Title: PSI 2018/2019 - Quantum Field Theory III - Lecture 4

Date: Feb 05, 2019 11:30 AM

URL: http://pirsa.org/19020025

Abstract:

Conformal symmetry $(P_\mu, M_{\mu\nu}, D, K_\mu)$ completely fixes position dependence of 2, 3pt functions

e.g. Scalar operators (i.e. spin $= 0$)

$$x^\mu \to \lambda x^\mu$$

$$\theta(x) \to \hat{\theta}(x) = \lambda^{-\Delta}\, \theta(\lambda^{-1} x)$$

$$\langle \theta_I(x_1)\, \theta_J(x_2) \rangle = \frac{\delta_{IJ}}{x_{12}^{2\Delta_I}}$$

$$x_{ij} \equiv |x_i - x_j|$$

$$\langle \theta_I(x_1)\, \theta_J(x_2)\, \theta_K(x_3) \rangle = \frac{C_{IJK}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\, x_{13}^{\Delta_1+\Delta_3-\Delta_2}\, x_{23}^{\Delta_2+\Delta_3-\Delta_1}}$$

$\Longrightarrow$ CFT data $\underbrace{\{\Delta_I\}}_{\text{scaling dimensions}}$, $\underbrace{\{C_{IJK}\}}_{\text{structure constants}}$

3 pt. functions

$\sigma(\lambda^{-1} x)$

4-point function.

Any spacetime dimension D

$$
\begin{array}{cccc}
x_1 & x_2 & x_3 & x_4 \\
 & \downarrow K_\mu & & \\
x_1' & x_2' & x_3' & \infty \\
 & \downarrow P_\mu & & \\
0 & x_2'' & x_3'' & \infty \\
 & \downarrow D & & \\
0 & x_2''' & x_3''' & \infty
\end{array}
$$

