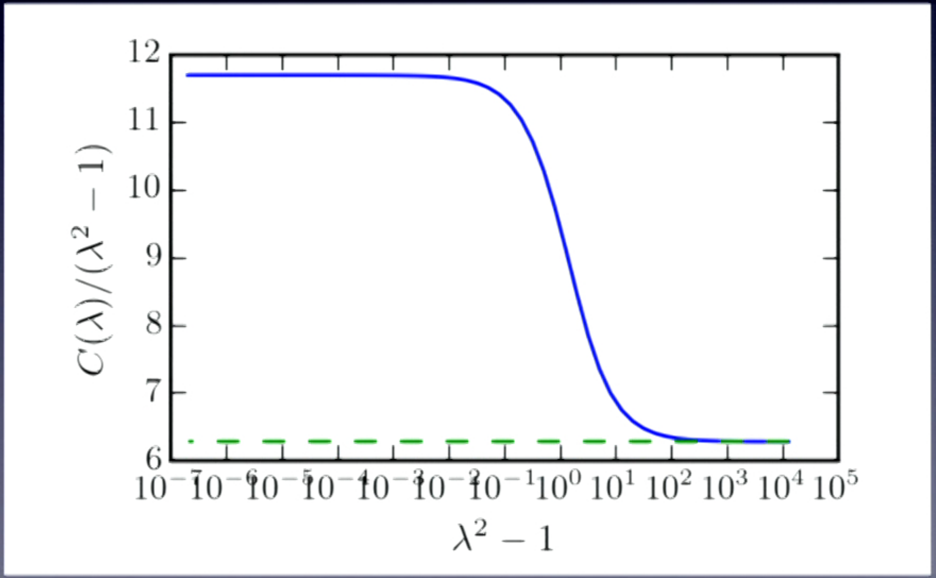
$$S_{\rm I} = S_{\rm E}[\phi_{\rm B}] - S_{\rm E}[\phi_{\rm fv}]$$

- Single negative eigenmode

$$\frac{\Gamma}{V} = \left(\frac{S_{\rm I}}{2\pi} \right)^{D/2} \sqrt{\frac{\det \delta^2 S_{\rm E}[\phi_{\rm fv}]}{\det' \delta^2 S_{\rm E}[\phi_{\rm B}]}} e^{-S_{\rm I}} \left(1 + \mathcal{O}(\hbar)\right)$$

Nucleation Rates

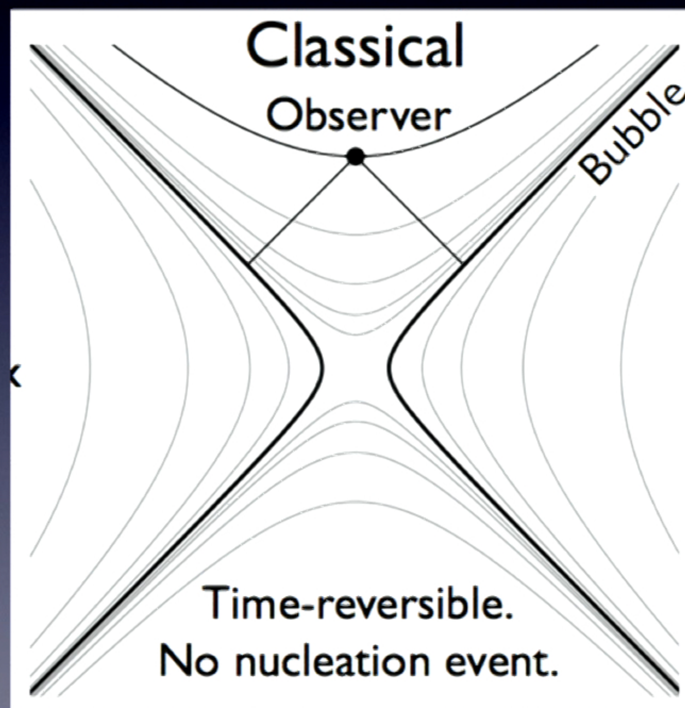
$$\frac{\Gamma}{V} \approx g(\lambda) \left[m_{\mathrm{eff}}^2\right]^{\frac{D}{2}} \left(\frac{S_{\mathrm{I}}}{2\pi}\right)^{\frac{D}{2}} e^{-S_{\mathrm{I}}}$$



$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2} \sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

Real-Time Interpretation

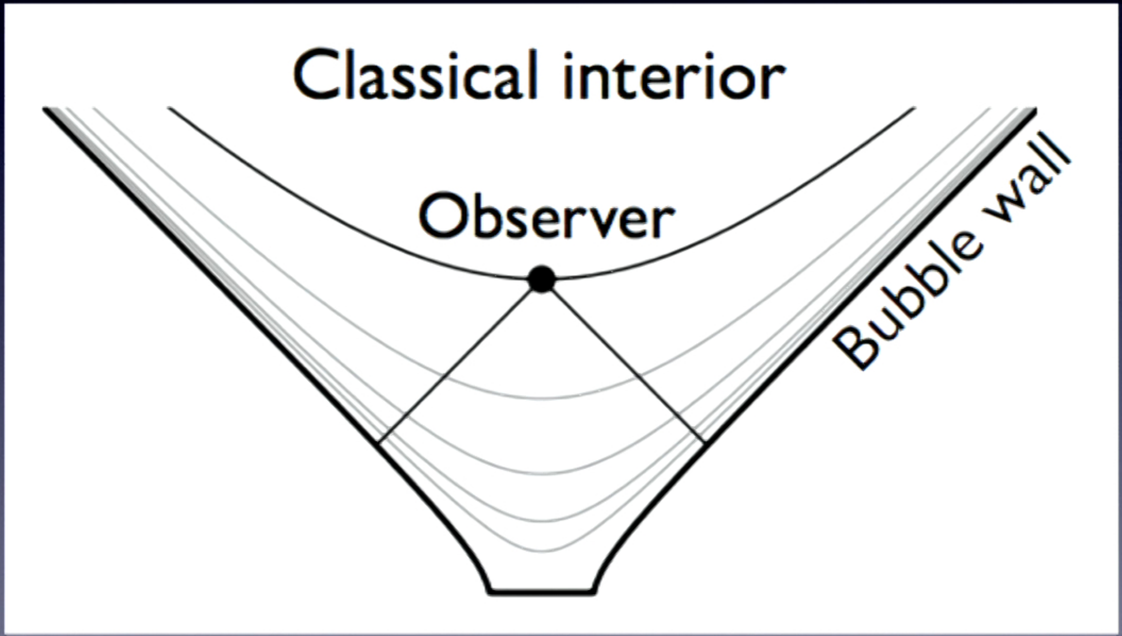
$$\phi(x,t) = \phi_{\rm I}(\sqrt{x^2 - t^2})$$



[Figure courtesy of Andrew Pontzen]

Ad-Hoc Nucleation

$$\phi(\mathbf{x}, t=0) = \phi_{\mathrm{I}}(|\mathbf{x}|)$$



No real-time classical description

[Figure courtesy of Andrew Pontzen]

Some Questions

- Time-dependent description of nucleation
  - Bubble precursor? Init. cond. at nucleation
- Bubble-bubble correlations
- Fast decay/large fluctuation limit?
- Time evolving background/potential
- Nonvacuum state

Some Questions

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  - Bubble precursor? Init ... t nucleation
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- Fast decay/lar ... ation limit?
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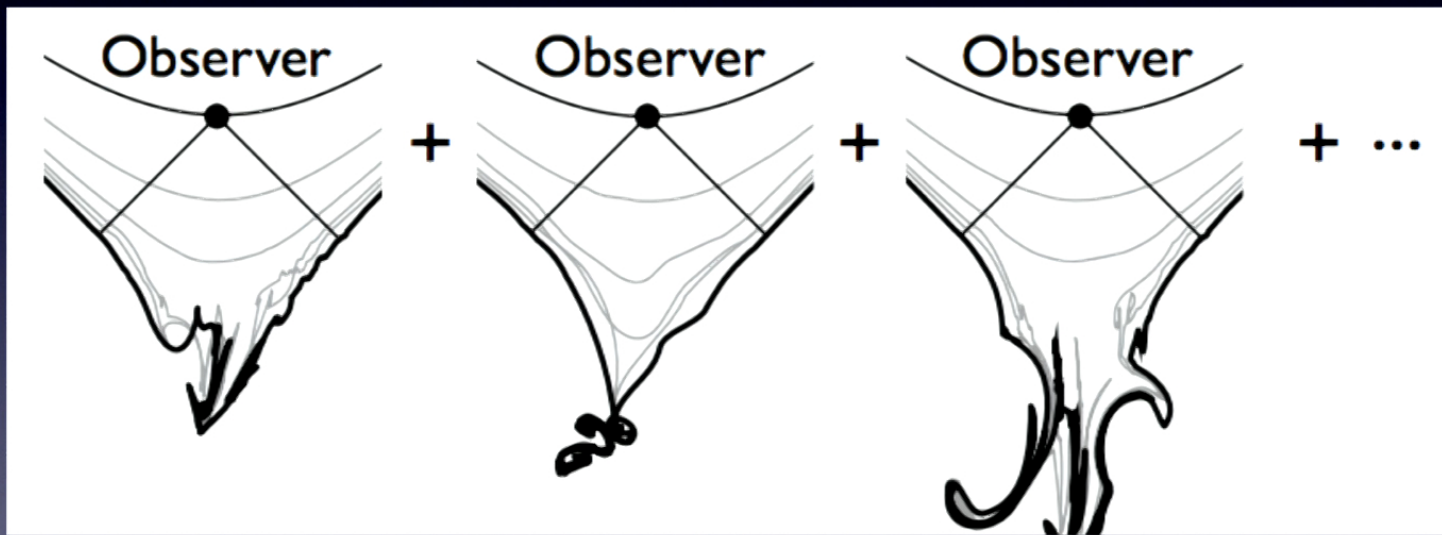
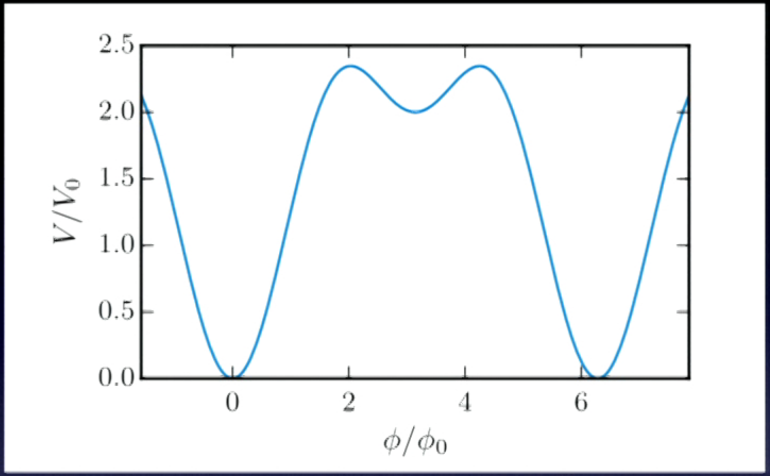
- Nonvacu ... state

QFT exponentially complex.
Need Approximations!

Full Evolution?

