

Title: PSI 2018/2019 - Quantum Field Theory III - Lecture 5

Date: Feb 07, 2019 11:30 AM

URL: <http://pirsa.org/19020027>

Abstract:

① CFT correlators are completely determined by 2 and 3pt functions
(follows from OPE)

② 4pt function imposes nonlinear constraints on the CFT data

$\{\Delta_I\}$



2 pt functions

$\{C_{IJK}\}$



3 pt functions

$$\langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle = \frac{1}{X_{12}^{2\Delta\phi} X_{34}^{2\Delta\phi}}$$

$$u = \frac{X_{12}^2 X_{34}^2}{X_{13}^2 X_{24}^2}$$

$$v = \frac{X_{14}^2 X_{23}^2}{X_{13}^2 X_{24}^2}$$

$\begin{matrix} \leftarrow \\ 1 \leftrightarrow 3 \end{matrix}$

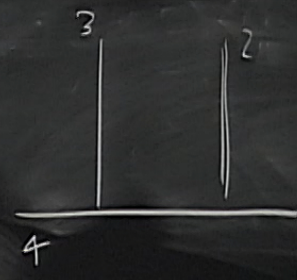
$$g(u, v) = \frac{1}{X_{23}^{2\Delta\phi} X_{14}^{2\Delta\phi}} g(v, u)$$

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$$v^{\Delta\phi} g(u, v) - u^{\Delta\phi} g(v, u) = 0$$

$$\textcircled{1} \quad \langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle$$

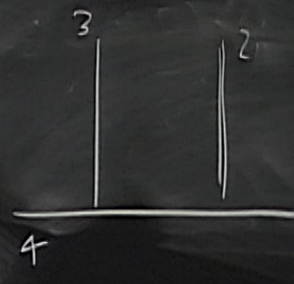
$$= \sum_a C_{\phi\phi\theta}^2 C^a(x_{12}, \partial_2) \langle \underbrace{\partial_a(x_2) \phi(x_3) \phi(x_4)}_{C_{\phi\phi\theta}} \rangle$$



$$\textcircled{1} \quad \langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle$$

$$= \sum_a C_{\phi\phi\theta}^2 C^a(x_{12}, a_2) \underbrace{\langle \partial_a(x_2) \phi(x_3) \phi(x_4) \rangle}_{C_{\phi\phi\theta}}$$

$$= \sum_a C_{\phi\phi\theta}^2 C^a(x_{14}, a_4) \underbrace{\langle \phi(x_2) \phi(x_3) \partial_a(x_4) \rangle}_{C_{\phi\phi\theta}}$$



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$\longleftrightarrow$
1 $\leftrightarrow$ 3

$$x_1 \rightarrow 0$$

$$x_2 = z$$

$$x_3 = (10 - z)$$

$$x_4 \rightarrow \infty$$

$$(u = |z|^2)$$

$$v = |1-z|^2$$

$$v^{\Delta\phi} g(u, v) - u^{\Delta\phi} g(v, u) = 0$$

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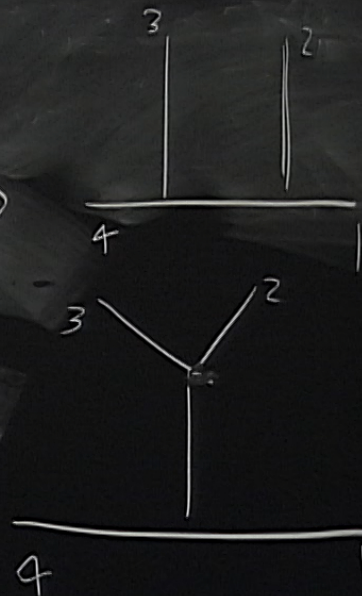
crossing symmetry

$$\textcircled{1} \quad \langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle$$

$$= \sum_a C_{\phi\phi\theta}^2 C^a(x_{12}, a_2) \langle \underbrace{\theta_a(x_2) \phi(x_3) \phi(x_4)}_{C_{\phi\phi\theta}} \rangle$$

$$\langle \phi(x_1) \phi(x_1) \phi(x_3) \phi(x_4) \rangle$$

$$= \sum_\theta C_{\phi\phi\theta}^2 C^a(x_{14}, a_4) \langle \underbrace{\phi(x_2) \phi(x_3) \theta_c(x_4)}_{C_{\phi\phi\theta}} \rangle$$



① $D=2$ Solved by pure thought these equations $\Rightarrow$ Minimal Model

② $D=3$: Solve for critical exponents of $D=3$ Ising Model
Ising model has critical point at $T=T_c$
 $\uparrow$ EFT.

