Title: Tensor models and combinatorics of triangulations in dimensions d>2

Speakers: Valentin Bonzom

Series: Quantum Gravity

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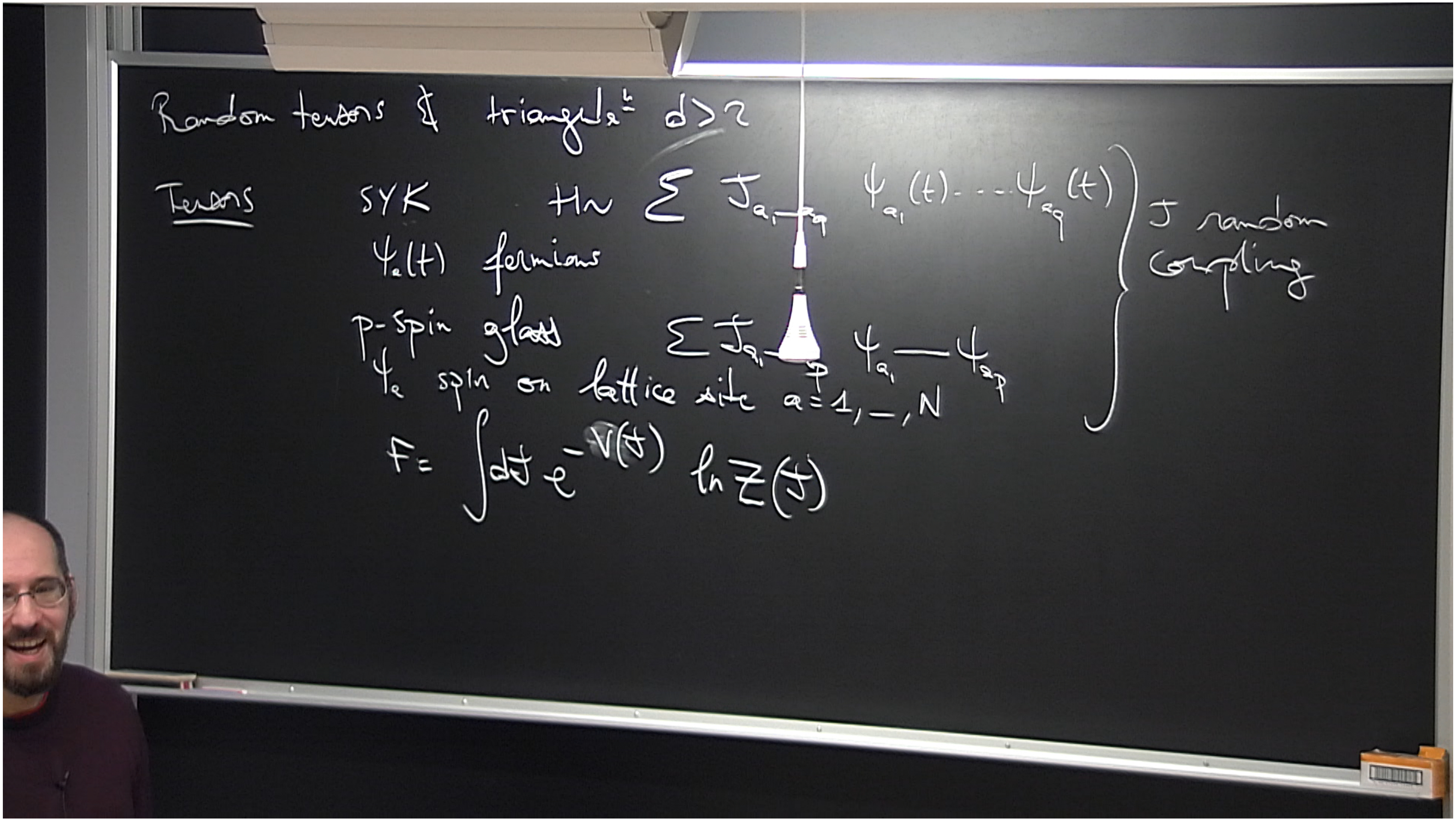
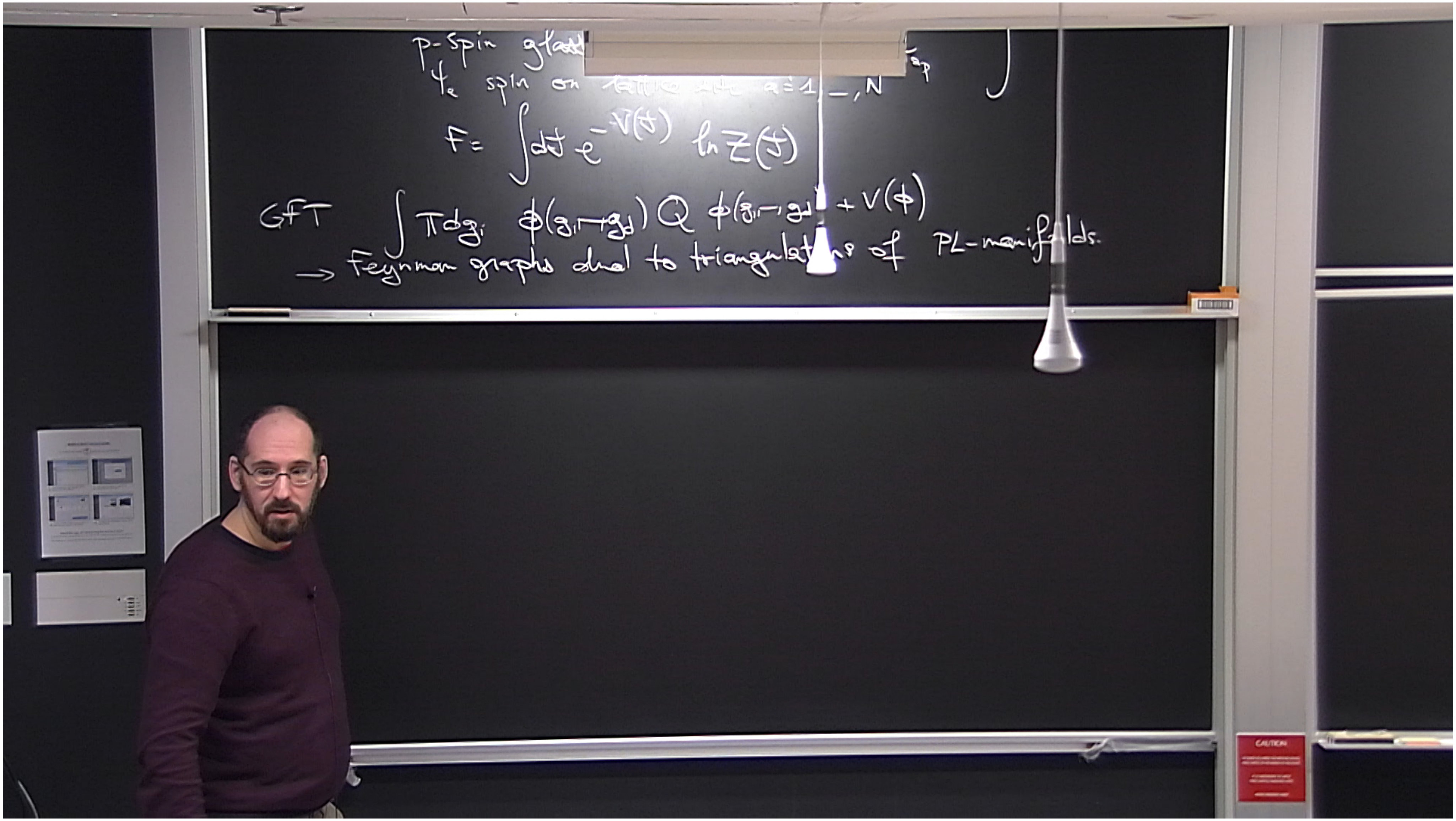
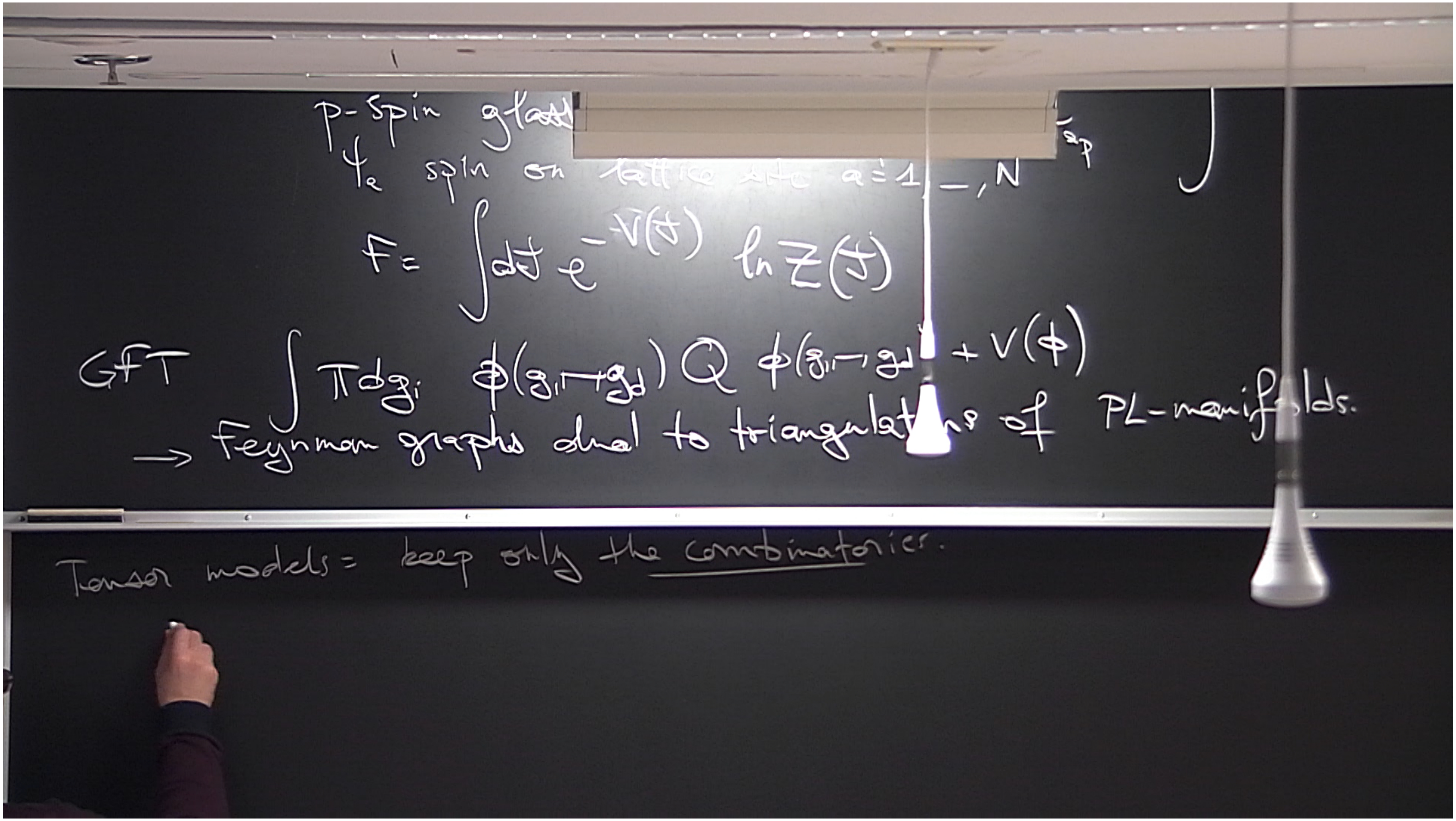
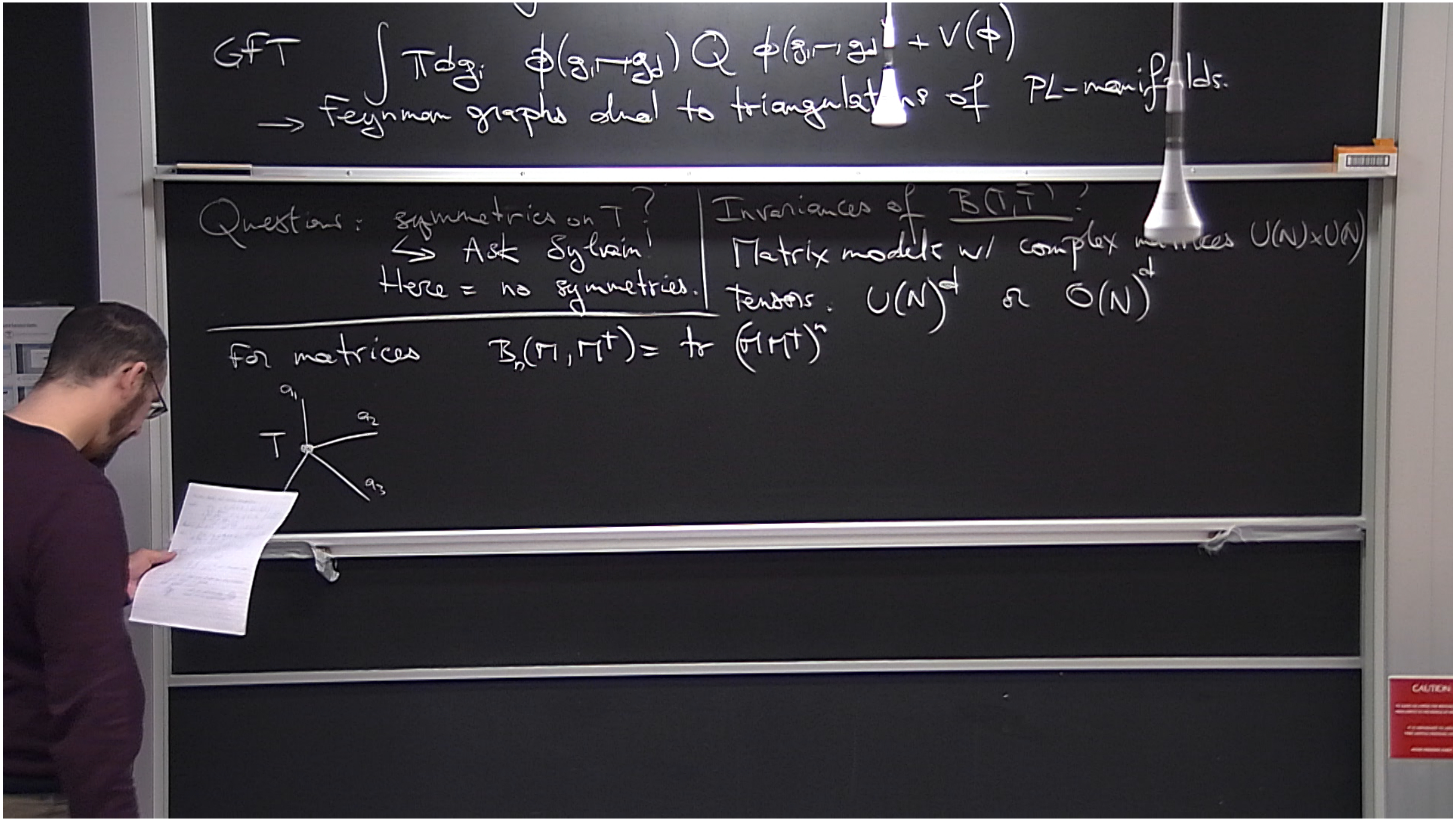
Abstract: Tensor models are generalizations of vector and matrix models. They have been introduced in quantum gravity and are also relevant in the SYK model. I will mostly focus on models with a U(N)^d-invariance where d is the number of indices of the complex tensor, and a special case at d=3 with O(N)^3 invariance. The interactions and observables are then labeled by (d-1)-dimensional triangulations of PL pseudo-manifolds. The main result of this talk is the large N limit of observables corresponding to 2-dimensional planar triangulations at d=3. In particular, models using such observables as interactions have a large N limit exactly solvable as it is Gaussian. If time permits, I will also discuss interesting questions in the field: models which are non-Gaussian at large N, the enumeration of triangulations of PL-manifolds, matrix model representation of some tensor models, etc.