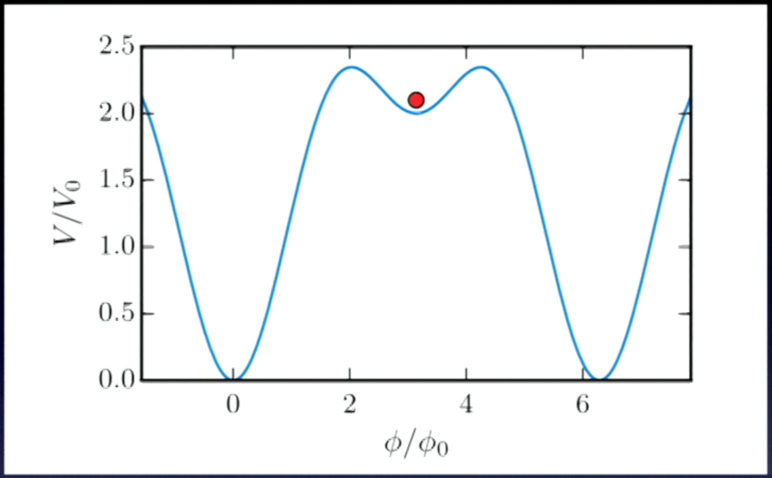


[Figure courtesy of Andrew Pontzen]



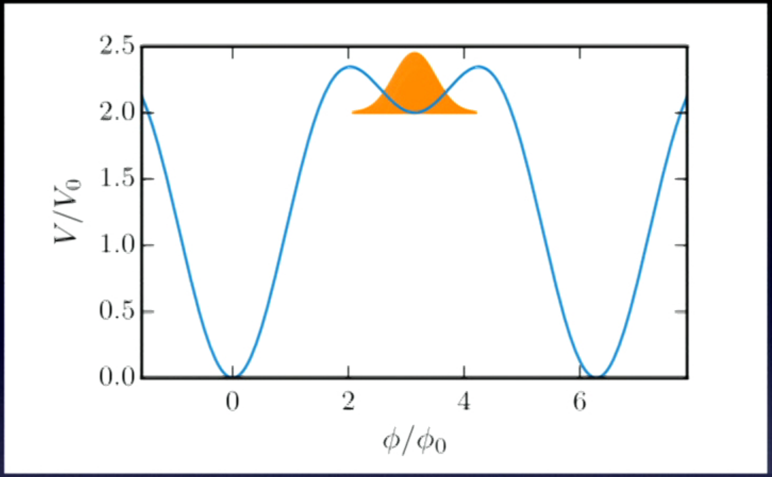
$$\phi =$$

$$\Pi =$$



$$\phi = \phi_{\rm fv}$$

$$\Pi = 0$$

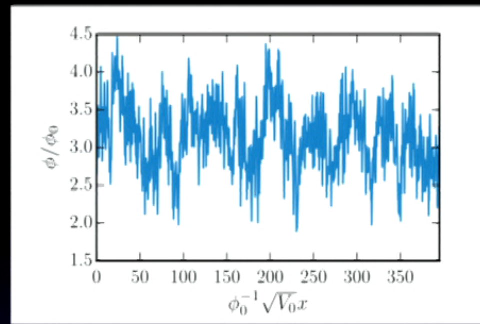


$$\phi = \phi_{\rm fv} + \delta\hat{\phi}(\mathbf{x}, t)$$

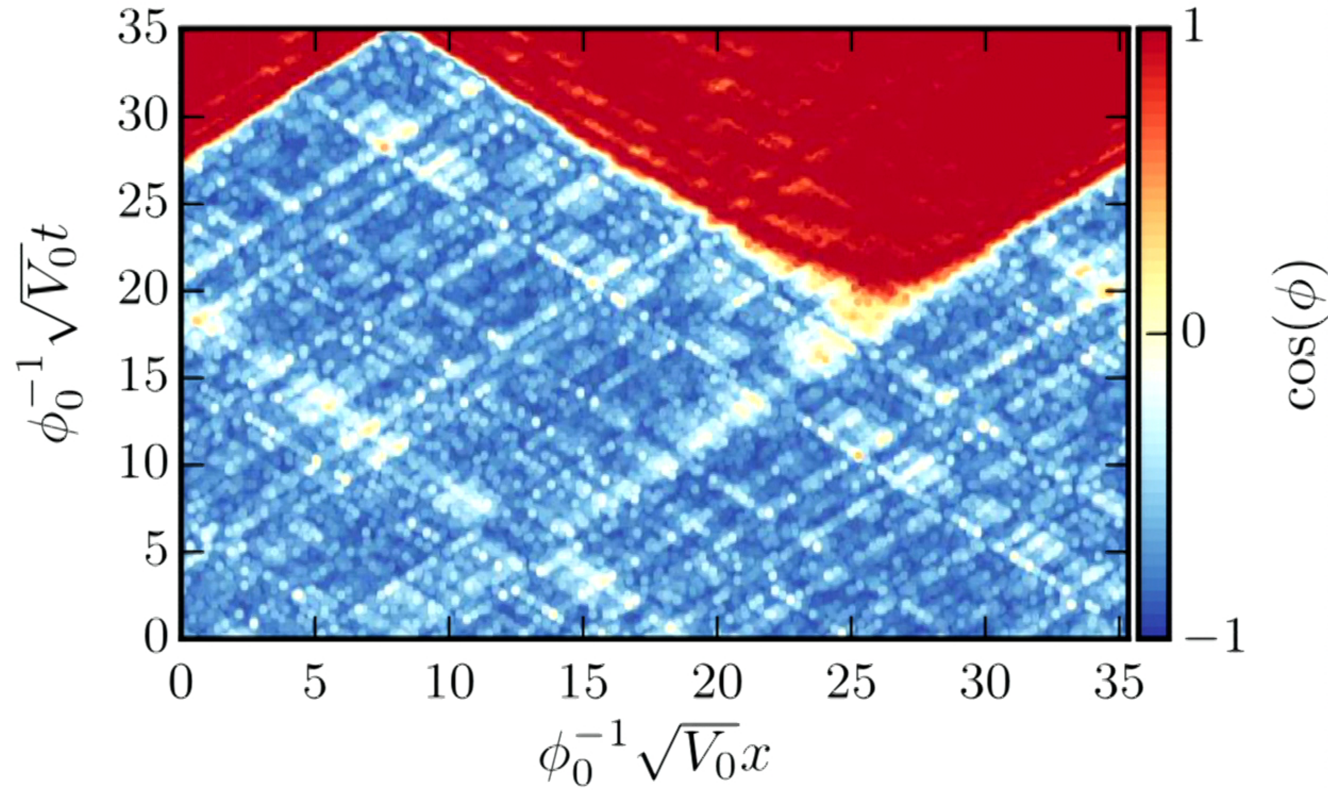
$$\Pi = 0 + \delta\hat{\Pi}(\mathbf{x}, t)$$

$$\langle \delta\tilde{\phi}_k \delta\tilde{\phi}^*_p \rangle = \frac{1}{2\omega_k}\delta(k-p) \qquad \langle \delta\tilde{\Pi}_k \delta\tilde{\Pi}^*_p \rangle = \frac{\omega_k}{2}\delta(k-p)$$

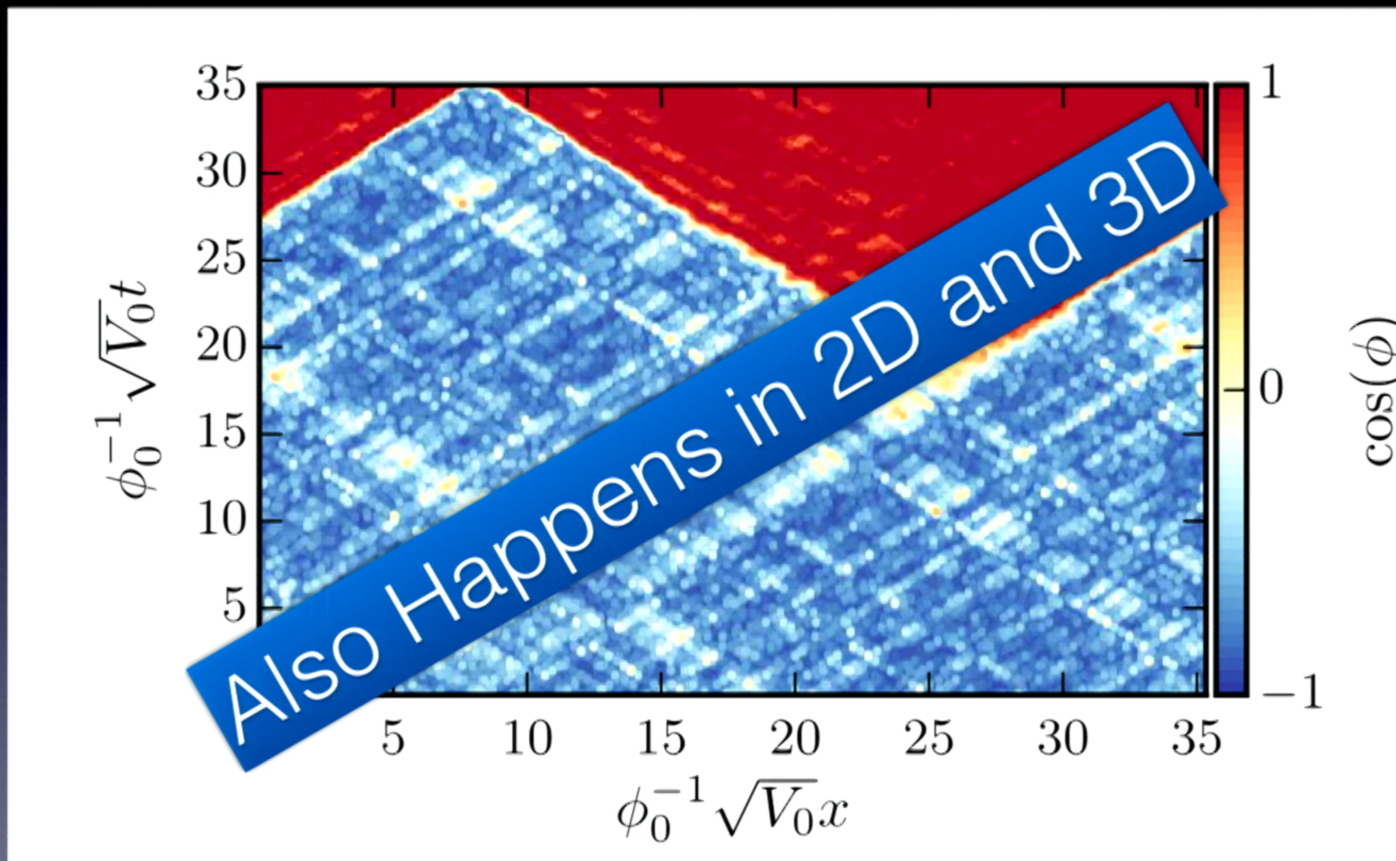
Quantum
Commutators



$$\ddot{\phi} - \nabla^2\phi + V'(\phi) = 0$$



Classically-Allowed Vacuum Decay



Classically-Allowed Vacuum Decay

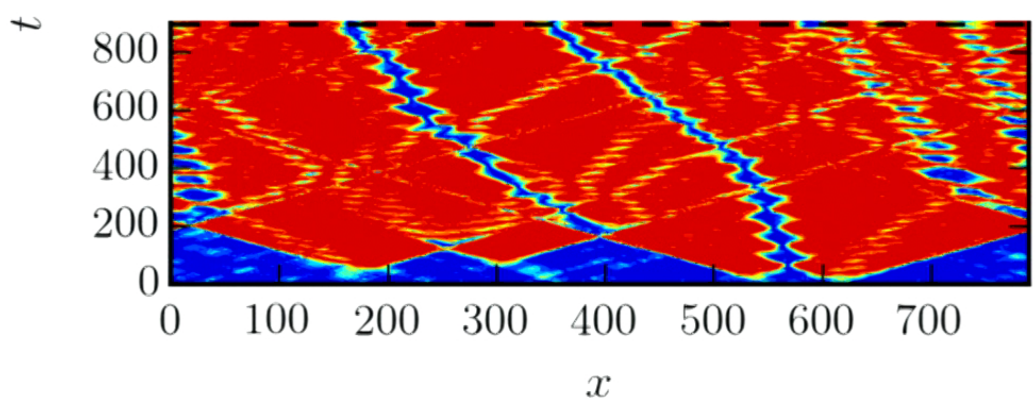
Numerical Artifact?