$|x_3'''| = 1 \xrightarrow{\quad M_{1j} \quad} j = 2,3.$

4 point function is not fixed

$\Uparrow$

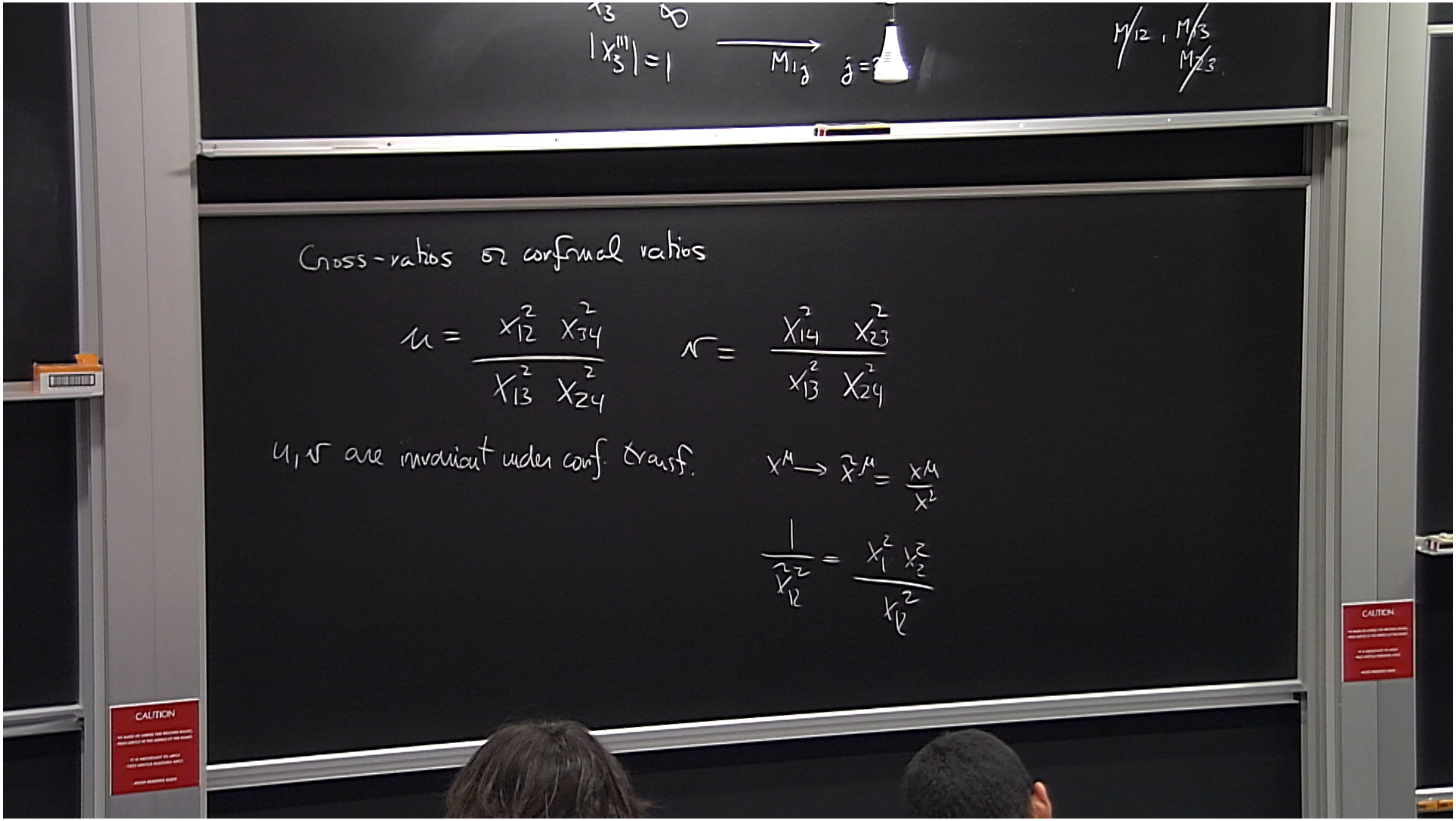
$\boxed{D=3}$

$0 \quad (x, y, 0) \quad (1\,0\,0) \quad \infty$

$\uparrow M_{23}$

$0 \quad x_2''' \quad (1\,0\,0) \quad \infty$

$\cancel{M_{12}}, \cancel{M_{13}}, \cancel{M_{23}}.$

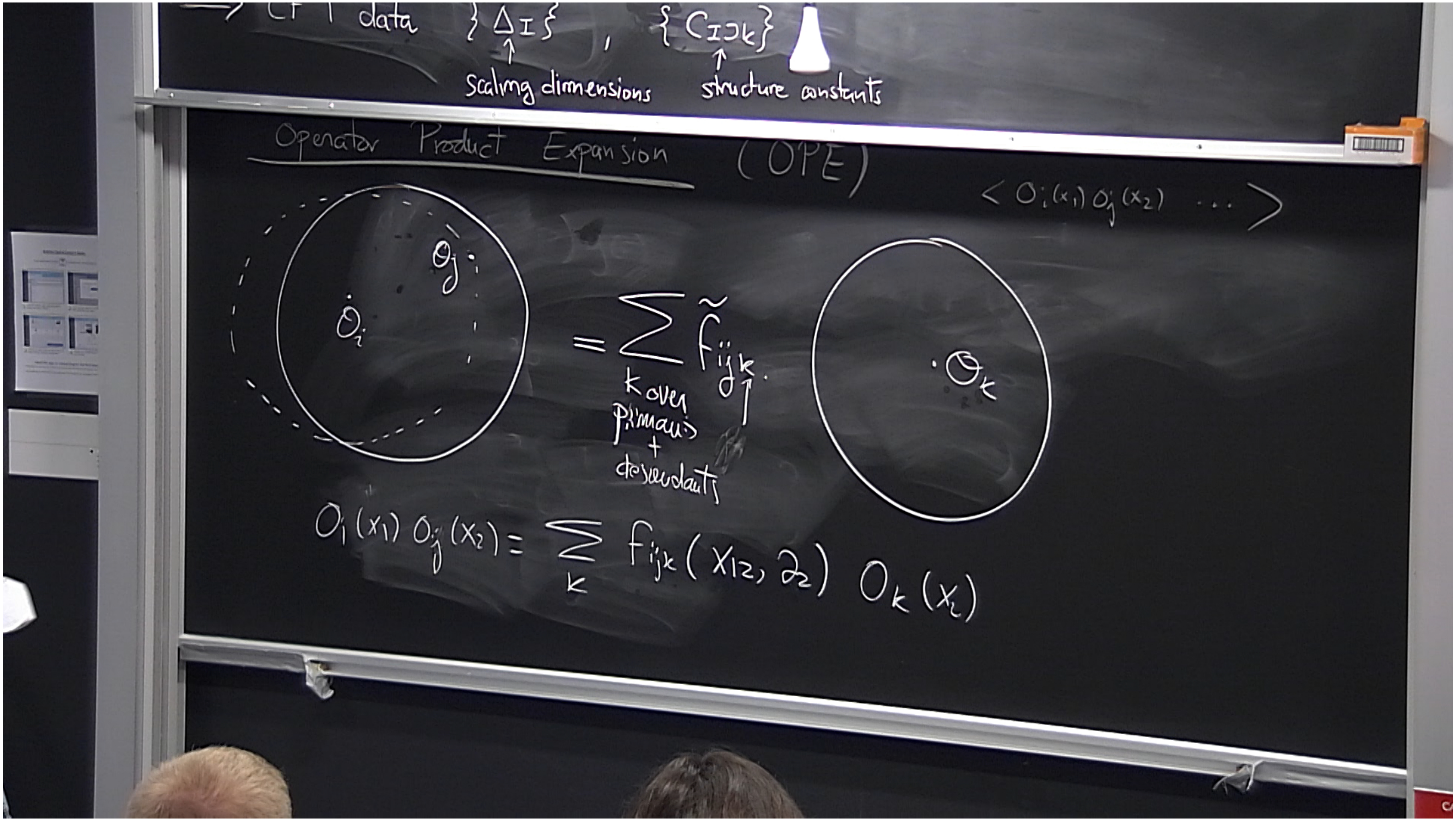

$|x_3'''| = 1$
M_{1j}
$j =$
$\cancel{M_{12}}$, $\cancel{M_{13}}$