① $D=2$

Solved by pure thought these equations $\Rightarrow$ Minimal Model

② $D=3$

Solve for critical exponents of $D=3$ Ising Model

Ising model has critical point at $T=T_c$

σ : spin operator

ε : energy operator

$$\Delta\sigma = 0.518\dots$$

$$\Delta\varepsilon = 1.412$$

$\uparrow$ EFT.

Anomalies

Symmetry $\implies (\partial^\mu j_\mu = 0)$ $Q = \int j^0$
Continuous

Is a $\hbar$ violation of a conservation law

$$\partial^\mu j_\mu \neq 0 = \mathcal{O}(\hbar)$$

Conformal Anomaly in $D=2$

conformal inv

$$T^M_M = 0$$

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle = \underset{\substack{\uparrow \\ \text{build most general structure}}}{P_\mu P_\nu P_\rho P_\sigma} F_1(p^2) + \eta_{\mu\nu} \eta_{\rho\sigma} F_2(p^2) + (\eta_{\mu\rho} \eta_{\nu\sigma} + \dots) F_3(p^2)$$

5 different structures.

$$\text{Translational inv} \Rightarrow P^M T_{M\mu} = 0$$

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle = \frac{1}{p^2} \left[\pi_{\mu\rho} \pi_{\nu\sigma} + \pi_{\mu\sigma} \pi_{\nu\rho} - \frac{2}{D-1} \pi_{\mu\nu} \pi_{\rho\sigma} \right] F(p^2) + \pi_{\mu\nu} \pi_{\rho\sigma} G(p^2)$$

Go to $D=2$

$$\hat{P}_\mu = \epsilon_{\mu\alpha} p^\alpha$$

$$\pi_{\mu\nu} = \hat{P}_\mu \hat{P}_\nu$$

$$\epsilon_{\mu\alpha} \epsilon_{\nu\beta} = \eta_{\mu\nu} \eta_{\alpha\beta} - \eta_{\mu\beta} \eta_{\alpha\nu}$$

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle = \frac{1}{p^2} \Pi_{\mu\nu} \Pi_{\rho\sigma}(p^2) = \frac{C \Pi_{\mu\nu} \Pi_{\rho\sigma}}{p^2}$$

① scaling symmetry

$$[T_{\mu\nu}(x)] = D$$

$$[T_{\mu\nu}(p)] = 0$$

$$\langle T_{\mu\nu}^M(p) T_{\rho\sigma}(-p) \rangle = C \Pi_{\rho\sigma}$$

$$\langle T_{\mu\nu}^M(x) T_{\rho\sigma}(0) \rangle = \frac{1}{p^2} (\partial_\mu \partial_\rho \partial_\sigma - \eta_{\rho\sigma} \square) \delta^2(x)$$

violation of conformal symmetry only at
coincident points

C: measures violation of conf. inv.

$$\langle T_{\mu}^{\mu}(0) \rangle_h \equiv \langle T_{\mu}^{\mu}(0) e^{\int h_{\rho\sigma} T_{\rho\sigma} d^2x} \rangle_{h=0}$$

$$= \int d^2x \langle T_{\mu}^{\mu}(0) T_{\rho\sigma}(x) \rangle h_{\rho\sigma}(x)$$

$$= \int d^2x c (\partial_{\rho} \partial_{\sigma} - \square \eta_{\rho\sigma}) \delta^2(x) h_{\rho\sigma}(x)$$

$$= c (\partial_{\rho} \partial_{\sigma} - \square \eta_{\rho\sigma}) h_{\rho\sigma}$$

$$T_{\mu}^{\mu} = c (\partial_{\rho} \partial_{\sigma} - \eta_{\rho\sigma} \square) h^{\rho\sigma} = c R / (4\pi G)$$

$$D=3$$

$$T_{\mu}^{\mu} = 0$$

$$D=4 \quad T_{\mu}^{\mu} = c \text{ Weyl}^2 - a \text{ Euler}$$

$$R_{\mu\nu\rho\sigma}^2 + \# R_{\mu\nu}^2 + \# R^2$$

$$T_{\mu}^{\mu} = c (\partial_{\rho} \partial_{\sigma} - \eta_{\rho\sigma} \square) h_{\rho\sigma} = c R|_{\eta_{\mu\nu} = \eta_{\mu\nu}}$$

D=5

$$T_{\mu}^{\mu} = 0$$

$$D=4 \quad T_{\mu}^{\mu} = c \text{ Weyl}^2 - a \text{ Euler}$$

$$R_{\mu\nu\rho\sigma}^2 + 4 R_{\mu\nu}^2 + R^2$$

$$CFT_{UV} + \int \lambda \Theta$$

CFT_{IR}

$$C_{UV} > C_{IR}$$

Same is true in D=4

$$a_{UV} > a_{IR}$$

< T_{UV}
 @ scaling

violation of
 coincident