- Spatial Discretization: Fourier pseudospectral (exponential convergence)
  Temporal Discretization: Gauss-Legendre (10th order in dt, symplectic)

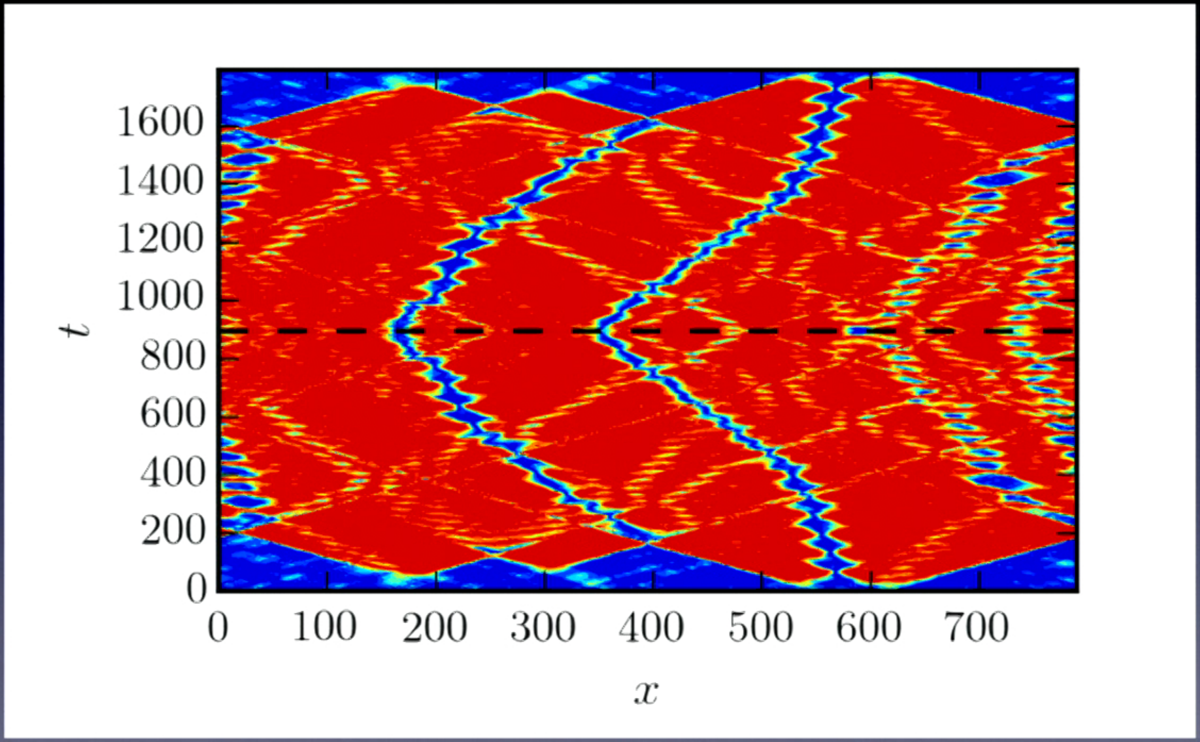
- Energy conservation: $\mathcal{O}(10^{-15})$
  Momentum conservation: $\mathcal{O}(10^{-15})$
  Pointwise convergence with dt step: $\mathcal{O}(10^{-15})$
  Pointwise convergence with dx step: $\mathcal{O}(10^{-15})$

NO

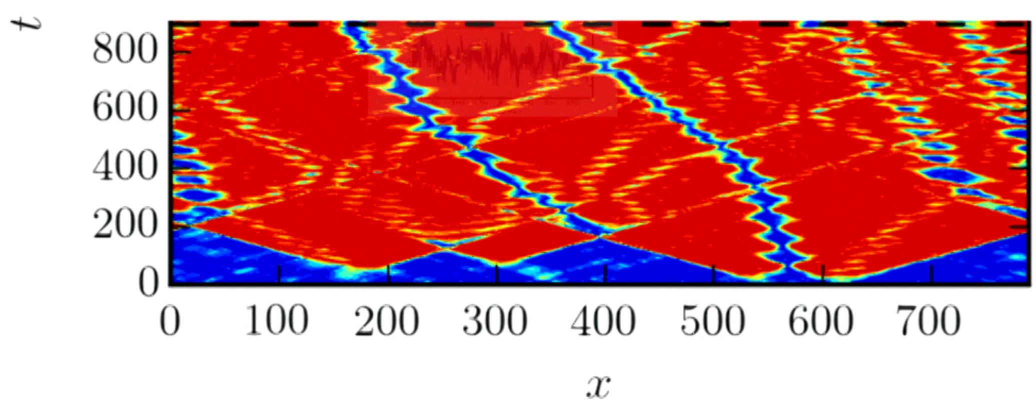
Numerical Reversibility



Numerical Reversibility



Destroyed by Addition of Noise



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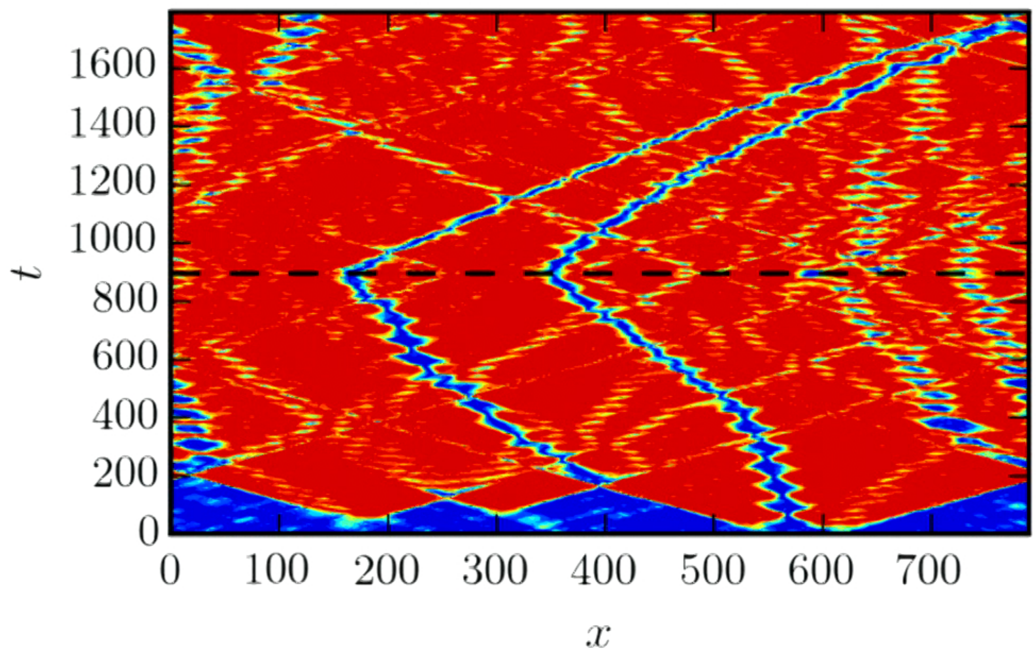


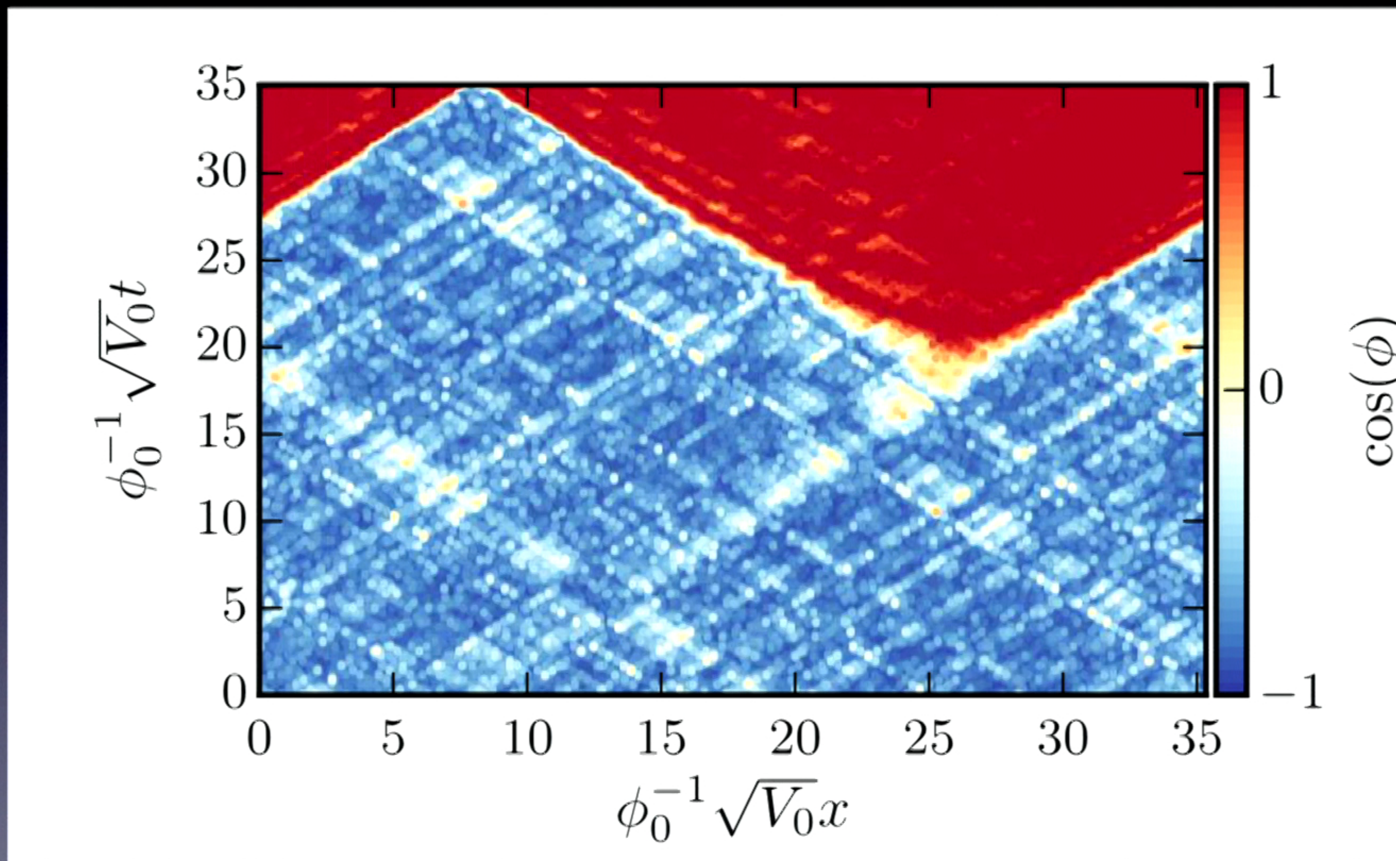
Decay Rates?