$\cancel{M_{23}}$
Cross-ratios or conformal ratios
$u = \frac{x_{12}^2 \, x_{34}^2}{x_{13}^2 \, x_{24}^2}$
$v = \frac{x_{14}^2 \, x_{23}^2}{x_{13}^2 \, x_{24}^2}$
u, v are invariant under conf. transf.
$x^\mu \to \tilde{x}^\mu = \frac{x^\mu}{x^2}$
$\frac{1}{\tilde{x}_{12}^2} = \frac{x_1^2 \, x_2^2}{x_{12}^2}$

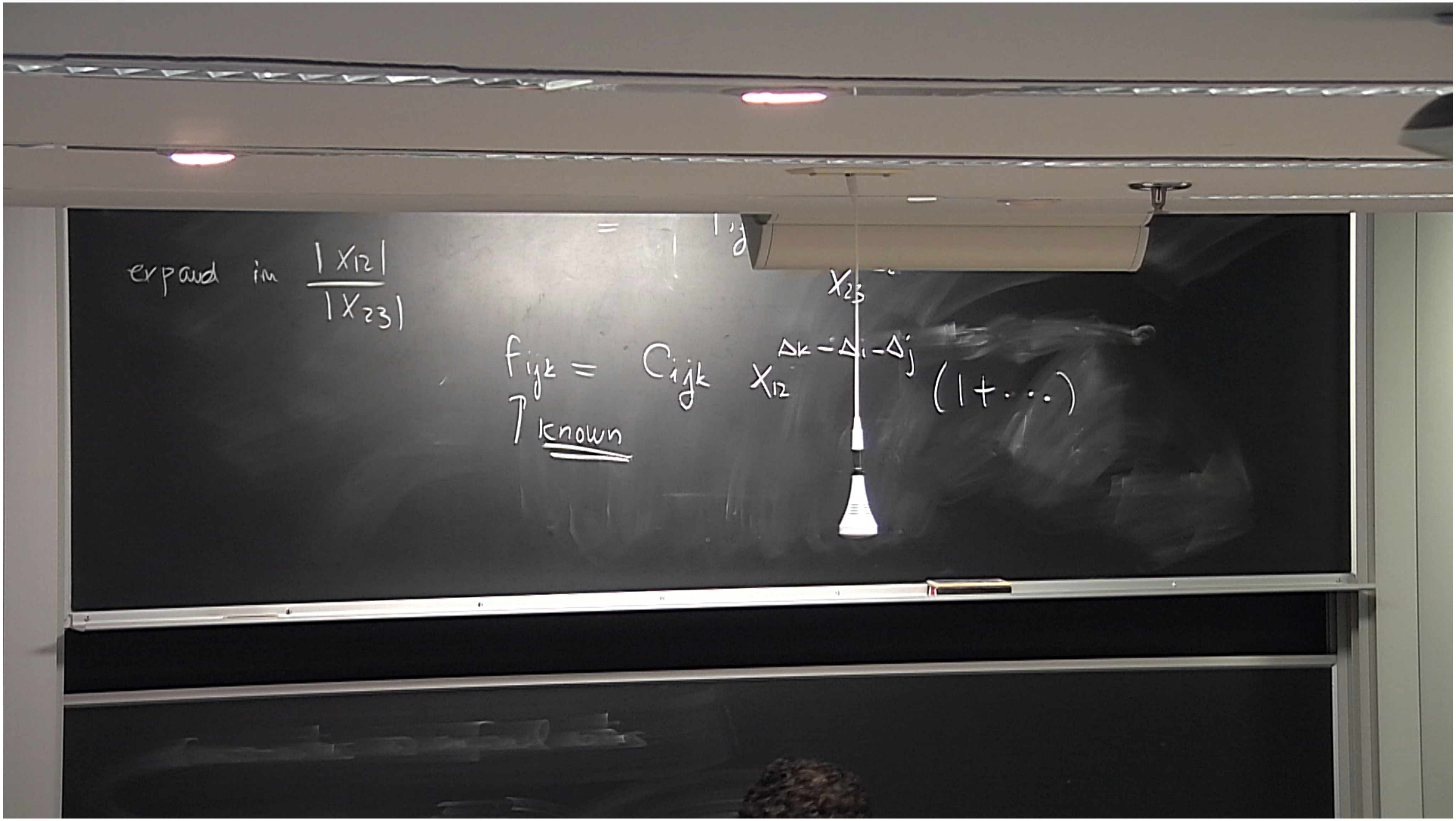
Most general 4pt. function of identical scalar operators