Random tensors & triangulat° $d>2$
Tensors
SYK $\mathrm{tr} \sum J_{a_1 \dots a_q} \psi_{a_1}(t) \dots \psi_{a_q}(t)$
J random coupling
$\psi_a(t)$ fermions
p-spin glass $\sum J_{a_1 \dots a_p} \psi_{a_1} \dots \psi_{a_p}$
ψ_a spin on lattice site $a = 1, \dots, N$
$F = \int dJ \, e^{-V(J)} \ln Z(J)$


p-spin
F = ∫dJ e^{-V(J)} ln Z(J)
GFT ∫ Π dg_i φ(g_1…g_d) Q φ(g_1…g_d) + V(φ)
→ Feynman graphs dual to triangulat of PL-manif lds.



p-spin
spin on lattice site a=1,...,N
F = ∫dJ e^{-V(J)} ln Z(J)
GFT ∫ Π dg_i φ(g_1 ... g_d) Q φ(g_1 ... g_d) + V(φ)
→ Feynman graphs dual to
of PL-
Tensor models = keep only the combinatorics.


GFT ∫ ∏ dg_i φ(g_1,…,g_d) Q φ(g_1,…,g_d) + V(φ)
→ Feynman graphs dual to triangulat ns of PL-mani lds.
Questions: symmetries on T?
⇔ Ask Sylvain!
Here = no symmetries.
Invariances of B(T,T̄)?
Matrix models w/ complex ma ices U(N)×U(N)
Tensors: U(N)^d or O(N)^d
For matrices B_n(M,M†) = tr (MM†)^n
T
a_1
a_2
a_3

$$F = \int dT \, e^{-V(T)} \ln Z(T)$$

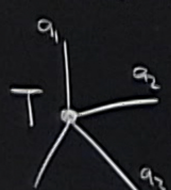
GFT $\displaystyle\int \prod dg_i \; \phi(g_1, \dots, g_d) \, Q \, \phi(g_1, \dots, g_d) + V(\phi)$

$\rightarrow$ Feynman graphs dual to triangulations of PL-manifolds.

Questions: symmetries on T?
$\Rightarrow$ Ask Sylvain!
Here = no symmetries.

Invariances of $B(T, \bar{T})$?
Matrix models w/ complex matrices $U(N) \times U(N)$
Tensors: $U(N)^d$ or $O(N)^d$

For matrices $B_n(M, M^\dagger) = \mathrm{tr}\,(MM^\dagger)^n$



To ensure $U(N)^d$-invariance, need to contract index in position i of T w/ index in same position of $\bar{T}$.

$$\sum_{a_i=1} T_{\dots a_i \dots} \, \bar{T}_{\dots a_i \dots}$$

$$F = \int dJ\, e^{-V(J)} \ln Z(J)$$

GFT $\int \prod dg_i\ \phi(g_1 \cdots g_d)\, Q\, \phi(g_1 \cdots g_d) + V(\phi)$

$\to$ Feynman graphs dual to triangulations of PL-manifolds.

$$S = \sum T_{a_1 \cdots a_d} \bar{T}_{a_1 \cdots a_d} + B(T, \bar{T})$$

$\Leftrightarrow$ $\ln Z$ to be GF of triangul^{n} of PL-manifolds.

Questions: symmetries on T? $\Leftrightarrow$ Ask Sylvain! Here = no symmetries.

Invariances of $B(T, \bar{T})$?