Prediction

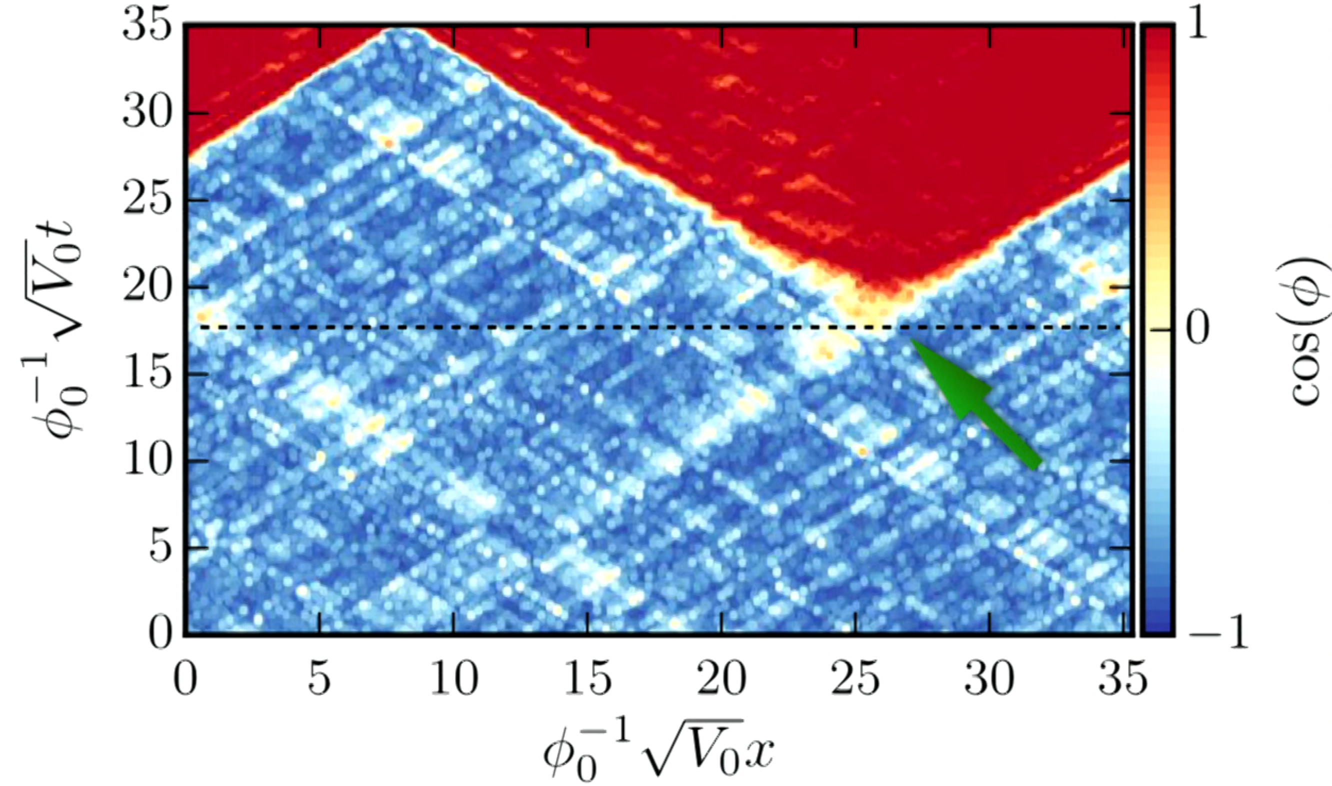
$$\frac{\Gamma_{\rm I}^{(1+1)}}{L} \approx g(\lambda, V_0, \phi_0) m_{\rm eff}^2 \phi_0^2 C(\lambda) e^{-2\pi\phi_0^2 C(\lambda)}$$

$\mathcal{O}(1)$ $\sim V''(\phi_{\rm fv})$ Instanton

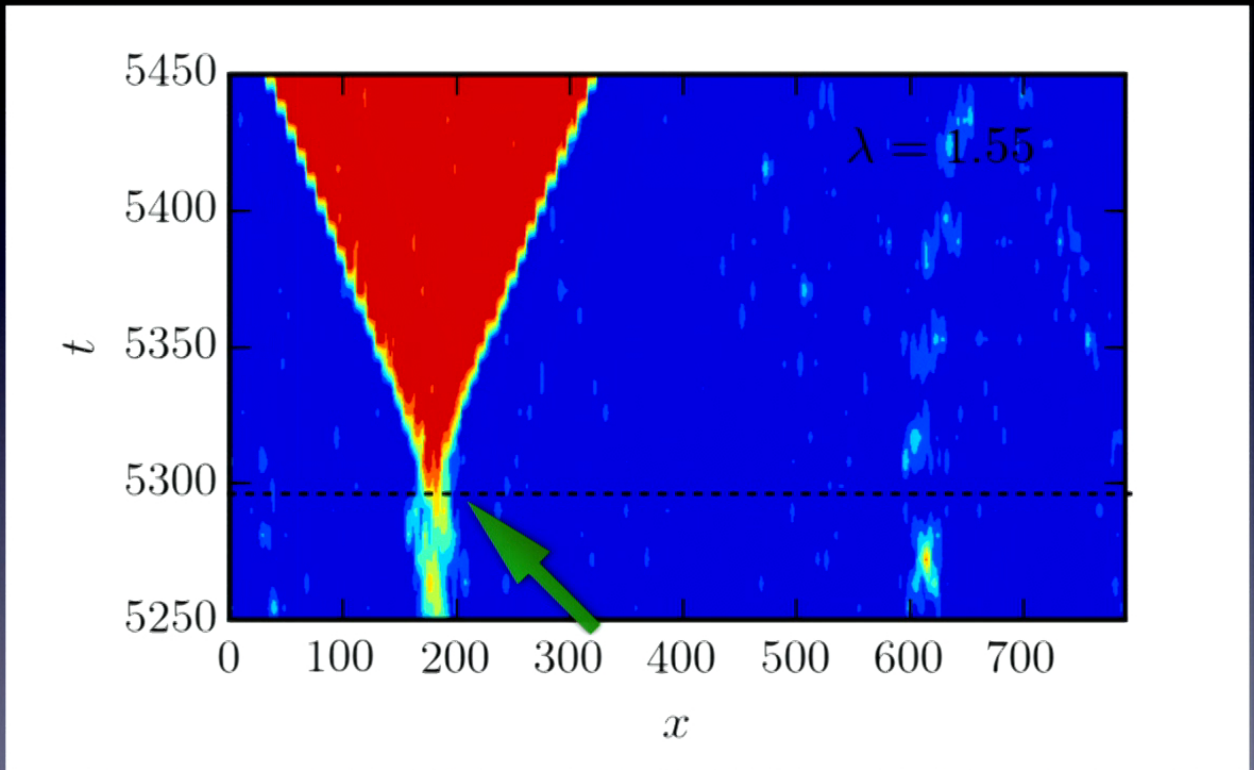
$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$



35
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
$\phi_0^{-1}\sqrt{V_0}t$
$\phi_0^{-1}\sqrt{V_0}x$
1
0
−1
$\cos(\phi)$