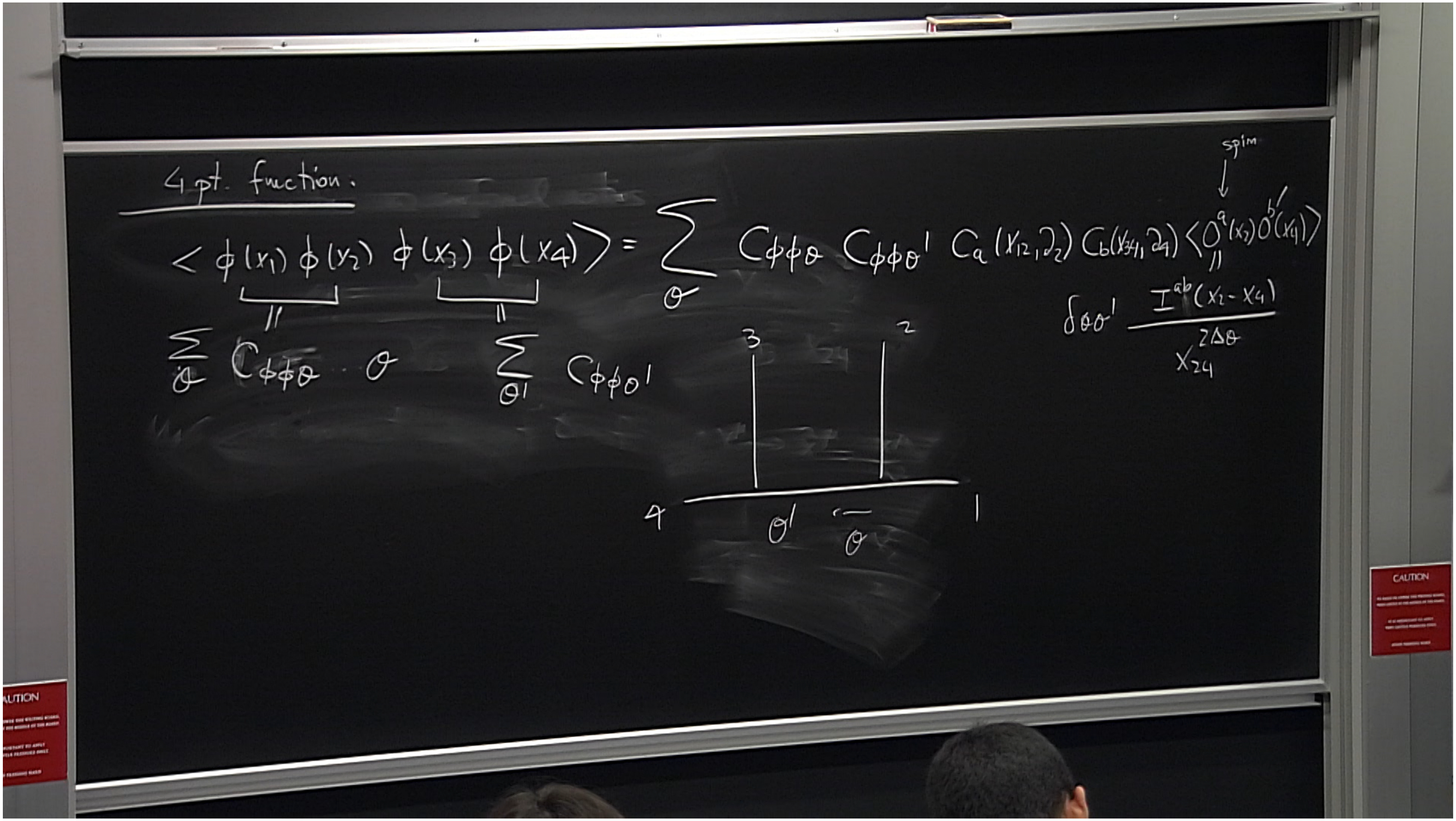
$$\langle \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4) \rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}}$$