Matrix models w/ complex matrices $U(N) \times U(N)$

Tensors: $U(N)^d$ or $O(N)^d$

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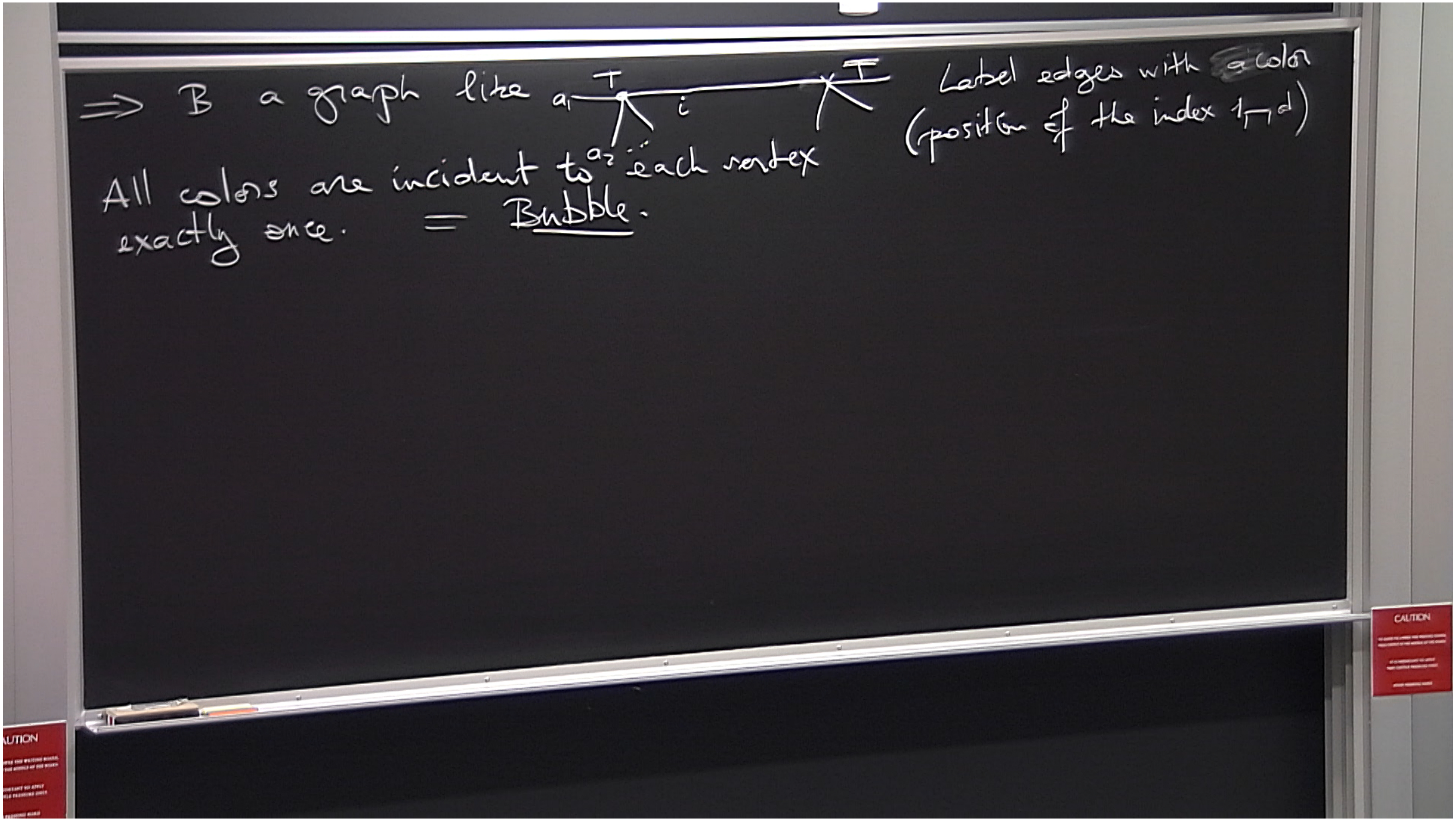
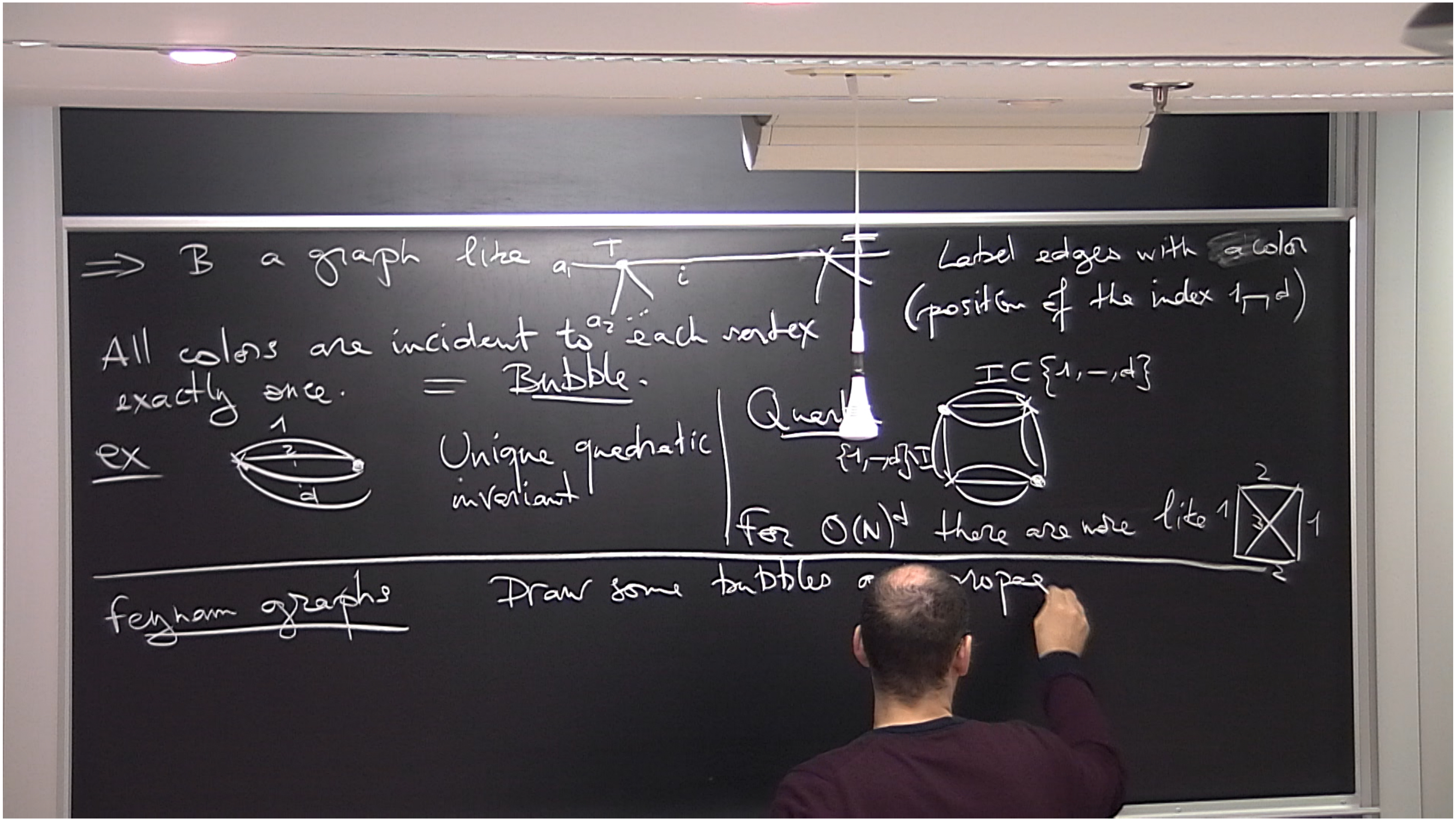
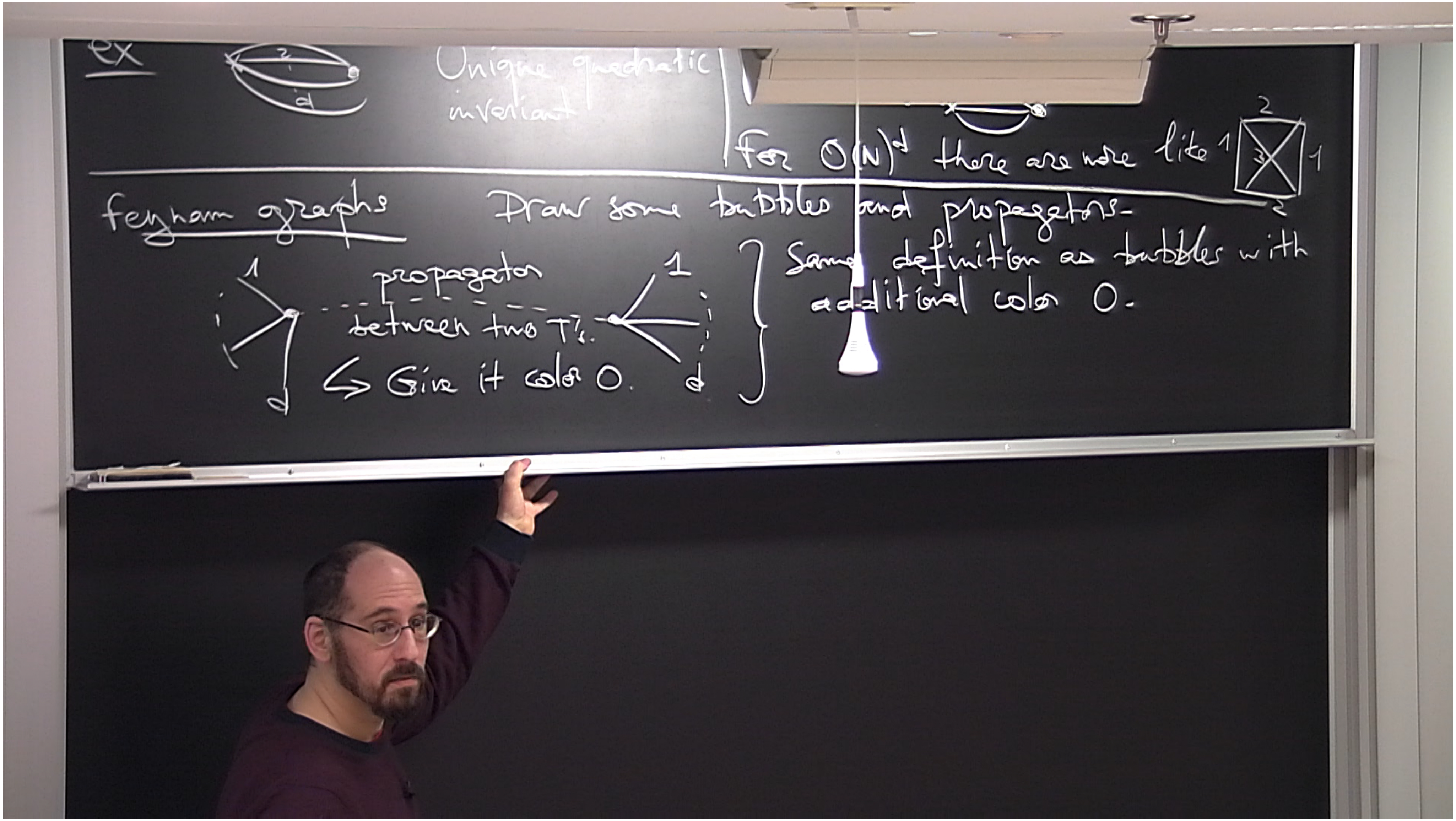
To ensure $U(N)^d$-invariance, need to contract index in position i of T w/ index in same position of $\bar{T}$.