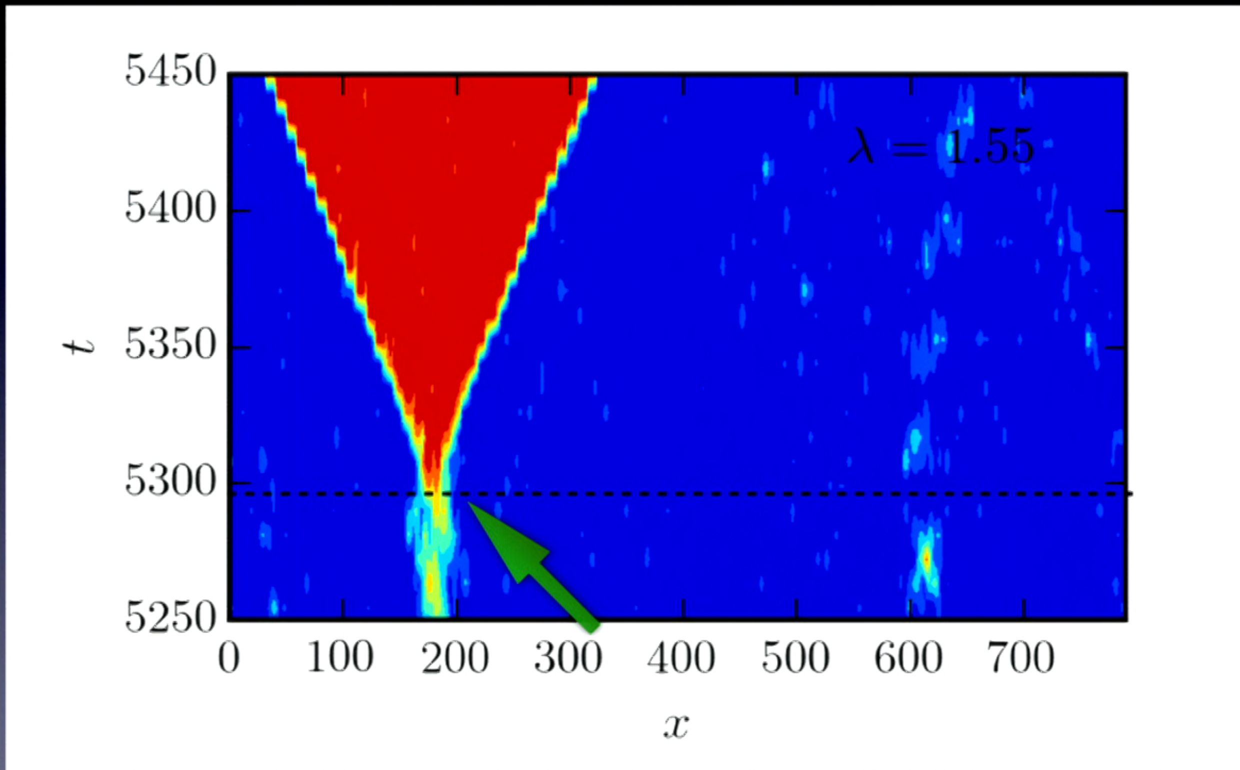


$\phi_0^{-1}\sqrt{V_0}t$
$\phi_0^{-1}\sqrt{V_0}x$
$\cos(\phi)$



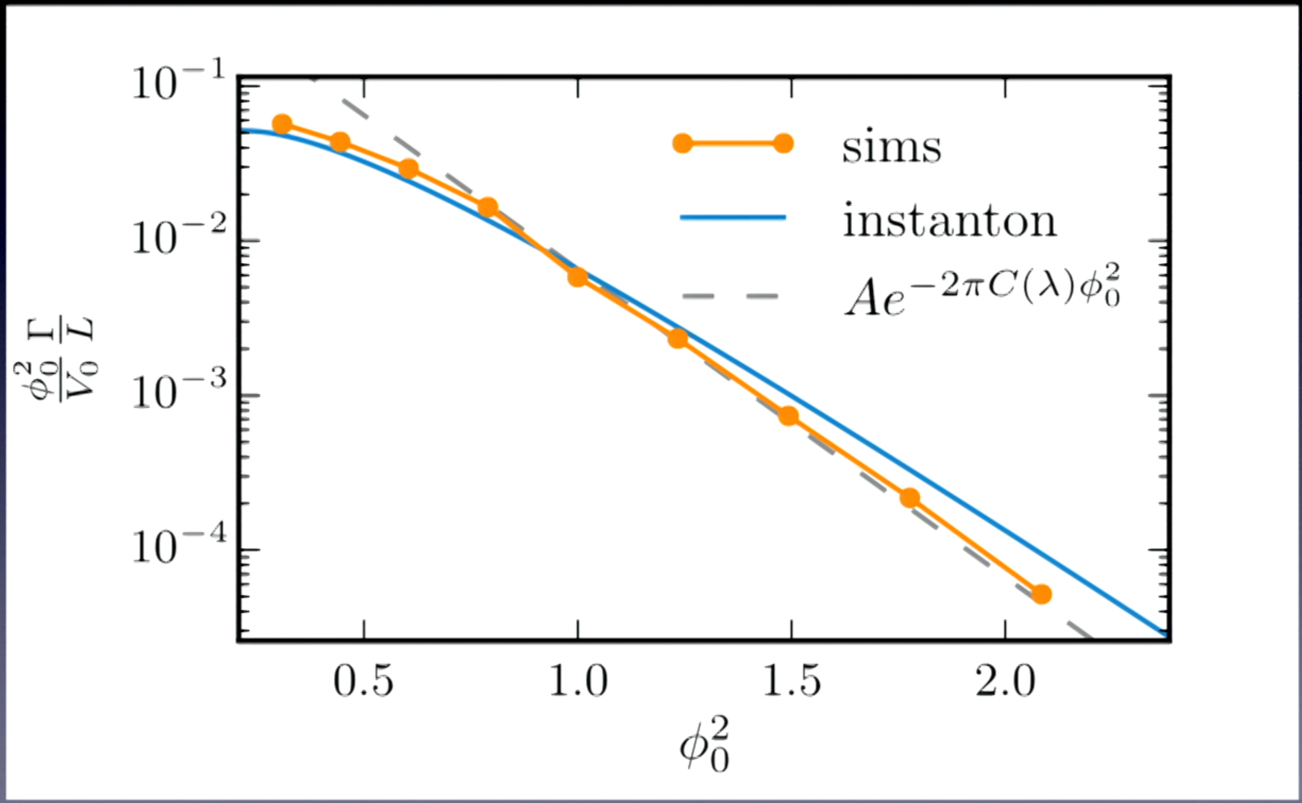
$t^{(i)}_{\text{decay}}$

Not Just Peaks in Initial Field!



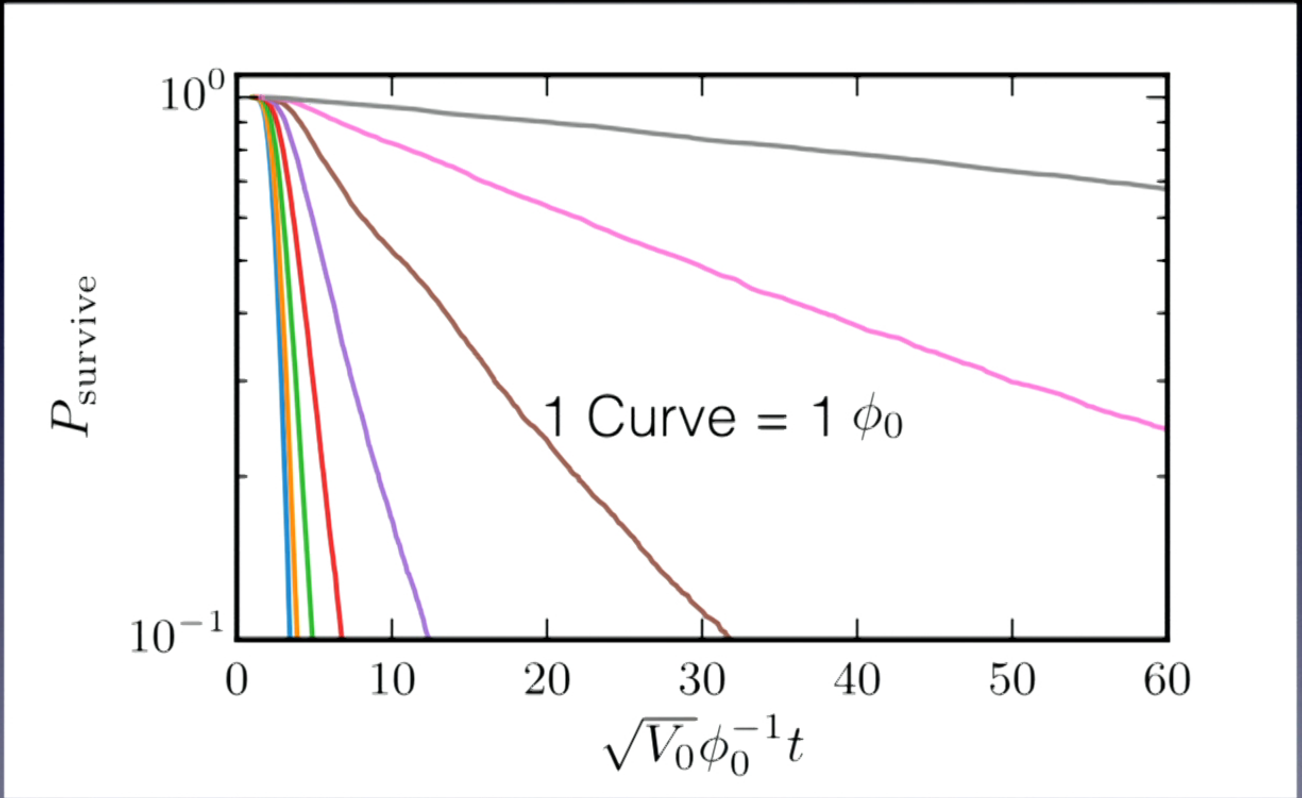
$t^{(i)}_{\mathrm{decay}}$

$$\frac{\Gamma_I^{(1+1)}}{L} = g(\lambda, \phi_0) m_{\rm eff}^2 \phi_0^2 e^{-2\pi\phi_0^2 C(\lambda)}$$

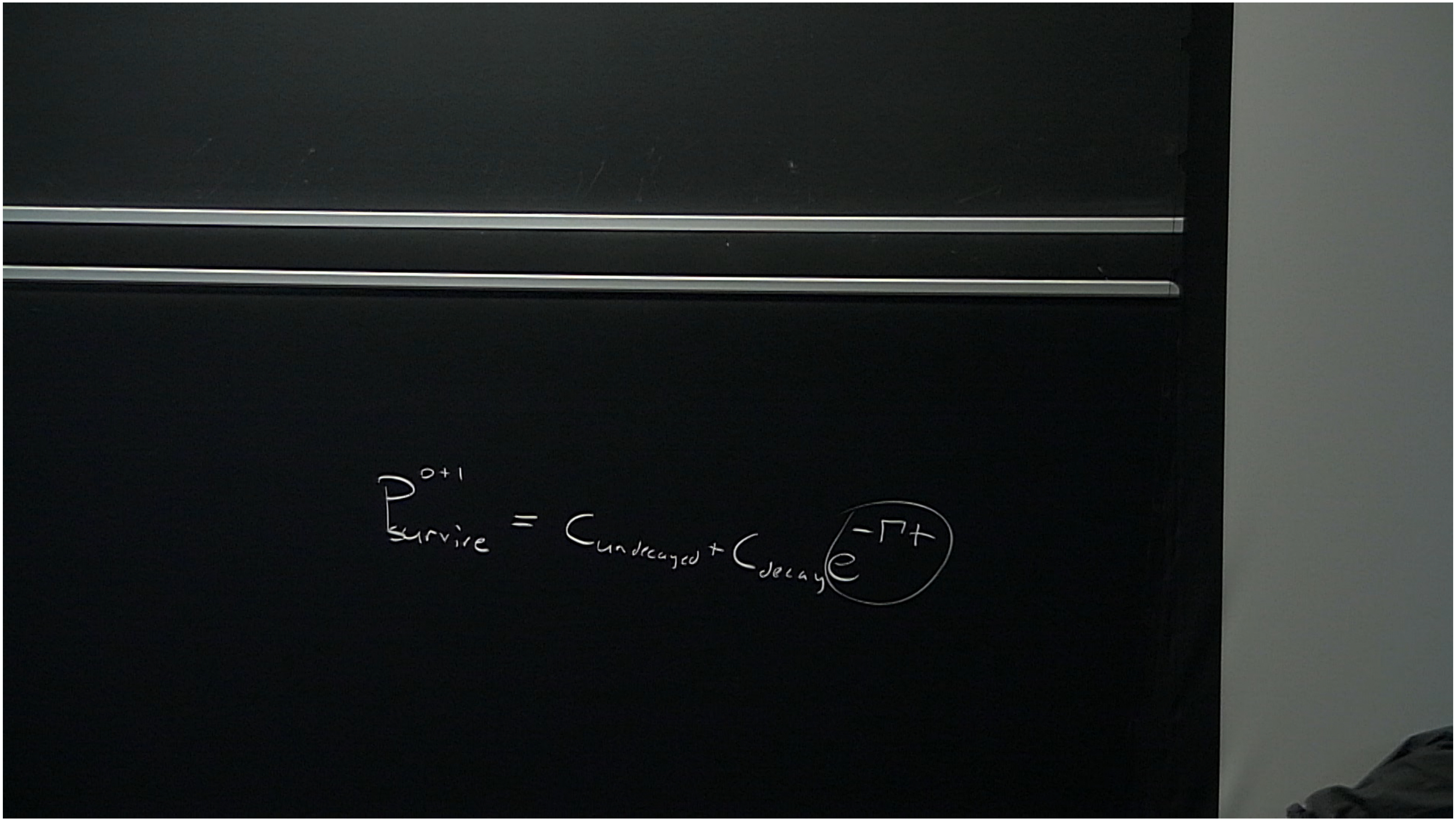


[JB, Johnson, Peiris, Pontzen, Weinfurtner, 1806.06069]

$$P_{\text{survive}} \sim e^{-\Gamma(t-t_0)}$$



Sanity Check : $\Gamma \propto L$


$P^{0+1}_{survive} = C_{undecayed} + C_{decay} e^{-\Gamma t}$

First Principles Derivation of Approximation

My Original Question

$$|\Psi\rangle$$

$$i\hbar\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$$

$$P[\phi, \Pi]$$

$$\frac{\partial\phi}{\partial t} = \frac{\delta H}{\delta \Pi}$$

$$\frac{\partial\Pi}{\partial t} = -\frac{\delta H}{\delta \phi}$$

Nonlinear, Nonperturbative, Nonequilibrium Phenomena

QFT in Phase Space

Consider the Wigner functional

$$W[\phi, \Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

Important Properties

$$\int \mathcal{D}\phi \mathcal{D}\Pi \; W[\phi, \Pi] = 1$$

$$\langle \hat{\mathcal{O}}(\hat{\phi}, \hat{\Pi}) \rangle = \int \mathcal{D}\phi \mathcal{D}\Pi \, W(\phi, \Pi) \mathcal{O}_{\mathrm{W}}(\phi, \Pi)$$