- $g(u,v)$ is **NOT** an independent function!
- $g(u,v)$ is completely determined by $\{\Delta_I\}$, $\{C_{IJK}\}$.
- BUT 4pt function imposes constraints on CFT data

$$\boxed{v^{\Delta\phi}\, g(u,v) = u^{\Delta\phi}\, g(v,u)}$$


data {ΔI}, {CIJK}
scaling dimensions
structure constants
Operator Product Expansion (OPE)
< Oi(x1) Oj(x2) ... >
Oj
Oi
= Σ f̃ijk
k over primaries + descendants
Ok
O1(x1) Oj(x2) = Σk fijk(x12, ∂2) Ok(x2)


expand in $\frac{|x_{12}|}{|x_{23}|}$
$f_{ijk} = C_{ijk}\, x_{12}^{\Delta_k - \Delta_i - \Delta_j} (1 + \dots)$
known


4 pt. function.
spin
$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \sum_{\mathcal{O}} C_{\phi\phi\mathcal{O}} C_{\phi\phi\mathcal{O}'} C_a(x_{12},\partial_2) C_b(x_{34},\partial_4) \langle \mathcal{O}^a(x_2)\mathcal{O}'^b(x_4)\rangle$
$\sum_{\mathcal{O}} C_{\phi\phi\mathcal{O}} \cdot \mathcal{O}$
$\sum_{\mathcal{O}'} C_{\phi\phi\mathcal{O}'}$
$\delta_{\mathcal{O}\mathcal{O}'} \frac{I^{ab}(x_2-x_4)}{x_{24}^{2\Delta_\mathcal{O}}}$
3
2
4
1
$\mathcal{O}'$
$\mathcal{O}$

$$\underbrace{\langle O_i(x_1)\, O_j(x_2)}_{\text{known}}\, O_k(x_3)\rangle = \sum_{k'} f_{ijk'}(x_{12}, \partial_k^{(2)})\, \underbrace{\langle O_{k'}(x_2)\, O_k(x_3)\rangle}_{= \frac{\delta_{kk'}}{x_{23}^{2\Delta_k}}}$$

$$= f_{ijk}(x_{12}, \partial_k^{(2)}) \frac{1}{x_{23}^{2\Delta_k}}$$

expand in $\dfrac{|x_{12}|}{|x_{23}|}$

$$f_{ijk} = C_{ijk}\, \underbrace{x_{12}^{\Delta_k - \Delta_i - \Delta_j} (1 + \dots)}_{= (1 + \alpha x^\mu \partial_\mu + \beta x^\mu x^\nu \partial_\mu \partial_\nu + \dots)}$$

known

Contrast $\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \dfrac{g(u,v)}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}}$

$$\Rightarrow \;= \frac{1}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} \cdot \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\; g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(x_i)$$

$$g_{\Delta,\ell}(y_i) = y_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\, C_a(x_{12},\partial_2)\, C_b(x_{34},\partial_4)\, \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta_{\mathcal{O}}}}$$