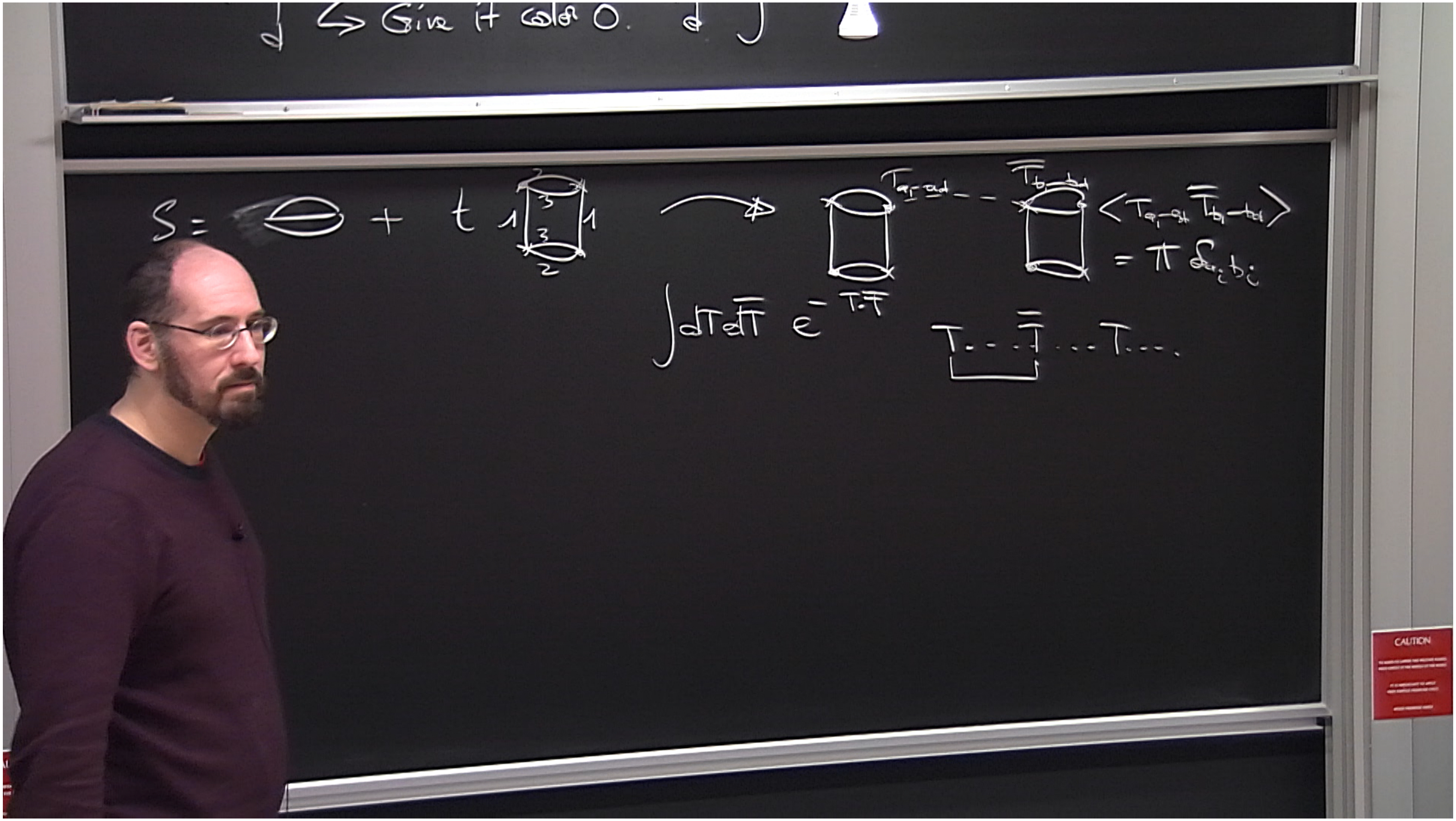
$$\sum_{a_i = 1} T_{\dots a_i \dots} \bar{T}_{\dots a_i \dots}$$