W ~ quantum probability distribution

(caveat: Not postive definite in general,
but is for Gaussian states)

Wigner Approach

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$$

Wigner Approach

$$W[\phi, \Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\Pi \frac{\delta}{\delta \phi} + \nabla^2 \phi \frac{\delta}{\delta \Pi} - \frac{2}{i\hbar} V(\phi) \sin\left(\overleftarrow{\nabla}_\phi \frac{i\hbar}{2} \overrightarrow{\frac{\partial}{\partial \Pi}}\right)\right)\right] W[\phi(x), \Pi(x); t] = 0$$

Wigner Approach

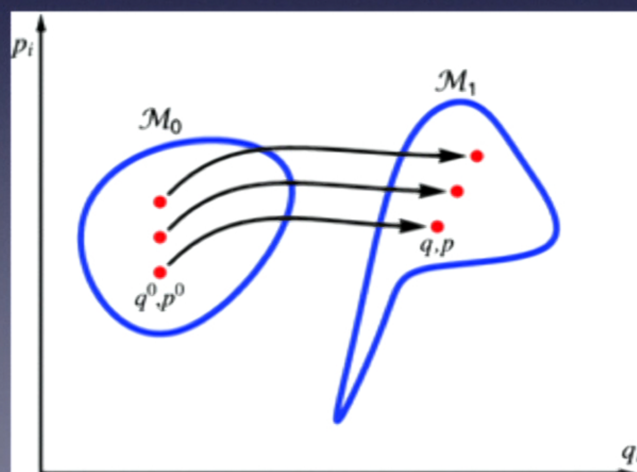
$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta\, e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi+\frac{\eta}{2}\middle|\Psi\right\rangle\left\langle\Psi\middle|\phi-\frac{\eta}{2}\right\rangle$$

$$\left[\frac{\partial}{\partial t}+\int d^d x\left(\dot{\phi}\frac{\delta}{\delta\phi}+\dot{\Pi}\frac{\delta}{\delta\Pi}\right)+\mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right]W[\phi(x),\Pi(x);t]=0$$

Classical Evolution

Quantum "Noise" (Interference)

Initial State (t=0) (Uncertainty Prin.)



Quantum Noise

$$(L_0 + \hbar^2 L_1)W = 0$$

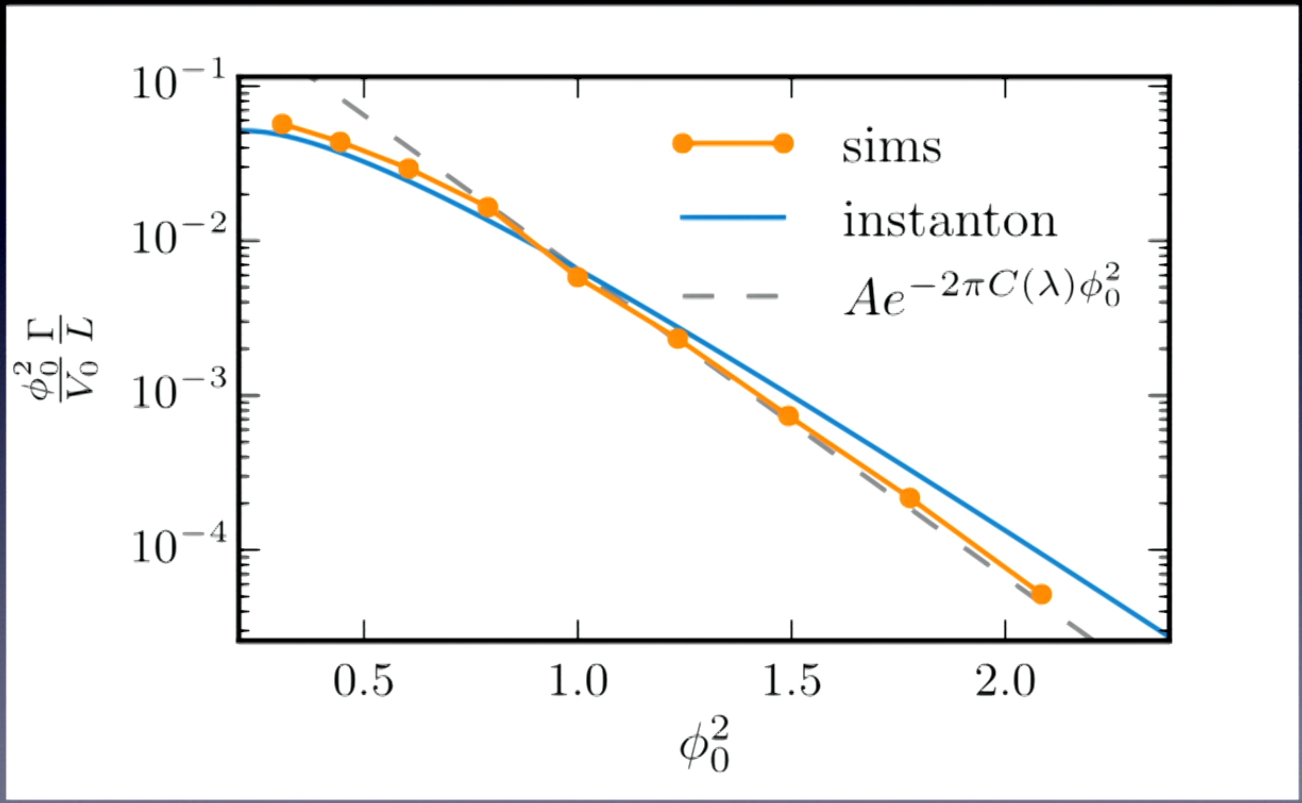
$$W = W_0 + \hbar^2 W_1$$

$$L_0 W_1 = L_1 W_0$$

Nonlinear Response

Stochastic Kick

Why The Discrepancy?



[JB, Johnson, Peiris, Pontzen, Weinfurtner, 1806.06069]

$\hbar$ Expansions

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x),\Pi(x);t] = 0$$

Leading Order

First Correction

$$\frac{\Gamma}{V} = \left(\frac{S_{\rm I}}{2\pi}\right)^{D/2}\sqrt{\frac{\det\delta^2 S_{\rm E}[\phi_{\rm fv}]}{\det'\delta^2 S_{\rm E}[\phi_{\rm B}]}}\, e^{-S_{\rm I}}\,(1+\mathcal{O}(\hbar))$$

Fluctuation Determinant

$$\left[-\frac{1}{r^{d-1}}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^{d-1}\frac{\mathrm{d}}{\mathrm{d}r}\right)+\frac{\ell(\ell+d-2)}{r^2}+V''(\phi_{\mathrm{B,fv}})\right]R_\ell=\lambda R_\ell$$

$\ell = 0$ 1 negative mode (instability)

$\ell = 1$ d+1 zero modes (spacetime translations)

$$\ln\left(\frac{\delta^2 S(\phi_{\mathrm{B}})}{\delta^2 S(\phi_{\mathrm{fv}})}\right)=\Gamma_{(\ell=0)}+\Gamma_{(\ell=1)}+\sum_{\ell=2}^{\infty}g_\ell\ln\Gamma_{(\ell)}$$

Gelfand-Yaglom Theorem

$$\hat{L}f = \left[\frac{d}{dx}\left(P(x)\frac{d}{dx}\right) + Q(x)\right] f = \lambda f$$

$$f(0) = f(L) = 0$$

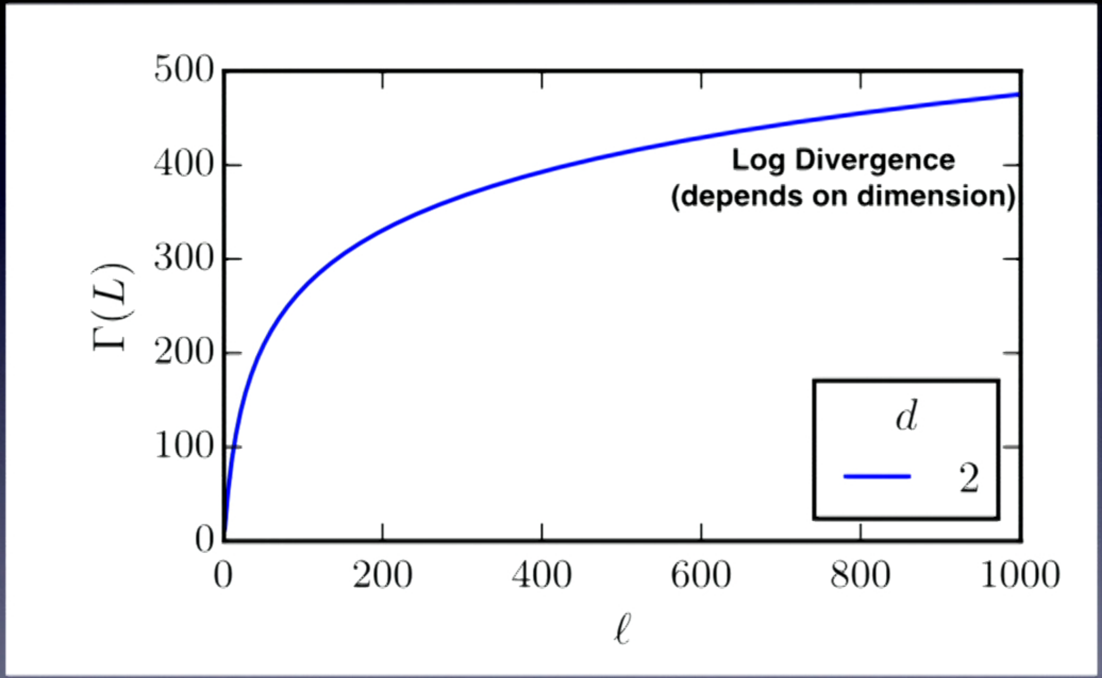
We can compute the determinant as

$$\det\left(\frac{\hat{L}}{\hat{L}_0}\right) = \frac{g(L)}{g_0(L)}$$

Where g satisfies the initial value problem

$$\hat{L}g = 0 \qquad g(0) = 0, \;\; g'(0) = 1$$

Fluctuations and Decay



Divergences appear that we must renormalise

Renormalization

Standard 1PI Effective Potential

$$V_{\mathrm{eff}}^{1\mathrm{PI}}(\bar{\phi}) = V(\bar{\phi}) + \frac{1}{2}\int \frac{\mathrm{d}^{d+1}k}{(2\pi)^{d+1}} \ln\left(\frac{V''(\bar{\phi}) + k_{\mathrm{E}}^2}{V''(\bar{\phi}_{\mathrm{fv}}) + k_{\mathrm{E}}^2}\right) + \dots$$