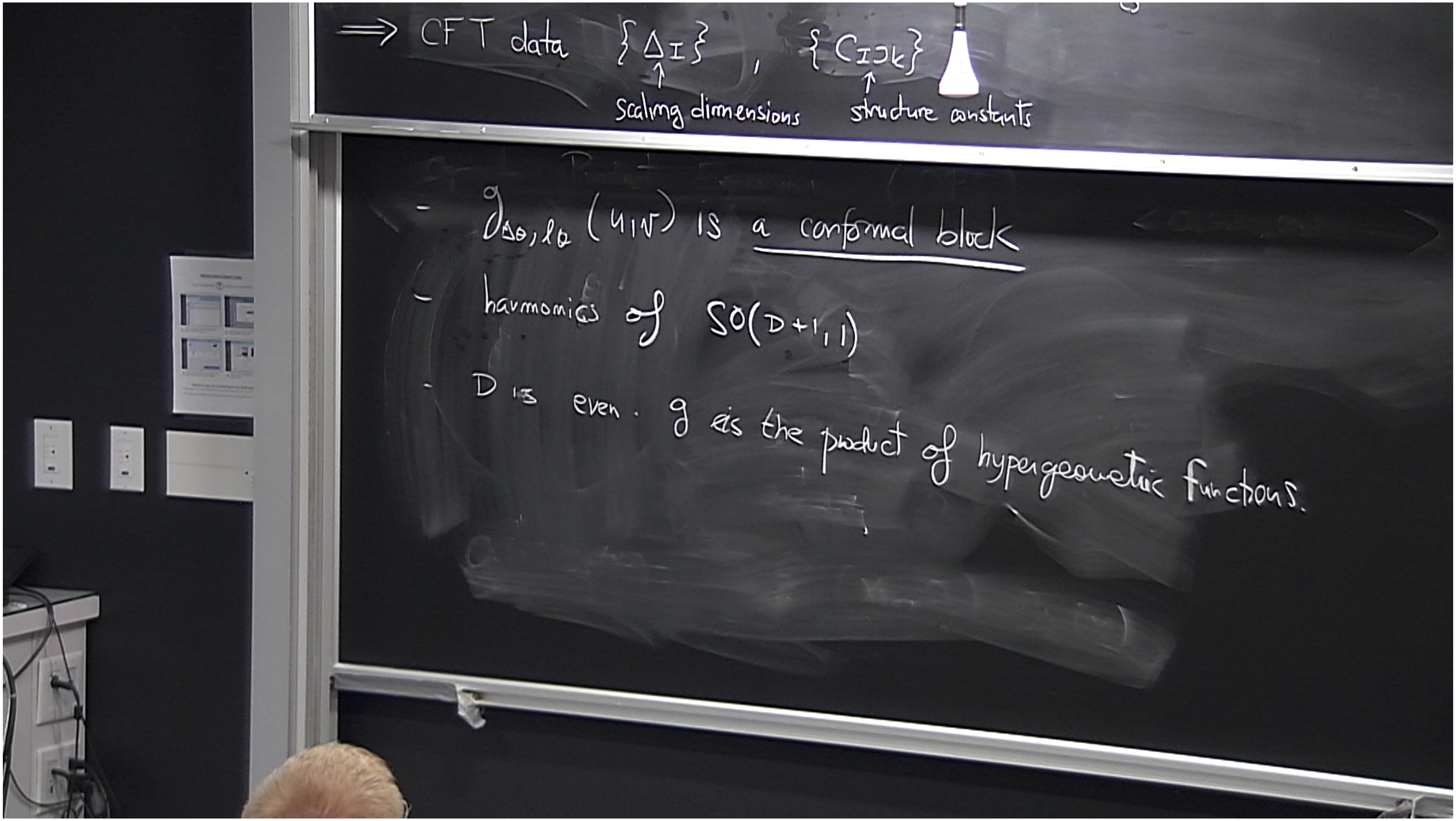
$$\boxed{g(u,v) = \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\; g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v)}$$