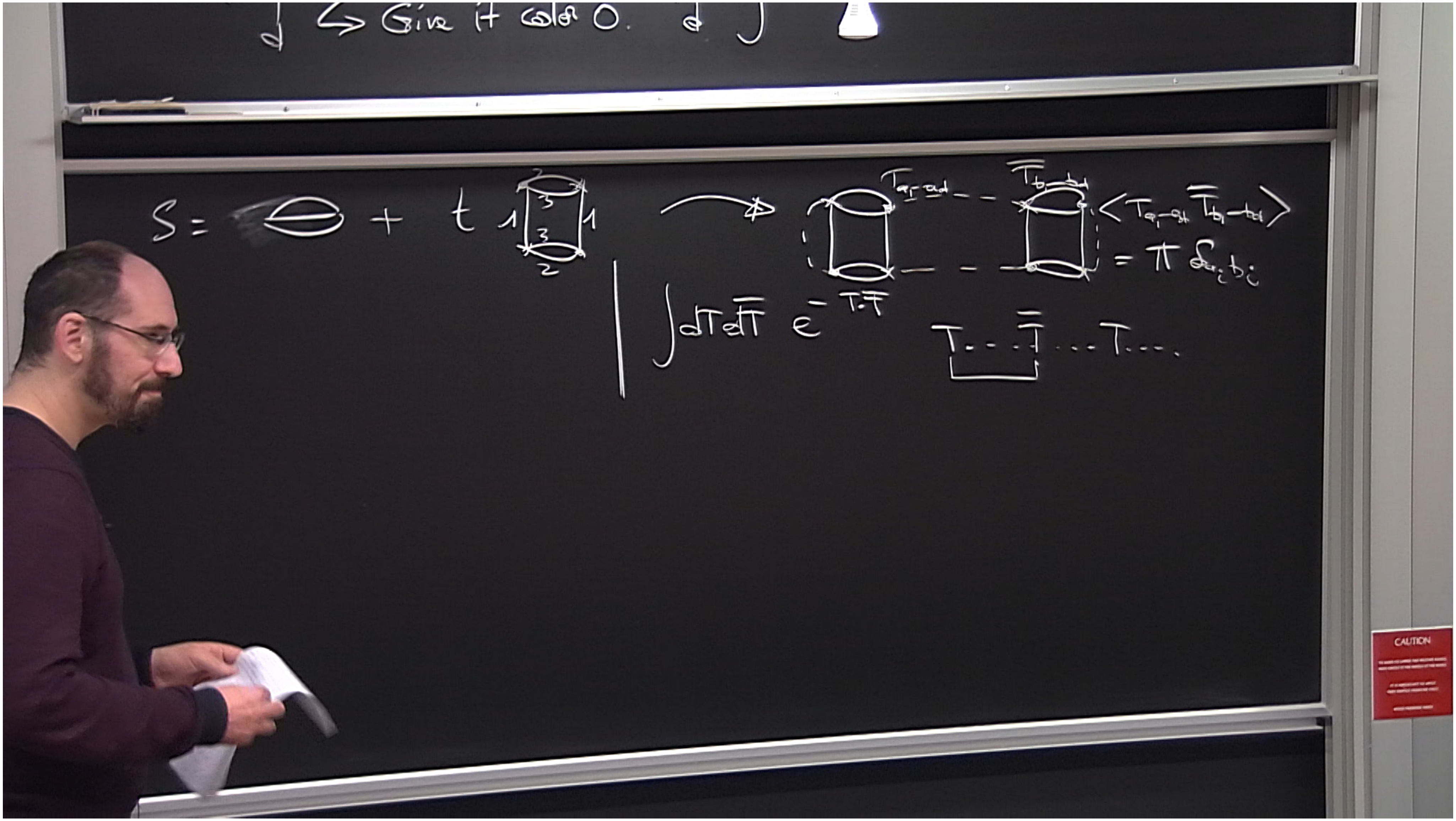


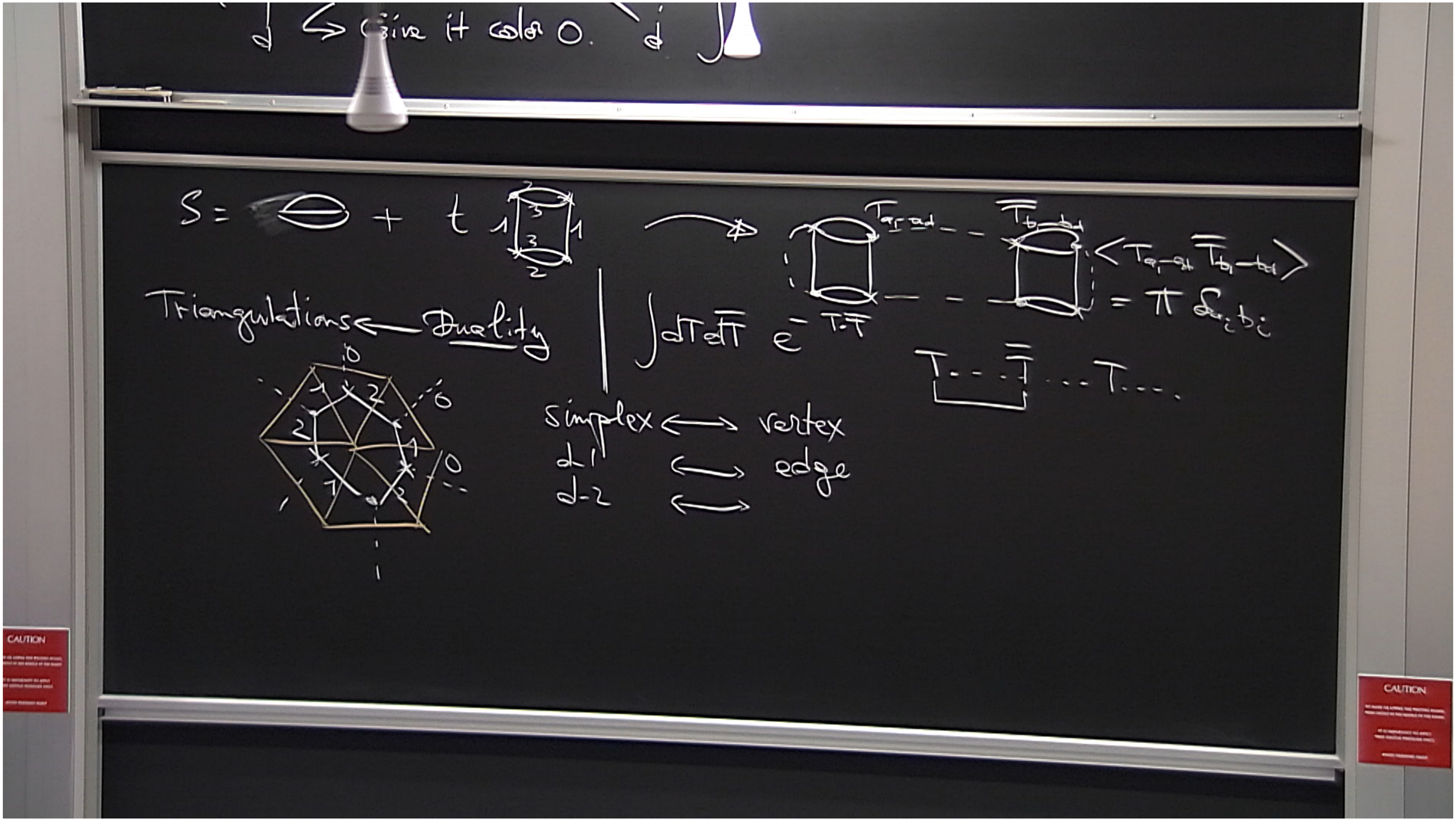
⇒ B a graph like
a_1
T
i
a_2 ...
$\bar{T}$
Label edges with a color
(position of the index 1,...,d)
All colors are incident to each vertex
exactly once. = Bubble.

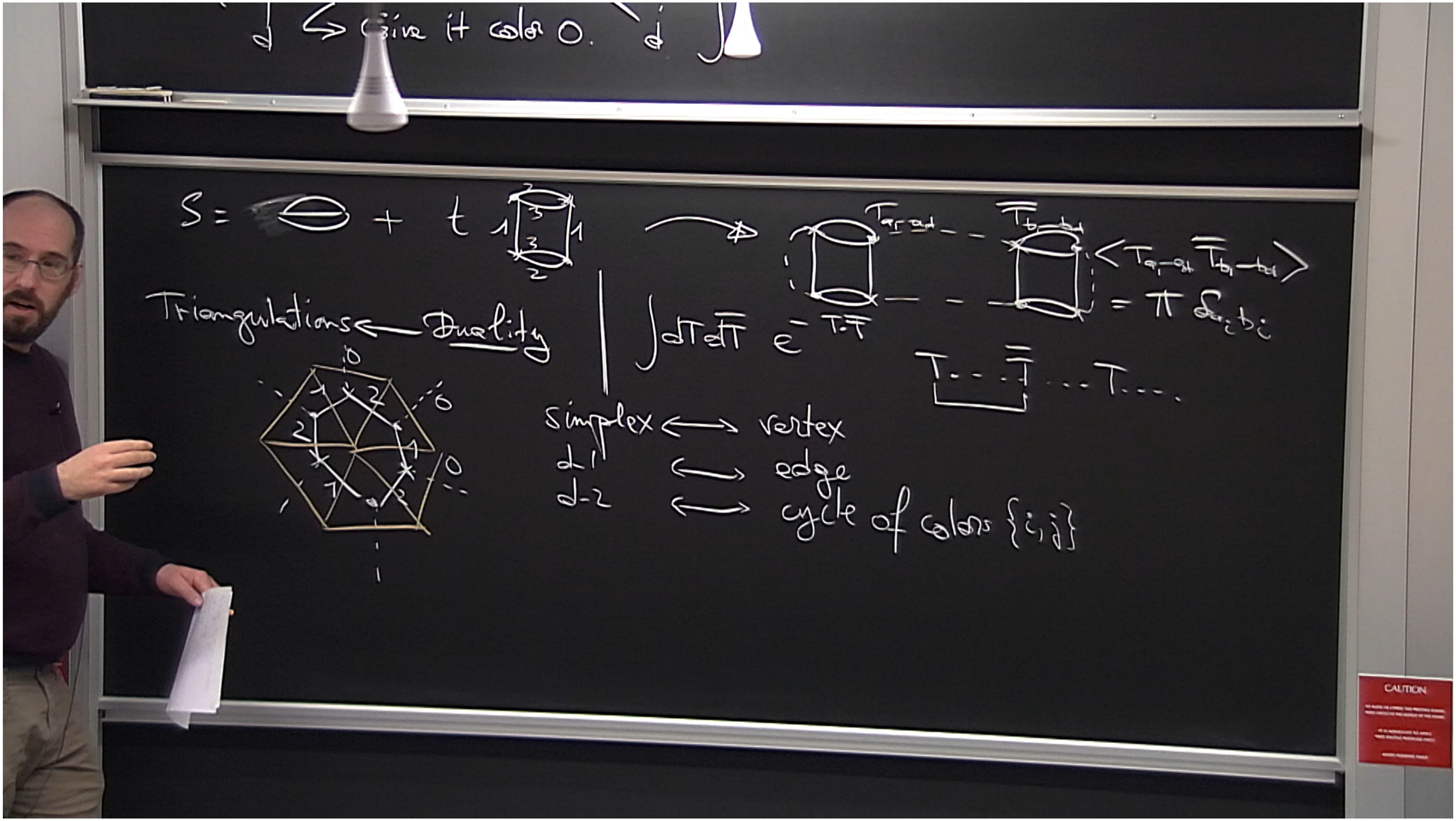

⇒ B a graph like
Label edges with a color
(position of the index 1,...,d)
All colors are incident to each vertex
exactly once. = Bubble.
ex
Unique quadratic invariant
I ⊂ {1,...,d}
For O(N)^d there are more like
feynman graphs
Draw some bubbles

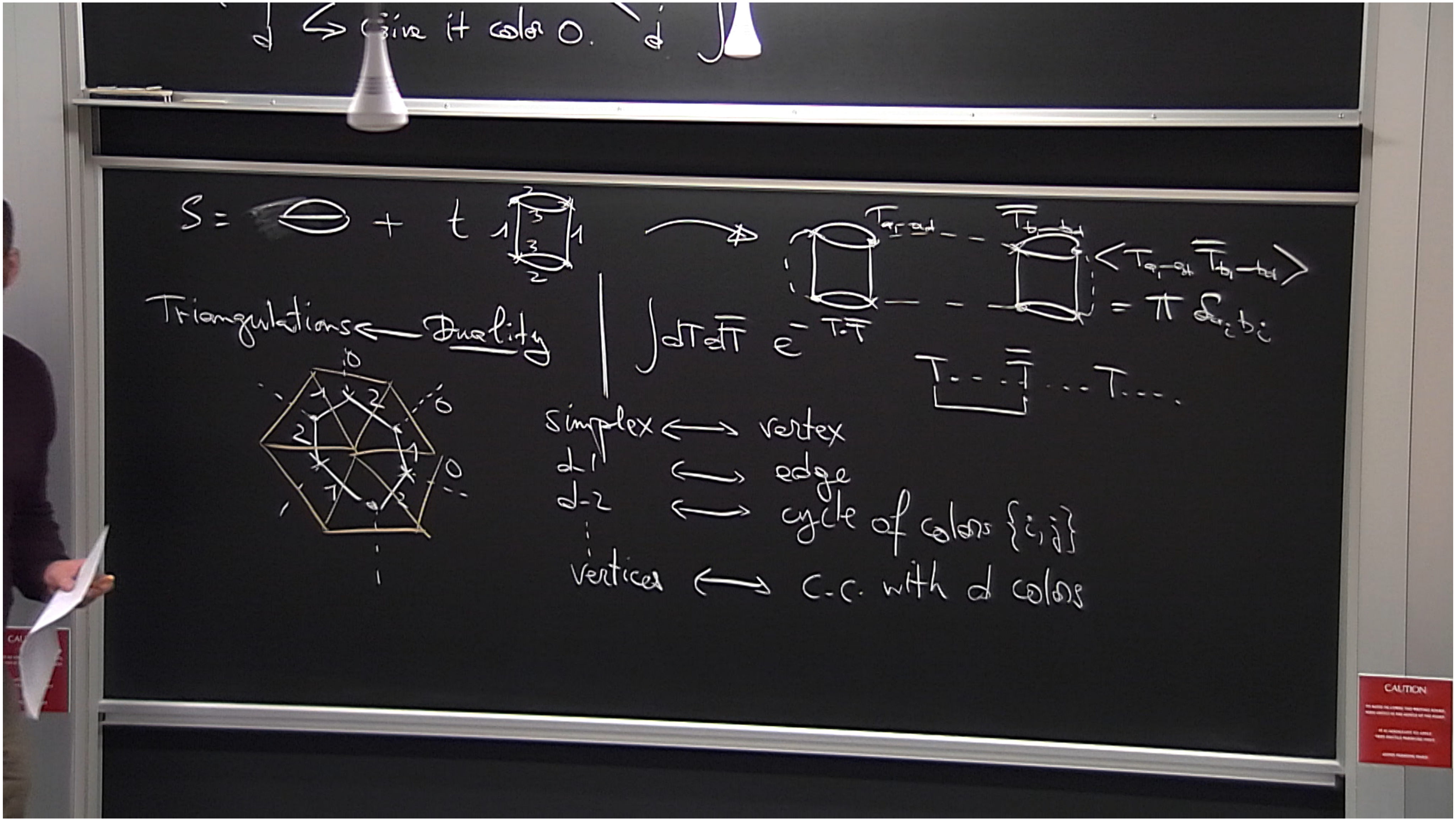

ex
Unique quadratic invariant
For O(N)^d there are more like
feynman graphs
Draw some bubbles and propagators.
propagator between two T's.
Give it color 0.
Same definition as bubbles with additional color 0.