(Implicit) Assumptions

- Homogeneous background:
- Linear fluctuations
- Vacuum fluctuation statistics

Lattice Effective Potential

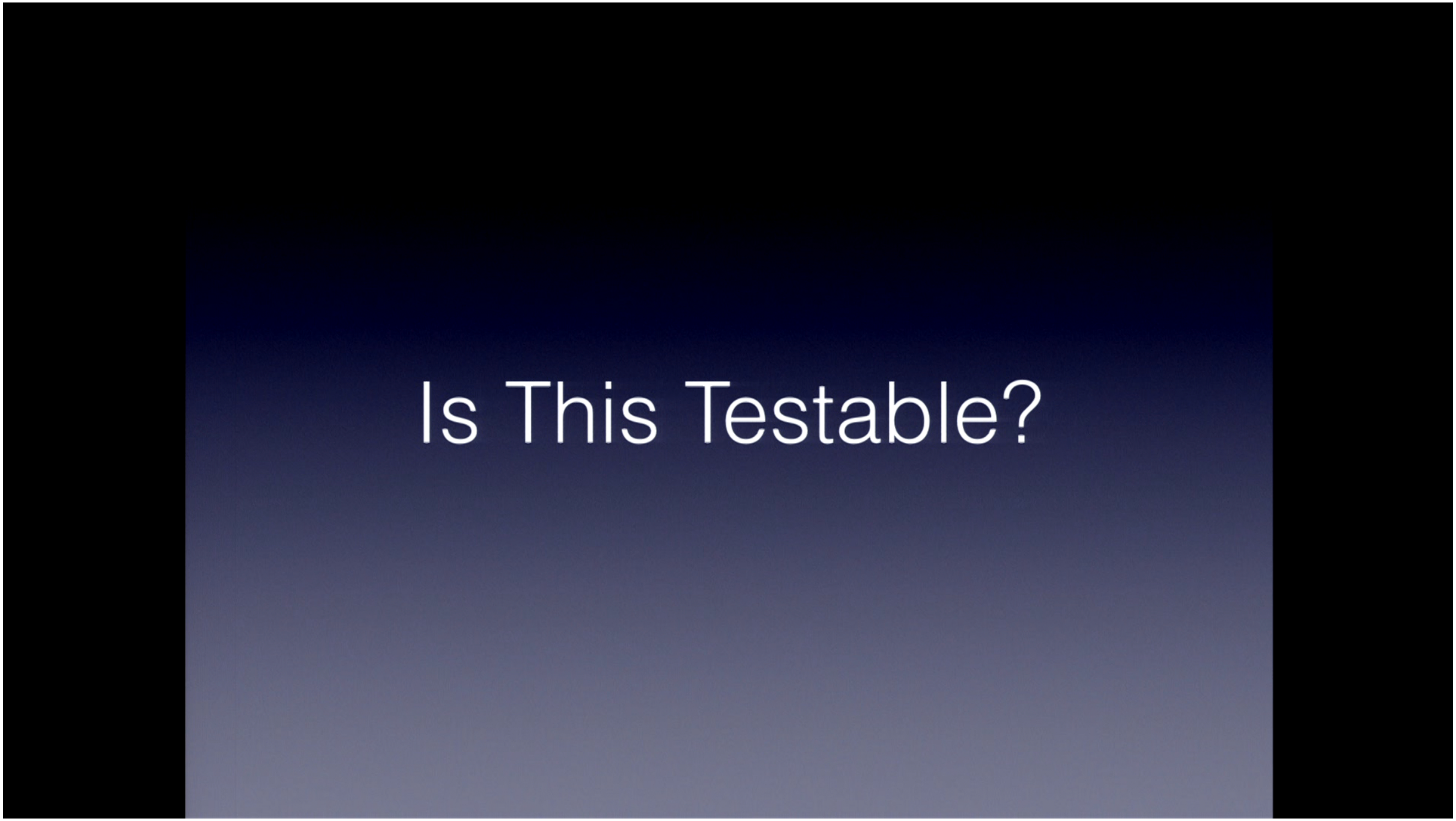
$$V_{\rm eff}^{\rm lat} \equiv \langle \rho \rangle = V(\bar{\phi}) + \frac{1}{2}\int \frac{{\rm d}^d k}{(2\pi)^3}\sqrt{k^2 + V''(\bar{\phi})} + \mathcal{O}\langle \delta\phi^3 \rangle \,.$$

$$\omega = \int d\omega^2 \frac{1}{2\omega} = \int_{-\infty}^{\infty} \frac{{\rm d}k_4}{2\pi}\int \frac{{\rm d}\omega^2}{\omega^2 + k_4^2} = \int \frac{dk_4}{2\pi}\ln(\omega_k^2 + k_4^2)$$

$$V_{\rm eff}^{\rm lat}(\bar{\phi}) = V(\bar{\phi}) + \frac{1}{2}\int \frac{{\rm d}^{d+1}k}{(2\pi)^{d+1}}\ln\left(\frac{V''(\bar{\phi}) + k^2 + k_4^2}{V''(\bar{\phi}_{\rm fv}) + k^2 + k_4^2}\right)$$

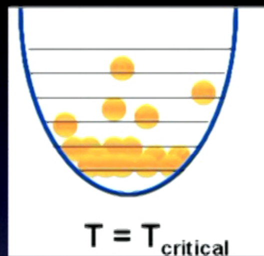
Also holds dynamically

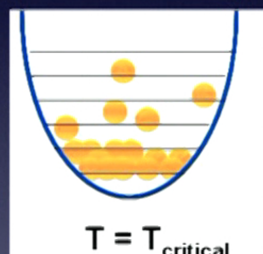
$$\ddot{\bar{\phi}} = -\langle V'(\bar{\phi} + \delta\phi)\rangle = -\frac{\partial V_{\rm eff}(\bar{\phi})}{\partial \bar{\phi}}$$

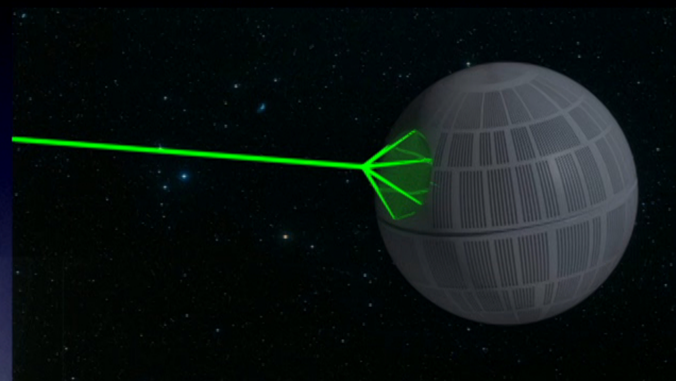

Is This Testable?

Analog Cold Atom BEC

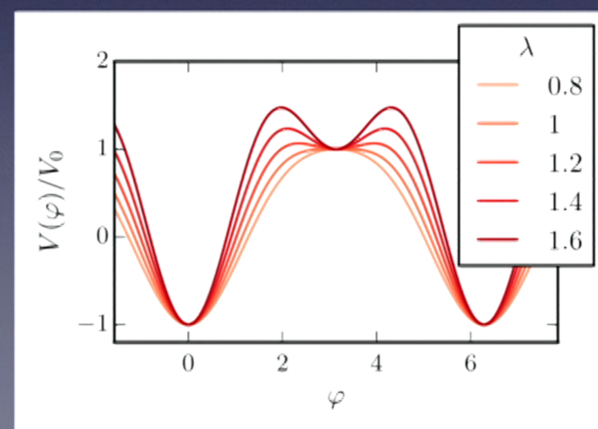
[JB, Johnson, Peiris, Weinfurtner, 1712.02356]