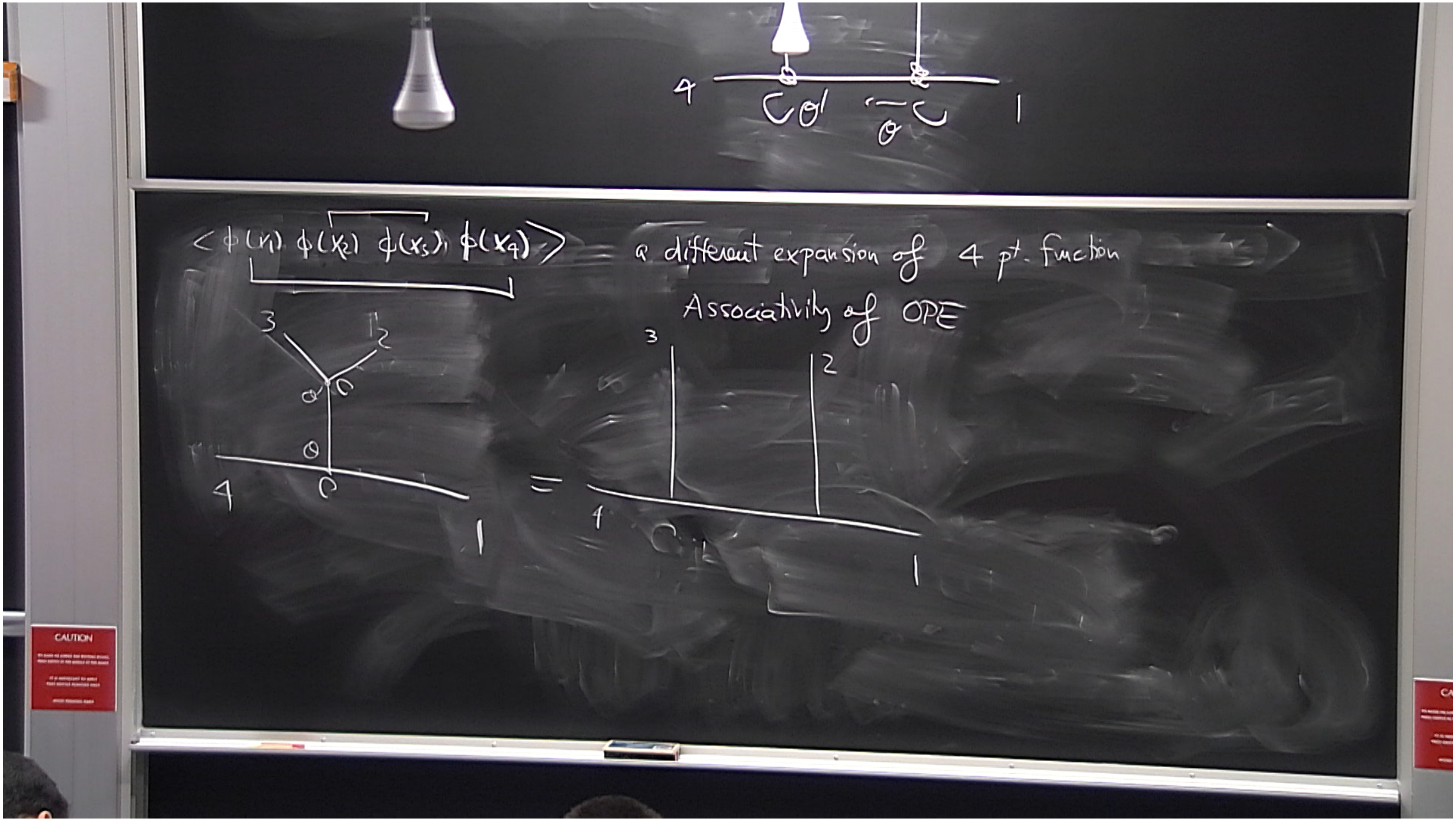
$$\Rightarrow \quad = \frac{1}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} \cdot \sum_{O} C_{\phi\phi O}^{2}\; g_{\Delta_O,\ell_O}(x_i)$$

$$g_{\Delta,\ell}(x_i) = x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\; C_a(x_{12},\partial_2)\, C_b(x_{34},\partial_4)\, \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta_O}}$$

$$\boxed{g(u,v) = \sum_{O} C_{\phi\phi O}^{2}\; g_{\Delta_O,\ell_O}(u,v)}$$

$$\boxed{v^{\Delta_\phi}\, g(u,v) = u^{\Delta_\phi}\, g(v,u)}$$


⟹ CFT data $\{\Delta_I\}$, $\{C_{IJk}\}$
scaling dimensions
structure constants
- $g_{\Delta_O,\ell_O}(u,v)$ is a conformal block
- harmonics of $SO(D+1,1)$
- D is even. g is the product of hypergeometric functions.



< φ(x1) φ(x2) φ(x3) φ(x4) >
a different expansion of 4 pt. function
Associativity of OPE
CAUTION