Give it color 0.
S =
CAUTION


Give it color 0.
S =
+ t
$\int dT d\bar{T}\, e^{-T\cdot\bar{T}}$
$= \prod \delta_{a_i b_i}$


Give it color 0.
S =
Triangulations
Duality
simplex
vertex
d-1
edge
d-2
CAUTION
CAUTION


Give it color 0.
S = + t
Triangulations ⟵ Duality
simplex ⟷ vertex
d-1 ⟷ edge
d-2 ⟷ cycle of colors {i, j}
CAUTION


Give it color 0.
Triangulations ⟵ Duality
simplex ⟷ vertex
d-1 ⟷ edge
d-2 ⟷ cycle of colors {i,j}
vertices ⟷ c.c. with d colors
CAUTION

Bubble is dual to: a vertex of the triangula$^{\underline{n}}$, and also

Bubble is dual to: a vertex of the triangulaᵗⁿ, and also to a (d-1)-dimensional triangulation of PL-pseudomanifold.

Bubbles are dual to building blocks obtained by a cone over their boundary.

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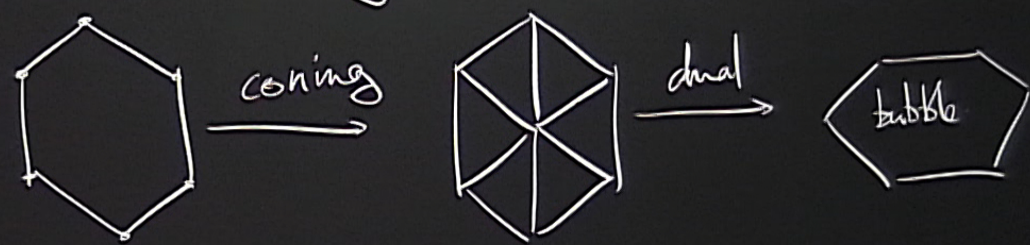
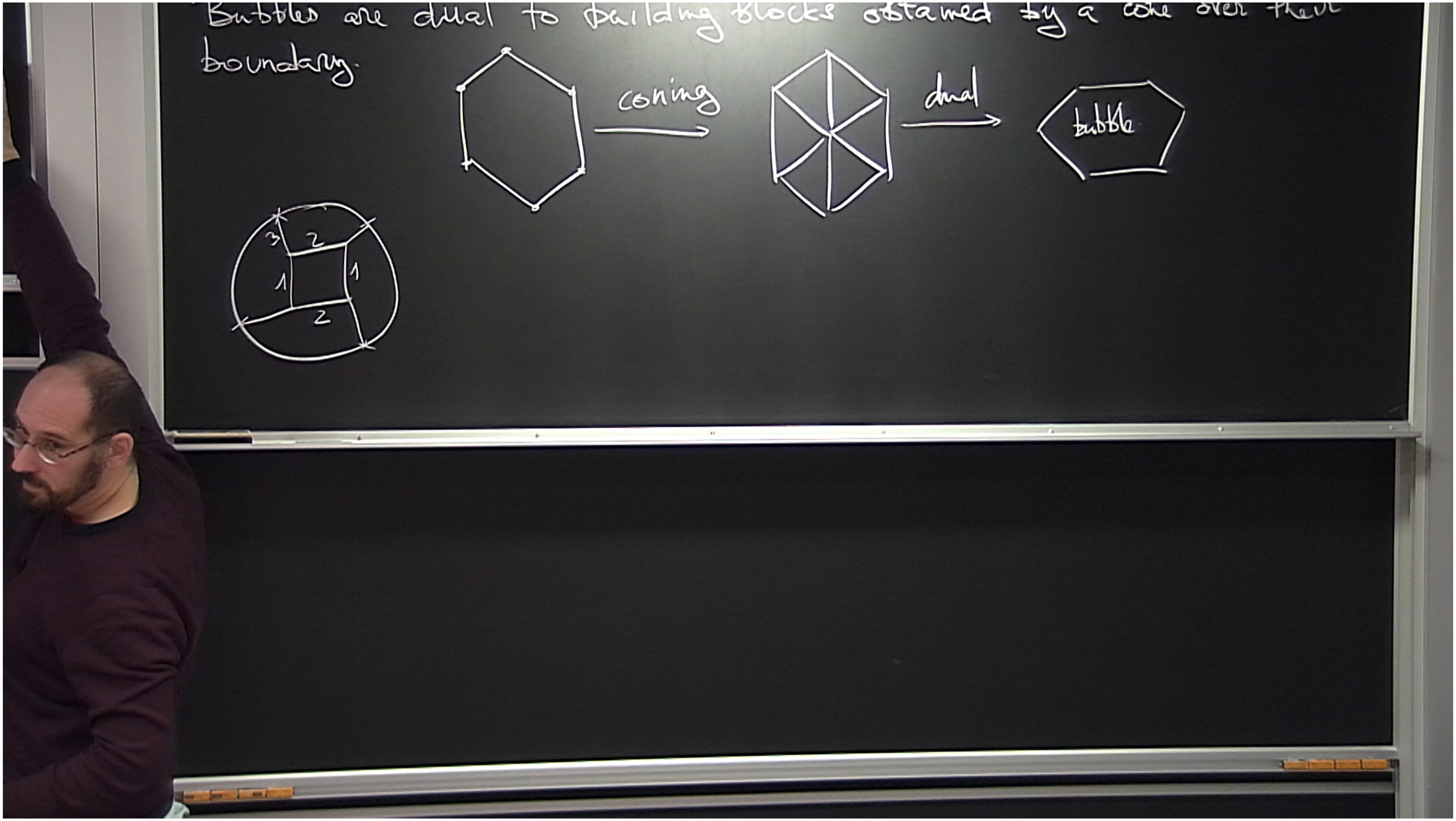
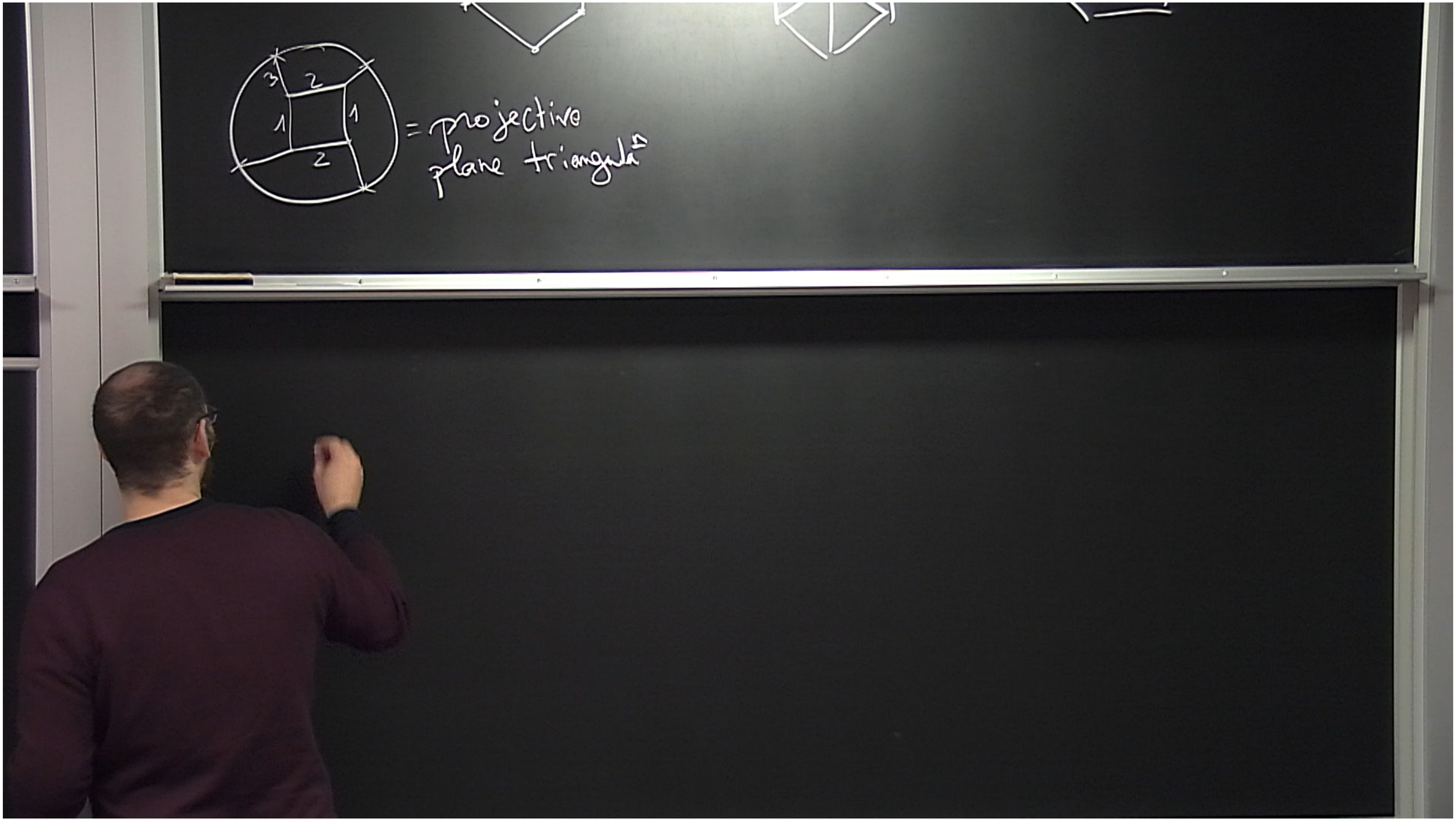
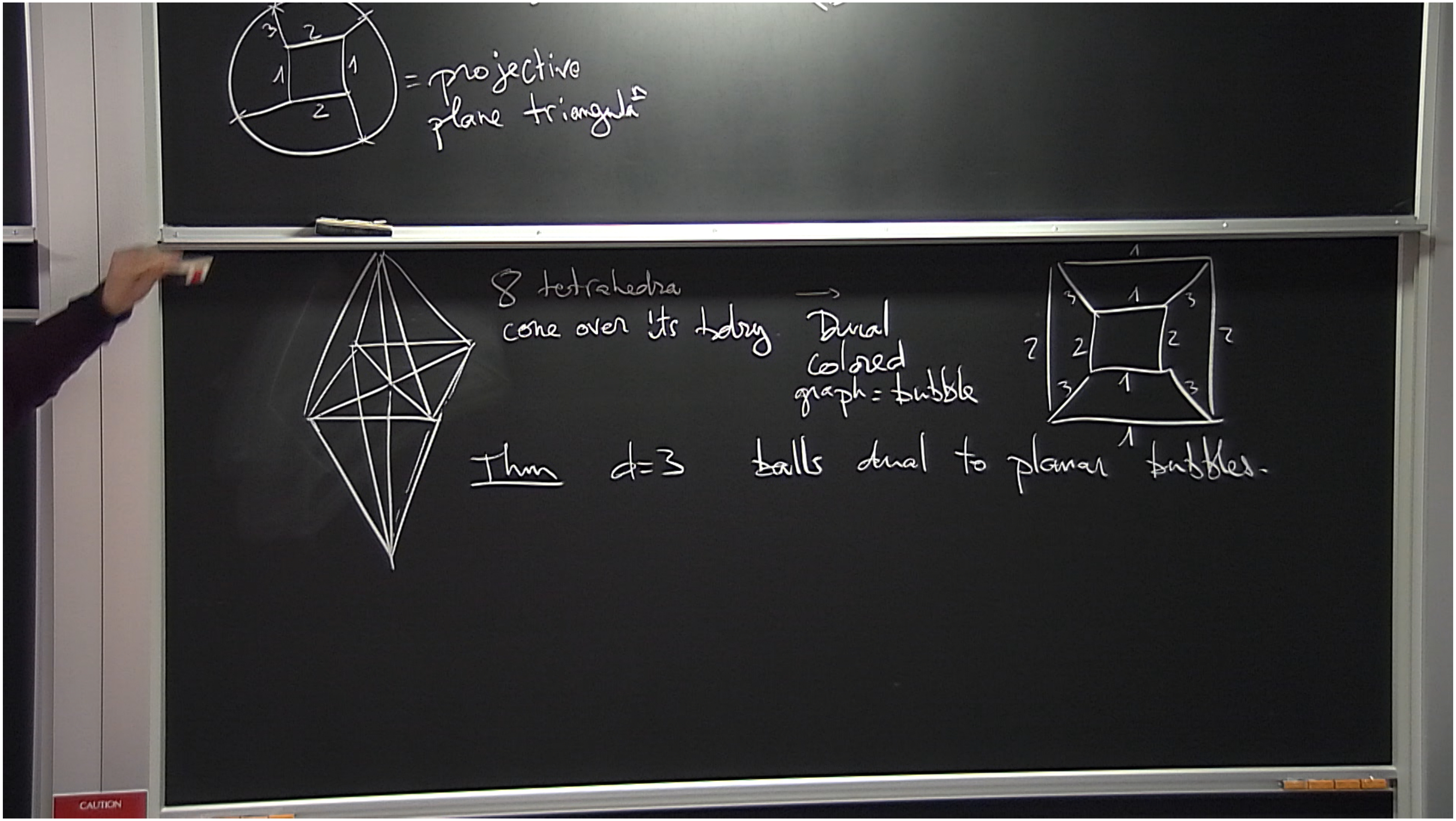
Bubbles are dual to building blocks obtained by a cone over their boundary.