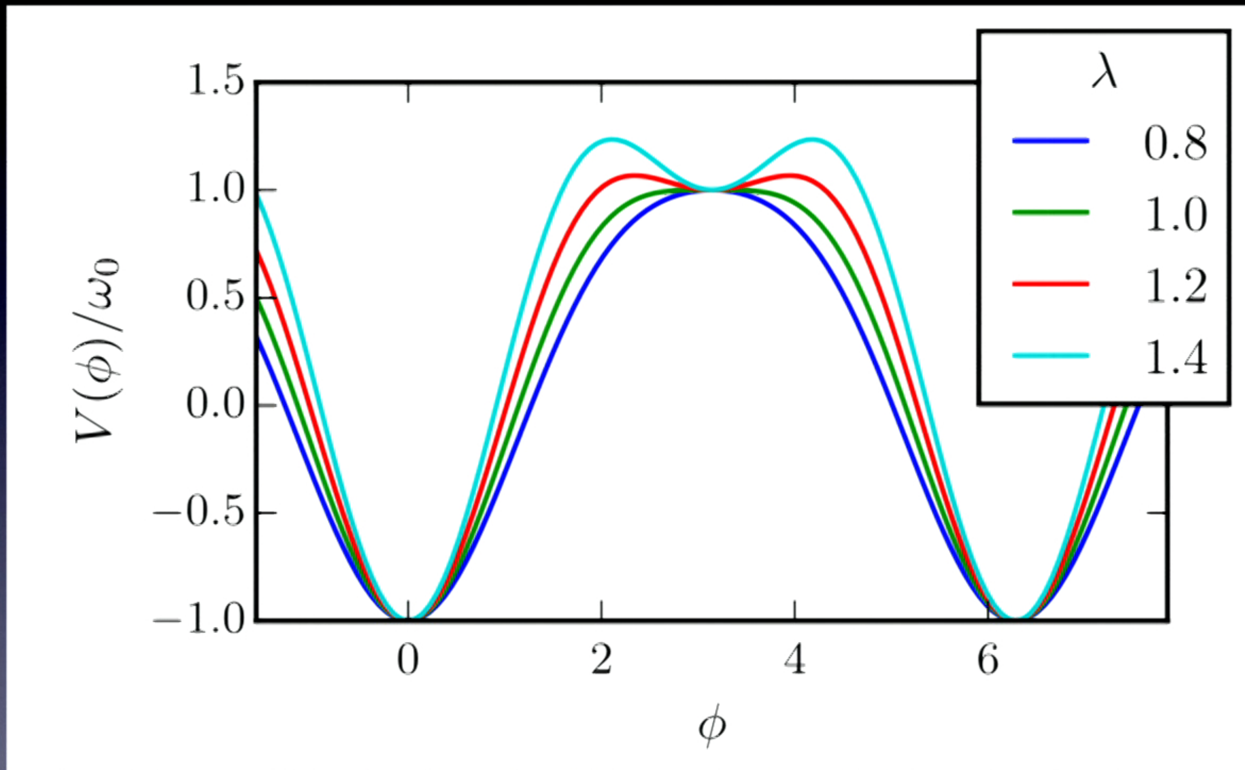




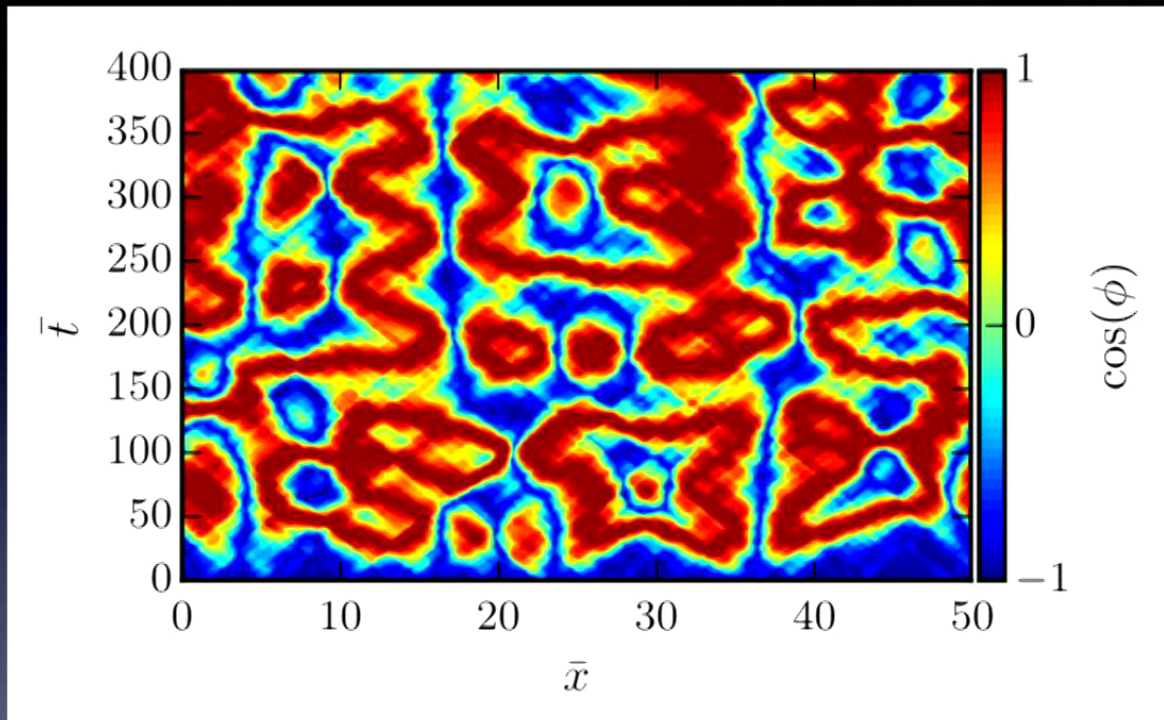
Dynamics of relative phase
is a relativistic field
with periodic potential



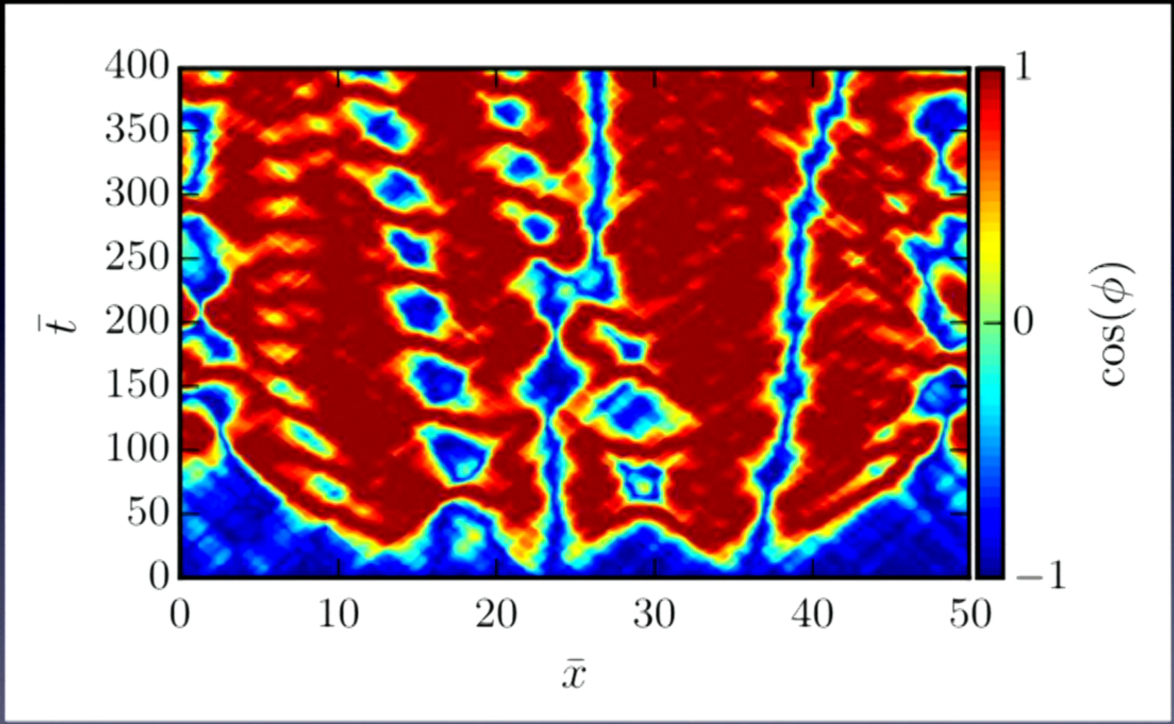
Time Averaged Potential



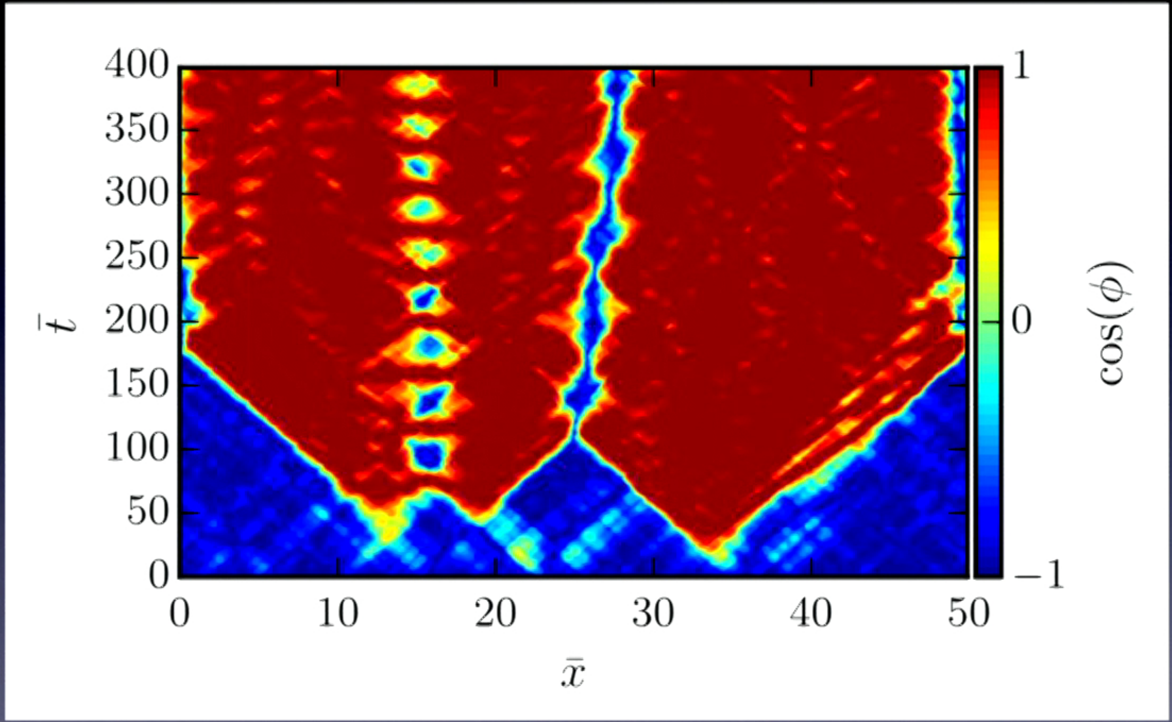
$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$



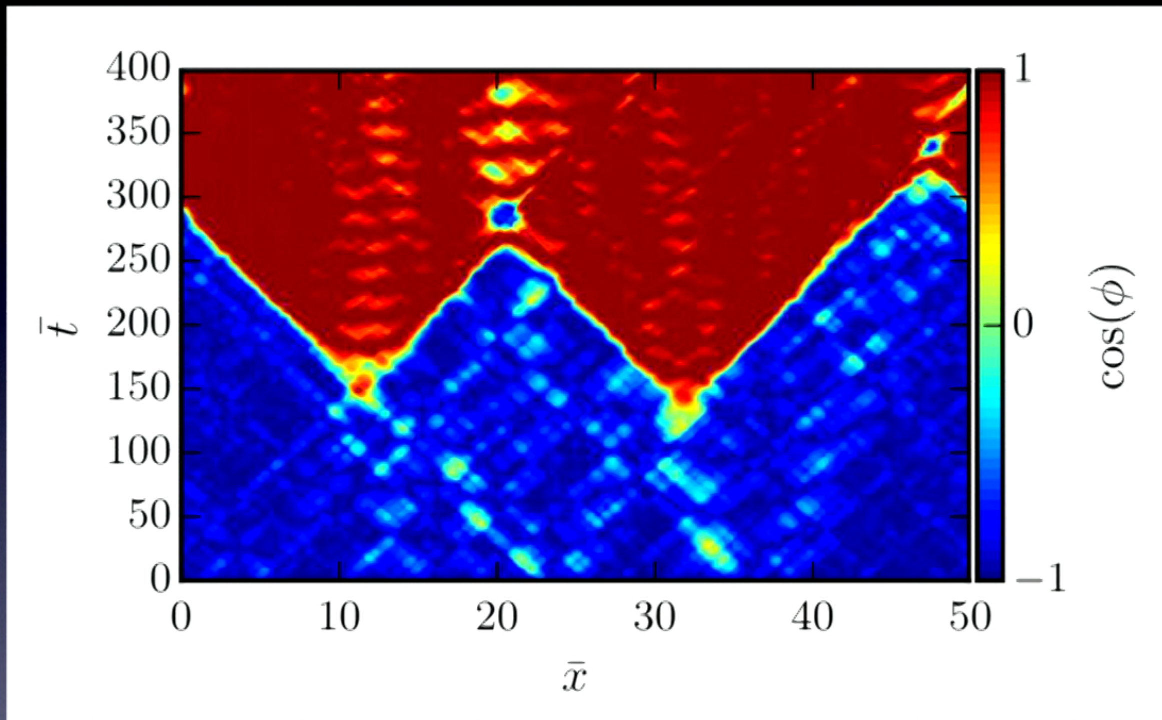
Spinodal Instability



Transition Regime



Rapid Nucleation



Slower Nucleation

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
  - Alternative description of instanton (no tunnelling)
  - Complimentary to instanton (Euclidean rate wrong)
  - (Magic cancellation of amplitudes)

Current/Future Work

- Real-time ↔ Instanton
    - Renormalisation, Fluc. Determinant, Wigner
- Mean bubble profile = instanton?
- Bubble-bubble correlations?
- Time-dependent background or potential
- Non-vacuum initial states (pure or mixed)
- Application to many fields
- Testability in BEC experiments?

THANK YOU