Bubbles are dual to building blocks obtained by a cone over their boundary.
coning
dual
bubble
3
2
1
1
2


3
2
1
1
2
= projective
plane triangula

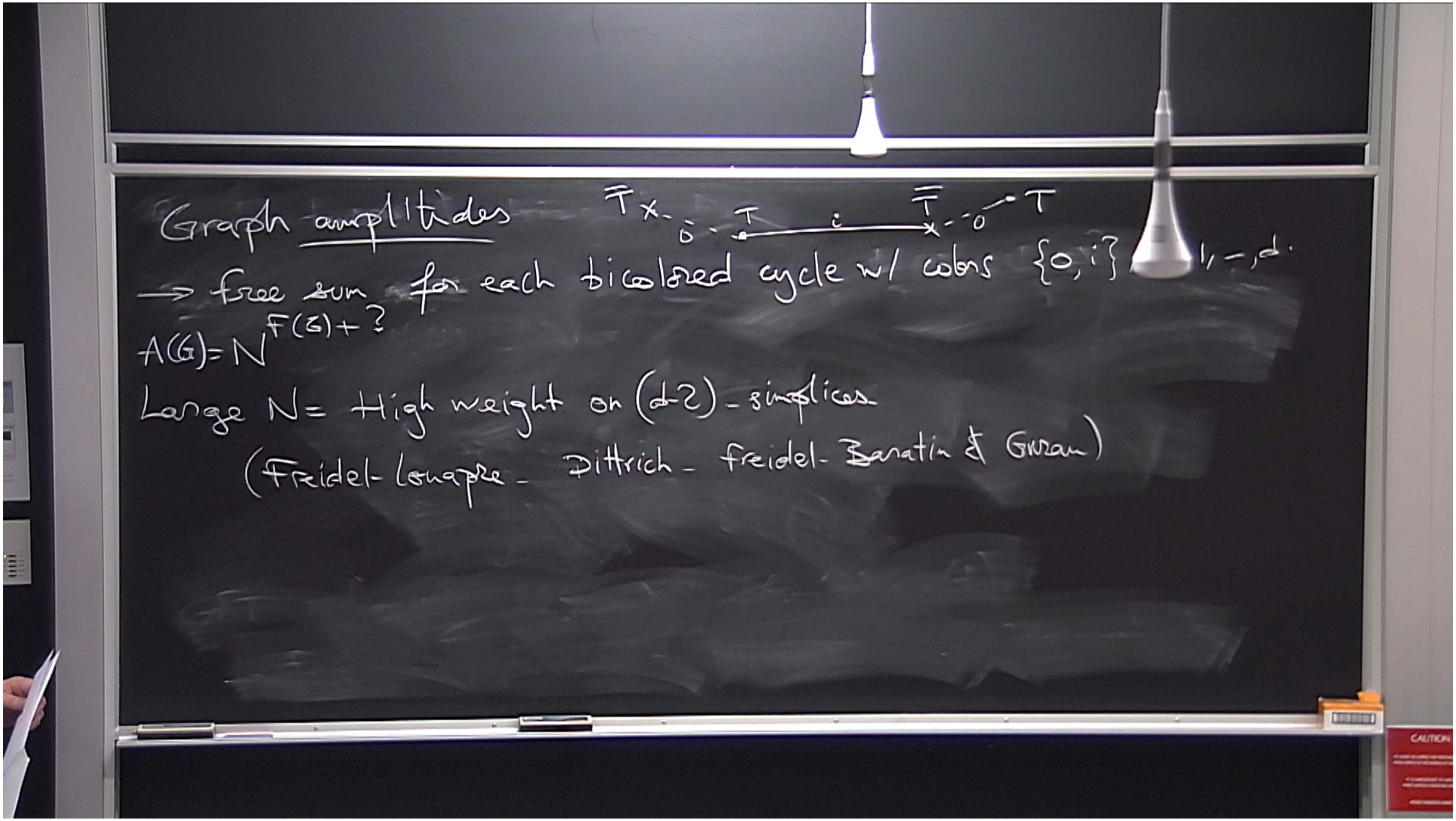

= projective
plane triangulaⁿ
8 tetrahedra
cone over its bdry
→ Dual
colored
graph = bubble
Thm d=3 balls dual to planar bubbles.
CAUTION

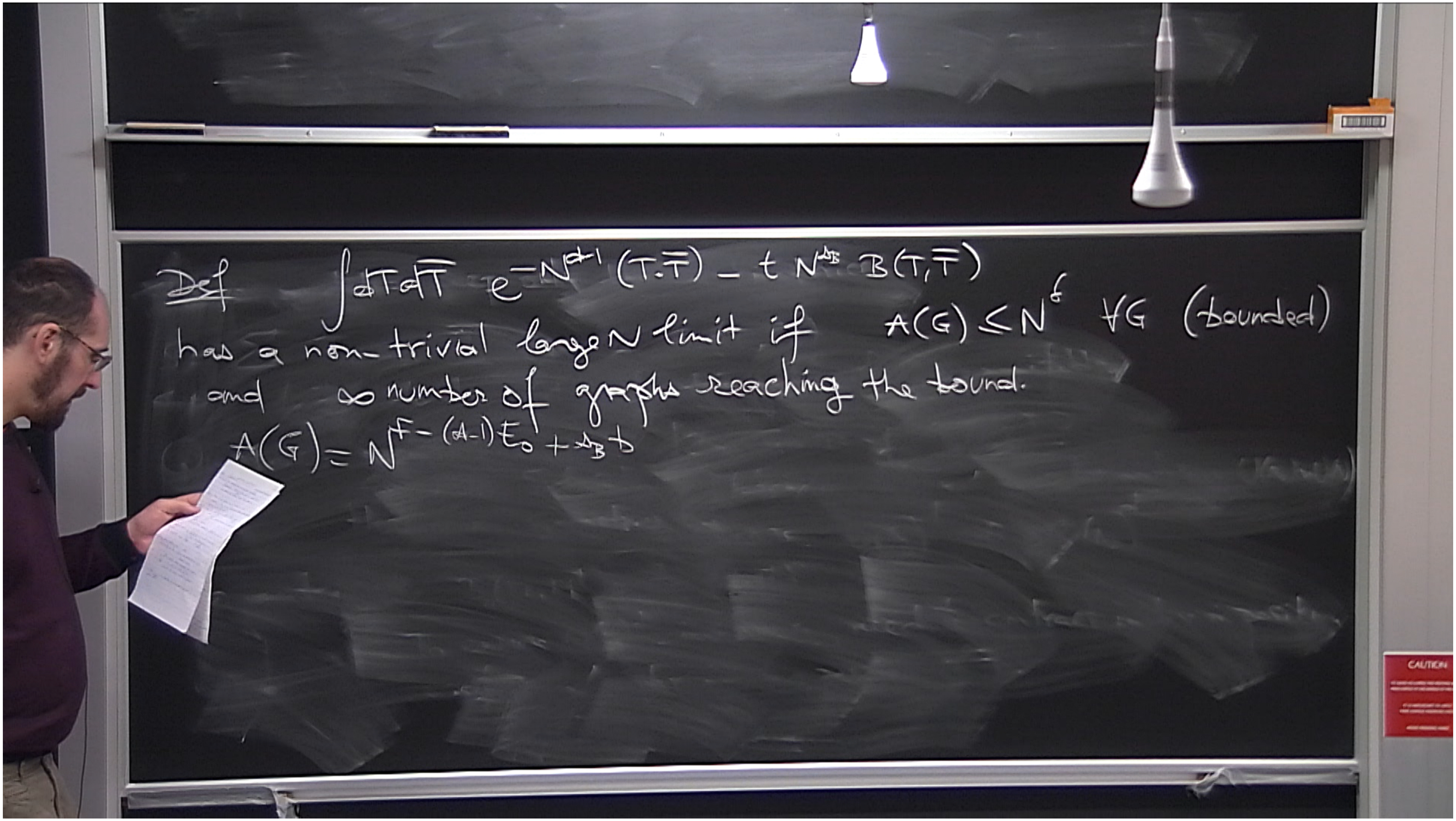
Graph amplitudes

→ free sum for each bicolored cycle w/ colors $\{0, i\}$ $1,\dots,d$.

$A(G) = N^{F(G)+?}$

Large N = High weight


Graph amplitudes
→ free sum for each bicolored cycle w/ colors {0, i} 1, ..., d.
A(G) = N^{F(G) + ?}
Large N = High weight on (d-2)-simplices
(Freidel-Louapre - Dittrich - Freidel - Baratin & Gurau)


Def $\int dT d\bar{T} \; e^{-N^{d-1}(T\cdot\bar{T}) - t N^{s_B} B(T,\bar{T})}$
has a non-trivial large N limit if $A(G) \le N^{f}$ $\forall G$ (bounded)
and ∞ number of graphs reaching the bound.
$A(G) = N^{F-(d-1)E_0 + s_B b}$

Def $\int dT\, d\bar{T}\; e^{-N^{d-1}(T\cdot\bar{T}) - t\, N^{\Delta_B} B(T,\bar{T})}$

has a non-trivial large N limit if $A(G) \le N^{f}$ $\forall G$ (bounded)

and ∞ number of graphs reaching the bound.

$$A(G) = N^{F-(d-1)E_0 + \Delta_B b} \rightsquigarrow |(d-2)\sigma| - \frac{(d-1)\,|\sigma(B)| - 2\Delta_B}{2|\sigma(B)|}\,|d\cdot\sigma|$$

Thm (Lionni) Δ_B unique (if it exists).

Def $\int dT\, d\bar{T}\; e^{-N^{d-1}(T\cdot\bar{T}) - t N^{\Delta_B} B(T,\bar{T})}$

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$$A(G) = N^{F-(d-1)E_0 + \Delta_B b} \rightsquigarrow \left| (d-2)\sigma \right| - \frac{(d-1)|\sigma(B)| - 2\Delta_B}{2|\sigma(B)|} \left| d-\sigma \right|$$

Thm (Lionni) Δ_B unique (if it exists).

(Gurau) $\Delta_B > d-1$ & $\Delta = d-1$ for melonic bubbles

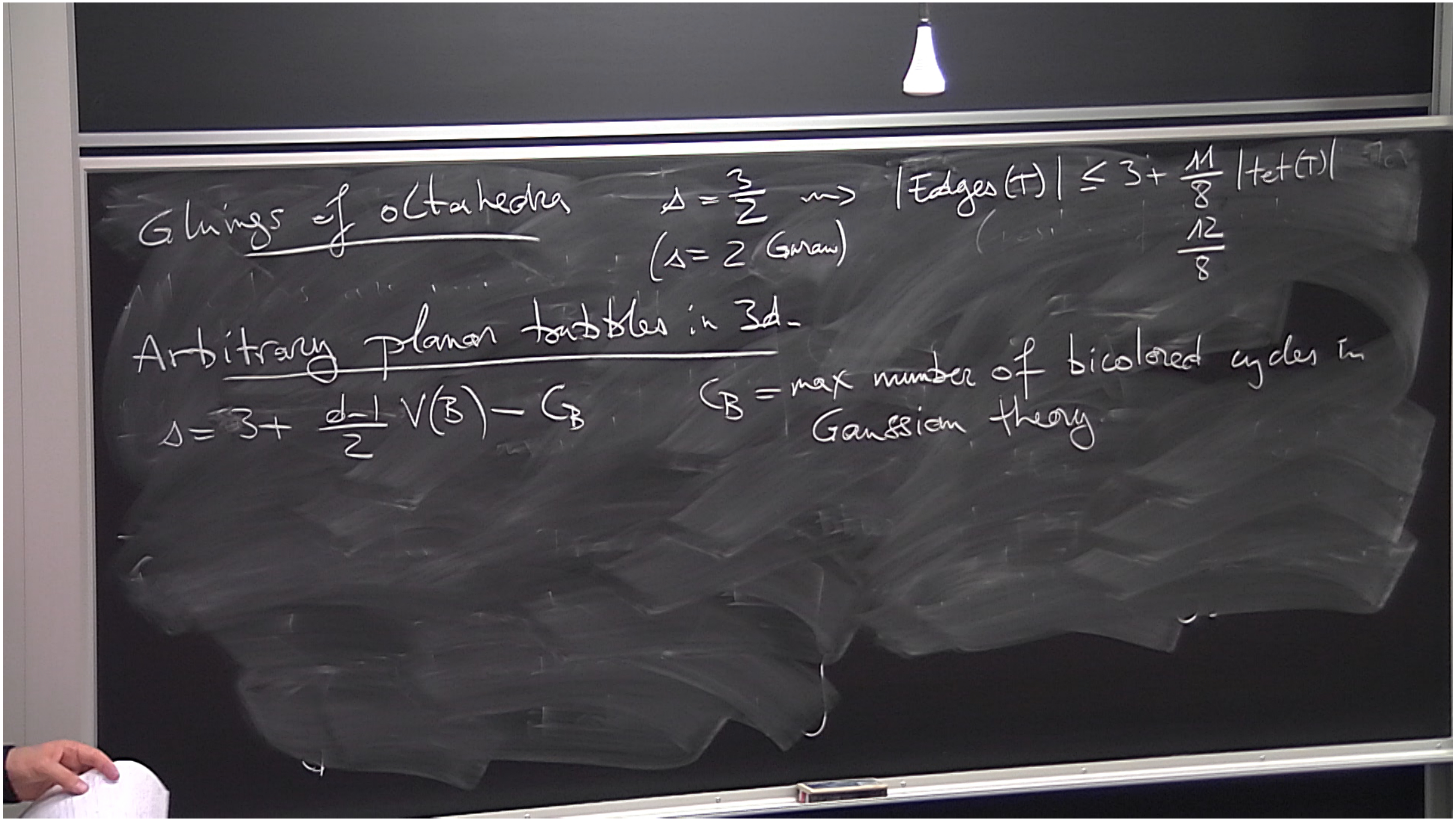
has a non-trivial large N limit if $A(G) \le N \quad \forall G$ (bounded)

and ∞ number of graphs reaching the bound.

$$A(G) = N^{F-(d-1)E_0 + \Delta_B b} \rightsquigarrow |(d-2)\sigma| - \frac{(d-1)|\sigma(B)| - 2\Delta_B}{2|\sigma(B)|}|d-\sigma|$$

Thm (Lionni) Δ_B unique (if it exists).

(Gurau) $\Delta_B \geq d-1$ & $\Delta = d-1$ for melonic bubbles


Gluings of octahedra
Δ = 3/2 ⇝ |Edges(T)| ≤ 3 + 11/8 |tet(T)|
(Δ = 2 Gurau)
12/8
Arbitrary planar bubbles in 3d.
Δ = 3 + (d−1)/2 V(B) − C_B
C_B = max number of bicolored cycles in Gaussian theory

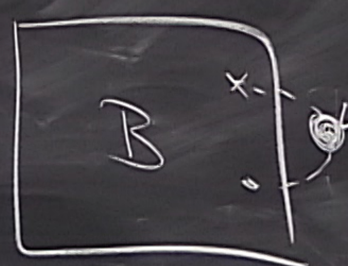
Gluings of octahedra $\quad \Delta = \frac{3}{2} \rightsquigarrow |\mathrm{Edges}(T)| \le 3 + \frac{11}{8}|\mathrm{tet}(T)|$

$(\Delta = 2$ Gurau$) \qquad \frac{12}{8}$

Arbitrary planar bubbles in 3d.

$$\Delta = 3 + \frac{d-1}{2} V(B) - C_B$$

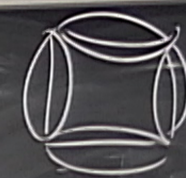
C_B = max number of bicolored cycles in Gaussian theory.



full large N 2-pt function.

$\Longrightarrow$ Large N limit is Gaussian.

Perspectives : * Different behaviors in even dim. due to symmetric bubbles.



* Find other families $\rightsquigarrow$ Look at combinatorial properties
* Use topology instead. $|\#$ triangulations of 3-manifolds$| \sim n^{n/6}$ (Chapuy – Perarnau)